Constraining the dense matter equation of state with gravitational wave astrophysics

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Plan

Outline

Spacetime dynamics

- Special relativity : metric tensor $\boldsymbol{g}$ = fixed bilinear form on the spacetime *affine space*
- General relativity : metric tensor $\boldsymbol{g}$ = field of bilinear forms on the spacetime *manifold*

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Einstein equation : $$\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}$$

- $\boldsymbol{R}$ = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : "$\boldsymbol{R} \sim \boldsymbol{g}\, \partial^2 \boldsymbol{g} + \boldsymbol{g}\, \partial \boldsymbol{g}\, \partial \boldsymbol{g}$"
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- $\boldsymbol{T}$ = *energy-momentum tensor* of matter = symmetric bilinear form such that
  - $E = \boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}})$ is the energy density of matter as measured by an observer $\mathcal{O}$ of 4-velocity $\vec{\boldsymbol{u}}$
  - $p_i = -\boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{e}}_i)$ component of the matter momentum density as measured by $\mathcal{O}$ in the direction $\vec{\boldsymbol{e}}_i$
  - $S_{ij} = \boldsymbol{T}(\vec{\boldsymbol{e}}_i, \vec{\boldsymbol{e}}_j)$ component i of the force exerted by matter on the unit surface normal to $\vec{\boldsymbol{e}}_j$

Comparing Newtonian and relativistic gravitation theories

Newtonian gravitation :

fundamental equation : Poisson equation for the gravitational potential Φ :

$$\Delta\Phi = 4\pi G\rho$$

- scalar equation
- linear equation
- elliptic equation
  ($\Rightarrow$ instantaneous propagation)
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Remark : for a weak gravitational field, one of the 10 components of Einstein equation reduces to the Poisson equation (and the other 9 reduced to $0 = 0$).

What is a strong gravitational field ?

Relativity parameter or compacity parameter of a self-gravitating body of mass M and mean radius R :

$$\Xi = \frac{GM}{c^2 R} \sim \frac{|E_{\rm grav}|}{Mc^2} \sim \frac{|\Phi_{\rm surf}|}{c^2} \sim \frac{v_{\rm esc}^2}{c^2}$$

- $E_{\rm grav}$: gravitational potential energy[1]
- $\Phi_{\rm surf}$: gravitational potential at the surface of the body
- $v_{\rm esc}$: escape velocity from the body's surface[2]

	Earth	Sun	white dwarf	neutron star	black hole
Ξ	10^{-10}	10^{-6}	10^{-3}	0.2	1

if $\Xi \gtrsim 0.1$, general relativity must be employed to describe the body (**compact object**)

[1]for a homogeneous ball : $E_{\rm grav} = -\frac{3}{5}\frac{GM^2}{R}$

[2]for a spherically symmetric body : $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$

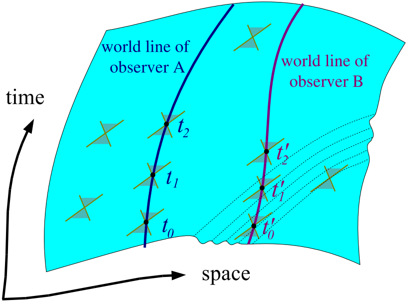
Gravitational waves

Linearization of Einstein equation in weak field :

$$\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}, \quad \boldsymbol{\eta} = \text{Minkowski metric}^3$$

$$\Longrightarrow \text{wave equation :} \quad \boxed{\Box \bar{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \quad \text{(Lorenz gauge)}$$

with $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\bar{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\,\boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



[3] $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ in Cartesian coordinates

Gravitational wave emission

- For a weakly relativistic source : quadrupole formula :

$$h_{ij}^{\rm TT}(t,\vec{x}) = \frac{2G}{c^4 r}\left[P_i^{\ k} P_j^{\ l} - \frac{1}{2}P_{ij}P^{kl}\right]\ddot{Q}_{ij}\left(t-\frac{r}{c}\right)$$

  - r : distance to the source
  - $P_{ij} = \delta_{ij} - x^i x^j / r^2$: transverse projector
  - $Q_{ij}(t) := \int_{\rm source} \rho(t,\vec{x})\left(x^i x^j - \frac{1}{3}\vec{x}\cdot\vec{x}\,\delta_{ij}\right) d^3\vec{x}$: mass quadrupole

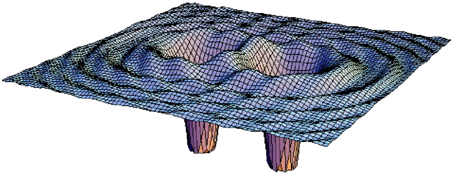
- GW luminosity :

$$L \sim \frac{c^5}{G}\, s^2\, \boldsymbol{\Xi}^2 \left(\frac{v}{c}\right)^6$$

  - s : asymmetry factor ($s=0$ fpr spherical symmetry)
  - $\boldsymbol{\Xi} := GM/(c^2 R)$: compacity parameter
  - v : characteristic velocity of matter in the source

NB : $c^5/G \simeq 4\ 10^{52}$ W !

