Observing black holes with gravitational waves

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Plan

1. Gravitational radiation from black holes
2. Black hole quasi-normal modes
3. Binary black hole coalescence
4. Inspiral of a star into a massive black hole
5. Gravitational radiation from microquasars and gamma-ray bursts

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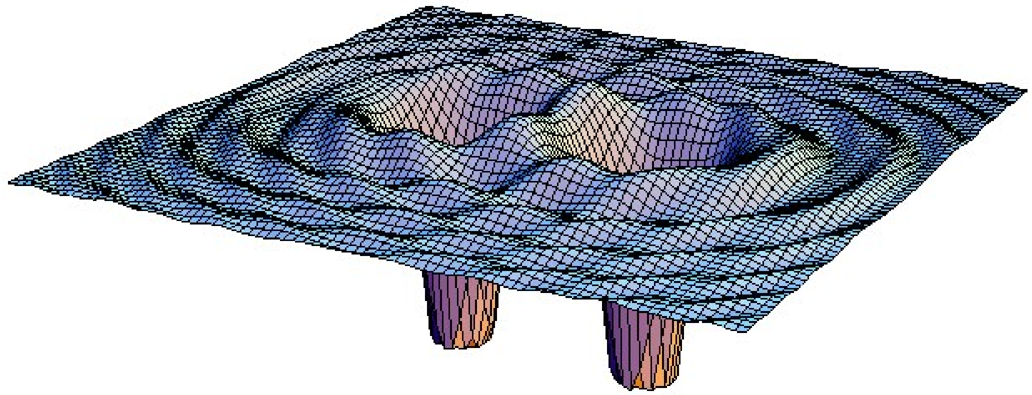
Gravitational radiation from black holes

Gravitational waves

...the only detectable radiation which comes directly from a black hole.

(Hawking radiation negligible)



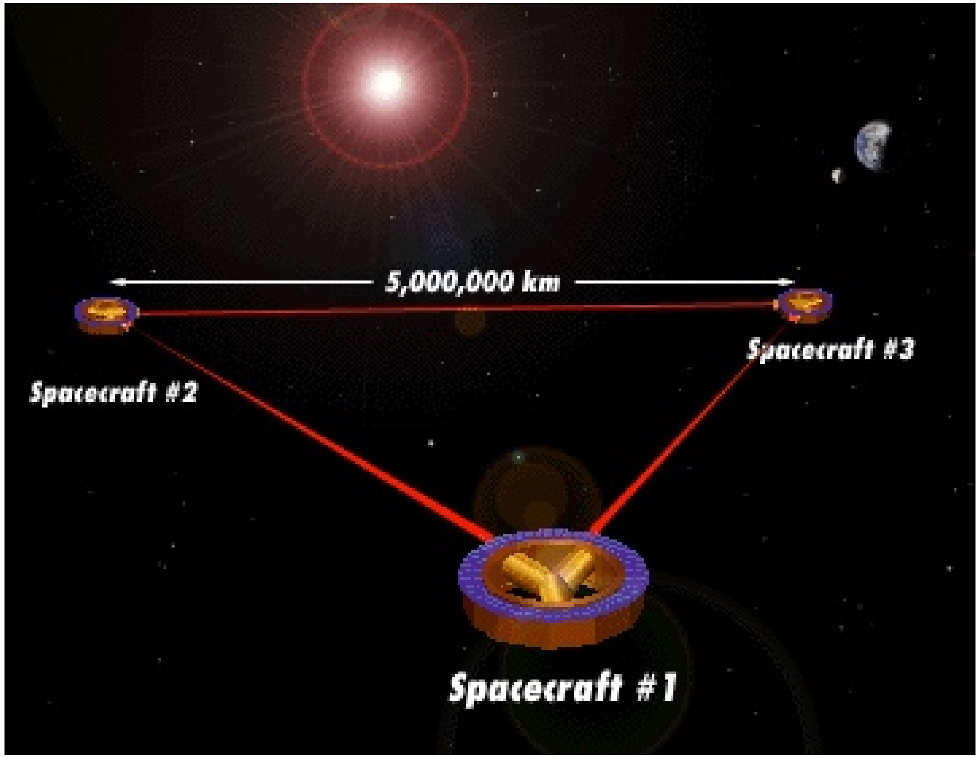
Black holes and gravitational waves are both pure spacetime structures.

Detection of gravitational radiation

Gravitational wave detectors are coming on line...

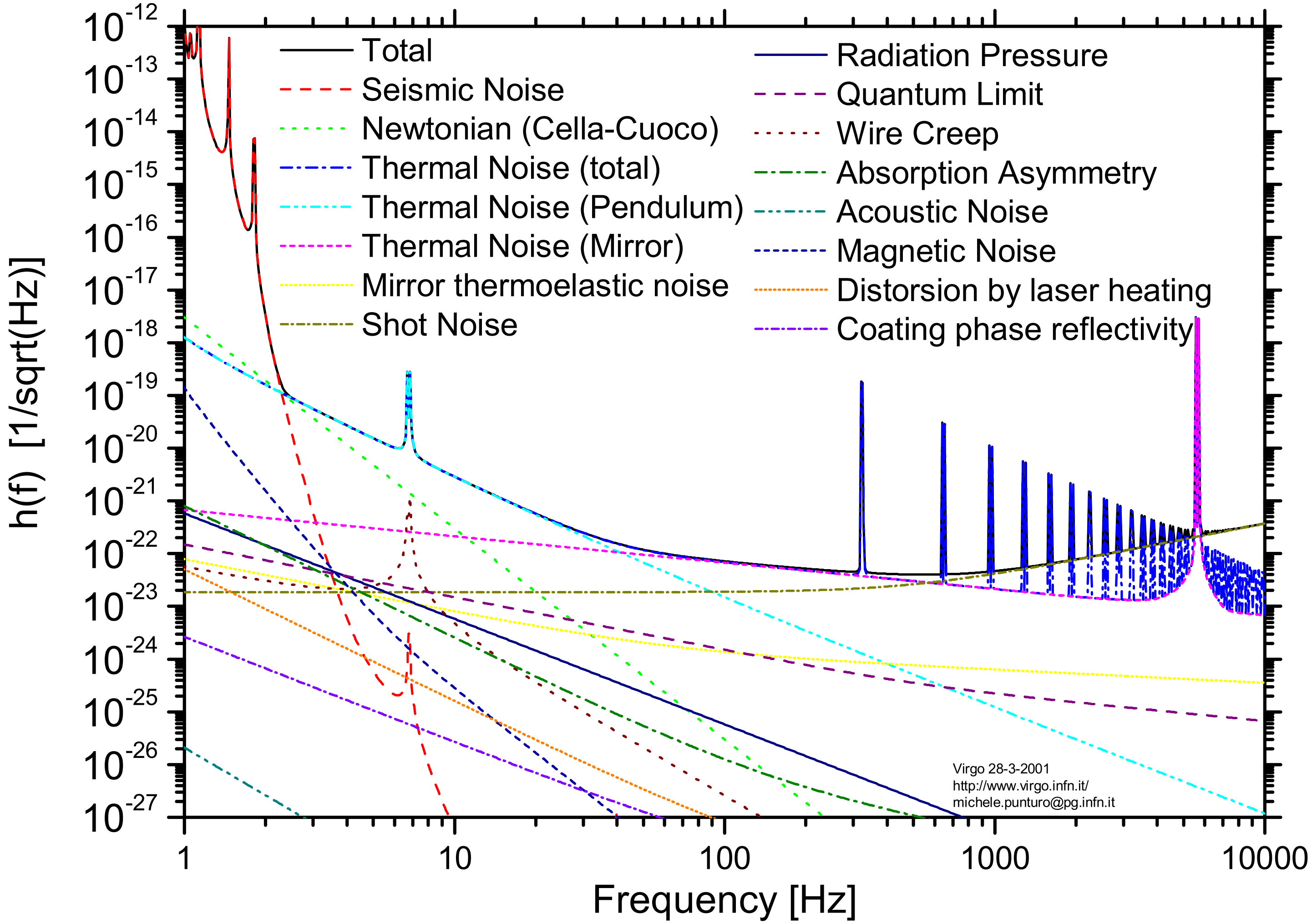
VIRGO, Cascina, Italy
$10\ \mathrm{Hz} < f < 10^3\ \mathrm{Hz}$
also LIGO, GEO600, TAMA

