

# A new observational era for black holes

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14 mars 2017

- 1 Black holes : a century-old history
- 2 Black holes in the sky
- 3 Observing black holes via gravitational waves : a dream come true
- 4 Testing general relativity with black holes

# Outline

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## A two centuries-old prehistory...

$$\boxed{V_{\text{esc}} > c} \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho > c^2 \iff \boxed{R > \sqrt{\frac{3c^2}{8\pi G\rho}}}$$

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John Michell (1784)

*"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"*

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## Pierre Simon de Laplace (1796)

*"Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles."*

[Exposition du système du monde (1796)]

# Limits of the Newtonian concept of a black hole

- No privileged role of the velocity of light in Newtonian theory : nothing forbids  $V > c$  : the “dark stars” are not causally disconnected from the rest of the Universe

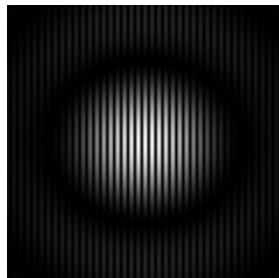
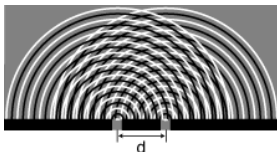
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 $\implies$  a *relativistic* theory of gravitation is necessary !
- No clear action of the gravitation field on electromagnetic waves in Newtonian gravity



[R. Taillet]

## 100 years ago : a relativistic theory of gravitation

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

## Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

In zwei vor kurzem erschienenen Mitteilungen<sup>1</sup> habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.

$$R - \frac{1}{2}Rg = \frac{8\pi G}{c^4} T$$

[A. Einstein, Sitz. Preuss. Akad. Wissenschaften Berlin, 844 (1915)]

# The Schwarzschild solution

- **Nov-Dec. 1915** : **Karl Schwarzschild** : first exact non-trivial solution of Einstein equation  $\implies$  spacetime metric outside a **spherical body** of mass  $M$

$$g_{\alpha\beta}dx^\alpha dx^\beta = - \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

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- **1923** : **George Birkhoff** : outside any *spherical* body, the metric is Schwarzschild metric

# The Schwarzschild solution : Lemaître breakthrough

- **1932 : Georges Lemaître** : the singularity at  $r = R_S$  is a coordinate singularity : the metric components are regular in Lemaître coordinates  $(\tau, \chi, \theta, \varphi)$  :

$$g_{\alpha\beta} dx^\alpha dx^\beta = -c^2 d\tau^2 + \frac{R_S}{r} d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$r = r(\tau, \chi) := \left[ \frac{3}{2} \sqrt{R_S} (c\tau - \chi) \right]^{2/3}$$

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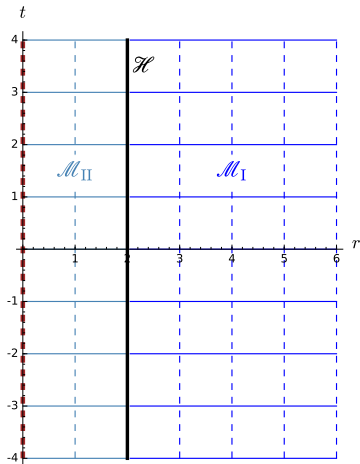
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- **1939 : Robert Oppenheimer & Hartland Snyder** : gravitational collapse of a homogeneous dust cloud (special case of Lemaître's general solution)  
 $\implies$  for an external observer,  $R \rightarrow R_S$  as  $t \rightarrow +\infty$

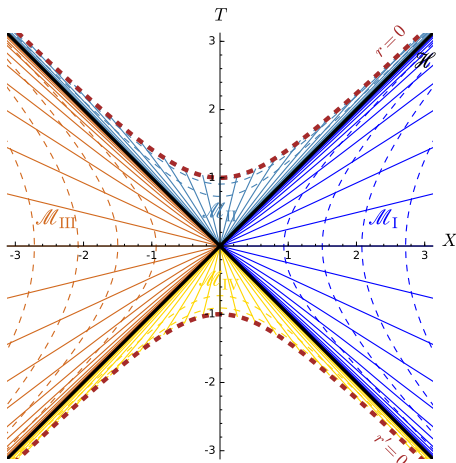


# The Schwarzschild solution : the complete picture

- 1950 : John L. Synge, 1960 : Martin Kruskal, George Szekeres : complete mathematical description of Schwarzschild spacetime ( $\mathbb{R}^2 \times \mathbb{S}^2$  manifold)



Schwarzschild-Droste coordinates  $(t, r)$



## The Schwarzschild spacetime : Carter-Penrose diagram

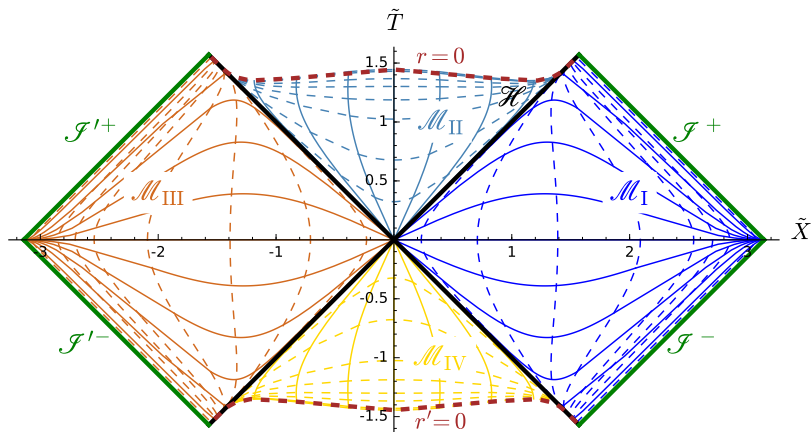


figure drawn with SageMath : <http://sagemanifolds.obspm.fr>

# Rotation enters the game : the Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta} dx^\alpha dx^\beta = - \left( 1 - \frac{2GM r}{c^2 \rho^2} \right) c^2 dt^2 - \frac{4GM a r \sin^2 \theta}{c^2 \rho^2} c dt d\varphi + \frac{\rho^2}{\Delta} dr^2 \\ + \rho^2 d\theta^2 + \left( r^2 + a^2 + \frac{2GM a^2 r \sin^2 \theta}{c^2 \rho^2} \right) \sin^2 \theta d\varphi^2$$

where

$$\rho^2 := r^2 + a^2 \cos^2 \theta, \quad \Delta := r^2 - \frac{2GM}{c^2} r + a^2, \quad a := \frac{J}{cM}$$

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Schwarzschild as the subcase  $a = 0$  :

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# Physical meaning of the parameters $M$ and $J$

- **mass  $M$**  : *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*  
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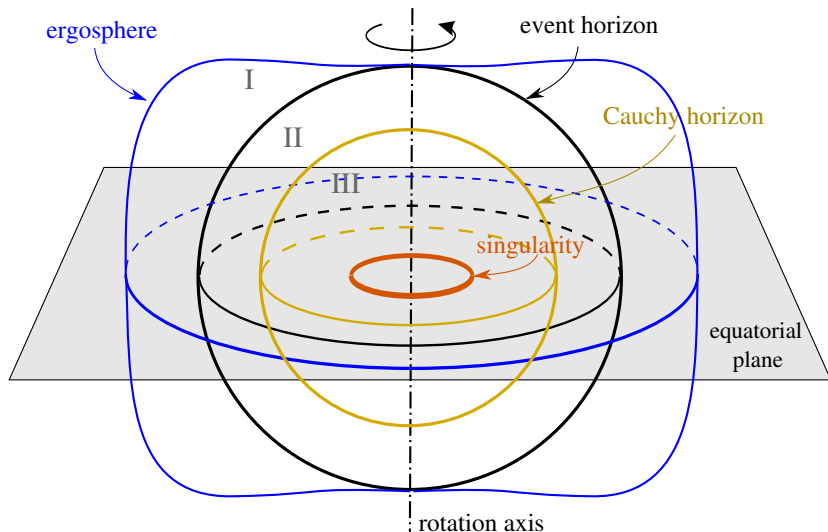
*Remark* : the **radius** of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the **area** of the event horizon,  $A$ .

