Black holes,
a century after the birth of General Relativity

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
92190 Meudon, France

eric.gourgoulhon@obspm.fr

http://luth.obspm.fr/~luthier/gourgoulhon/

Solvay Colloquium
Université Libre de Bruxelles
15 March 2016

Outline

Outline

A two centuries-old prehistory...

$$\boxed{V_{\rm esc} > c} \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho > c^2 \iff \boxed{R > \sqrt{\frac{3c^2}{8\pi G\rho}}}$$

A two centuries-old prehistory...

$$\boxed{V_{\rm esc} > c} \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho > c^2 \iff \boxed{R > \sqrt{\frac{3c^2}{8\pi G\rho}}}$$

John Michell (1784)

"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"

[Phil. Trans. R. Soc. Lond. **74**, 35 (1784)]

A two centuries-old prehistory...

$$\boxed{V_{\rm esc} > c} \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho > c^2 \iff \boxed{R > \sqrt{\frac{3c^2}{8\pi G\rho}}}$$

John Michell (1784)

"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"

[Phil. Trans. R. Soc. Lond. **74**, 35 (1784)]

Pierre Simon de Laplace (1796)

"Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles."

[Exposition du système du monde (1796)]

Limits of the Newtonian concept of a black hole

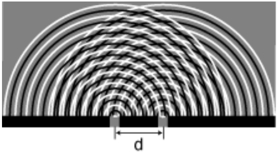
- No privileged role of the velocity of light in Newtonian theory : nothing forbids $V > c$: the "dark stars" are not causally disconnected from the rest of the Universe

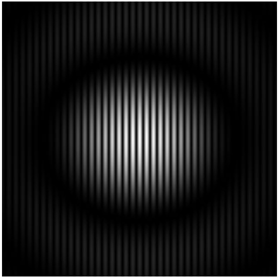
Limits of the Newtonian concept of a black hole

- No privileged role of the velocity of light in Newtonian theory : nothing forbids $V > c$: the "dark stars" are not causally disconnected from the rest of the Universe
- $V_{\rm esc} \sim c \Longrightarrow$ gravitational potential energy $\sim$ mass energy Mc^2 $\Longrightarrow$ a *relativistic* theory of gravitation is necessary !

Limits of the Newtonian concept of a black hole

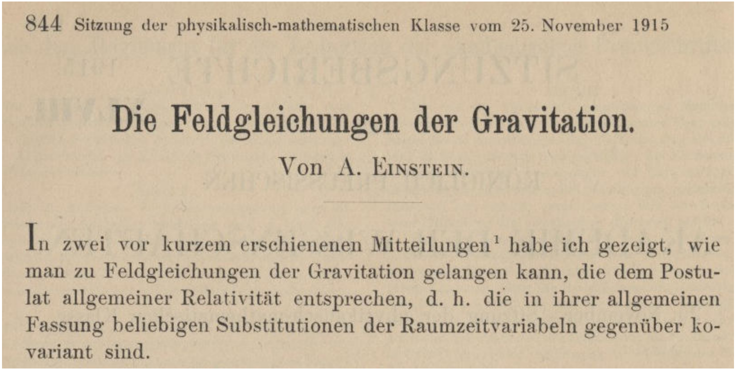
- No privileged role of the velocity of light in Newtonian theory : nothing forbids $V > c$: the "dark stars" are not causally disconnected from the rest of the Universe
- $V_{\rm esc} \sim c \Longrightarrow$ gravitational potential energy $\sim$ mass energy Mc^2
$\Longrightarrow$ a *relativistic* theory of gravitation is necessary !
- No clear action of the gravitation field on electromagnetic *waves* in Newtonian gravity





[R. Taillet]

100 years ago : a relativistic theory of gravitation


844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

$$\boldsymbol{R} - \frac{1}{2} R \, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}$$

[A. Einstein, Sitz. Preuss. Akad. Wissenschaften Berlin, 844 (1915)]

The Schwarzschild solution

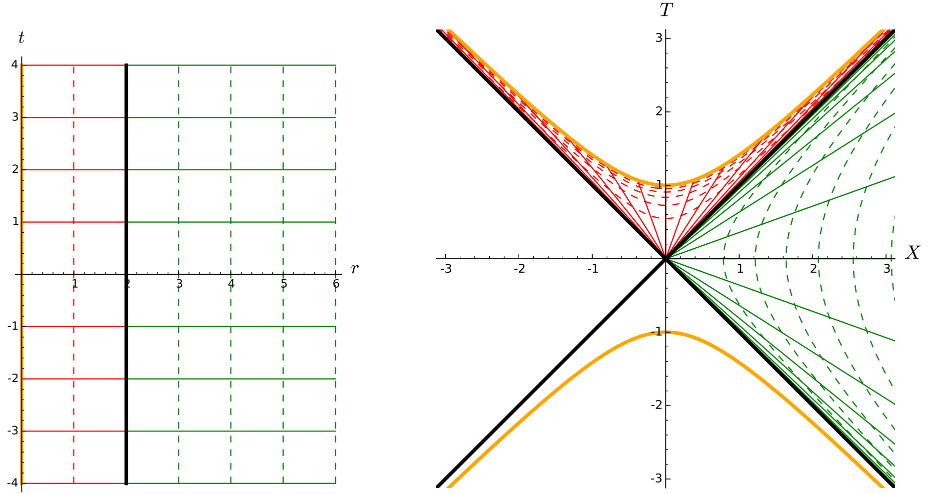
- Nov-Dec. 1915 : Karl Schwarzschild : first exact non-trivial solution of Einstein equation $\Longrightarrow$ spacetime metric outside a **spherical body** of mass M

$$g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta = -\left(1-\frac{2GM}{c^2 r}\right)c^2\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

- 1916 : Johannes Drostes : circular orbit of photons at $r = 3GM/c^2$
- 1920 : Alexander Anderson : light cannot emerge from the region $r < R_\mathrm{S} := \dfrac{2GM}{c^2}$ (*"shrouded in darkness"*)
- 1923 : George Birkhoff : outside any *spherical* body, the metric is Schwarzschild metric
- 1932 : **Georges Lemaître** : the singularity at $r = R_\mathrm{S}$ is a coordinate singularity
- 1939 : Robert Oppenheimer & Hartland Snyder : first solution describing a gravitational collapse $\Longrightarrow$ for a external observer, $R \to R_\mathrm{S}$ as $t \to +\infty$

The Schwarzschild solution : the complete picture

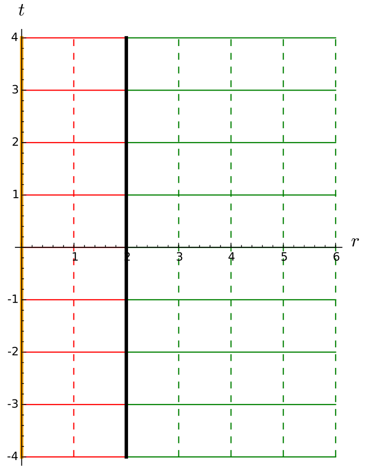
- 1960 : Martin Kruskal, John A. Wheeler : complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times \mathbb{S}^2$ manifold)



