LISA: a primordial black hole detector?

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based on a collaboration with

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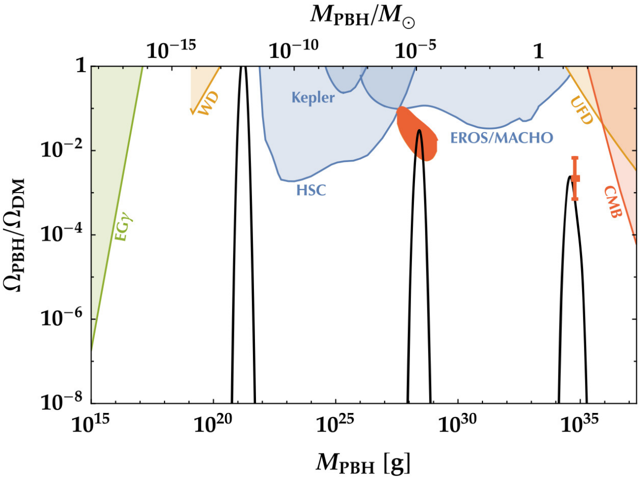
2$^{\rm e}$ Assemblée Générale du GdR *Ondes Gravitationnelles*
Institut de Physique des 2 Infinis, Lyon
10-11 October 2019

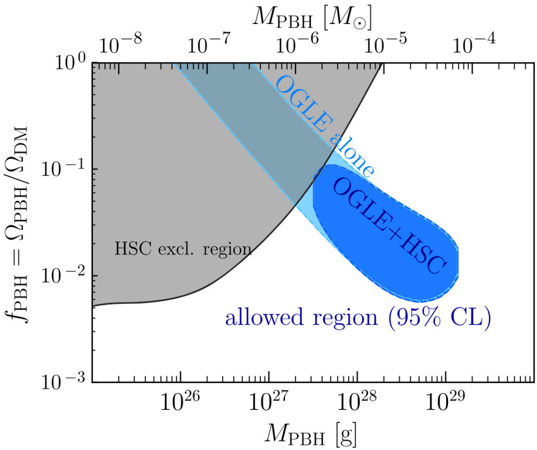
Primordial black holes (PBH)



Predicted PBH mass spectrum from multiphase inflation
+ observational upper bounds

[Tada & Yokoyama, PRD **100**, 023537 (2019)]

Observation of 6 ultrashort microlensing events (OGLE)



PBH abundance assuming that the 6 ultrashort OGLE events are due to PBHs

[Niikura et al., PRD **99**, 083503 (2019)]

6 microlensing events in OGLE-IV survey (2011-15) with ultrashort timescale:

$t_E \in [0.1\ {\rm d},\ 0.1\ {\rm d}]$

$\Longrightarrow \mu \in [0.5 M_\oplus,\ 20 M_\oplus]$

[Mróz et al., Nature **548**, 183 (2017)]

- free floating planets?
- primordial black holes?

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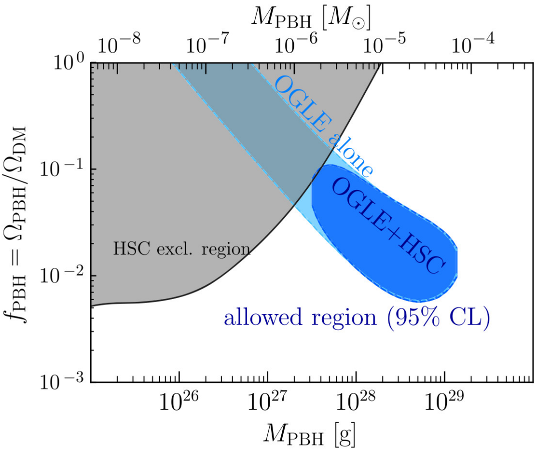

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Planet 9 could be a primordial BH of mass $\mu \sim 5 - 15\ M_\oplus$
Capture probability similar to that of a free floating planet
[Scholtz & Unwin, arXiv:1909.11090]

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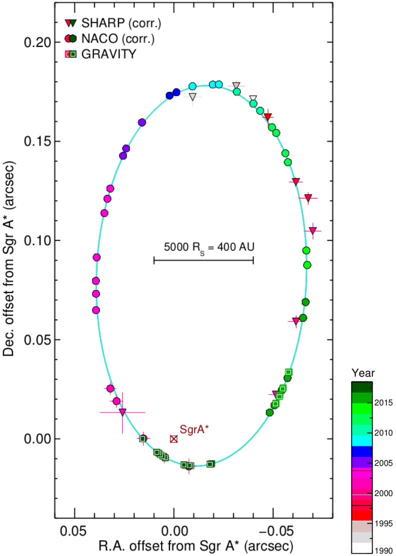
What about the supermassive BH at the center of our galaxy?