The “radius” can be defined from it : for a Schwarzschild black hole :

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left( \frac{M}{M_{\odot}} \right) \text{ km}$$



## Kerr spacetime



Slice  $t = \text{const}$  and  $\theta = \pi/2$  of the Kerr spacetime

# The Golden Age of black hole theory

- 1964 : Edwin Salpeter, Yakov Zeldovich : quasars (just discovered !) shine thanks to accretion onto a supermassive black hole
- 1965 : Roger Penrose : if a trapped surface is formed in a gravitational collapse and matter obeys some energy condition, then a singularity will necessarily appear
- 1967 : John A. Wheeler coined the word *black hole*
- 1969 : Roger Penrose : energy can be extracted from a rotating black hole
- 1972 : Stephen Hawking : law of area increase  $\implies$  **BH thermodynamics**
- 1975 : Stephen Hawking : **Hawking radiation**
- 1965-1972 : **the no-hair theorem**

# The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

*Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a **Kerr-Newmann black hole**, which is an **electro-vacuum solution** of Einstein equation described by only 3 parameters :*

- the total mass  $M$
- the total specific angular momentum  $a = J/(Mc)$
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⇒ *“a black hole has no hair”* (John A. Wheeler)

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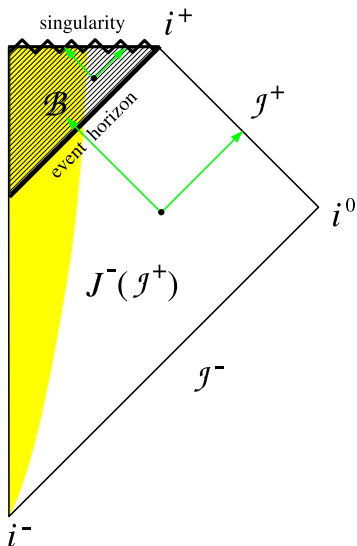
Astrophysical black holes have to be electrically neutral :

- $Q = 0$  : **Kerr solution (1963)**

Other special cases :

- $a = 0$  : **Reissner-Nordström solution (1916, 1918)**
- $a = 0$  and  $Q = 0$  : **Schwarzschild solution (1916)**
- $a = 0$ ,  $Q = 0$  and  $M = 0$  : **Minkowski metric (1907)**

# General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

**black hole :**  $\mathcal{B} := \mathcal{M} - J^-(\mathcal{I}^+)$

where

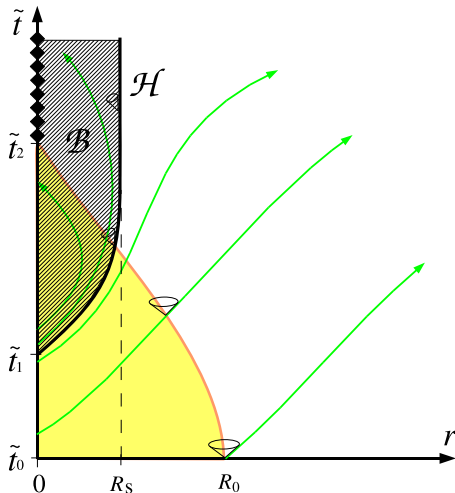
- $(\mathcal{M}, g)$  = asymptotically flat manifold
- $\mathcal{I}^+$  = future null infinity
- $J^-(\mathcal{I}^+)$  = causal past of  $\mathcal{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

**event horizon :**  $\mathcal{H} := \partial J^-(\mathcal{I}^+)$   
(boundary of  $J^-(\mathcal{I}^+)$ )

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# Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : **the singularity** (*NB : this is not the primary definition of a black hole*). The singularity is inaccessible to observations, being hidden by the event horizon.

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- The singularity marks the **limit of validity of general relativity** : to describe it, a quantum theory of gravitation would be required.
- The event horizon  $\mathcal{H}$  is a **global structure** of spacetime : no physical experiment whatsoever can detect the crossing of  $\mathcal{H}$ .

# Main properties of black holes (2/2)

- Viewed by a distant observer, the horizon approach is perceived with an **infinite redshift**, or equivalently, by an **infinite time dilation**
- A black hole **is not an infinitely dense object** : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its “mean density” by  $\bar{\rho} = M/(4/3\pi R^3)$ , then
  - for the Galactic centre BH (Sgr A\*) :  $\bar{\rho} \sim 10^6 \text{ kg m}^{-3} \sim 2 \cdot 10^{-4} \rho_{\text{white dwarf}}$
  - for the BH at the centre of M87 :  $\bar{\rho} \sim 2 \text{ kg m}^{-3} \sim 2 \cdot 10^{-3} \rho_{\text{water}} !$

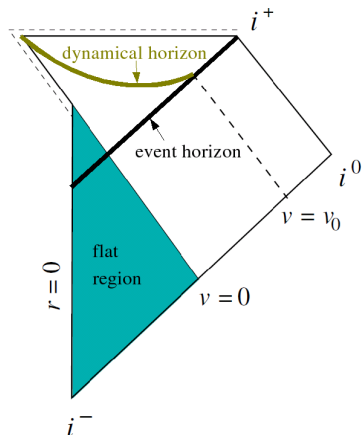
$\implies$  a black hole is a **compact object** :  $\frac{M}{R}$  large, not  $\frac{M}{R^3} !$
- Due to the non-linearity of general relativity, **black holes can form in spacetimes without any matter**, by collapse of gravitational wave packets.