Schwarzschild-Droste coordinates (t, r)

The Schwarzschild solution : the complete picture

- 1960 : Martin Kruskal, John A. Wheeler : complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times \mathbb{S}^2$ manifold)



Schwarzschild-Droste coordinates (t, r)

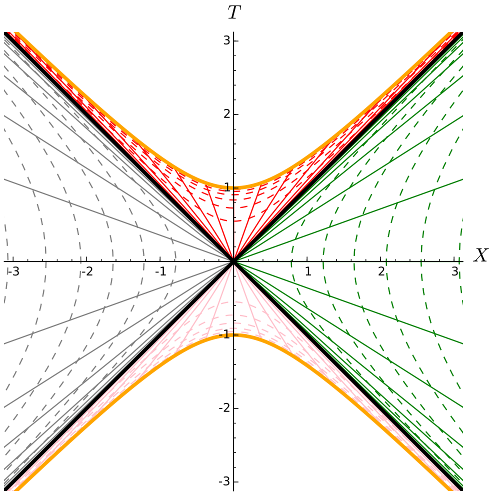


figure : http://sagemanifolds.obspm.fr

The Schwarzschild spacetime : Carter-Penrose diagram

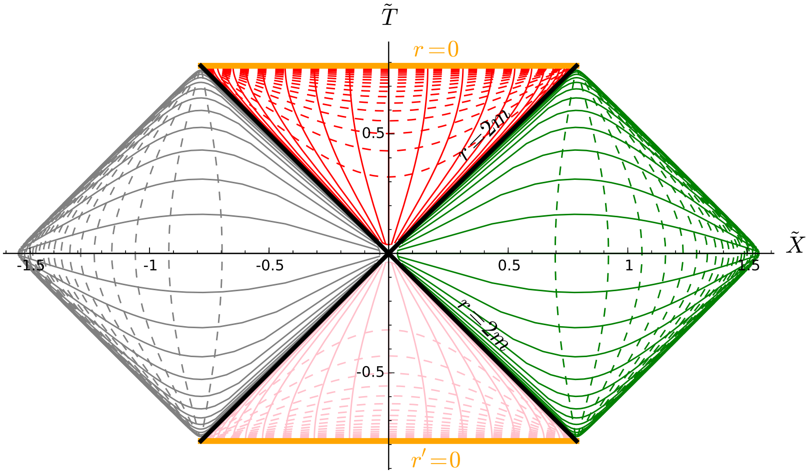


figure : http://sagemanifolds.obspm.fr

Rotation enters the game : the Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\, dx^\alpha\, dx^\beta = -\left(1 - \frac{2GMr}{c^2\rho^2}\right) c^2 dt^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\, c\, dt\, d\varphi + \frac{\rho^2}{\Delta}\, dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2GMa^2 r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\, d\varphi^2$$

where

$$\rho^2 := r^2 + a^2\cos^2\theta, \qquad \Delta := r^2 - \frac{2GM}{c^2} r + a^2, \qquad a := \frac{J}{cM}$$

$\rightarrow$ 2 parameters : M : gravitational mass ; J : angular momentum

Rotation enters the game : the Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GMr}{c^2\rho^2}\right)c^2\mathrm{d}t^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\,c\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2GMa^2r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\,\mathrm{d}\varphi^2$$

where

$$\rho^2 := r^2 + a^2\cos^2\theta, \qquad \Delta := r^2 - \frac{2GM}{c^2}r + a^2, \qquad a := \frac{J}{cM}$$

$\rightarrow$ 2 parameters : M : gravitational mass ; J : angular momentum

Schwarzschild as the subcase $a=0$:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GM}{c^2r}\right)c^2\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2\right)$$

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
$\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

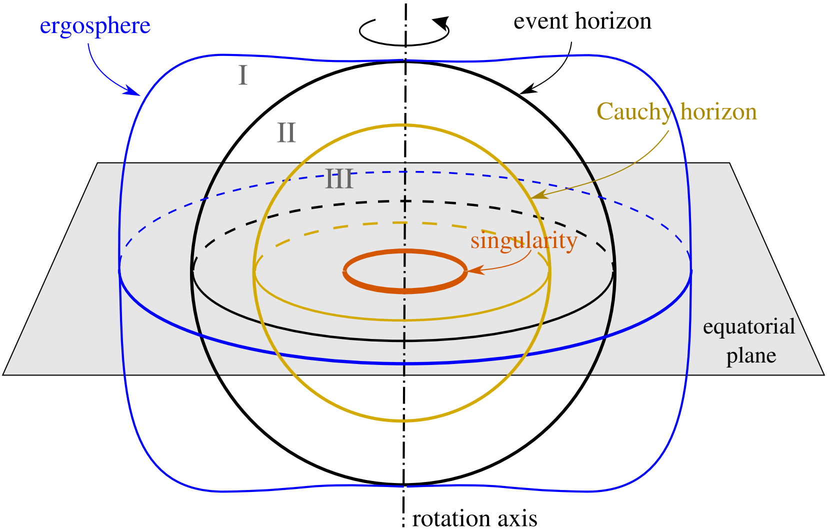
Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

Remark : the radius of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole "centre" and the event horizon. A well defined quantity is the area of the event horizon, A.
The radius can be then defined from it : for a Schwarzschild black hole :

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot} \right) \text{ km}$$

Kerr spacetime



Slice $t = \text{const}$ and $\theta = \pi/2$ of the Kerr spacetime

The Golden Age of black hole theory

- 1964 : Edwin Salpeter, Yakov Zeldovich : quasars (just discovered !) shine thanks to accretion onto a supermassive black hole
- 1965 : Roger Penrose : if a trapped surface is formed in a gravitational collapse and matter obeys some energy condition, then a singularity will appear
- 1967 : John A. Wheeler coined the word *black hole*
- 1969 : Roger Penrose : energy can be extracted from a rotating black hole
- 1972 : Stephen Hawking : law of area increase $\Longrightarrow$ **BH thermodynamics**
- 1975 : Stephen Hawking : **Hawking radiation**
- 1965-1972 : **the no-hair theorem**