The massive black hole at the Galactic center, Sgr A*



- distance: $d = 8.12 \text{ kpc}$
- mass:

$$\begin{aligned} M &= 4.10 \times 10^6 \, M_\odot \\ &= 20.2 \text{ s} \qquad\qquad (c = G = 1) \\ &= 6.06 \times 10^9 \text{ m} \\ &= 4.05 \times 10^{-2} \text{ au} \\ &= 1.96 \times 10^{-7} \text{ pc} \end{aligned}$$

- spin $J = aM$ unknown yet...

← Orbit of star S2 around Sgr A*

S2: main-sequence B star

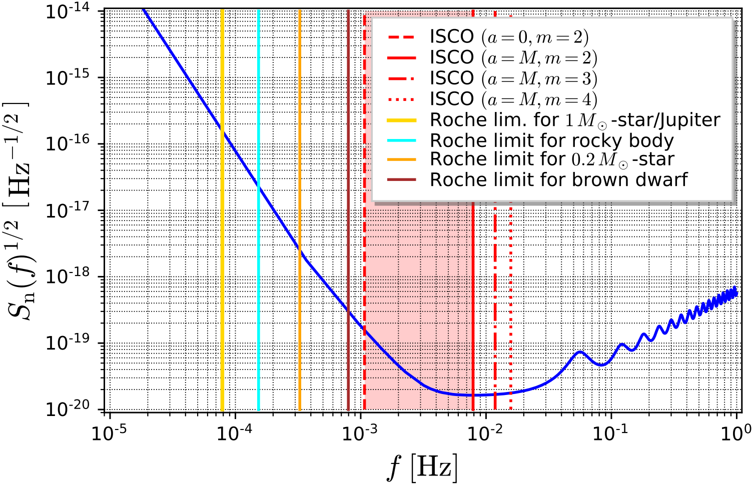
orbital period: $P = 16.05 \text{ yr}$

periastron (May 2018):

- $r_{\rm per} = 120 \text{ au} = 3 \times 10^3 M$
- $v_{\rm per} = 7650 \text{ km s}^{-1} = 0.025\, c$

[GRAVITY team, A&A **615**, L15 (2018)]

GW frequencies from Sgr A* close orbits are in LISA band



ISCO = *Innermost Stable Circular Orbit*: $r_0 = 6M\ (a = 0) \rightarrow r_0 = M\ (a = M)$
ISCO for $a = M$: $f_{m=2} = 7.9\ \mathrm{mHz} \leftarrow$ coincides with LISA max. sensitivity!

GWs from close circular orbits around Sgr A*

Starting from [Freitag, ApJ **583**, L21 (2003)], various studies about GWs from stars and compact objects orbiting Sgr A* and their detectability by LISA.
All studies have been performed in a Newtonian framework (quadrupole formula).
Now, for orbits close to the ISCO, relativistic effects are expected to be important.

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Our study [Gourgoulhon, Le Tiec, Vincent & Warburton, A&A **627**, A92 (2019)]

Fully relativistic framework:

- Sgr A* modeled as a Kerr BH and GWs computed via the theory of perturbations of Kerr metric
- tidal effects via the theory of Roche potential in Kerr metric developed by Dai & Blandford (2013) [MNRAS **434**, 2948]

Current limitation: circular equatorial orbits

Waveforms from circular orbits

computed as linear perturbations of Kerr metric (Teukolsky 1973)

Detweiler (1978)

$$h_+ - \mathrm{i}h_\times = \frac{2\mu}{r}\sum_{\ell=2}^{\infty}\sum_{\substack{m=-\ell\\ m\neq 0}}^{\ell} \frac{Z^\infty_{\ell m}(r_0)}{(m\omega_0)^2}\, {}_{-2}S^{am\omega_0}_{\ell m}(\theta,\varphi)\, \mathrm{e}^{-\mathrm{i}m(\omega_0(t-r_*)+\varphi_0)}$$

μ: mass of orbiting object; (t, r, θ, φ): Boyer-Lindquist coordinates of the observer

${}_{-2}S^{am\omega_0}_{\ell m}(\theta,\varphi)$: spheroidal harmonics of spin weight -2