# Teleological nature of event horizons

The standard definition of a black hole is **highly non-local** : determination of  $\mathcal{J}^-$  ( $\mathcal{I}^+$ ) requires the knowledge of the entire future null infinity. Moreover this is *not locally linked with the notion of strong gravitational field* :

**Example of event horizon in a flat region of spacetime** :

Vaidya metric, describing incoming radiation from infinity :



$$ds^2 = - \left( 1 - \frac{2m(v)}{r} \right) dv^2 + 2dv dr + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

with

$$m(v) = 0 \quad \text{for } v < 0$$

$$dm/dv > 0 \quad \text{for } 0 \leq v \leq v_0$$

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[Ashtekar & Krishnan, LRR 7, 10 (2004)]

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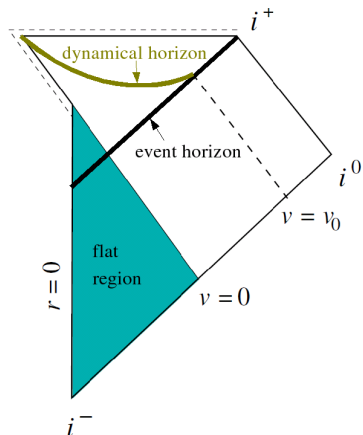
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**⇒ no local physical experiment can locate the event horizon**

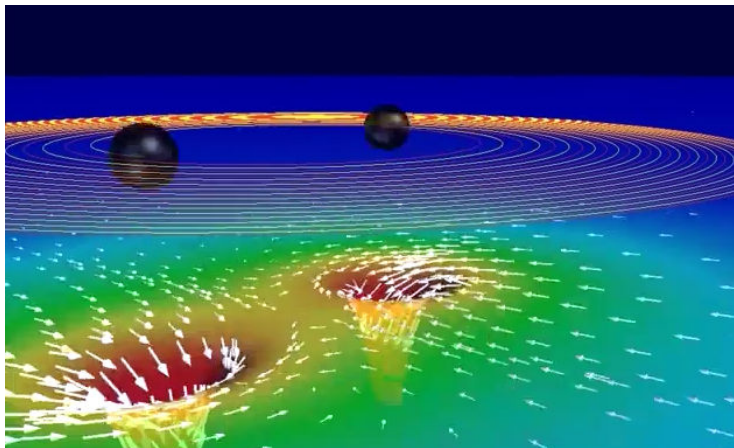
# Quasi-local approaches to black holes

**New paradigm** for the theoretical approach to black holes : instead of *event horizons*, black holes are described by

- **trapping horizons** (Hayward 1994)
- **isolated horizons** (Ashtekar et al. 1999)
- **dynamical horizons** (Ashtekar and Krishnan 2002)
- **slowly evolving horizons** (Booth and Fairhurst 2004)

All these concepts are **local** and are based on the notion of **trapped surfaces**

# The 2000's : the triumph of numerical relativity



[Caltech/Cornell SXS]

[Scheel et al., PRD **79**, 024003 (2009)]

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# Known black holes

Three kinds of black holes are known in the Universe :

- **Stellar black holes** : supernova remnants :

$$M \sim 10 - 30 M_{\odot} \text{ and } R \sim 30 - 90 \text{ km}$$

$$\text{example : Cyg X-1 : } M = 15 M_{\odot} \text{ and } R = 45 \text{ km}$$



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- **Supermassive black holes**, in galactic nuclei :

$$M \sim 10^5 - 10^{10} M_{\odot} \text{ and } R \sim 3 \times 10^5 \text{ km} - 200 \text{ UA}$$

$$\text{example : Sgr A* : } M = 4.3 \times 10^6 M_{\odot} \text{ and } R = 13 \times 10^6 \text{ km} = 18 R_{\odot} = 0.09 \text{ UA} = \frac{1}{4} \times \text{radius of Mercury's orbit}$$

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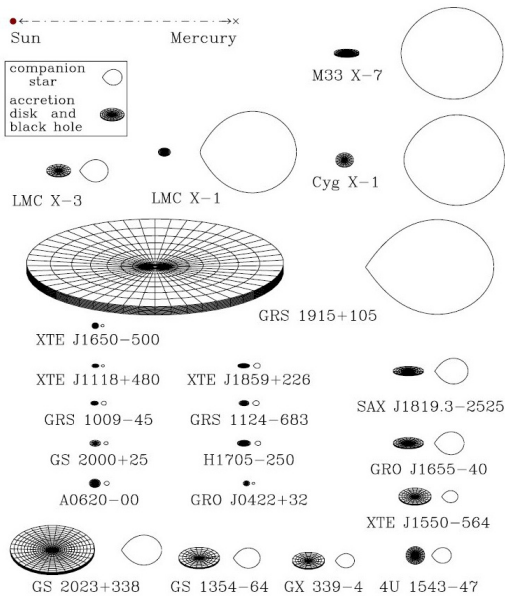
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- **Intermediate mass black holes**, as ultra-luminous X-ray sources :

$$M \sim 10^2 - 10^4 M_{\odot} \text{ and } R \sim 300 \text{ km} - 3 \times 10^4 \text{ km}$$

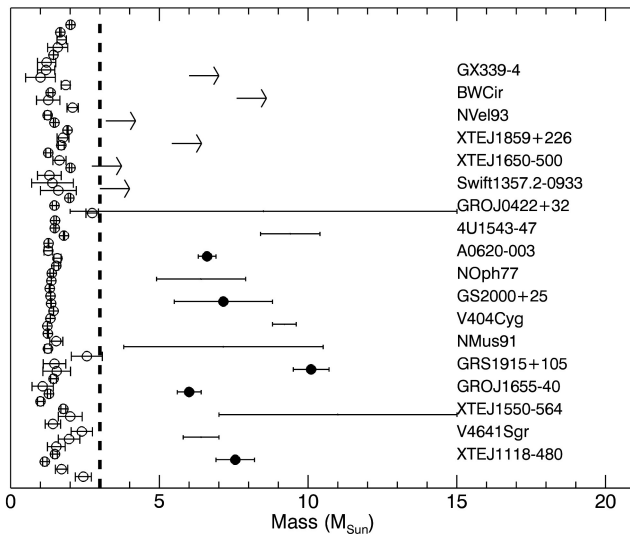
example : ESO 243-49 HLX-1 :  $M > 500 M_{\odot}$  and  $R > 1500 \text{ km}$

# Stellar black holes in X-ray binaries



[McClintock et al. (2011)]

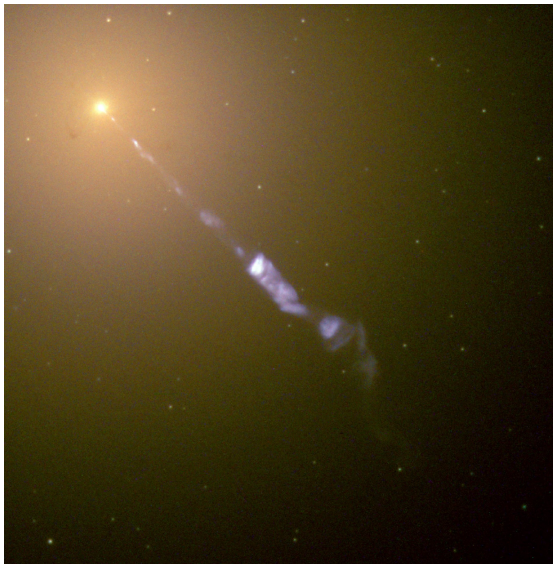
## Stellar black holes in X-ray binaries



Dynamically measured masses of black holes in transient low-mass X-ray binaries (right), compared with measured masses of neutron stars (left)

[Corral-Santana et al., A&A 587, A61 (2016)]