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

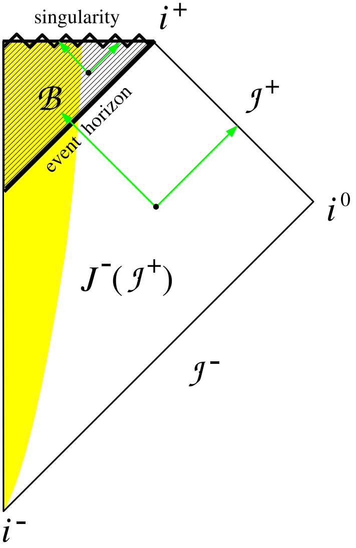
Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

where

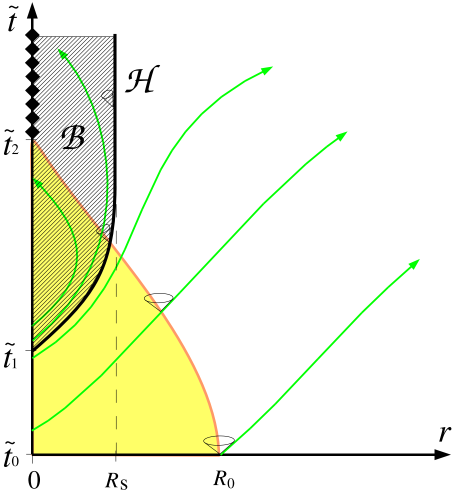
- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathscr{I}^+)$
(boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

where

- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathscr{I}^+)$ (boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.

Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.
- The singularity marks the limit of validity of general relativity : to describe it, a quantum theory of gravitation would be required.

Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.
- The singularity marks the limit of validity of general relativity : to describe it, a quantum theory of gravitation would be required.
- The event horizon $\mathcal{H}$ is a global structure of spacetime : no physical experiment whatsoever can detect the crossing of $\mathcal{H}$.

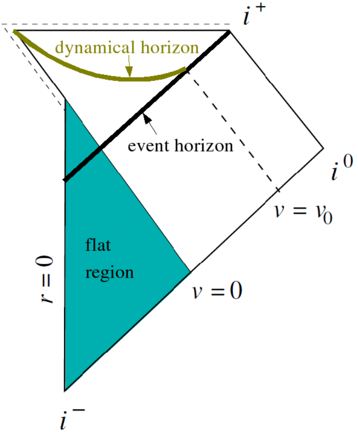
Main properties of black holes (2/2)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its "mean density" by $\bar{\rho} = M/(4/3\pi R^3)$, then
  - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-4}\ \rho_{\text{white dwarf}}$
  - for the BH at the centre of M87 : $\bar{\rho} \sim 2 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-3}\ \rho_{\text{water}}$!

  $\Longrightarrow$ a black hole is a compact object : $\dfrac{M}{R}$ large, not $\dfrac{M}{R^3}$!
- Due to the non-linearity of general relativity, black holes can form in spacetimes without any matter, by collapse of gravitational wave packets.

Teleological nature of event horizons

The standard definition of a black hole is highly non-local : determination of $\dot{J}^-(\mathscr{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is *not locally linked with the notion of strong gravitational field :*



[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

Example of event horizon in a **flat** region of spacetime :

Vaidya metric, describing incoming radiation from infinity :

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\,dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

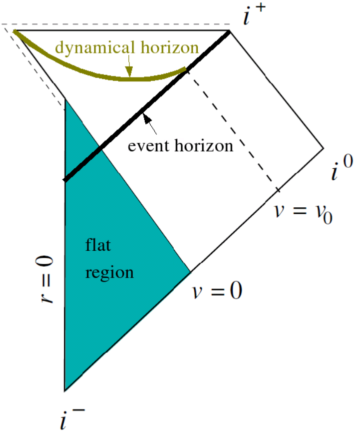
with $m(v) = 0$ for $v < 0$

$dm/dv > 0$ for $0 \leq v \leq v_0$

$m(v) = M_0$ for $v > v_0$

Teleological nature of event horizons

The standard definition of a black hole is highly non-local : determination of $\dot{J}^-(\mathscr{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is *not locally linked with the notion of strong gravitational field :*



[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

Example of event horizon in a **flat** region of spacetime :

Vaidya metric, describing incoming radiation from infinity :

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\,dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with
$m(v) = 0$ for $v < 0$
$dm/dv > 0$ for $0 \leq v \leq v_0$
$m(v) = M_0$ for $v > v_0$

$\Rightarrow$ no local physical experiment can locate the event horizon

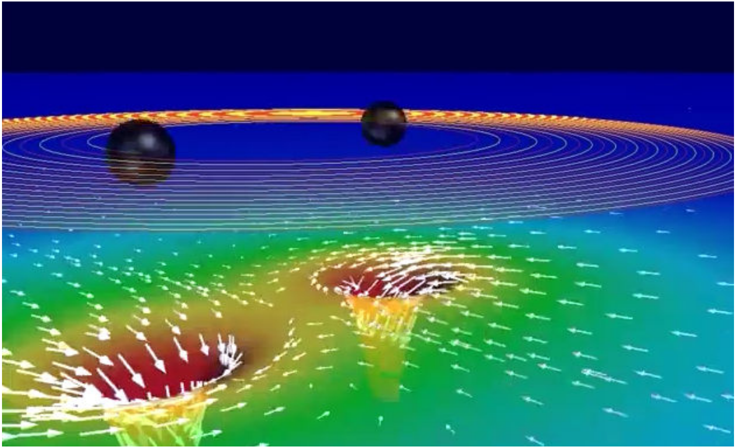
Quasi-local approaches to black holes

New paradigm for the theoretical approach to black holes : instead of *event horizons*, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are **local** and are based on the notion of trapped surfaces

The 2000's : the triumph of numerical relativity



[Caltech/Cornell SXS]
[Scheel et al., PRD **79**, 024003 (2009)]

Outline

Known black holes

Three kinds of black holes are known in the Universe :

- **Stellar black holes** : supernova remnants :
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example : Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

Known black holes

Three kinds of black holes are known in the Universe :

- **Stellar black holes** : supernova remnants :
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example : Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

- **Supermassive black holes**, in galactic nuclei :
  $M \sim 10^5 - 10^{10}\ M_\odot$ and $R \sim 3 \times 10^5\ \mathrm{km} - 200\ \mathrm{UA}$

  example : Sgr A* : $M = 4.3 \times 10^6\ M_\odot$ and
  $R = 13 \times 10^6\ \mathrm{km} = 18\, R_\odot = 0.09\ \mathrm{UA} = \frac{1}{4}$ × radius of Mercury's orbit

Known black holes

Three kinds of black holes are known in the Universe :

- **Stellar black holes** : supernova remnants :
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example : Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