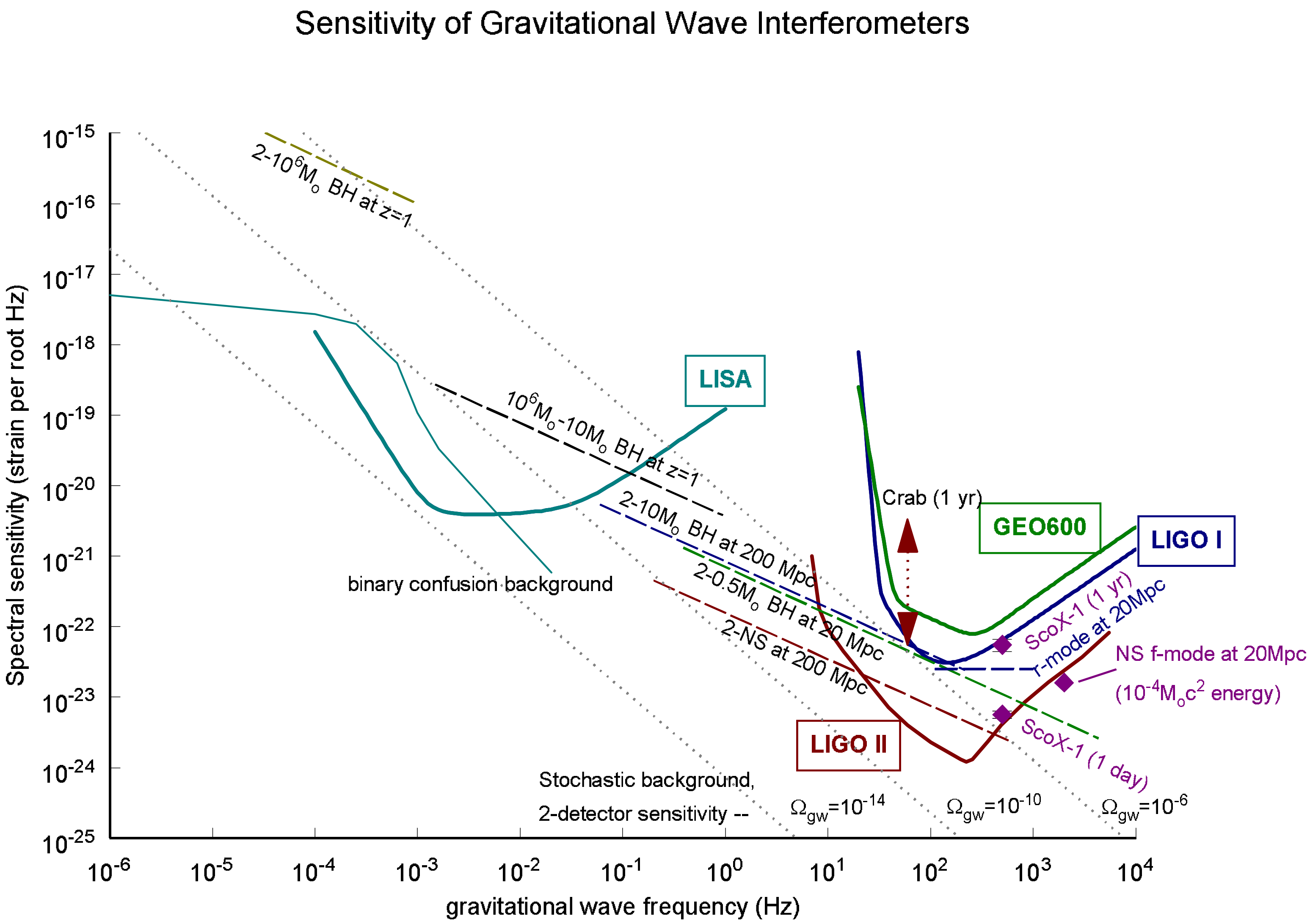
...or will be launched in the not too far future (2011)



LISA (ESA/NASA)
$10^{-4}\ \mathrm{Hz} < f < 10^{-1}\ \mathrm{Hz}$

Expected noise density $S(f)^{1/2}$ for the VIRGO detector





[Schutz, CQG **16**, A131 (1999)]

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Oscillations of black holes

Emission of gravitational waves by a single black hole

An excited black hole loses his hair by emitting gravitational waves

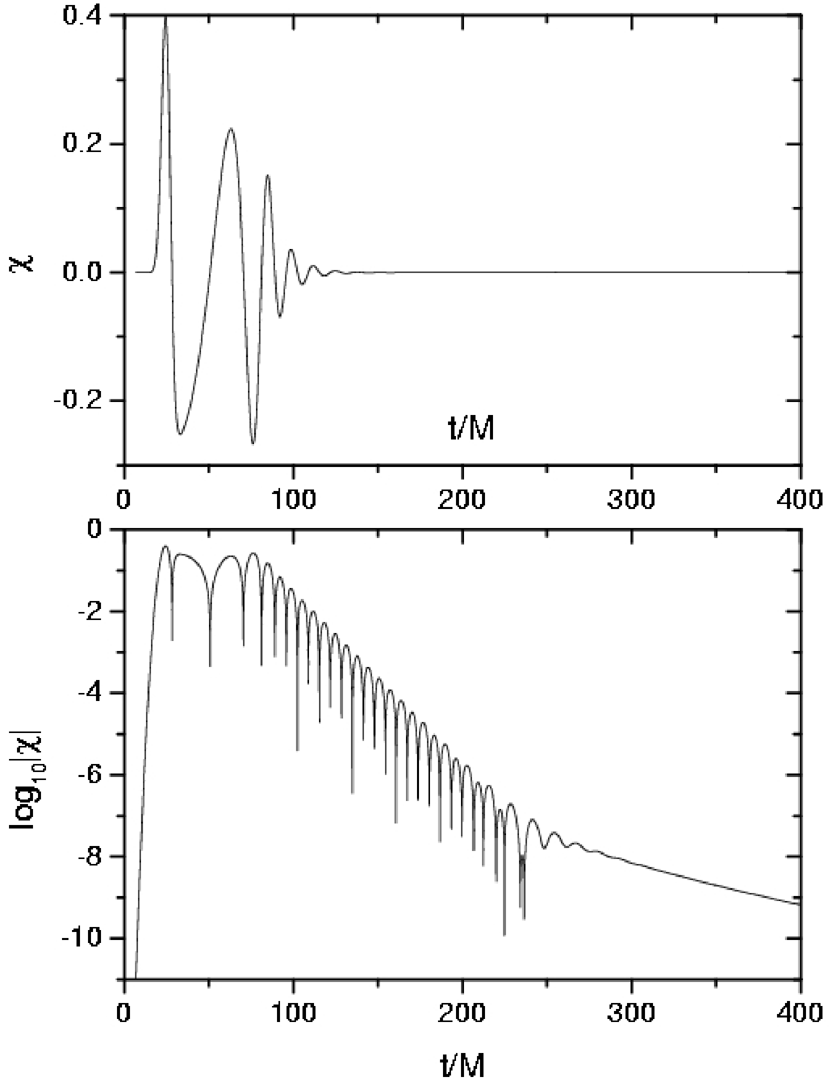
Occurrence of an excited black hole:

- end product of supernova explosion or coalescence of binary black hole or neutron star

- excitation by infalling matter (star or accreted blob of gas)

Final outcome: Kerr black hole (no hair theorem)

Black hole quasi-normal modes



[from Kokkotas & Schmidt, LRR **2**, 2 (1999)]

Strongly damped quasi-periodic oscillations

Most slowly damped mode ($\ell = m = 2$):

$$h_+ - i h_\times \propto h_0 \exp(i\omega t - t/\tau)$$

$$h_0 \simeq 4 \times 10^{-23} \left(\frac{\delta E}{10^{-6} M}\right)^{1/2} \left(\frac{M}{10\, M_\odot}\right) \left(\frac{15 \text{ Mpc}}{r}\right)$$

$$\omega \simeq \frac{1}{M}\left[1 - 0.63\,(1 - a/M)^{0.3}\right]$$

$$\tau \simeq \frac{4}{\omega\,(1 - a/M)^{0.45}}$$

[Echeverria, PRD **40**, 3194 (1989)]

$$M = 10\, M_\odot \Rightarrow \begin{cases} f = 1.2 \text{ kHz} \\ \tau = 0.55 \text{ ms} \end{cases}$$ (VIRGO)

$$M = 10^6\, M_\odot \Rightarrow \begin{cases} f = 12 \text{ mHz} \\ \tau = 55 \text{ s} \end{cases}$$ (LISA)

Black-hole "spectroscopy"

Deducing black hole parameters from QNM detection

The QNM wave parameters (f, τ) of the most slowly damped mode are a unique and invertible function of the black hole mass and spin (M, a) [Detweiler, ApJ **239**, 292 (1977)].

1-sigma uncertainties in terms of the signal-to-noise ratio S/N of the matched filter detection [Echeverria, PRD **40**, 3194 (1989)] :

$$\frac{\Delta M}{M} \simeq 2(1 - a/M)^{0.45} \left(\frac{S}{N}\right)^{-1} \qquad \text{and} \qquad \Delta(a/M) \simeq 6(1 - a/M)^{1.06} \left(\frac{S}{N}\right)^{-1}$$

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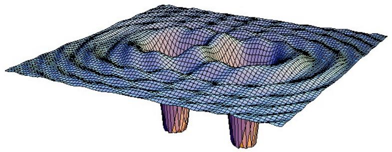
Coalescence of binary black holes

Binary black holes



From the GW detection point of view: the most promising source
From the theoretical point of view:

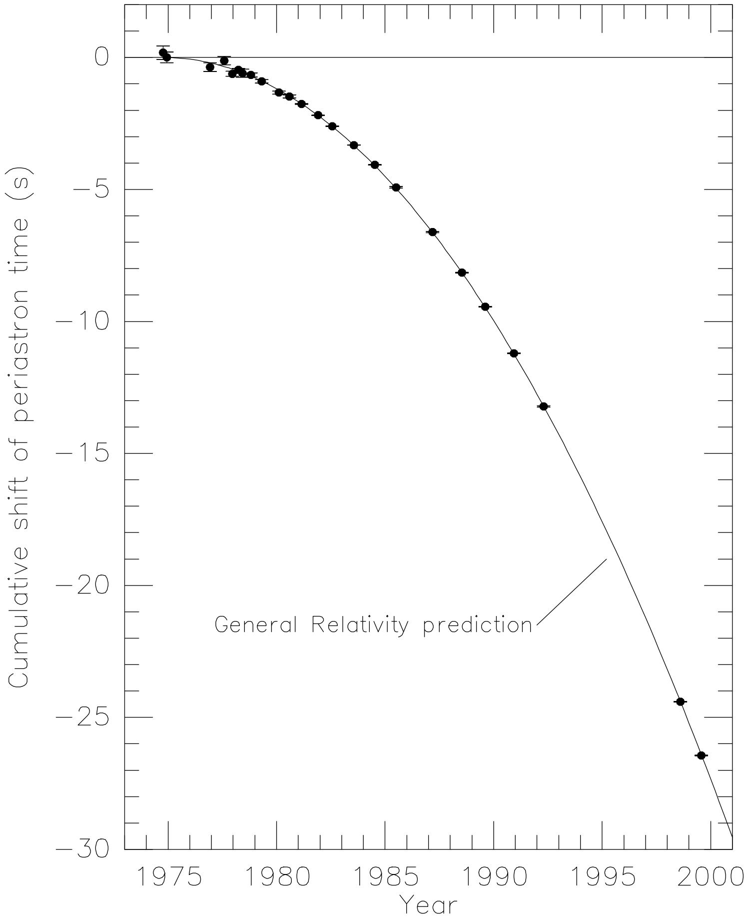
- Binary BH = the two body problem in General Relativity
- Extreme GR $\Longrightarrow$ probes the limit of GR (as weak field limit of string theory)

From the astrophysical point of view:

- Rate of binary black hole coalescence $\Longrightarrow$ massive star evolution
- Inspiral GW signal $\Longrightarrow$ precise measure of Hubble constant H_0
- GW observations of supermassive BH at high $z \Longrightarrow$ large structure formation

Evolution of binary black holes

Contrary to Newtonian 2-body problem, no stationary solution for 2 bodies in GR :
Energy and angular momentum loss due to gravitational radiation $\Longrightarrow$ shrink of the orbits