# Supermassive black holes in active galactic nuclei (AGN)

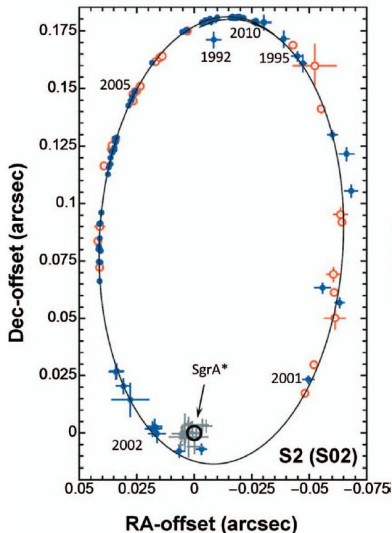


Jet emitted by the nucleus of the giant elliptical galaxy M87, at the centre of Virgo cluster [HST]

$$M_{\text{BH}} = 3 \times 10^9 M_{\odot}$$

$$V_{\text{jet}} \simeq 0.99 c$$

## The black hole at the centre of our galaxy : Sgr A\*



[ESO (2009)]

Measure of the mass of Sgr A\* black hole by stellar dynamics :

$$M_{\text{BH}} = 4.3 \times 10^6 M_{\odot}$$

← Orbit of the star S2 around Sgr A\*

$$P = 16 \text{ yr}, \quad r_{\text{per}} = 120 \text{ UA} = 1400 R_{\text{S}},$$

$$V_{\text{per}} = 0.02 c$$

[Genzel, Eisenhauer &amp; Gillessen, RMP 82, 3121 (2010)]

# Can we see a black hole from the Earth ?

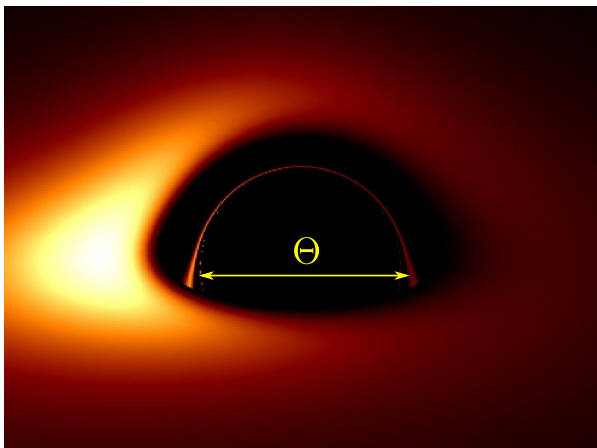


Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass  $M$  seen from a distance  $d$  :

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \simeq 2.60 \frac{2R_S}{d}$$

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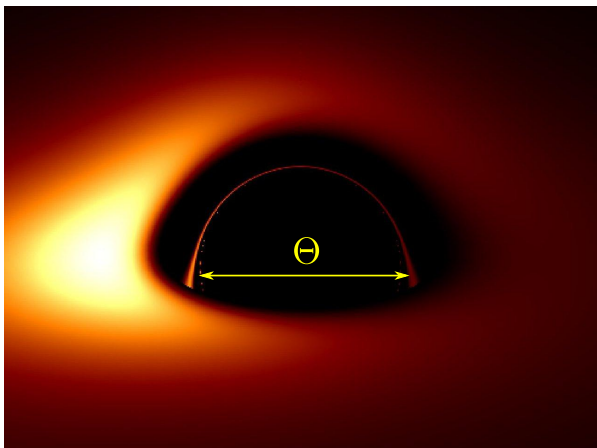


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Largest black holes in the Earth's sky :

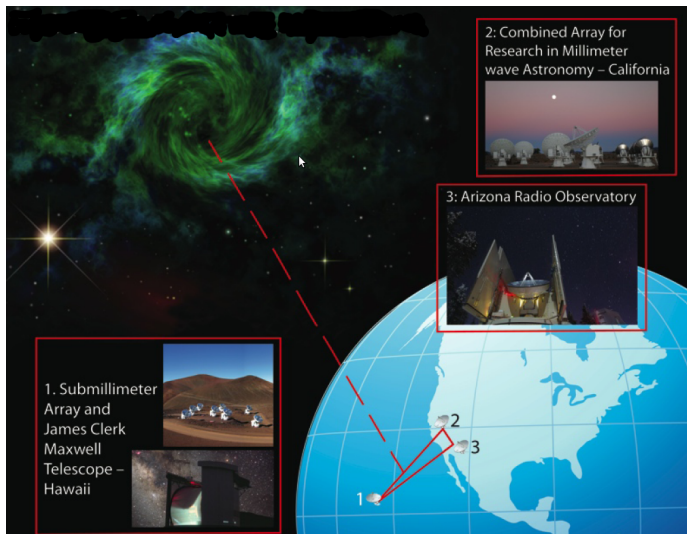
**Sgr A\*** :  $\Theta = 53 \mu\text{as}$

**M87** :  $\Theta = 21 \mu\text{as}$

**M31** :  $\Theta = 20 \mu\text{as}$

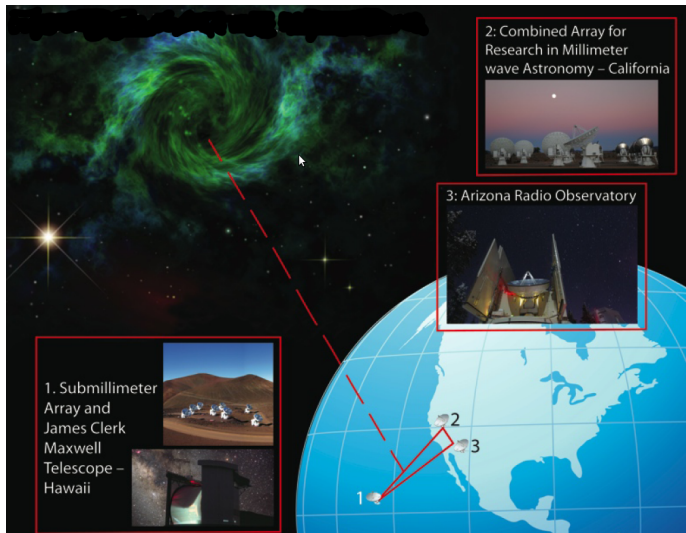
*Remark* : black holes in X-ray binaries are  $\sim 10^5$  times smaller, for  $\Theta \propto M/d$



Reaching the  $\mu\text{as}$  resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

Reaching the  $\mu\text{as}$  resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A\* is only  $37 \mu\text{as}$

[Doeleman et al., *Nature* 455, 78 (2008)]

Existing American VLBI network [Doeleman et al. 2011]

# The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : **1.3 mm** → **0.8 mm**
- increase the number of stations; in particular add ALMA