Gravitational waves



Bi-dimensional spacelike section of a spacetime generated by a binary system of black holes

gravitational waves = perturbations in spacetime curvature

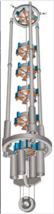
- reveal the **dynamics** of spacetime
- are generated by acceleration of matter
- far from the sources, propagate with the velocity of light
- NB : **electromagnetic waves** (radio waves, IR, optical, UV, X and gamma) are perturbations of the electromagnetic field which propagate *within* spacetime, whereas **gravitational waves** are waves of spacetime *itself*

Detection of gravitational waves

LIGO : USA, Louisiana

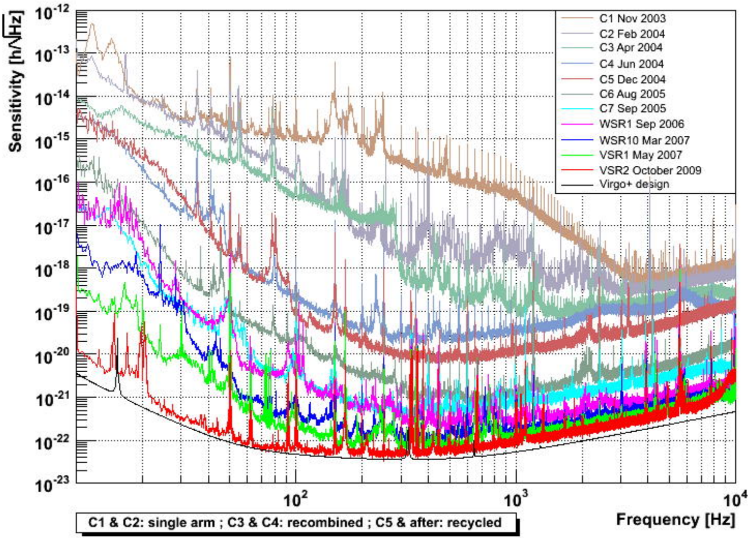
LIGO : USA, Washington

VIRGO : France/Italy/Poland (Pisa)



Interferometers VIRGO (3 km) and LIGO (4 km) are currently acquiring data.

VIRGO sensitivity curve



Event rates

Binary coalescences :

		NS-NS	BH-NS	BH-BH
predicted rate[(1)]	$[\mathrm{yr}^{-1} L_{10}^{-1}]$	$5\ 10^{-5}$	$2\ 10^{-6}$	$4\ 10^{-7}$
observed rate[(2)]	$[\mathrm{yr}^{-1} L_{10}^{-1}]$	$< 4\ 10^{-2}$	$< 2\ 10^{-2}$	$< 2\ 10^{-3}$
detection range	LIGO S5[(2)]	30 Mpc	50 Mpc	80 Mpc

$L_{10} = 10^{10} L_\odot$ (blue solar luminosity) ; our galaxy : $\sim 1.7\, L_{10}$

(1) [Kalogera, Belczynski, Kim, O'Shaughnessy & Willems, Phys. Rep. **442**, 75 (2007)]

(2) from 1st year of LIGO S5 data, Nov. 2005 - Nov. 2006

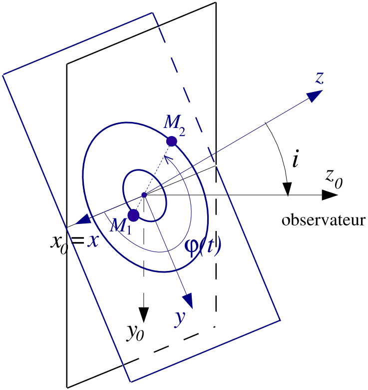
[Abbott et al., PRD **79**, 122001 (2009)]

Core collapse supernovae :

rate $\sim 2\ 10^{-2}\ \mathrm{yr}^{-1} L_{10}^{-1}$

Outline

Gravitational radiation from a binary system



Masses : M_1 and M_2

Chirp mass : $\mathcal{M} = \left[\frac{(M_1 M_2)^3}{M_1 + M_2} \right]^{1/5}$

Orbital period : P

Distance to the binary : d

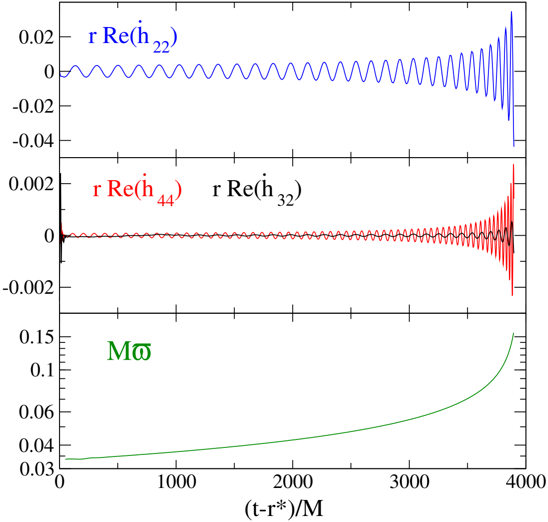
Inclination angle : i

GW for a circular orbit at the 0-PN level
(from quadrupole formula) :

$$h_+ = \frac{2}{c^4 d} (G\mathcal{M})^{5/3} \left(\frac{2\pi}{P} \right)^{2/3} (1 + \cos^2 i) \ \cos\left(4\pi \frac{t}{P} + \varphi_0 \right)$$