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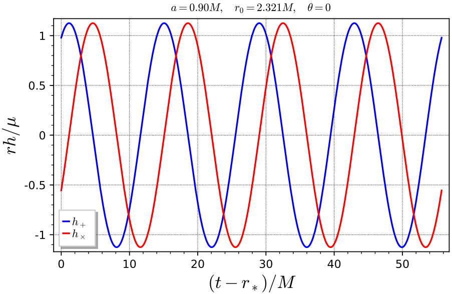
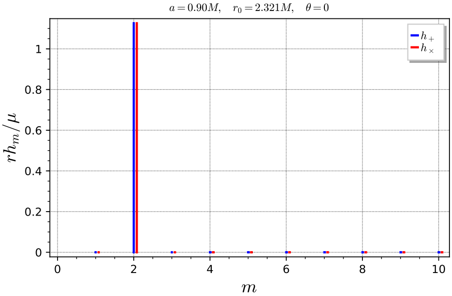
Example for $a = 0.9\,M$, $r_0 = r_{\rm ISCO}(a)$ and viewing angle $\theta = 0$ (face-on)





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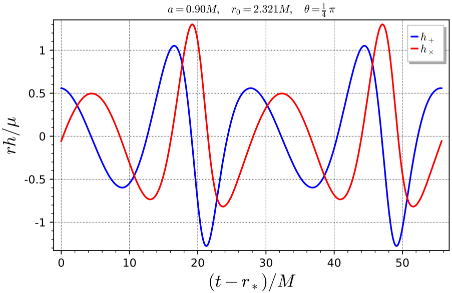
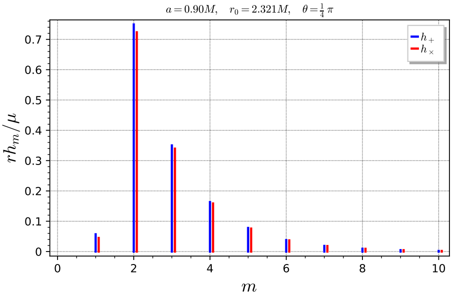
Example for $a = 0.9\,M$, $r_0 = r_{\rm ISCO}(a)$ and viewing angle $\theta = \pi/4$





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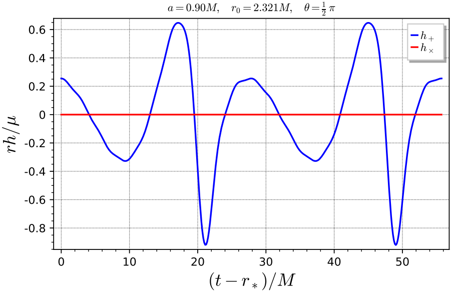
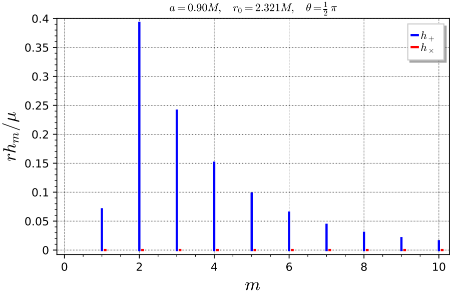
Example for $a = 0.9\,M$, $r_0 = r_{\mathrm{ISCO}}(a)$ and viewing angle $\theta = \pi/2$ (edge-on)





Implementation: the kerrgeodesic_gw package

All computations (GW waveforms, SNR in LISA, energy fluxes, inspiralling time, etc.) have been implemented as a Python package for the open-source mathematics software system SageMath:

`kerrgeodesic_gw`

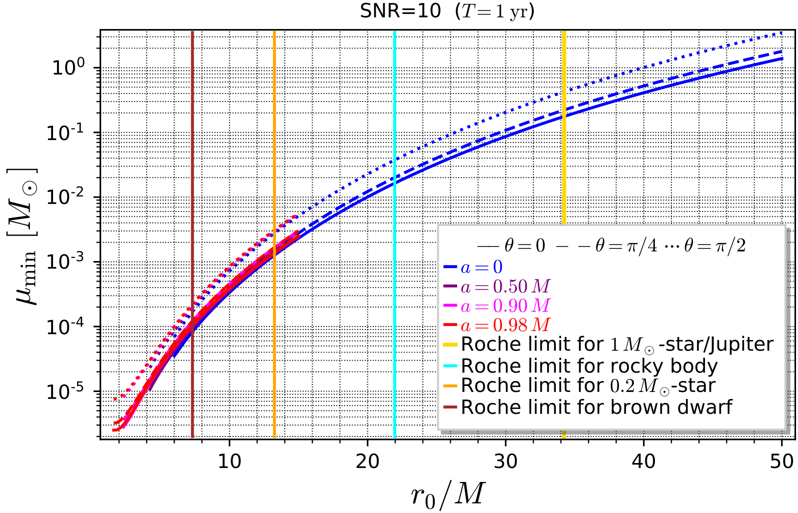
`kerrgeodesic_gw` is

- entirely open-source:
  `https://github.com/BlackHolePerturbationToolkit/kerrgeodesic_gw`
- is distributed via PyPi (Python Package Index):
  `https://pypi.org/project/kerrgeodesic-gw/`
- is part of the *Black Hole Perturbation Toolkit*:
  `http://bhptoolkit.org/`

Minimal detectable mass by LISA

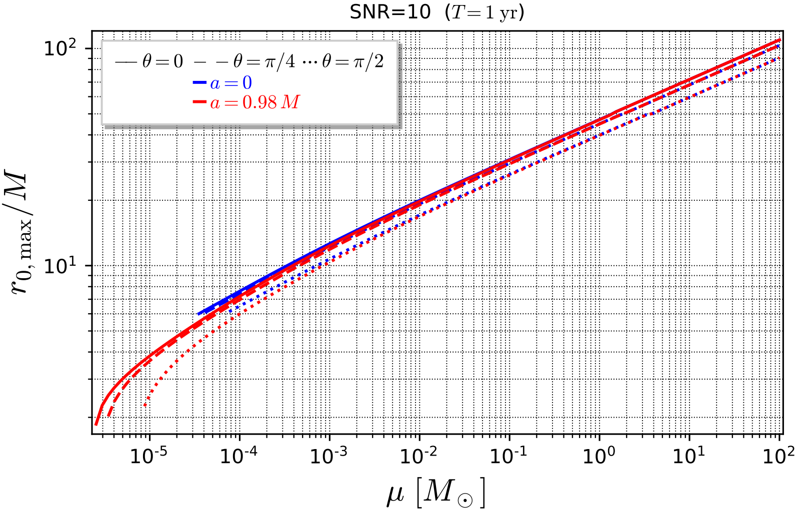
as a function of the radius r_0 of the circular orbit

Detection criteria: SNR $\geq$ 10
Observation time: $T = 1$ yr



Maximum orbital radius for LISA detection

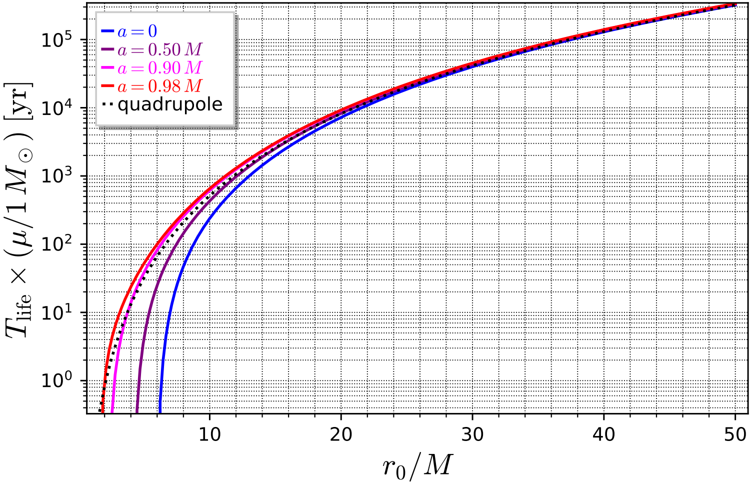
Maximum orbital radius $r_{0,\max}$ for a SNR $= 10$ detection by LISA in 1 yr of data, as a function of the mass μ of the object orbiting Sgr A*:



Detection probability governed by the life time of orbits

gravitational radiation reaction $\Longrightarrow$ slow inspiral motion

$T_{\rm life}(r_0)$: time for a compact object to reach the ISCO starting from circular orbit of radius r_0