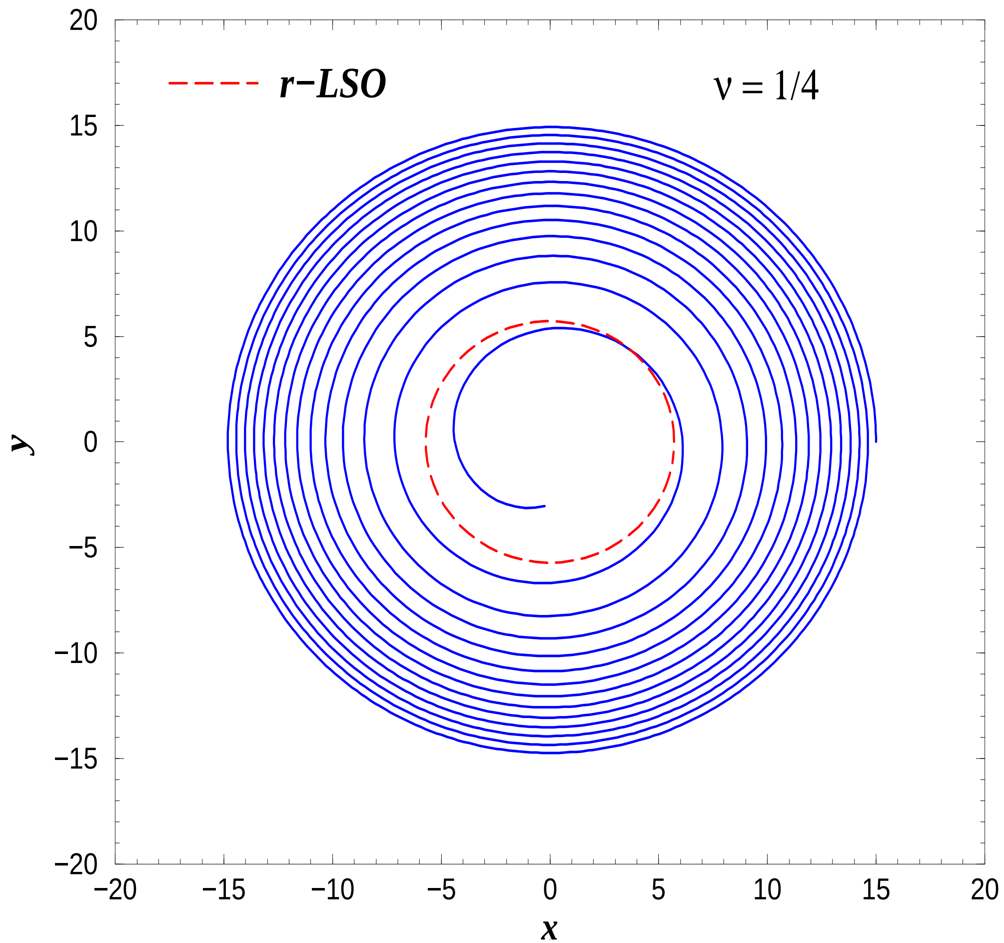


[from Lorimer (2001)]

$\longleftarrow$ Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$) of the binary pulsar PSR B1913+16 due to gravitational radiation reaction $\Longrightarrow$ merger in 140 Myr.

Another effect of gravitational wave emission:
circularisation of the orbits: $e \rightarrow 0$

Inspiraling motion

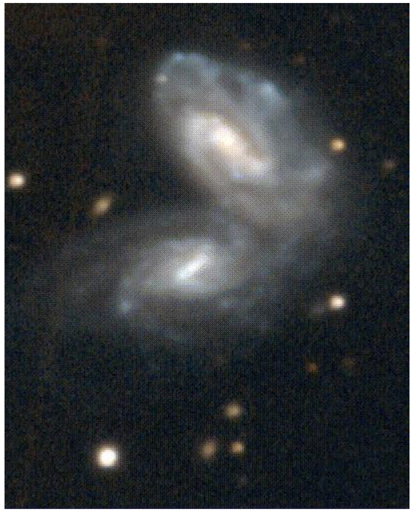


2-PN Effective One Body computation

[Buonanno & Damour, PRD **62**, 064015 (2000)]]

Two types of binary BH coalescence

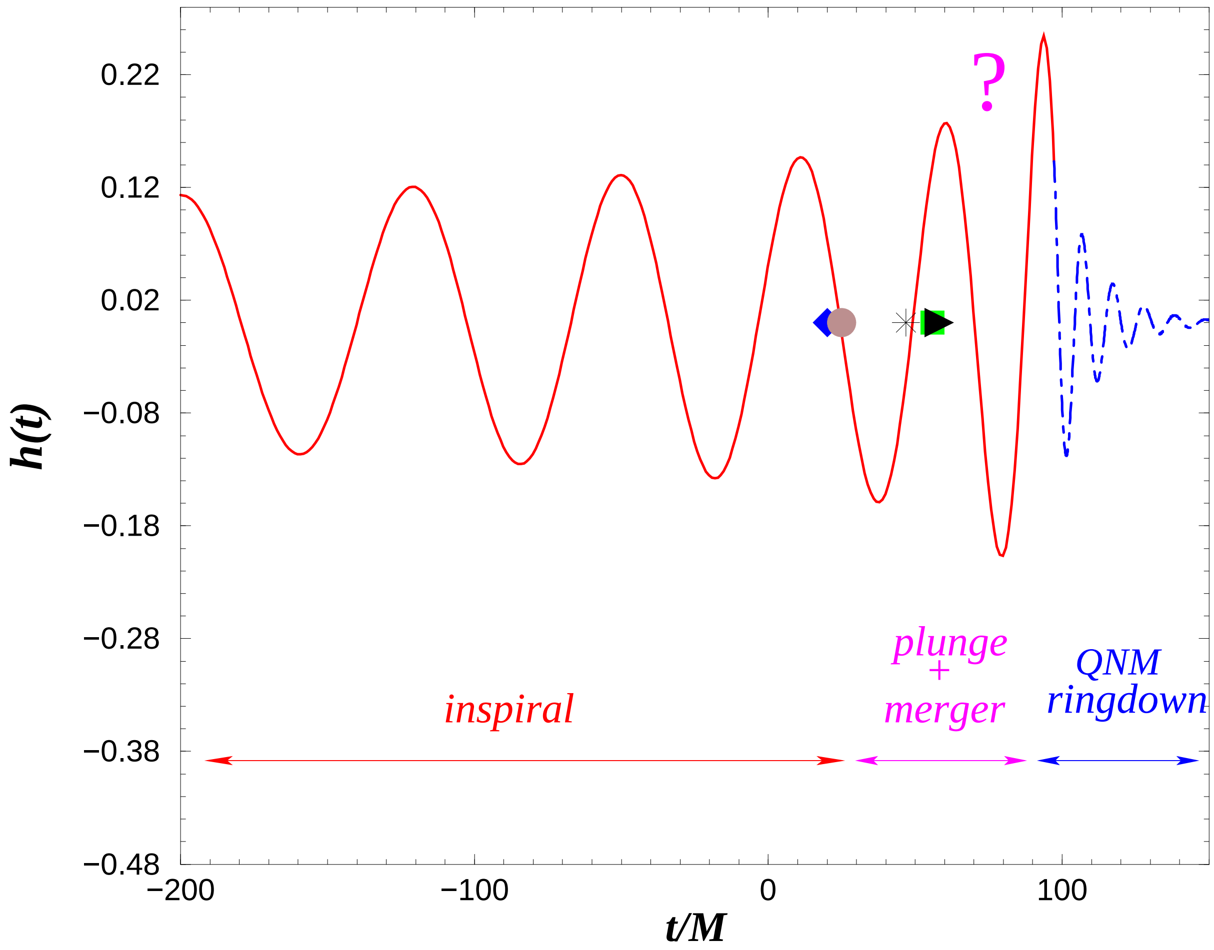
(1) Coalescence of stellar BH: from massive star evolution
event rate: • up to $\sim 20/\mathrm{Myr}$ per galaxy
[Belczynski, Kalogera, Bulik, ApJ **572**, 407 (2002)]
• $1.6 \times 10^{-7}\ \mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ from binary BH formation in globular clusters [Portegies Zwart & McMillan, ApJ **528**, L17 (2000)]