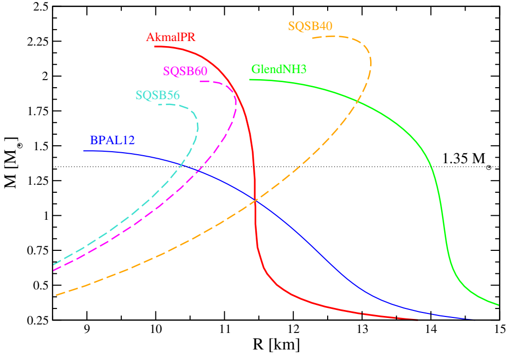
$$h_\times = \frac{4}{c^4 d} (G\mathcal{M})^{5/3} \left(\frac{2\pi}{P} \right)^{2/3} \cos i \ \sin\left(4\pi \frac{t}{P} + \varphi_0 \right)$$

Chirp signal



[Boyle et al., PRD **78**, 104020 (2008)]

A panel of different EOS



3 nuclear matter EOS :

• BPAL12 : phenomenological soft extreme of nucleonic EOS [Bombacci et al. 1995]

• AkmalPR : n,p,e,μ with 2-body (Argonne A18) and 3-body (Urbana UIX) nucleon interactions [Akmal, Pandharipande & Ravenhall 1998]

• GlendNH3 : n,p,e,μ with hyperons for $\rho > 2\rho_{\rm nuc}$ [Glendenning 1985]

3 strange matter EOS :

MIT bag model

•SQSB56 : $m_{\rm s}c^2 = 200$ MeV, $\alpha = 0.2$, $B = 56\ {\rm MeV/fm^3}$

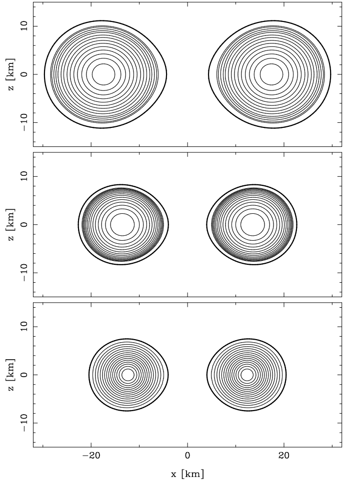
•SQSB60 : $m_{\rm s}c^2 = 0$, $\alpha = 0$, $B = 60\ {\rm MeV/fm^3}$

•SQSB40 : $m_{\rm s}c^2 = 100$ MeV, $\alpha = 0.6$, $B = 40\ {\rm MeV/fm^3}$

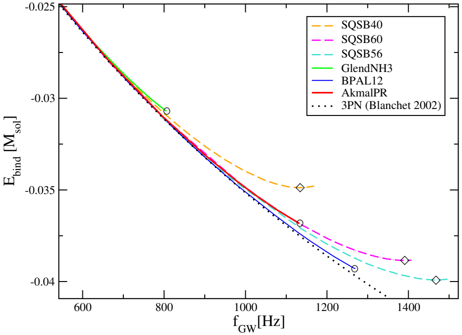
Inspiralling sequences for different EOS

Mass-shedding limit

for $M_1 = M_2 = 1.35\, M_\odot$ and GlendNH3, AkmalPR and BPLA12 EOS :



Binding energy along the sequence :



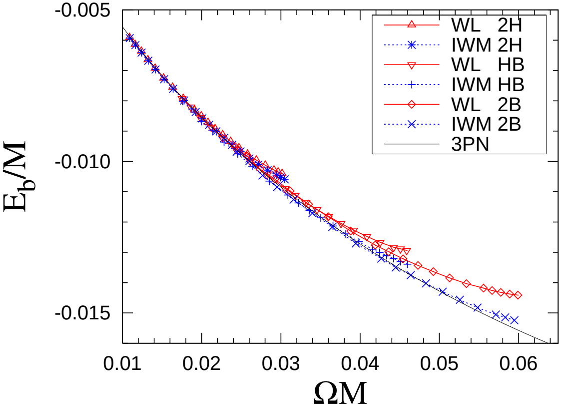
[Bejger, Gondek-Rosińska, Gourgoulhon, Haensel, Taniguchi & Zdunik, A&A **431**, 297 (2005)]

[Limousin, Gondek-Rosińska & Gourgoulhon, PRD **71**, 064012 (2005)]

[Gondek-Rosińska, Bejger, Bulik, Gourgoulhon, Haensel, Limousin, Taniguchi & Zdunik, ASR **39**, 271 (2007)]

Beyond the IWM approximation (1/2)

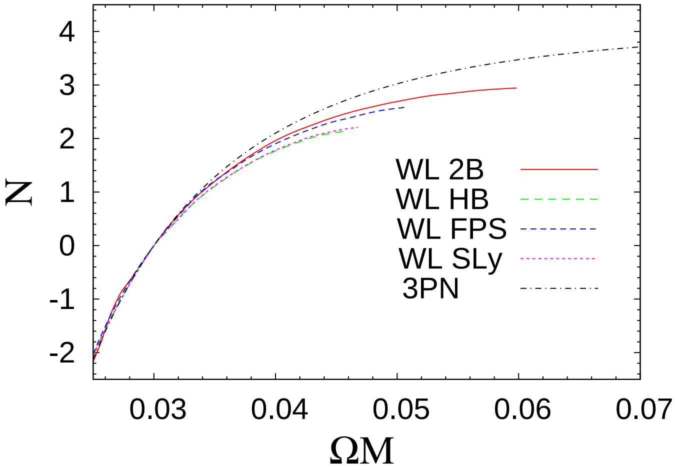
IWM approximation : conformally flat 3-metric, solving 5 Einstein equations
WL approximation : waveless scheme, full 3-metric, solving 10 Einstein equations
[Uryu, Limousin, Friedman, Gourgoulhon & Shibata, PRD **80**, 124004 (2009)]