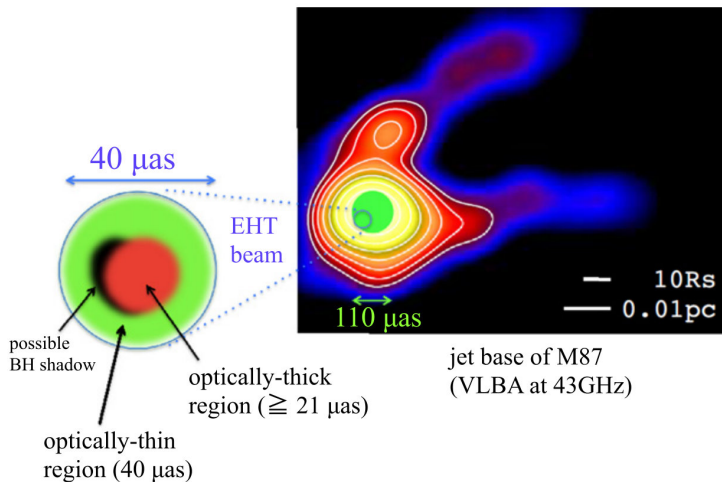


Atacama Large Millimeter Array (ALMA)

part of the **Event Horizon Telescope (EHT)** to be completed by 2020

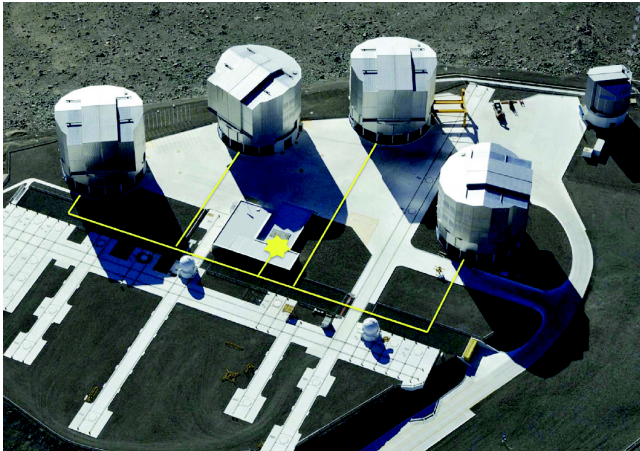
**August 2015 : VLBI observations involving ALMA and VLBA**

## VLBA and EHT observations of M87



[Kino et al., ApJ 803, 30 (2015)]

# Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits :  $10 \mu\text{as}$

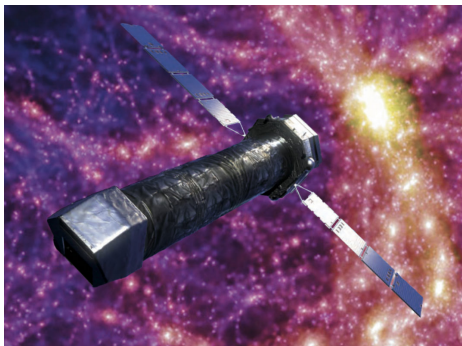
# Near-infrared optical interferometry : GRAVITY



[MPE/GRAVITY team]

July 2015 : GRAVITY  
shipped to Chile and  
successfully assembled  
at the Paranal  
Observatory  
Fall 2016 : observations  
have started !

# Observing black holes at high energy : the Athena mission



[Athena team]

**X-ray observatory Athena** : the future L2 mission of ESA (launch  $\sim$  2028)

Among the scientific objectives :

- Determine the formation and early growth of supermassive black holes, via the observation of a large sample of AGN at  $z \sim 6 - 8$
- Measure the spins of supermassive black holes
- Measure the spins of stellar black holes

# Outline

- 1 Black holes : a century-old history
- 2 Black holes in the sky
- 3 Observing black holes via gravitational waves : a dream come true**
- 4 Testing general relativity with black holes

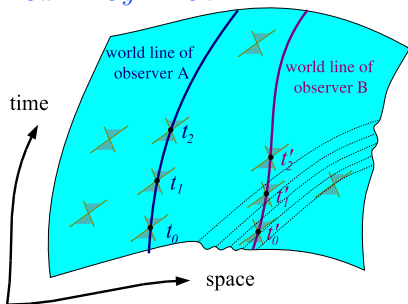


# Gravitational waves

Linearization of Einstein equation in weak field :  $g = \eta + h$ ,  
 $\eta$  = Minkowski metric<sup>1</sup>

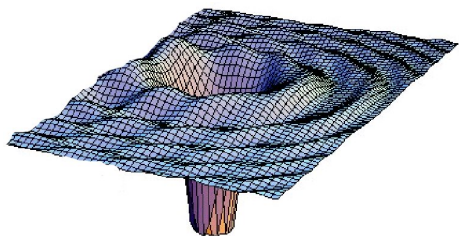
$$\Rightarrow \text{wave equation : } \square \bar{h} = -\frac{16\pi G}{c^4} T \quad (\text{Lorenz gauge})$$

$$\text{with } \square = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad \bar{h} = h - \frac{1}{2} h \eta \quad \text{and} \quad h = \text{Trace}(h).$$



1.  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  en Cartesian coordinates

# Black holes and gravitational waves



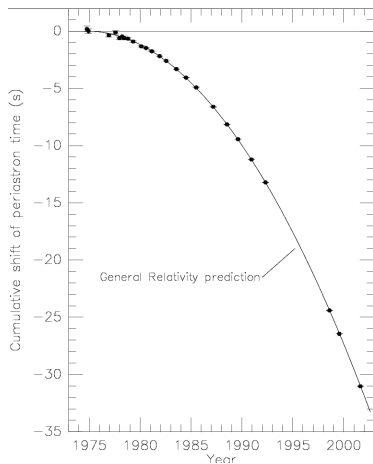
Link between black holes and gravitational waves :

Both are **spacetime distortions** :

- extreme distortions (black holes)
- small distortions (gravitational waves)

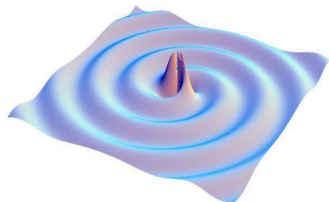
In particular, black holes and gravitational waves are both **vacuum solutions** of Einstein equation

# Observational evidence for gravitational waves



[Weisber & Taylor (2002)]

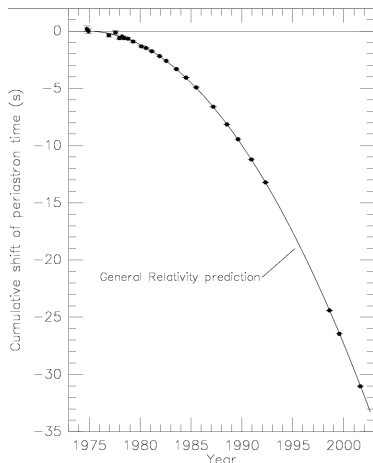
Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period

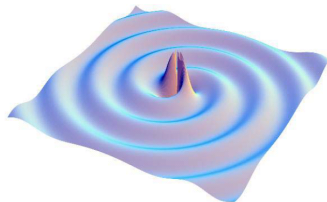
$P = 7$  h 45 min of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*  $\implies$  coalescence in 140 millions year.