(2) Coalescence of supermassive BH: from galaxy encounters
event rate : possibly large

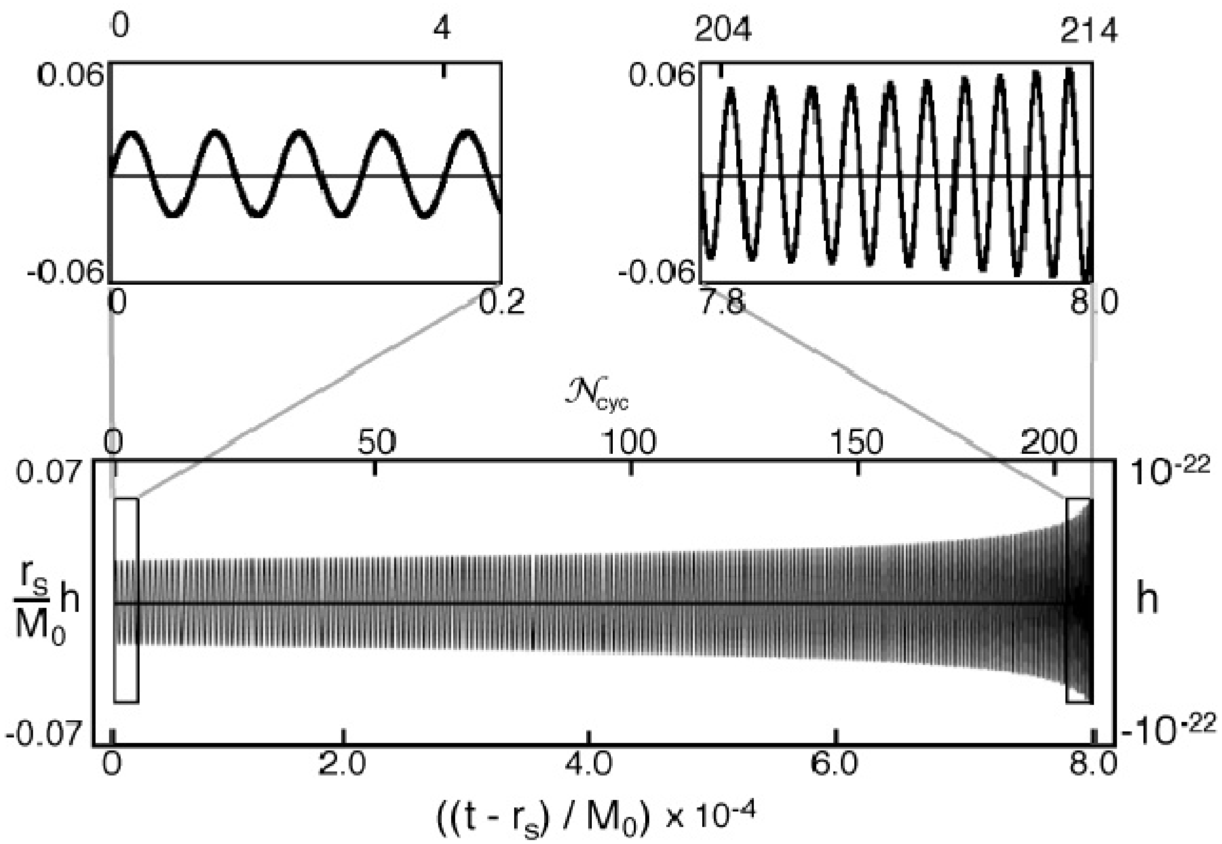
NB: Same physics (scaling with M)

Gravitational waveform



[from Buonanno & Damour, PRD **62**, 064015 (2000)]

Inspiral waveform



Chirp signal:

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

$$f = K_0\, \mathcal{M}^{-5/8} (t_{\rm coal} - t)^{-3/8}$$

with the "chirp mass":

$$\mathcal{M} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

and the constant:

$$K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G}\right)^{5/8}$$

[from Duez, Baumgarte & Shapiro, PRD **63**, 084030 (2001)]

More precise formulae:

- More harmonics in $h_+(t)$ and $h_\times(t)$ (up to 6 at the 2.5PN level)
- Orbital phase ($\Longrightarrow$ number of cycles) at the 3.5PN level:

$$
\begin{aligned}
\phi(t) = -\frac{1}{\nu}\Big\{ & \tau^{5/8} + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\tau^{3/8} - \frac{3}{4}\pi\tau^{1/4} \\
+ & \left(\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2\right)\tau^{1/8} + \left(-\frac{38645}{172032} - \frac{15}{2048}\nu\right)\pi\ln\left(\frac{\tau}{\tau_0}\right) \\
& + \left(\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}C + \frac{107}{448}\ln\left(\frac{\tau}{256}\right)\right. \\
& + \left[-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\theta\right]\nu + \frac{154565}{1835008}\nu^2 \\
& \left.-\frac{1179625}{1769472}\nu^3\right)\tau^{-1/8} + \left(\frac{188516689}{173408256} + \frac{140495}{114688}\nu - \frac{122659}{516096}\nu^2\right)\pi\tau^{-1/4}\Big\}
\end{aligned}
$$

[Blanchet, Faye, Iyer & Joguet, PRD **65**, 061501(R) (2002)]

Chirp time

Characteristic evolution time at the frequency f:

$$\tau := \frac{f}{\dot{f}} = \frac{8}{3}(t_{\rm coal} - t) = \frac{5}{96\pi^{8/3}} \frac{c^5}{G^{5/3}} \mathcal{M}^{-5/3} f^{-8/3}$$

- for stellar black holes ($M_1 = M_2 = 10\, M_\odot \Rightarrow \mathcal{M} = 8.7\, M_\odot$):

$$\tau = 100 \text{ s} \left(\frac{10\,\mathrm{Hz}}{f}\right)^{8/3} \left(\frac{8.7\, M_\odot}{\mathcal{M}}\right)^{5/3}$$

- for supermassive black holes ($M_1 = M_2 = 10^6\, M_\odot \Rightarrow \mathcal{M} = 8.7 \times 10^5\, M_\odot$):

$$\tau = 116 \text{ d} \left(\frac{10^{-4}\,\mathrm{Hz}}{f}\right)^{8/3} \left(\frac{8.7 \times 10^5 M_\odot}{\mathcal{M}}\right)^{5/3}$$