$M = 1.35\, M_\odot$
piecewise polytropic EOS
[Read et al., PRD **79**, 124033 (2009)]
$\gamma = 1.35 \to 3$
2H : $M/R = 0.13$
HB : $M/R = 0.17$
2B : $M/R = 0.21$

Beyond the IWM approximation (2/2)

Number of orbital cycles

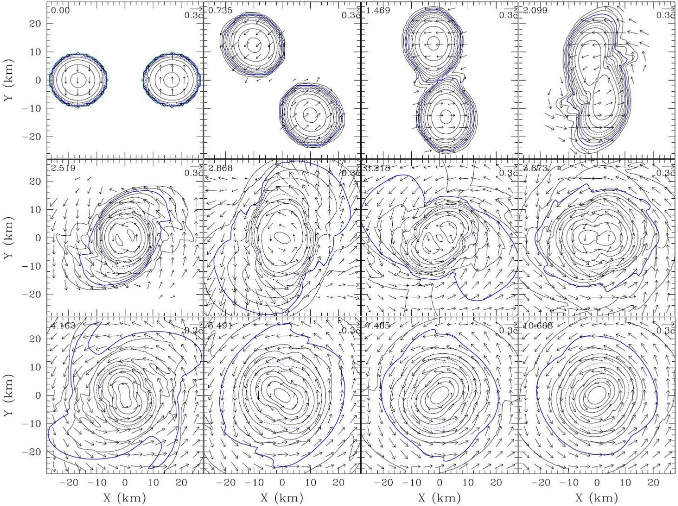


[Uryu, Limousin, Friedman, Gourgoulhon & Shibata, PRD **80**, 124004 (2009)]

The merger

Small mass : hypermassive neutron star remnant (bar shape, short living)

EOS : Akmal, Pandharipande & Ravenhall (1998), $M_1 = M_2 = 1.3\, M_\odot$

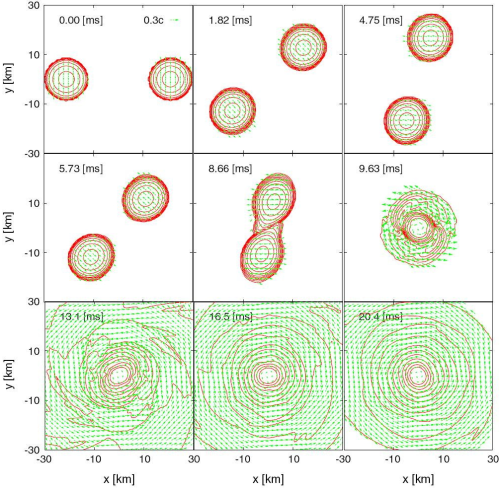


[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

The merger

Slightly larger mass : hypermassive neutron star remnant (bar shape, short living)

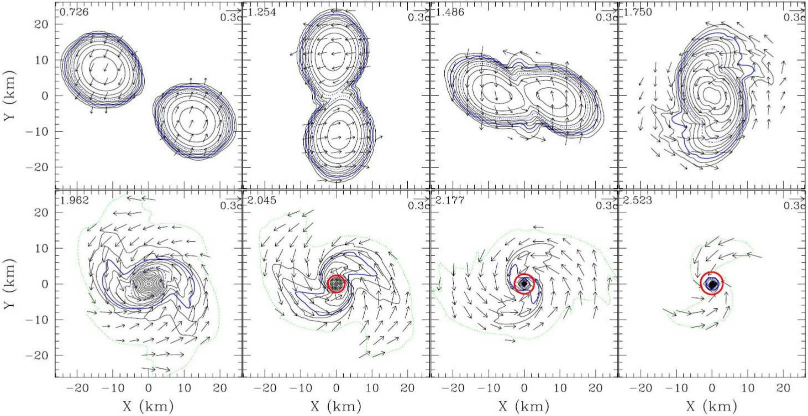
EOS : Akmal, Pandharipande & Ravenhall (1998), $M_1 = M_2 = 1.4\, M_\odot$



[Kiuchi, Sekiguchi, Shibata & Taniguchi, PRD **80**, 064037 (2009)]