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Emission of gravitational waves by the neutron star binary system PSR B1509-58 (*binary pulsar*)



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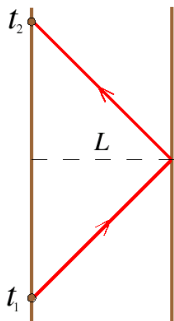
**Nobel Prize in Physics to R. Hulse & J. Taylor (1993)**

# Measurable effects of a gravitational wave passage



Measure of the distance  $L$  between two free masses by a “radar” method :

$$L = \frac{1}{2} c(t_2 - t_1)$$



Variation of length  $L$  when a gravitational wave passes by :

$$\delta L \simeq h L$$

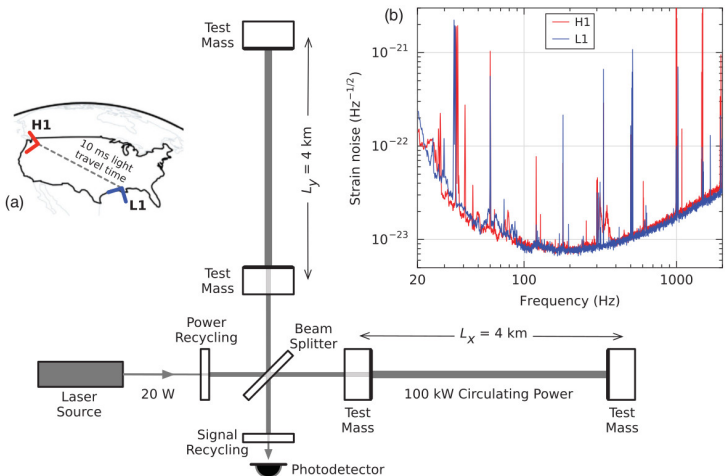
$h$  = amplitude of the gravitational wave

In practice,  $h$  is so small that our senses are not sensitive to it :

for the most important **astrophysical sources** :

$$h \sim 10^{-21} !!!$$

# Advanced LIGO detectors



[Abbott et al., PRL 116, 061102 (2016)]

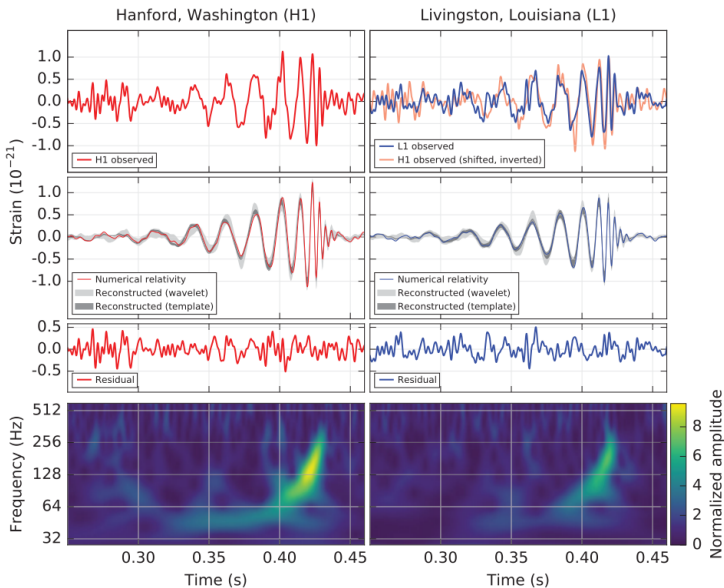
# Advanced ground-based GW detectors



- **Adv. LIGO** : started Sept. 2015
- **Adv. Virgo** : will start in spring 2017
- **KAGRA** (Japan) : 2018

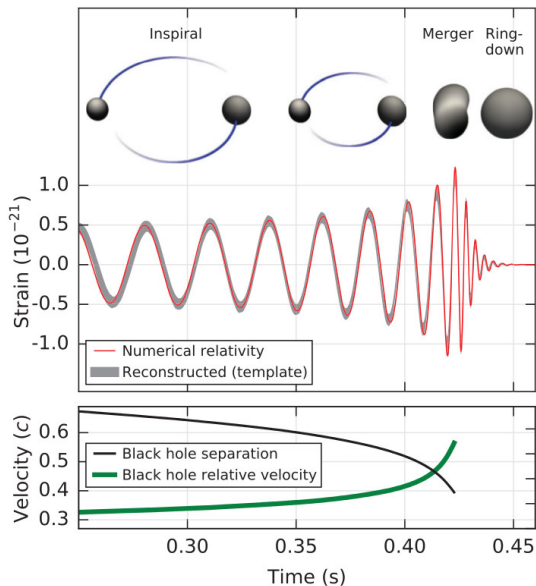
Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

September 14, 2015, 09:50:45 UTC





## GW150914 event



Signal :

$$\Delta t = 0.2 \text{ s}$$

$$f : 35 \rightarrow 250 \text{ Hz}$$

$$h_{\text{max}} = 1.0 \cdot 10^{-21}$$

Matched filter :

$$S/N = 24$$

$$F_{\text{false}} = 1/203,000 \text{ yr}$$

$$M_1 = 36 \pm 5 M_{\odot}$$

$$M_2 = 29 \pm 4 M_{\odot}$$

$$d = 410 \pm 180 \text{ Mpc}$$

$$z = 0.09 \pm 0.04$$

$$M_{\text{final}} = 62 \pm 4 M_{\odot}$$

$$\Rightarrow E_{\text{rad}}^{\text{GW}} = 3.0 \pm 0.5 M_{\odot} c^2$$

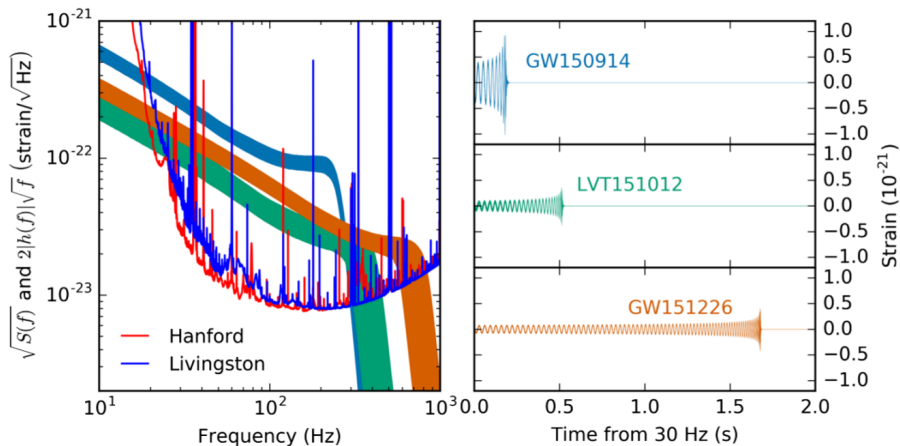
$$a_1 < 0.7, a_2 < 0.9$$

$$a_{\text{final}} = 0.67 \pm 0.07$$

[Abbott et al., PRL 116, 061102

(2016)]

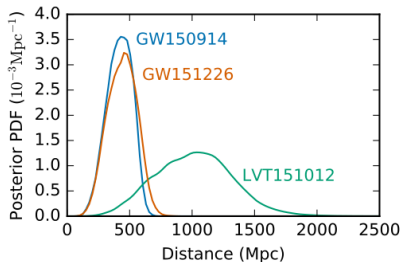
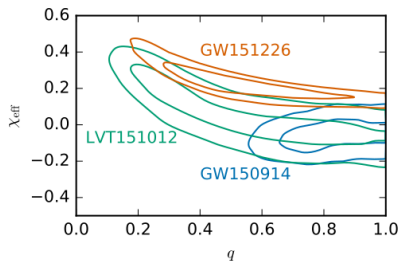
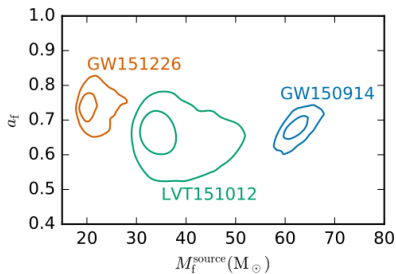
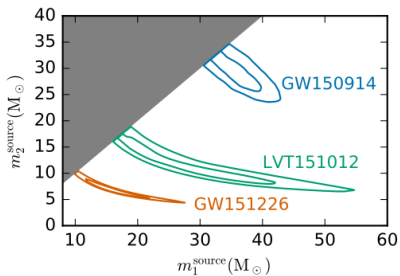
## LIGO events in run O1 (12 Sept. 2015 - 19 Jan. 2016)