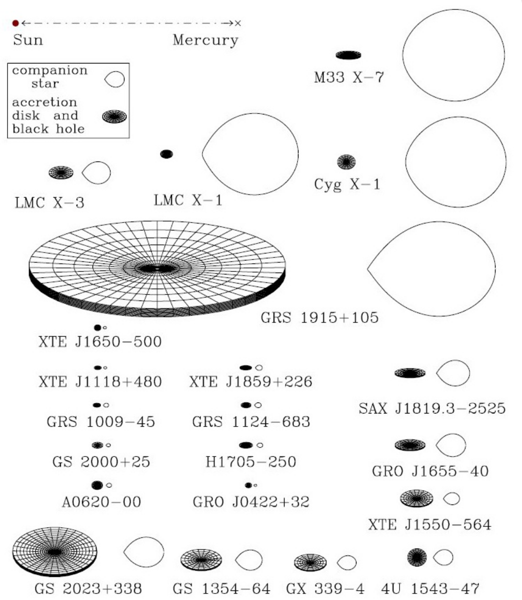
- **Supermassive black holes**, in galactic nuclei :
  $M \sim 10^5 - 10^{10}\ M_\odot$ and $R \sim 3 \times 10^5\ \mathrm{km} - 200\ \mathrm{UA}$

  example : Sgr A* : $M = 4.3 \times 10^6\ M_\odot$ and
  $R = 13 \times 10^6\ \mathrm{km} = 18\, R_\odot = 0.09\ \mathrm{UA} = \frac{1}{4}$ $\times$ radius of Mercury's orbit

- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?) :
  $M \sim 10^2 - 10^4\ M_\odot$ and $R \sim 300\ \mathrm{km} - 3 \times 10^4\ \mathrm{km}$

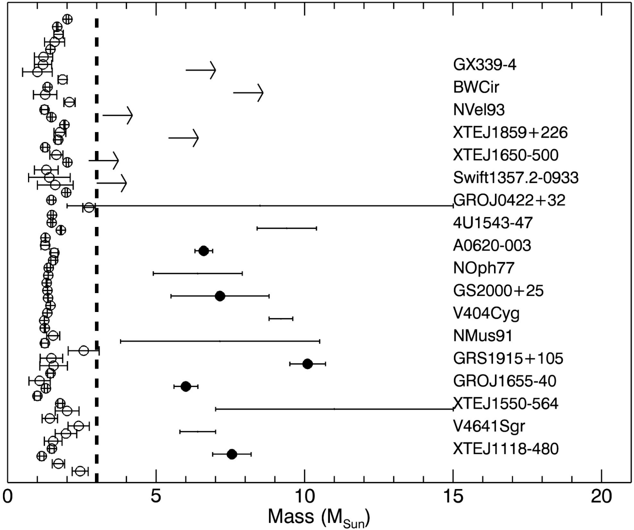
  example : ESO 243-49 HLX-1 : $M > 500\ M_\odot$ and $R > 1500\ \mathrm{km}$

Stellar black holes in X-ray binaries



[McClintock et al. (2011)]

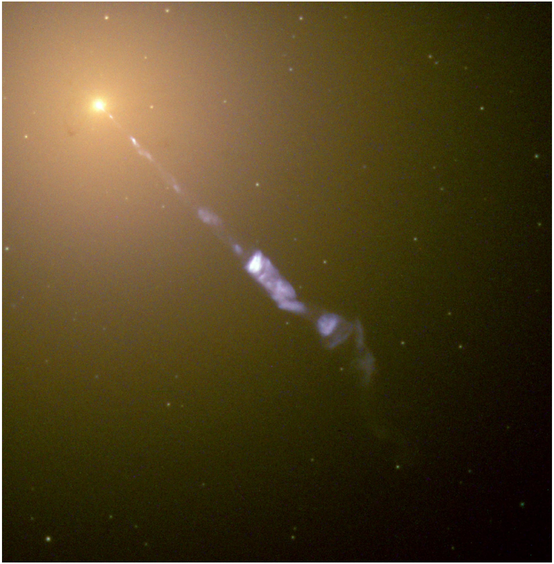
Stellar black holes in X-ray binaries



Dynamically measured masses of black holes in transient low-mass X-ray binaries (right), compared with measured masses of neutron stars (left)

[Corral-Santana et al., A&A, in press, arXiv:1510.08869]

Supermassive black holes in active galactic nuclei (AGN)

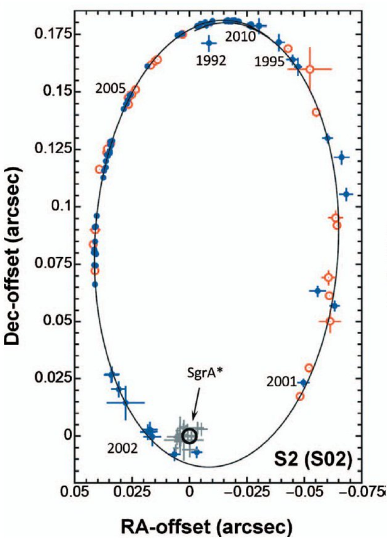


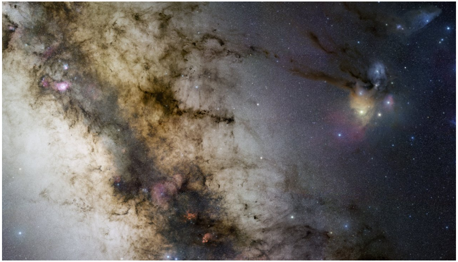
Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

$M_{\rm BH} = 3 \times 10^9\, M_\odot$

$V_{\rm jet} \simeq 0.99\, c$

The black hole at the centre of our galaxy : Sgr A*





[ESO (2009)]

Measure of the mass of Sgr A* black hole by stellar dynamics :

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*

$P = 16\,{\rm yr}$, $r_{\rm per} = 120\,{\rm UA} = 1400\,R_{\rm S}$, $V_{\rm per} = 0.02\,c$

[Genzel, Eisenhauer & Gillessen, RMP **82**, 3121 (2010)]

Can we see a black hole from the Earth ?

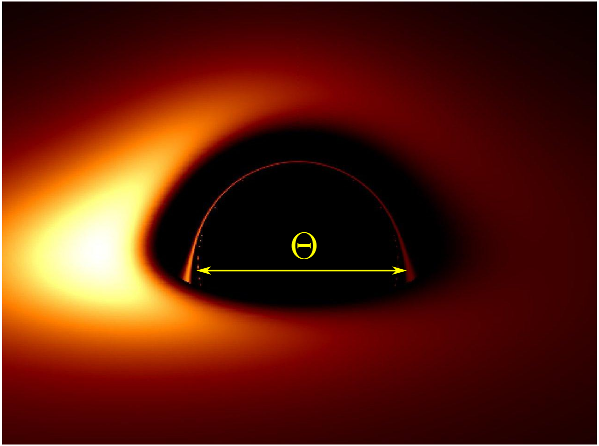


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}$$

Can we see a black hole from the Earth ?