Time spent in LISA band

Inspiral time from orbit r_0 to orbit r_1 due to reaction to gravitational radiation:

$$T_{\rm ins}(r_0, r_1) = \frac{M^2}{2\mu} \int_{r_1/M}^{r_0/M} \frac{1 - 6/x + 8\bar{a}/x^{3/2} - 3\bar{a}^2/x^2}{\left(1 - 3/x + 2\bar{a}/x^{3/2}\right)^{3/2}} \frac{{\rm d}x}{x^2(\tilde{L}_\infty(x) + \tilde{L}_{\rm H}(x))}$$

where $\tilde{L}_{\infty,{\rm H}}(x) := (M/\mu)^2 L_{\infty,{\rm H}}(xM)$ and L_∞ (resp. $L_{\rm H}$) is the total GW power emitted at infinity (resp. through the BH event horizon) by a particle of mass μ orbiting at $r = xM$

Compact object

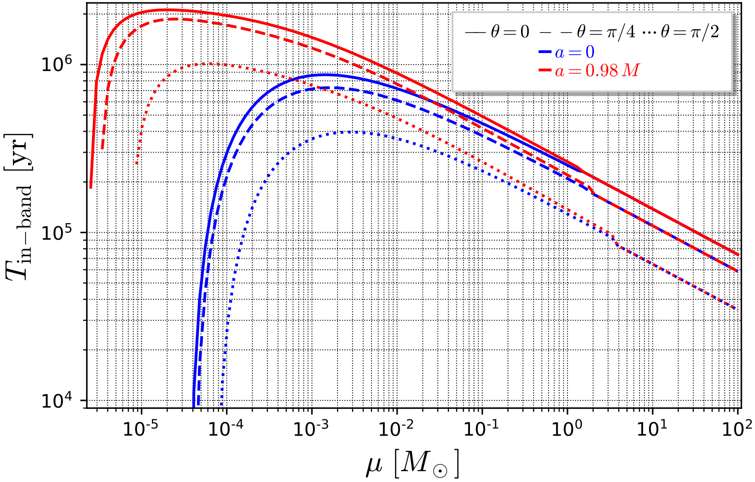
$$T_{\text{in-band}} = T_{\rm ins}(r_{0,\max}, r_{\rm ISCO}) = T_{\rm life}(r_{0,\max})$$

Main-sequence stars and brown dwarfs

$$T_{\text{in-band}} \geq T^{\rm ins}_{\text{in-band}} = T_{\rm ins}(r_{0,\max}, r_{\rm Roche})$$

Time in LISA band for an inspiralling compact object

as a function of the compact object mass μ



[Gourgoulhon, Le Tiec, Vincent & Warburton, A&A **627**, A92 (2019)]

Conclusions

For face-on orbits ($\theta = 0$):

- if Sgr A* is a slow rotator, primordial BHs with $2 \times 10^{-4} M_\odot \leq \mu \leq 0.1 M_\odot$ spend more than $5 \times 10^5\ \mathrm{yr}$ in LISA band with $\mathrm{SNR}_{1\,\mathrm{yr}} \geq 10$

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- if Sgr A* is a fast rotator ($a \sim 0.9M$), primordial BHs with

  $$3 \times 10^{-6} M_\odot \leq \mu \leq 5 \times 10^{-3} M_\odot \quad \Longleftrightarrow \quad 1 M_\oplus \leq \mu \leq 5 M_\mathrm{Jup}$$

  ($\sim$ the OGLE range!)

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If the capture rate of such primordial BHs by Sgr A* is higher than $10^{-6}\ \mathrm{yr}^{-1}$, then the probability of detection by LISA is ~ 1.

Appendix: time in LISA band for brown dwarfs and main-sequence stars

Results for

- inclination angle $\theta = 0$
- BH spin $a = 0$ (outside parentheses) and $a = 0.98M$ (inside parentheses)

	brown dwarf	red dwarf	Sun-type	$2.4\, M_\odot$-star
$\mu/M_\odot$	0.062	0.20	1	2.40
$\rho/\rho_\odot$	131.	18.8	1	0.367
$r_{0,\max}/M$	28.2 (28.0)	35.0 (34.9)	47.1 (47.0)	55.6 (55.6)
$f_{m=2}(r_{0,\max})$ [mHz]	0.105 (0.106)	0.076 (0.076)	0.049 (0.049)	0.038 (0.038)
$r_{\rm Roche}/M$	7.31 (6.93)	13.3 (13.0)	34.2 (34.1)	47.6 (47.5)
$T^{\rm ins}_{\textsf{in-band}}$ [10^5 yr]	4.98 (5.55)	3.72 (3.99)	1.83 (1.89)	0.938 (0.945)

Brown dwarfs stay for $\sim 5 \times 10^5$ yr in LISA band