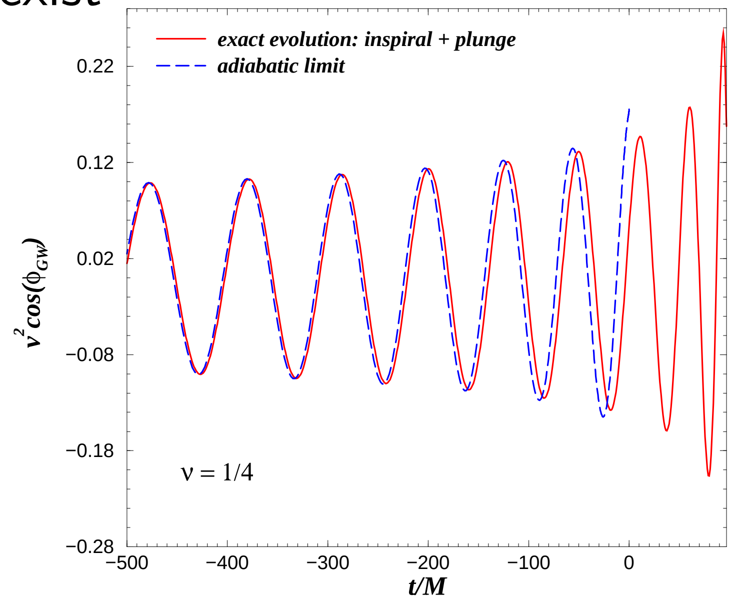
NB: $h \tau f^2 = \frac{K}{r}$ with K independent of $\mathcal{M}$ $\Longrightarrow$ standard candle

End of inspiral: the last stable orbit

Very small mass ratio (Schwarzschild spacetime) : there exists an *innermost stable circular orbit (ISCO)* :

$$R_{\rm ISCO}^{\rm Schw} = 6M \qquad \Omega_{\rm ISCO}^{\rm Schw} = 6^{-3/2} M^{-1} \simeq 0.068\, M^{-1}$$

Equal mass ratio : gravitational radiation dissipation $\Longrightarrow$ strictly circular orbits do not exist



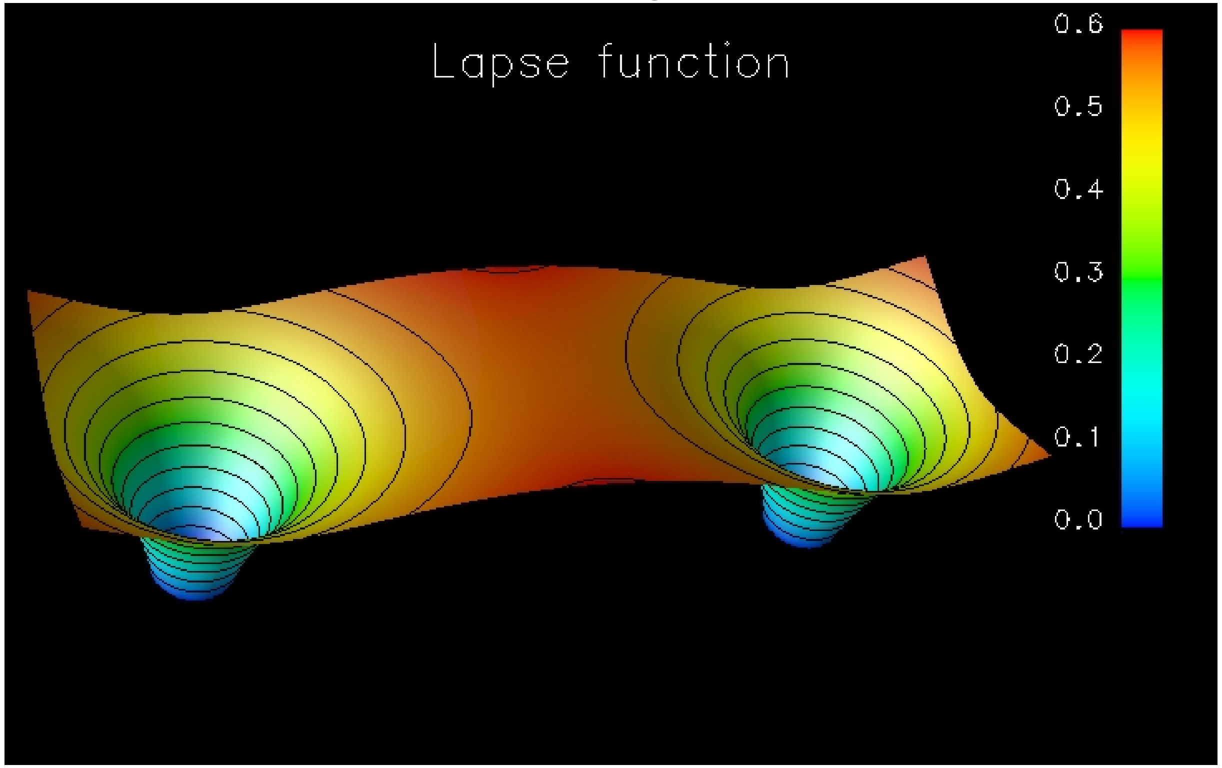
The ISCO is then defined in terms of the conservative part in the equation of motions, which give rise to circular orbits (adiabatic approximation). Consider a sequence of circular orbits of smaller and smaller radius, mimicking the inspiral. The ISCO is defined as the *turning point* in the *binding energy* of this sequence.

$\longleftarrow$ [Buonanno & Damour, PRD **62**, 064015 (2000)]

Quasi-equilibrium sequences of binary black hole on circular orbits

Computations performed in Meudon by means of multi-domain spectral methods (LORENE C++ based library)

ISCO configuration

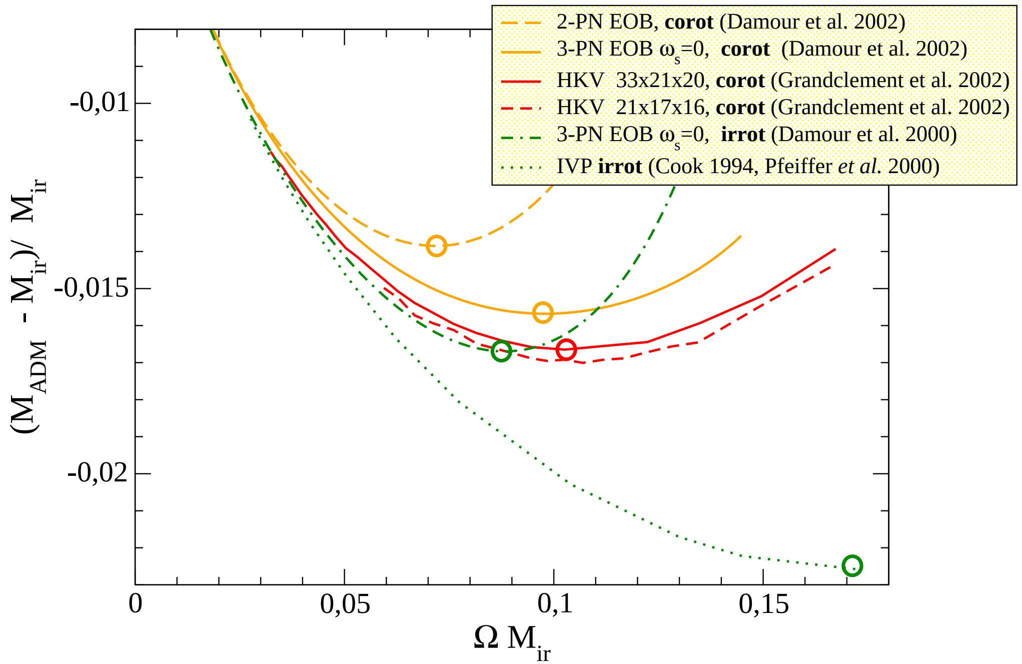


[Gourgoulhon, Grandclément, Bonazzola, PRD **65**, 044020 (2002)]