[Abbott et al., PRX 6, 041015 (2016)]

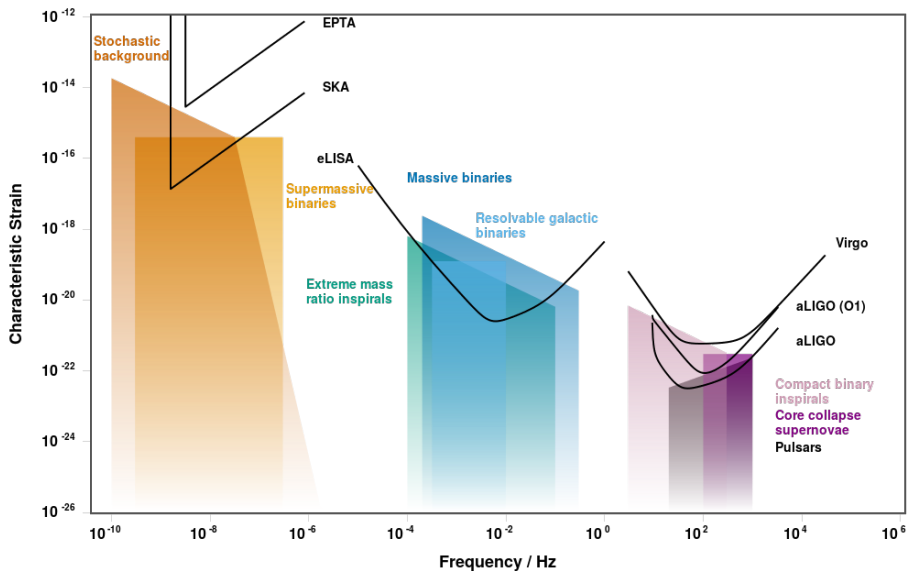
NB : LVT = LIGO-Virgo Trigger (not significant enough to be a detection)

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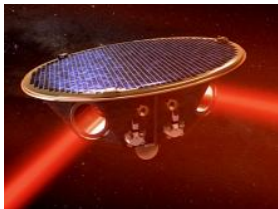
[Abbott et al., PRX 6, 041015 (2016)]

## GW detectors in different bandwidths



# Space detector LISA (ESA)

Interferometric gravitational wave detector in solar orbit

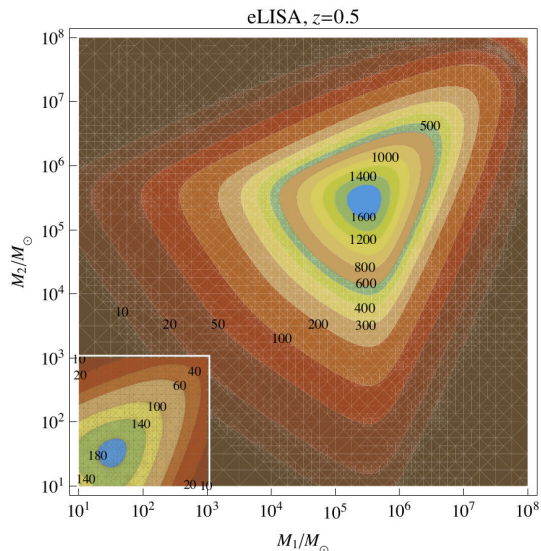


[eLISA / NGO]

- theme selected by ESA in 2013 for the L3 mission
- launch ~ 2034
- technology demonstrator **LISA Pathfinder** launched on 3 December 2015 ; successful results announced in June 2016 !



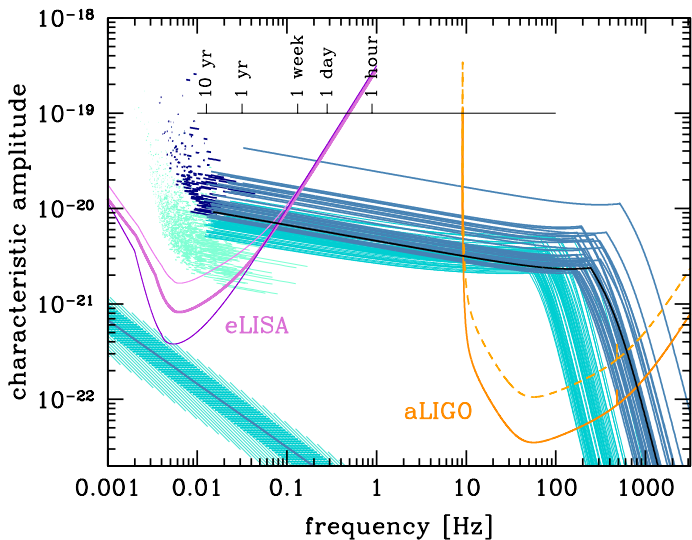
## LISA observations of massive binary BH mergers



Signal-to-noise ratio for  
gravitational waves from the  
inspiral of a BH binary at  
 $z = 0.5$

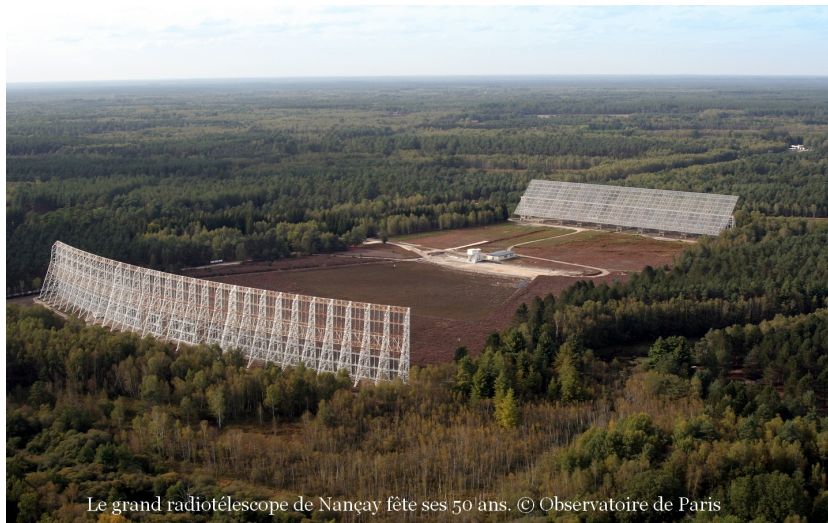
[Barusse et al., J. Phys. Conf. Ser. **610**, 012001 (2015)]

## The promise of multi-band gravitational wave astronomy



[Sesana, PRL 116, 231102 (2016)]