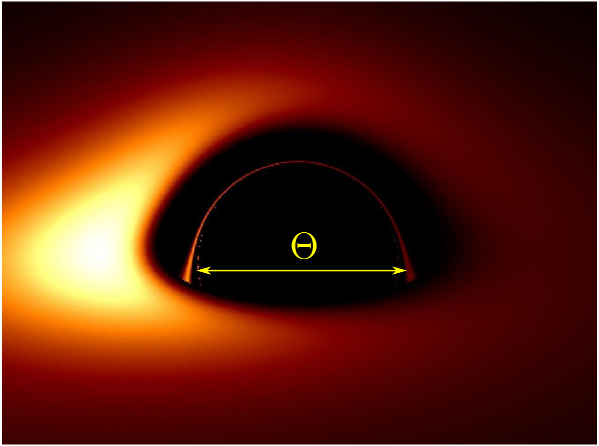


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

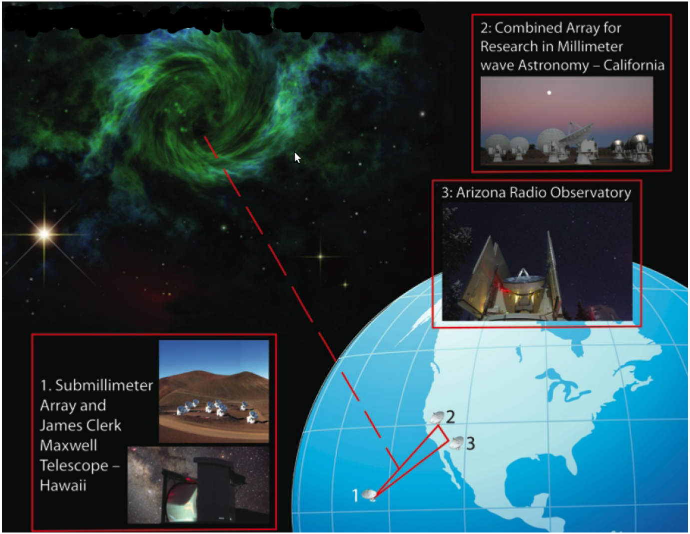
$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}$$

Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu\mathrm{as}$
M87 : $\Theta = 21\ \mu\mathrm{as}$
M31 : $\Theta = 20\ \mu\mathrm{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

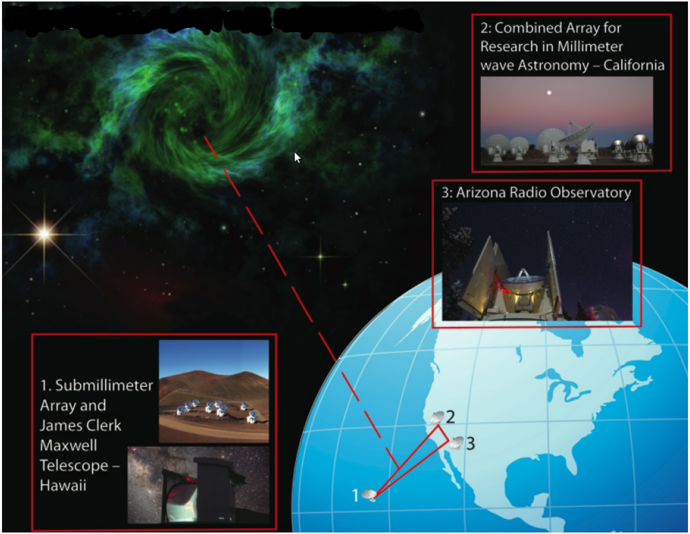
Reaching the μas resolution with VLBI



Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

Reaching the μas resolution with VLBI



Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu$as

[Doeleman et al., Nature **455**, 78 (2008)]

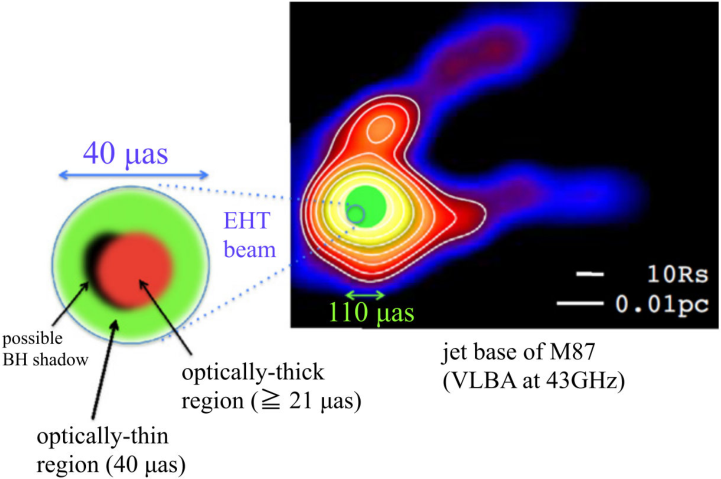
The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : $1.3\ \mathrm{mm} \rightarrow 0.8\ \mathrm{mm}$
- increase the number of stations ; in particular add ALMA

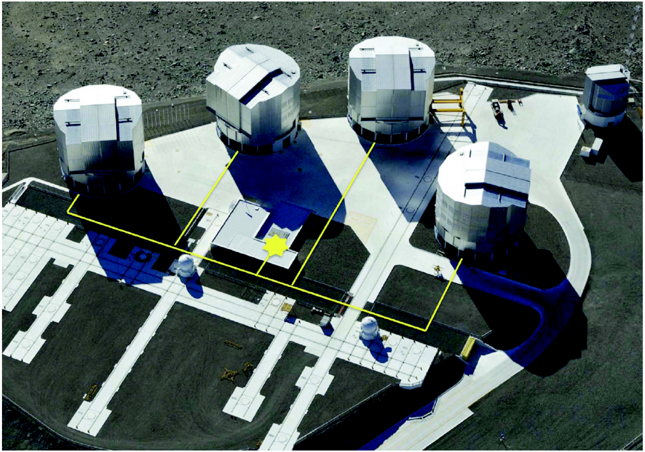
Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
August 2015 : VLBI observations involving ALMA and VLBA

VLBA and EHT observations of M87



[Kino et al., ApJ **803**, 30 (2015)]

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu as$

Near-infrared optical interferometry : GRAVITY

[MPE/GRAVITY team]

July 2015 : GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory
Commissioning with the four 8-m VLT Unit Telescope : first half 2016.