The merger
Larger mass : prompt black hole formation

EOS : Akmal, Pandharipande & Ravenhall (1998), $M_1 = M_2 = 1.5\, M_\odot$

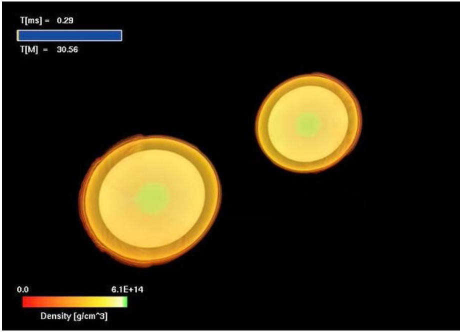


[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

The merger

Larger mass : prompt black hole formation

EOS : polytropic $\gamma = 2$, $M_1 = M_2 = 1.5\, M_\odot$



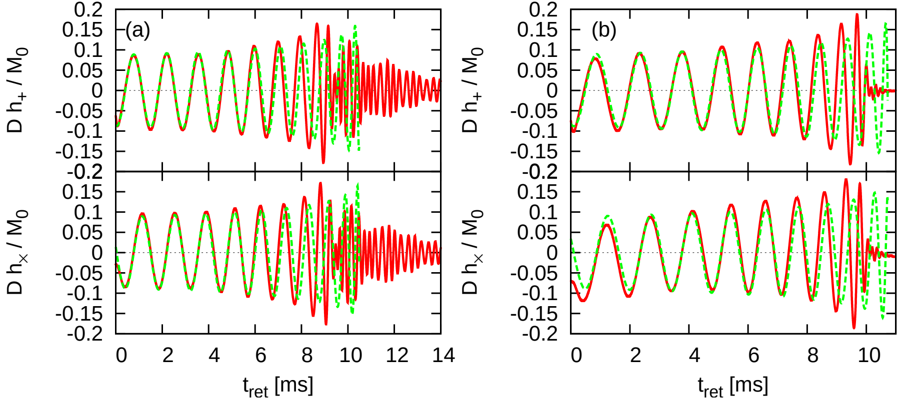
[Baiotti, Giacomazzo & Rezzola, PRD **78**, 084033 (2008)]

[movie from `numrel@aei`]

Gravitational wave signal

$M_1 = M_2 = 1.3\, M_\odot$ no BH

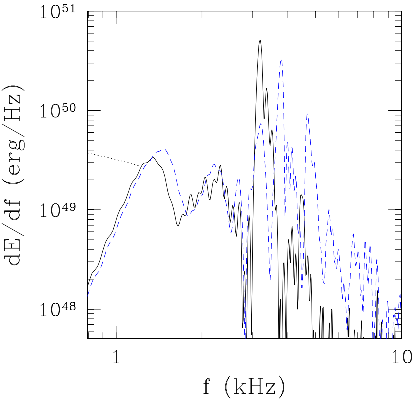
$M_1 = M_2 = 1.5\, M_\odot$ prompt BH formation



dashed line = point particles, 3.5 PN Taylor T4

[Kiuchi, Sekiguchi, Shibata & Taniguchi, PRD **80**, 064037 (2009)]

GW Fourier spectrum



← EOS : APR
$M_1 = M_2 = 1.3\, M_\odot$ (solid)
$M_1 = M_2 = 1.4\, M_\odot$ (dashed)
dotted line : 2-PN

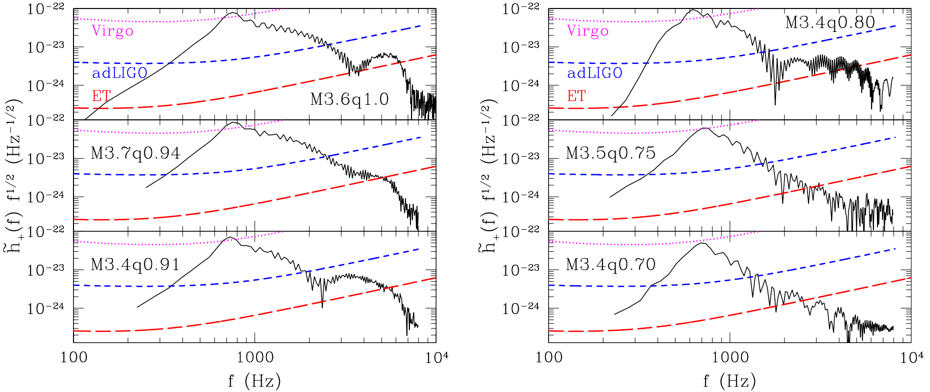
$M_{\rm crit}$: total mass for prompt black hole formation
$\exists$ peak at $f \sim 2-3\ {\rm kHz} \Rightarrow$ $M_{\rm tot} < M_{\rm crit}$
No peak $\Rightarrow$ prompt BH formation
$\Rightarrow$ **soft EOS**
FPS EOS : $M_{\rm crit} = 2.5\ M_{\rm sol}$
SLy EOS : $M_{\rm crit} = 2.7\ M_{\rm sol}$
APR EOS : $M_{\rm crit} = 2.9\ M_{\rm sol}$
In addition, the frequency of the peak depends on the EOS

[Shibata, PRL **94**, 201101 (2005)]

[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

GW Fourier spectrum

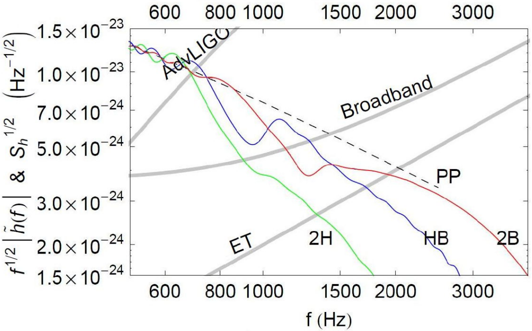
Prompt black hole formation : no peak



distance $d = 100\ \mathrm{Mpc}$

[Rezzolla, Baiotti, Link & Font, arXiv/1001.3074]