[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

Comparison with Post-Newtonian computations

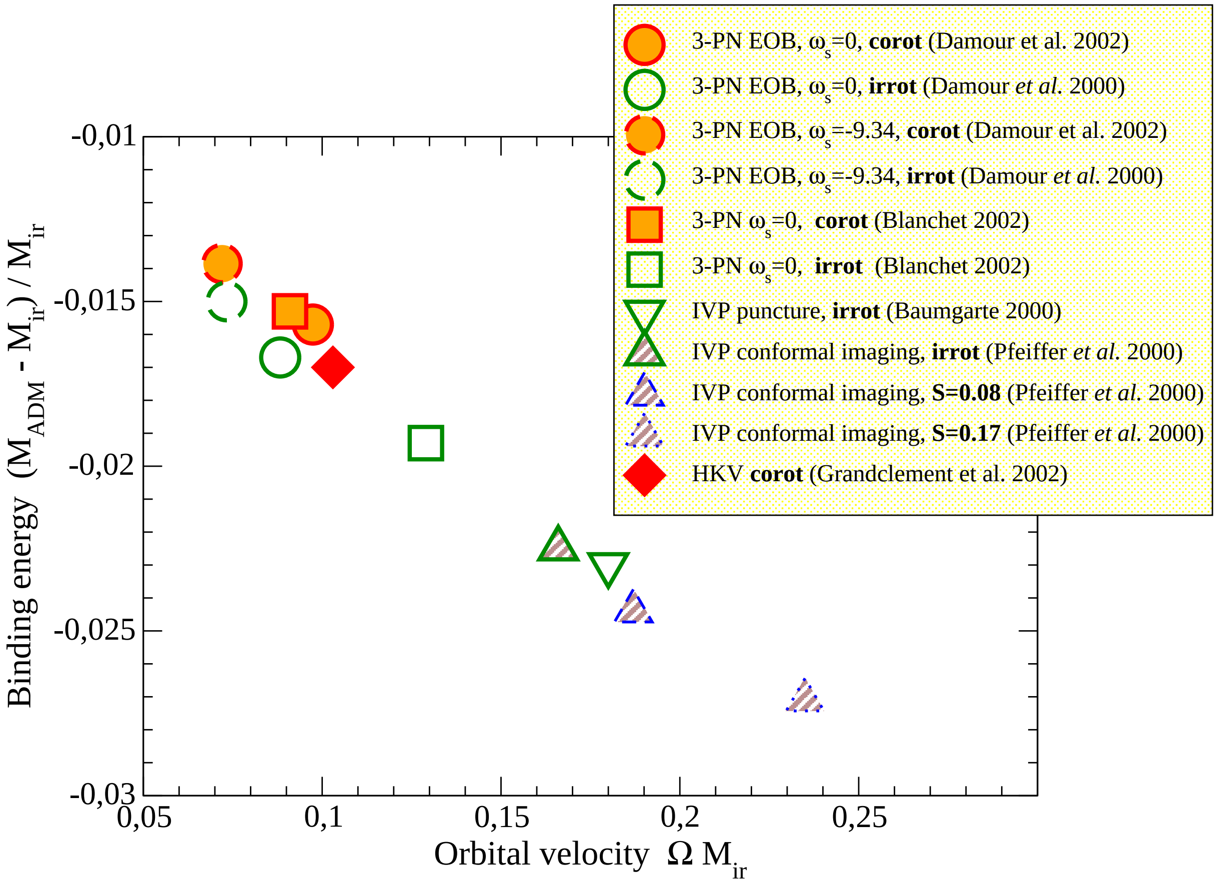
Binding energy along an evolutionary sequence of equal-mass binary black holes



[from Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Location of the ISCO

Comparison with Post-Newtonian computations



Gravitational wave frequency:

$$f = 320 \, \frac{\Omega M_{\rm ir}}{0.1} \, \frac{20 \, M_\odot}{M_{\rm ir}} \ {\rm Hz}$$

[from Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Energy emitted by gravitational radiation

Absolute upper bounds:

Hawking (1971) : $\frac{E_{\rm rad}}{M} < 0.5$ for merger of maximaly rotating Kerr BH, such that the final BH does not rotate

$\frac{E_{\rm rad}}{M} < 0.29$ for merger of non-rotating BH

Inspiral stage: $\frac{E_{\rm rad}}{M} \simeq 0.017$

Plunge + merger phase: $\frac{E_{\rm rad}}{M} \sim 0.1$?? [Flanagan & Hughes, PRD **57**, 4535 (1998)]

Ringdown phase: $\frac{E_{\rm rad}}{M} \simeq 0.03$?

[Brandt & Seidel, PRD **52**, 870 (1995)], [Flanagan & Hughes, PRD **57**, 4535 (1998)]

Range of detection and expected event rate

Stellar BH $(2 \times 10\, M_\odot)$:
Detection range:

- first generation (LIGO-I, VIRGO): $d_{\max} \simeq 100$ Mpc
- second generation: $d_{\max} \simeq 1$ Gpc

Expected event rate:

- first generation (LIGO-I, VIRGO): ~ 1 per year
- second generation: daily

Supermassive BH $(2 \times 10^6\, M_\odot)$:
$d_{\max} >$ Hubble radius for LISA $\Longrightarrow$ expected rate: a few per year up to 10^3 per year

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Compact star inspiral into massive black holes