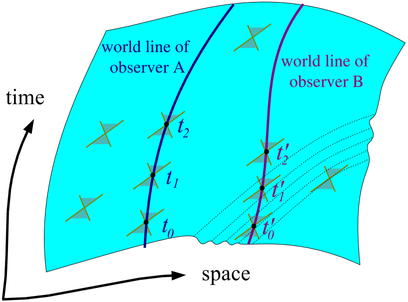
Outline

Gravitational waves

Linearization of Einstein equation in weak field : $\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$,
$\boldsymbol{\eta}$ = Minkowski metric [1]

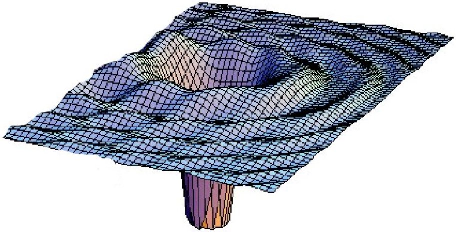
$$\Longrightarrow \text{wave equation : } \boxed{\Box\, \bar{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \quad \text{(Lorenz gauge)}$$

with $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\bar{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\, \boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



1. $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ en Cartesian coordinates

Black holes and gravitational waves



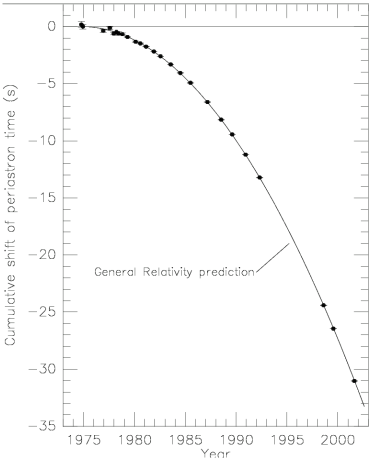
Link between black holes and gravitational waves :

Both are spacetime distortions :

- extreme distortions (black holes)
- small distortions (gravitational waves)

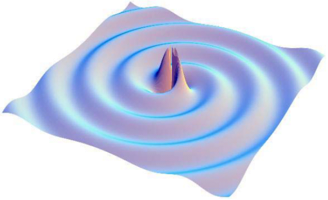
In particular, black holes and gravitational waves are both vacuum solutions of Einstein equation

Observational evidence for gravitational waves



[Weisber & Taylor (2002)]

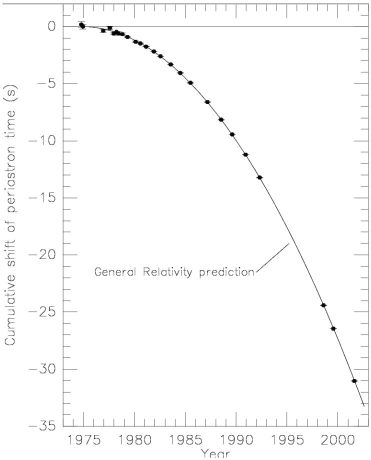
Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period

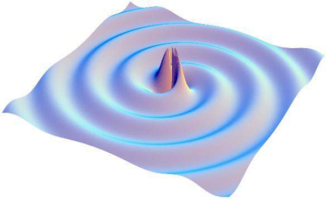
$P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*
$\Longrightarrow$ coalescence in 140 millions year.

Observational evidence for gravitational waves



[Weisber & Taylor (2002)]

Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period

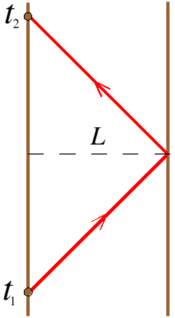
$P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*
$\Longrightarrow$ coalescence in 140 millions year.

Nobel Prize in Physics to R. Hulse & J. Taylor (1993)

Measurable effects of a gravitational wave passage

Measure of the distance L between two free masses by a "radar" method :

$$L = \frac{1}{2} c(t_2 - t_1)$$



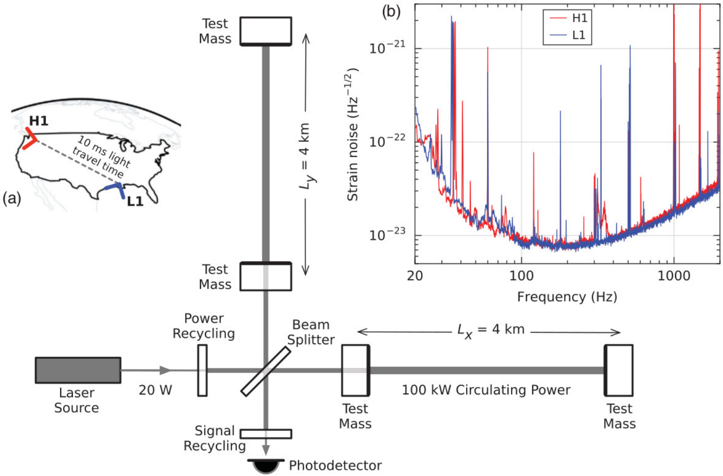
Variation of length L when a gravitational wave passes by :

$$\boxed{\delta L \simeq h\, L}$$

h = amplitude of the gravitational wave

In practice, h is so small that our senses are not sensitive to it :
for the most important **astrophysical sources :**
$h \sim 10^{-21}$! ! !

Advanced LIGO detectors



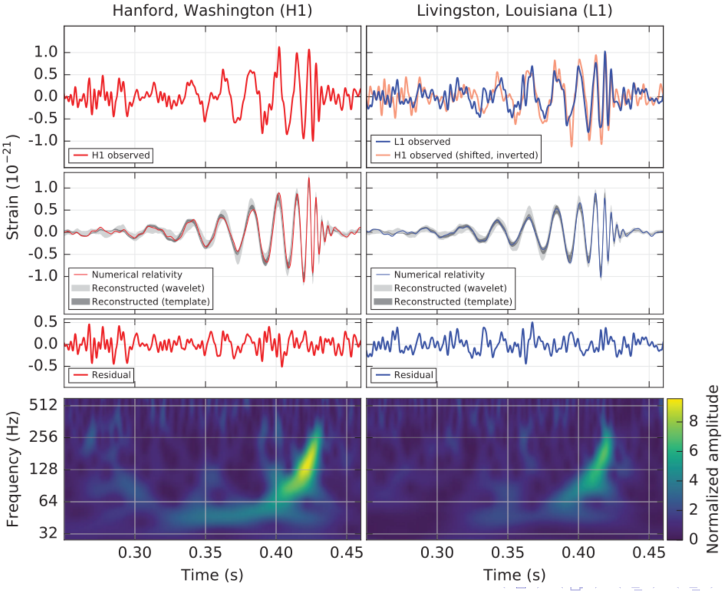
[Abbott et al., PRL **116**, 061102 (2016)]