Measuring the EOS stiffness



Measuring departure from point-particle limit

$\Rightarrow$ the GW phase accumulates more rapidly for smaller value of NS compactness

$\delta R \sim 1\ \mathrm{km} \times (100\ \mathrm{Mpc}/d)$

in broadband advanced LIGO

$M_1 = M_2 = 1.35 M_\odot$, $d = 100$ Mpc

PP = point particle, 2H : $M/R = 0.13$

HB : $M/R = 0.17$, 2B : $M/R = 0.21$

[Markakis, Read, Shibata, Uryu, Creighton, Friedman & Lackey, J. Phys.: Conf. Ser. **189** 012024 (2009)]

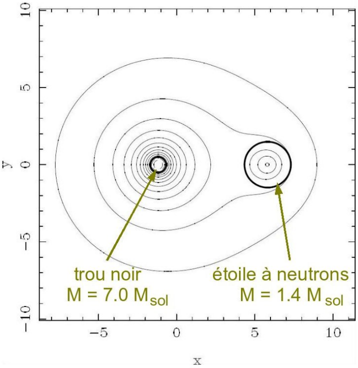
[Read, Markakis, Shibata, Uryu, Creighton & Friedman, PRD **79**, 124033 (2009)]

Outline

Black hole-neutron star binaries

The most favorable binary coalescence for VIRGO / LIGO ?

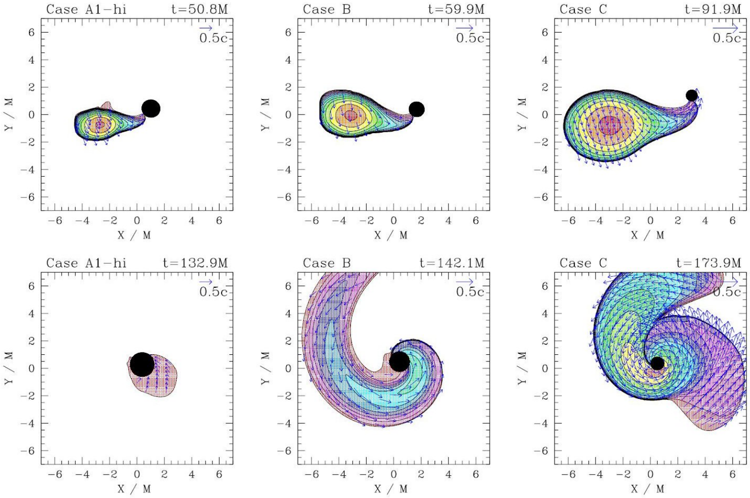
Sources of short gamma-ray burst ?



[Grandclément, PRD **74**, 124002 (2006)]

Black hole-neutron star merger

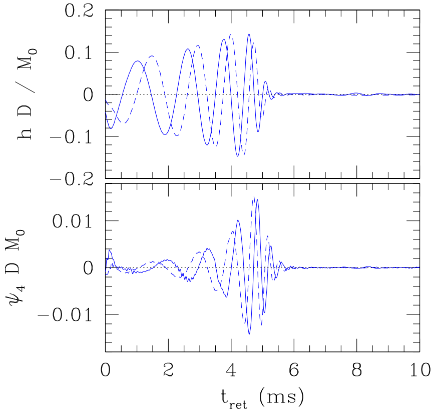
EOS : polyt. $\gamma = 2$, $M/R = 0.145$, mass ratio 3 (A), 2 (B) and 1 (C) :



[Etienne, Faber, Liu, Shapiro, Taniguchi & Baumgarte, PRD **77**, 084002 (2008)]

Gravitational wave signal

EOS : polyt. $\gamma = 2$, mass ratio 3



[Shibata & Taniguchi, PRD **77**, 084015 (2008)]

GW Fourier spectrum

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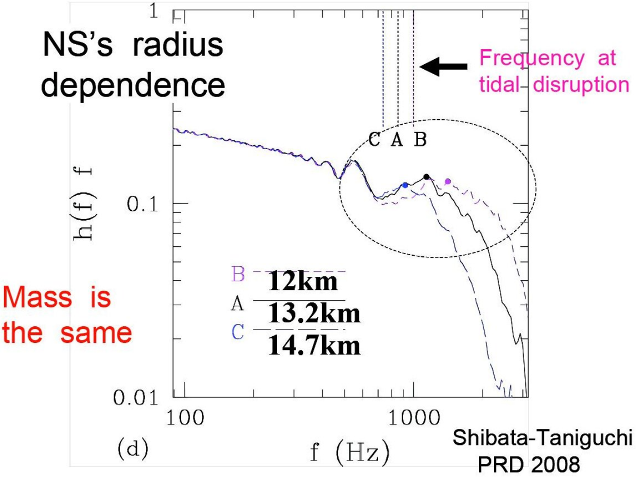
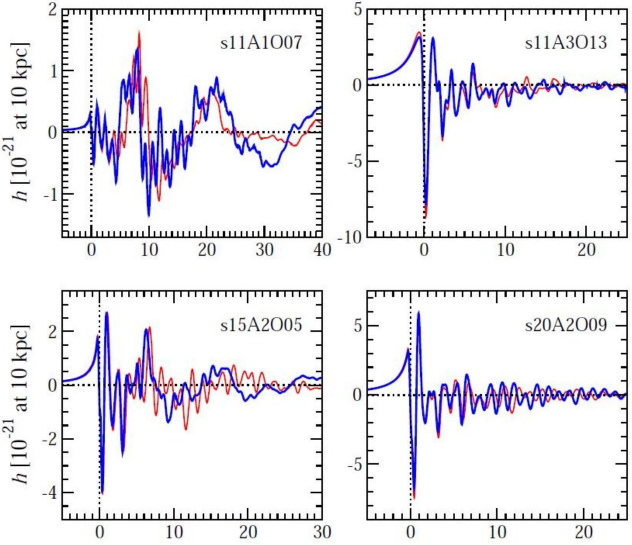