# Detecting gravitational waves by pulsar timing

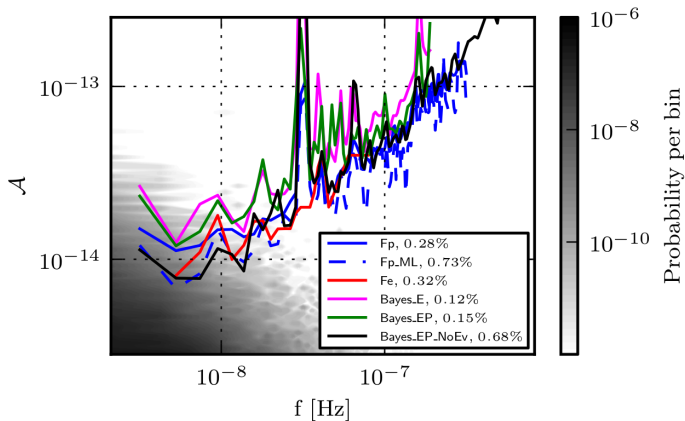


Le grand radiotélescope de Nançay fête ses 50 ans. © Observatoire de Paris



# EPTA results on supermassive BH binaries

EPTA : European Pulsar Timing Array



[Babak et al., MNRAS 455, 1665 (2016)]

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# Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor  $g$
- field equation involving only derivatives of  $g$  up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$  ( $\implies$  weak equivalence principle)

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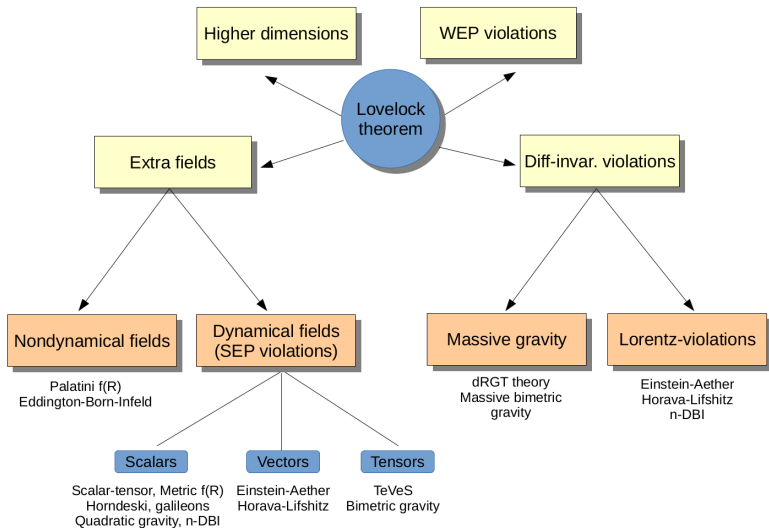
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

## Extensions of general relativity



[Berti et al., CGQ 32, 243001 (2015)]

# Test : are astrophysical black holes Kerr black holes ?

- GR  $\implies$  Kerr BH (no-hair theorem)
- extension of GR  $\implies$  BH may deviate from Kerr

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## Observational tests

### Search for

- **stellar orbits** deviating from Kerr timelike geodesics (**GRAVITY**)
- **accretion disk spectra** different from those arising in Kerr metric (**X-ray observatories, e.g. Athena**)
- **images of the black hole silhouette** different from that of a Kerr BH (**EHT**)

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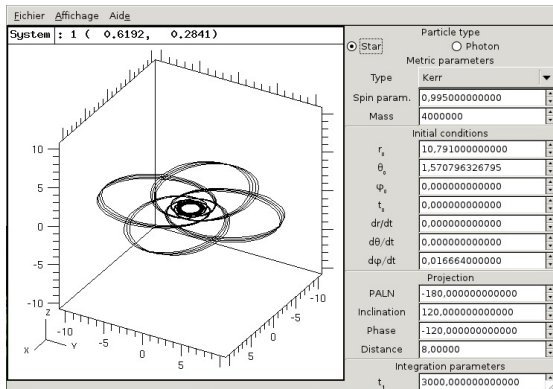
**Need for a good and versatile geodesic integrator**

to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric



## Gyoto code

Main developers : T. Paumard &amp; F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : <http://gyoto.obspm.fr/>

[Vincent, Paumard, Gourgoulhon &amp; Perrin, CQG 28, 225011 (2011)]

[Vincent, Gourgoulhon &amp; Novak, CQG 29, 245005 (2012)]

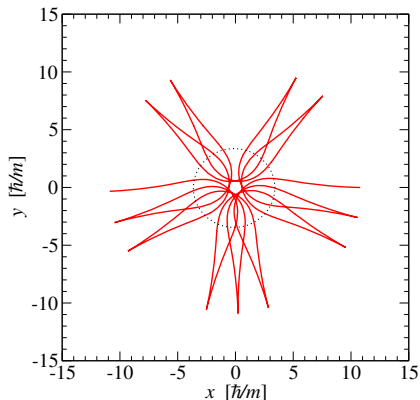
# An example : rotating boson stars

**Boson star** = localized configurations of a self-gravitating massive complex scalar field  $\Phi \equiv$  “Klein-Gordon geons”

[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

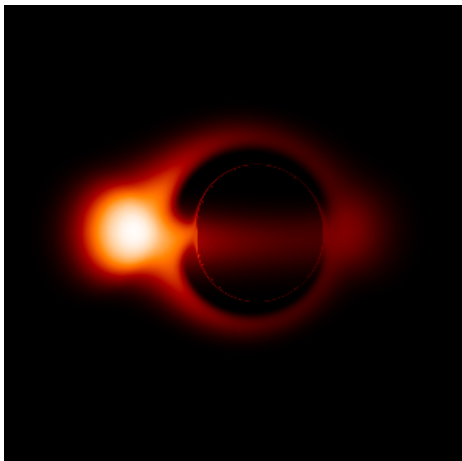
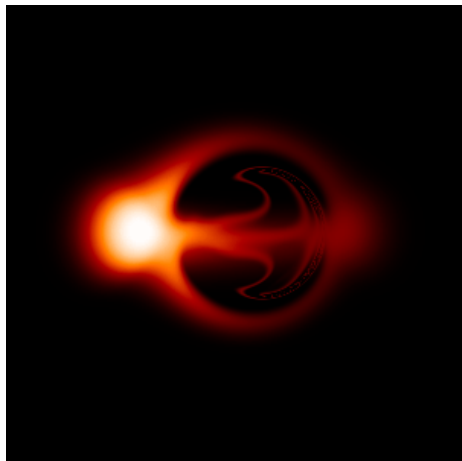
- Solutions of the *Einstein-Klein-Gordon* system computed by means of **Kadath** [Grandclément, JCP 229, 3334 (2010)]
- Timelike geodesics computed by means of **Gyoto**



Zero-angular-momentum orbit around a rotating boson star based on a free scalar field  $\Phi = \phi(r, \theta)e^{i(\omega t + 2\varphi)}$  with  $\omega = 0.75 m/\hbar$ .

[Grandclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]

## Image of an accretion torus

Kerr BH  $a/M = 0.9$ Boson star  $k = 1, \omega = 0.70 m/\hbar$ 

[Vincent, Meliani, Grandclément, Gourgoulhon &amp; Straub, CQG 33, 105015 (2016)]

# Conclusions

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The GW150914 event was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.