Advanced ground-based GW detectors

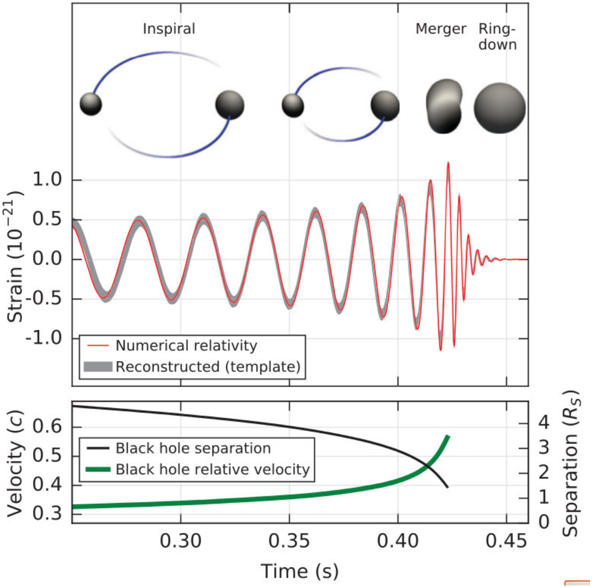
Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

- Adv. LIGO : started Sept. 2015
- Adv. Virgo : will start in fall 2016
- KAGRA (Japan) : 2018

September 14, 2015, 09:50:45 UTC



GW150914 event



Signal :

$\Delta t = 0.2\,\mathrm{s}$

$f: \ 35 \to 250\,\mathrm{Hz}$

$h_{\rm max} = 1.0\ 10^{-21}$

Matched filter :

$S/N = 24$

$F_{\rm false} = 1/203,000\ \mathrm{yr}$

$M_1 = 36 \pm 5\, M_\odot$

$M_2 = 29 \pm 4\, M_\odot$

$d = 410 \pm 180\,\mathrm{Mpc}$

$z = 0.09 \pm 0.04$

$M_{\rm final} = 62 \pm 4\, M_\odot$

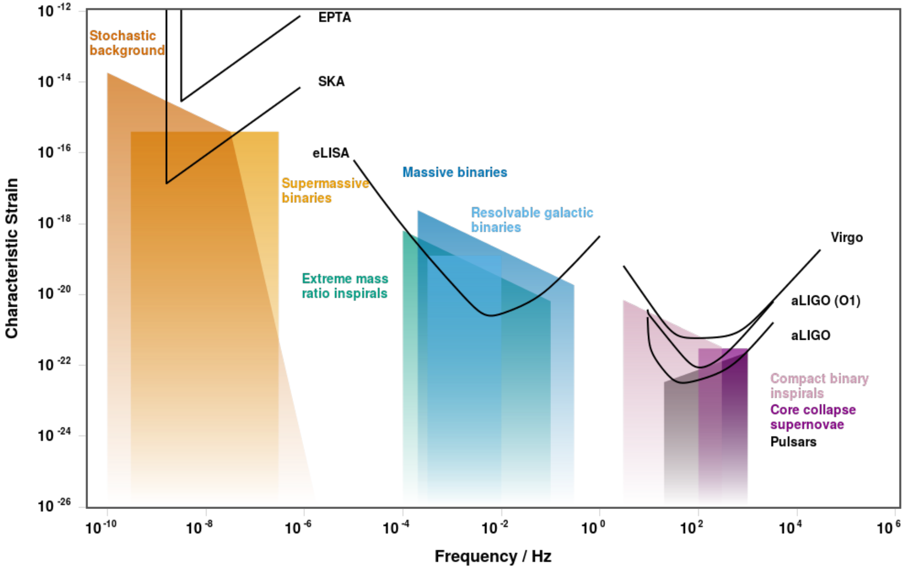
$\Rightarrow\ E^{\rm GW}_{\rm rad} = 3.0 \pm 0.5\, M_\odot c^2$

$a_1 < 0.7, \ a_2 < 0.9$

$a_{\rm final} = 0.67 \pm 0.07$

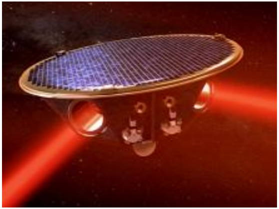
[Abbott et al., PRL **116**, 061102 (2016)]

GW detectors in different bandwidths



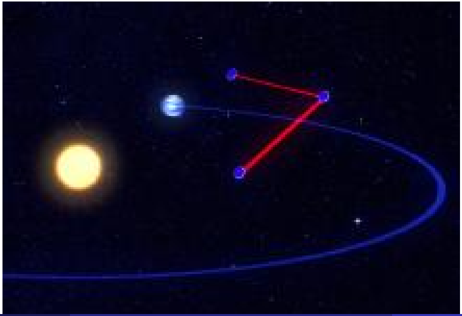
Space detector eLISA (ESA)

Interferometric gravitational wave detector in solar orbit

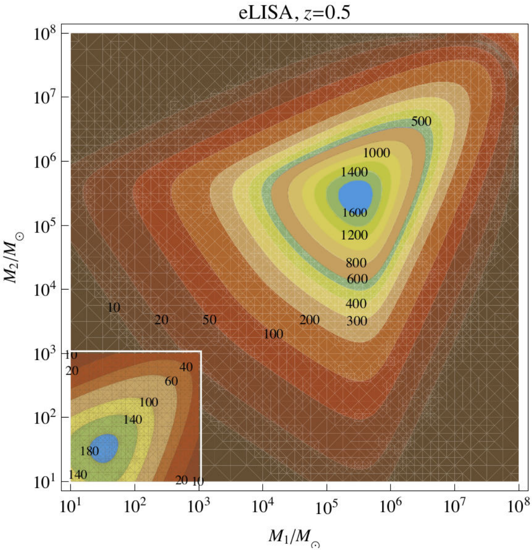


[eLISA / NGO]

- theme selected by ESA in 2013 for the L3 mission
- launch around 2028
- technology demonstrator LISA Pathfinder launched on 3 December 2015



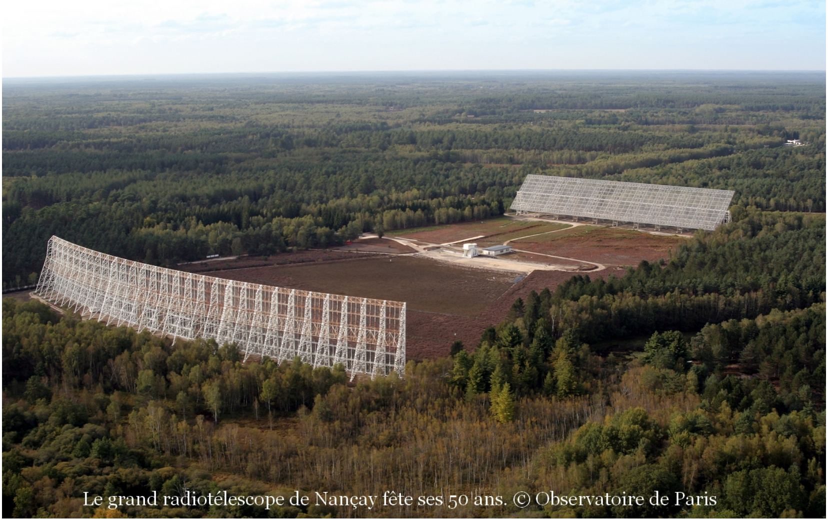
eLISA observations of massive binary BH mergers