[Shibata & Taniguchi, PRD **77**, 084015 (2008)]

Outline

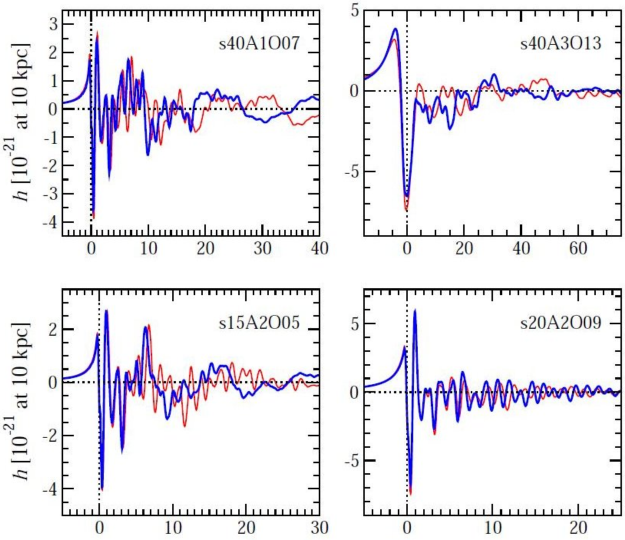
Neutron star formation in core-collapse supernovae



- EOS : red : Shen (1998) blue : Lattimer & Swesty (1991)
- Progenitor mass : s11 $= 11\,M_\odot$, s15 $= 15\,M_\odot$, s20 $= 20\,M_\odot$, s40 $= 40\,M_\odot$
- Rotation profile : A1 = uniform, A2 = moderately differential, A3 = strongly differential
- $T/|W|$: O1 = small, O15 = large

[Dimmelmeier, Ott, Marek & Janka, PRD **78**, 064056 (2008)]

Neutron star formation in core-collapse supernovae



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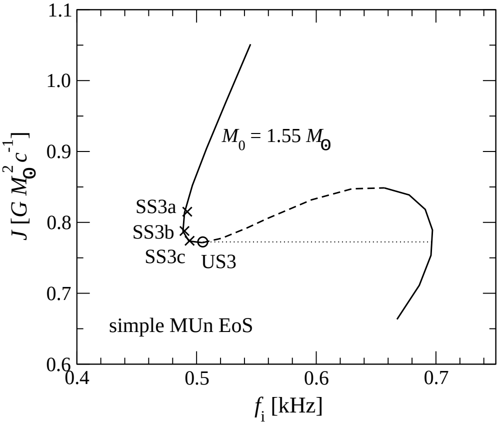
Phase-transition-induced mini-collapse of neutron stars

Back bending instability



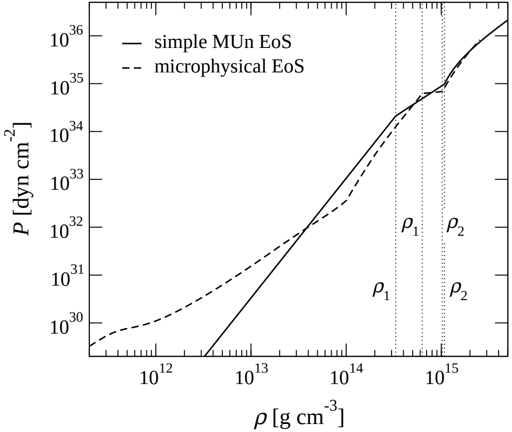
Electromagnetic radiation of an isolated rotating neutron star
$\Rightarrow$ angular momentum loss
$\Rightarrow$ increase of central pressure

If $\exists$ phase transition at high pressure
EOS softening $\Rightarrow$ *back bending* in the (Ω, J) curve
$\Rightarrow$ migration from unstable to stable configuration
(minicollapse)

[Dimmelmeier, Bejger, Haensel & Zdunik, MNRAS **396**, 2269 (2009)]

Phase-transition-induced mini-collapse of neutron stars

Two examples of EOS with phase transition :