Inspiral into a massive black hole

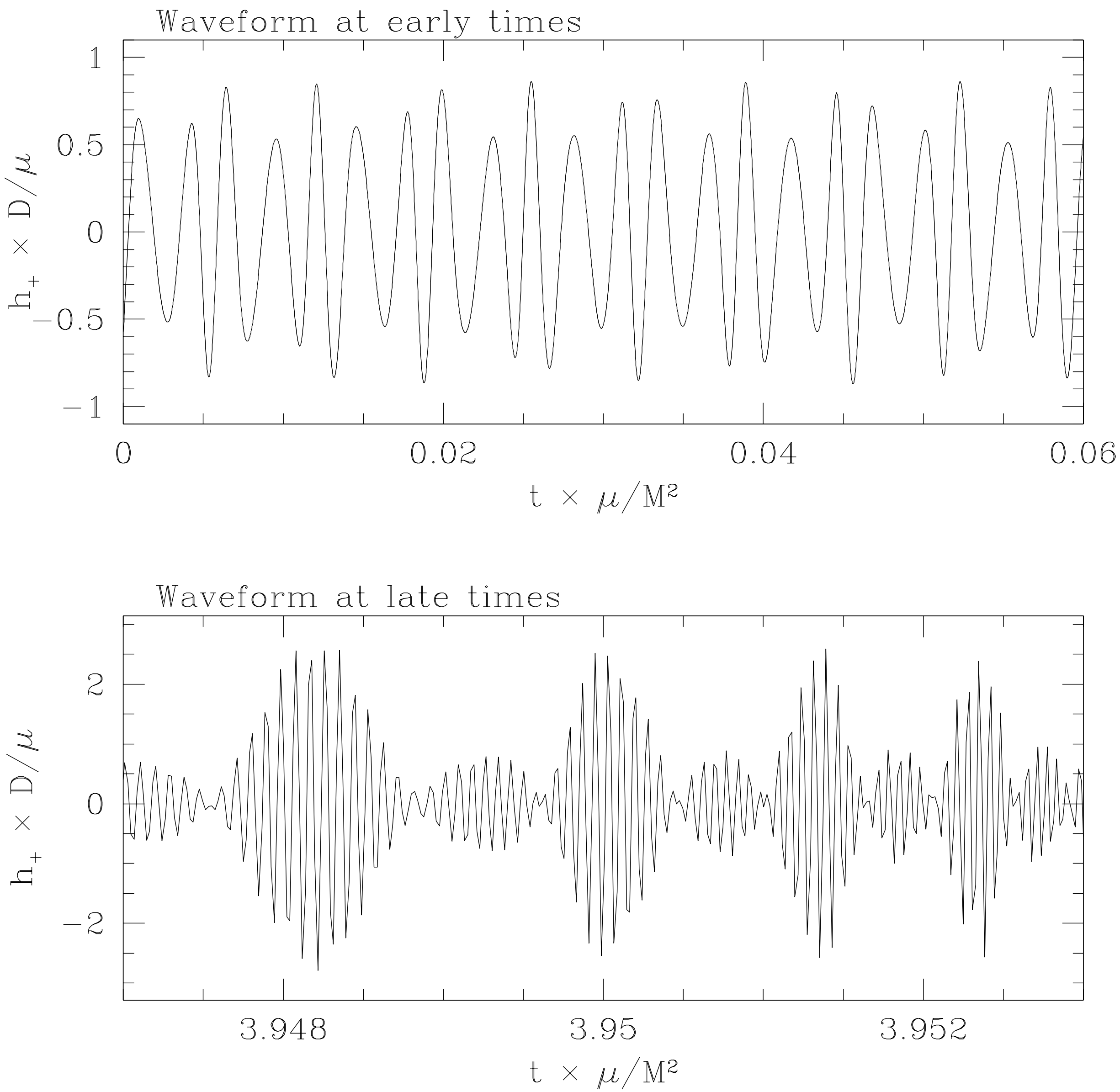
Capture of stellar-mass compact objects (NS or BH) by massive black holes residing in galactic nuclei.

Gravitational radiation carries orbital energy away from the system $\Rightarrow$ orbit shrinks

Eventually last stable orbit reached $\Rightarrow$ plunge and absorption by the central black hole

Emitted gravitational waves in the frequency range of LISA

Inspiral gravitational waveform



h_+ waveform for an inspiral trajectory which begins at an inclination of 40^o about an $a = 0.998\,M$ black hole, viewed in the hole's equatorial plane

[from Hughes, PRD **64**, 064004 (2001)]

Ultimate proof of black hole existence

Inspiral of a $m = 5\,M_\odot$ into a rapidly spinning $(a \simeq M)$ $M = 10^6\,M_\odot$ black hole:

- Time elapsed from orbital radius $r = 8M$ to the ISCO: ~ 1 yr
- Number of gravitational-wave cycles: 10^5
- Frequency band swept by the signal: $3\ \mathrm{mHz} \leq f \leq 30\ \mathrm{mHz}$
- Detection range by LISA (signal-to-noise ratio > 10): ~ 1 Gpc

Measure of large number of cycles $\Rightarrow$ **detailed map of the central object spacetime**

Comparison with Kerr spacetime $\Rightarrow$ **ultimate proof of existence of black holes in our universe**

Expected event rate for LISA: $1 - 10\ \mathrm{yr}^{-1}$ out to 1 Gpc.

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Gravitational waves from black hole environment (microquasars and gamma-ray bursts)

Jet emission

Gravitational wave emission from a blob of matter (mass m) accelerated to a Lorentz factor γ within a time $\Delta t_{\rm acc}$ [Segalis & Ori, PRD **64**, 064018 (2001)] :

$$\text{amplitude:} \quad h_+ = \frac{2G\,\gamma m}{c^2\,d}(1+\cos\theta) \qquad \text{frequency:} \quad f \sim \frac{1}{(1-\cos\theta)\Delta t_{\rm acc}}$$

where the angle θ between the jet and the observer direction is assumed to be much larger than γ^{-1}

Note: this gravitational emission is not produced by the black hole "by itself" but by some matter around it.

Jets from microquasars and GRB

Microquasars:

$$h_+ \sim 10^{-25} \left(\frac{\gamma}{100}\right) \left(\frac{m}{10^{-10} M_\odot}\right) \left(\frac{10 \text{ kpc}}{d}\right)$$

Gamma-ray bursts ("cannonball model"):

$$h_+ \sim 10^{-24} \left(\frac{\gamma}{10^3}\right) \left(\frac{m}{10^{-5} M_\odot}\right) \left(\frac{100 \text{ Mpc}}{d}\right)$$

Conclusions

Gravitational wave detection is about to open a new window onto the Universe. This new window will notably profit to black hole observations:

- quasi-normal mode ringing of a new born black hole (from gravitational collapse or coalescence of binary compact objects) $\Rightarrow$ measure of the mass and spin of the black hole

- coalescence of stellar binary black holes $\Rightarrow$ stellar evolution, measure of cosmological parameters

- coalescence of massive binary black holes $\Rightarrow$ galaxy formation in the early Universe, measure of cosmological parameters

- inspiral of a compact object into a massive black hole $\Rightarrow$ ultimate proof of black hole existence (Kerr metric)

For these detections to be possible, an a priori theoretical knowledge of the signal is necessary (detection via matched filtering).

Appendix

Signal in an interferometric detector

Gravitational wave strain:

$$h(t) = F_{+}(\theta,\phi,\psi)\,h_{+}(t) + F_{\times}(\theta,\phi,\psi)\,h_{\times}(t)$$

θ, ϕ : direction of the source with respect to the detector arms
ψ : polarization angle of the wave with respect to the detector orientation
$F_{+},\ F_{\times}$: beam-pattern functions

Detector's output:

$$o(t) = h(t) + n(t)$$

with the noise $n(t)$ in most cases larger than $h(t)$ $\Longrightarrow$ signal filtering necessary

Optimal signal filtering

Characterization of the noise: the r.m.s. noise in a bandwidth $[f, f+df]$ is $\sqrt{\langle n(t)^2\rangle} =: \sqrt{S(f)\,df}$, where $S(f)$ is the noise power spectral density. A stationary Gaussian noise is fully characterized by $S(f)$.

Signal filtering: $C := \int_{-\infty}^{+\infty} o(t)\,F(t)\,dt$ (F: filter)

Signal-to-noise ratio: $\frac{S}{N} := \frac{\langle C\rangle}{\sqrt{\langle C^2\rangle_{h=0}}}$

Wiener theorem: SNR maximal $\Leftrightarrow$ $\tilde{F}(f) = \frac{\tilde{h}(f)}{S(f)}$ (*optimal* or *matched* filter)
Then

$$\frac{S}{N} = 2\left(\int_0^{\infty} \frac{\left|\tilde{h}(f)\right|^2}{S(f)}\,df\right)^{1/2}$$

$\Longrightarrow$ a priori knowledge of $h(t)$ is required

Estimation of SNR

For detection of a quasi-periodic signal of amplitude h in a bandwith $\Delta f \sim f$:

$$\frac{S}{N} \sim \frac{h\sqrt{\mathcal{N}}}{S(f)^{1/2}\sqrt{f}}$$

where $\mathcal{N}$ is the number of cycles spent within Δf.