Signal-to-noise ratio for gravitational waves from the inspiral of a BH binary at $z = 0.5$

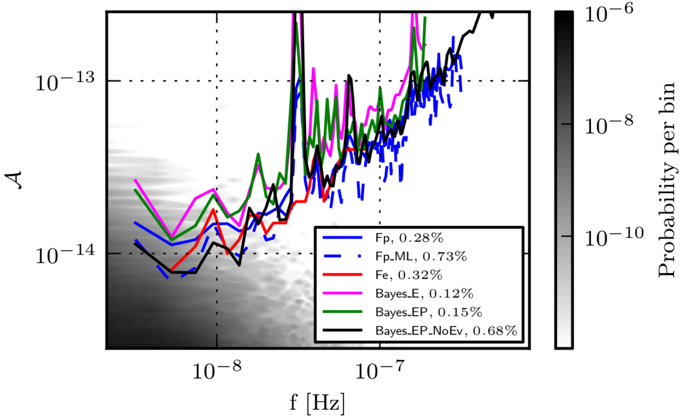
[Barausse et al., J. Phys. Conf. Ser. **610**, 012001 (2015)]

Detecting gravitational waves by pulsar timing



EPTA results on supermassive BH binaries

EPTA : European Pulsar Timing Array



[Babak et al., MNRAS **455**, 1665 (2016)]

Outline

Is general relativity unique ?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $\boldsymbol{g}$
- field equation involving only derivatives of $\boldsymbol{g}$ up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ ($\Longrightarrow$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

Is general relativity unique ?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $\boldsymbol{g}$
- field equation involving only derivatives of $\boldsymbol{g}$ up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ ($\Longrightarrow$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

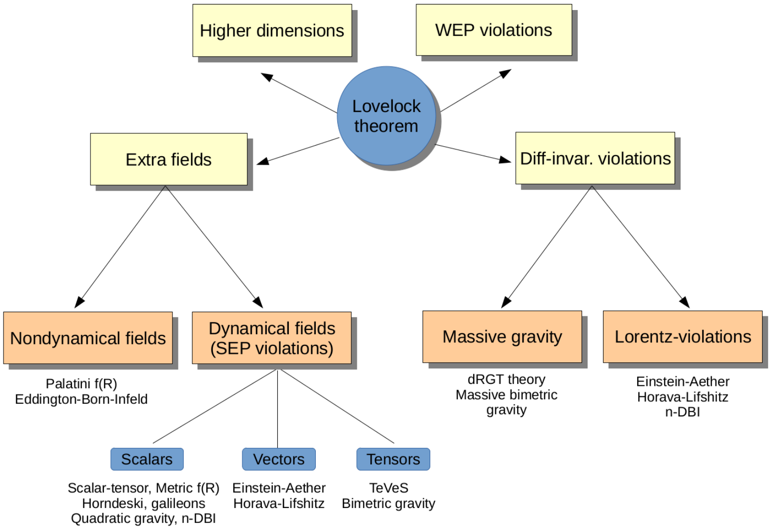
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

Extensions of general relativity



[Berti et al., CGQ **32**, 243001 (2015)]

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

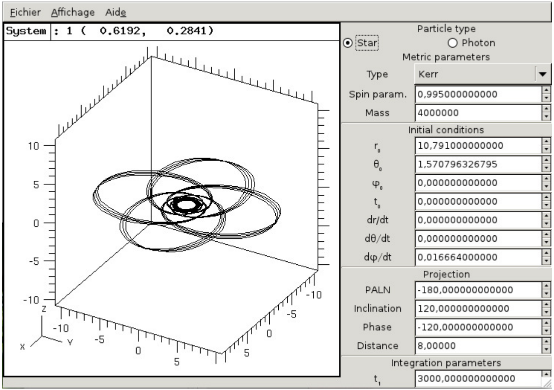
Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)

Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

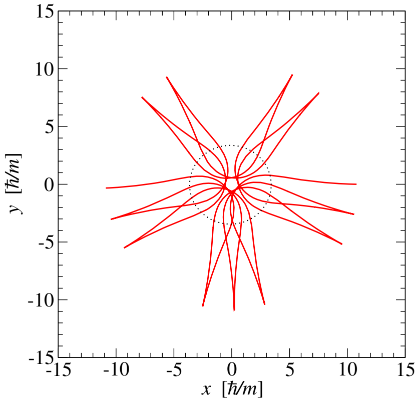
[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

An example : rotating boson stars

Boson star = localized configurations of a self-gravitating massive complex scalar field Φ ≡ *"Klein-Gordon geons"*
[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of Kadath
  [Grandclément, JCP **229**, 3334 (2010)]
- Timelike geodesics computed by means of Gyoto

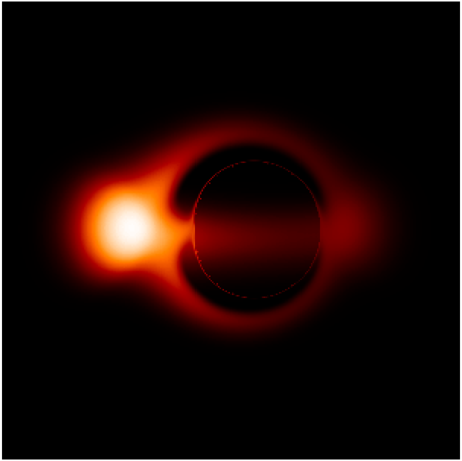


Zero-angular-momentum orbit around a rotating boson star based on a free scalar field $\Phi = \phi(r, \theta)e^{i(\omega t + 2\varphi)}$ with $\omega = 0.75\, m/\hbar$.

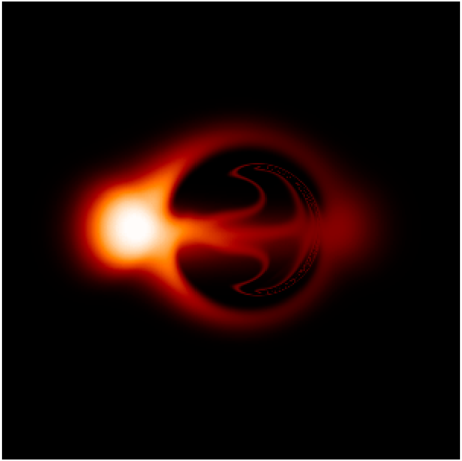
[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

Image of an accretion torus

Kerr BH $a/M = 0.9$



Boson star $k = 1$, $\omega = 0.70\, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, arXiv:1510.04170]

Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime.

Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime.

The GW150914 event was both the first direct detection of gravitational waves and the first observation of a the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.