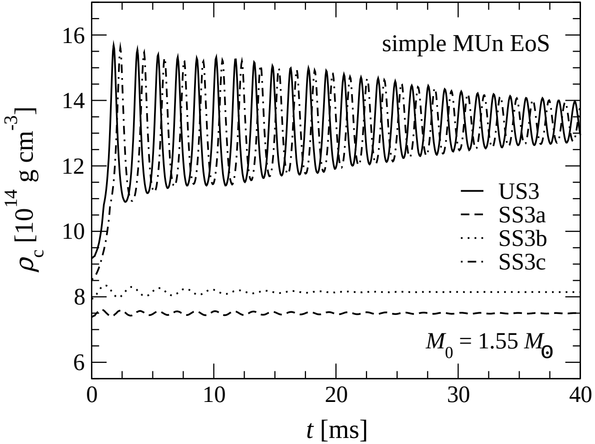


- MUn EoS : transition from normal baryon matter ($\rho < \rho_1$) to quark matter ($\rho > \rho_2$) via a mixed baryon-quark phase
- microphysical EoS : first order phase transition from normal baryon matter to kaon-condensed matter

[Dimmelmeier, Bejger, Haensel & Zdunik, MNRAS **396**, 2269 (2009)]

Phase-transition-induced mini-collapse of neutron stars

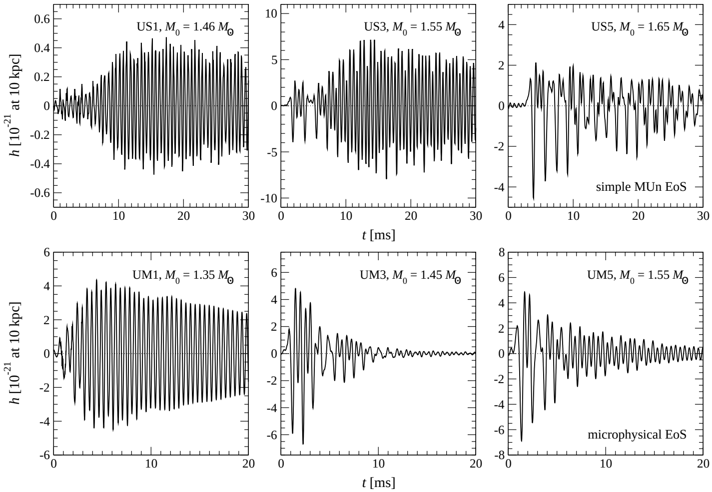
Evolution of central density during the migration from the unstable configuration to the stable one :



[Dimmelmeier, Bejger, Haensel & Zdunik, MNRAS **396**, 2269 (2009)]

Phase-transition-induced mini-collapse of neutron stars

Gravitational wave signal



← Mixed phase transition to quark matter

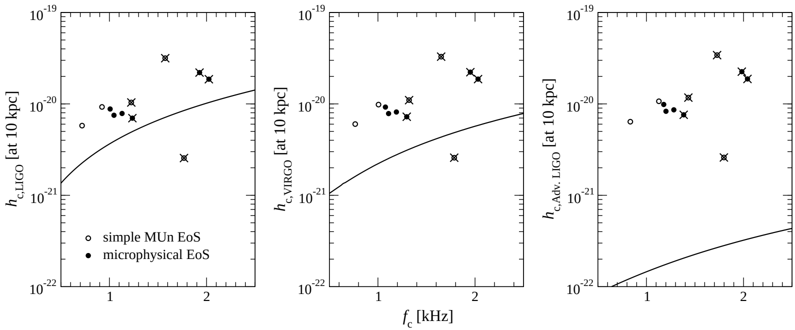
← First-order phase transition to kaon condensate (damping due to compression of matter through phase transition)

[Dimmelmeier, Bejger, Haensel & Zdunik, MNRAS **396**, 2269 (2009)]

Phase-transition-induced mini-collapse of neutron stars

Detectability

- Long quasi-periodic signal
- Promising candidates : young magnetars (strong angular momentum loss)
- Event rate : $10^{-2}\ \mathrm{yr}^{-1}$ in our Galaxy (VIRGO, LIGO) ; $1\ \mathrm{yr}^{-1}$ in the Virgo cluster (ET)

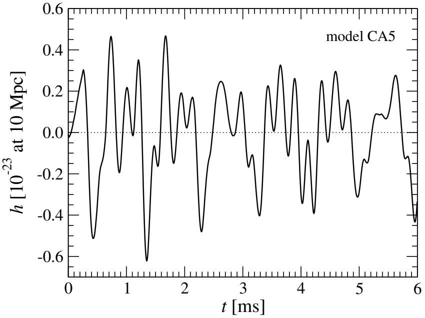


[Dimmelmeier, Bejger, Haensel & Zdunik, MNRAS **396**, 2269 (2009)]

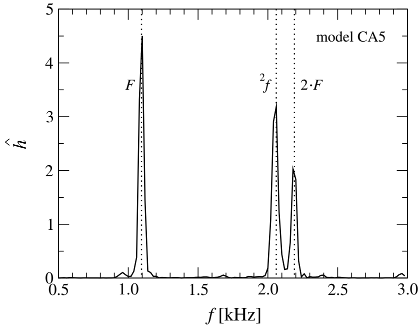
Phase-transition-induced mini-collapse of neutron stars

Another study : phase transition from hadronic matter to deconfined quark matter in the core ⇒ compact hybrid quark star

Waveform



Fourier spectrum



[Abdikamalov, Dimmelmeier, Rezzolla & Miller, MNRAS **392**, 52 (2009)]