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SCALE-RELATIVITY, FRACTAL SPACE-TIME AND GRAVITATIONAL STRUCTURES

L. NOTTALE
CNRS, DAEC, Observatoire de Paris-Meudon, 92195 Meudon Cedex, France
E-mail: laurent.nottale@obspm.fr

G. SCHUMACHER
Observatoire de la Côte d'Azur, Département Fresnel, UMR 6528 du CNRS,
Av. Copernic, 06130 Grasse, France. Email: schumacher@obs-azur.fr

The theory of scale relativity extends Einstein's principle of relativity to scale transformations of resolutions. It is based on the giving up of the axiom of differentiability of the space-time continuum. Three consequences arise from this withdrawal: (i) The geometry of space-time must be fractal, i.e., explicitly resolution-dependent. (ii) The geodesics of the non-differentiable space-time are themselves fractal and in infinite number. (iii) Time reversibility is broken at the infinitesimal level. These three effects are finally combined in terms of a new tool, the scale-covariant derivative, which transforms classical mechanics into a generalized, quantum-like mechanics. We describe virtual families of geodesics in terms of probability densities, interpreted as a tendency for the system to make structures. In the present contribution, we apply our scale-covariant procedure to the equations of motion of test-particles in a gravitational potential. A generalized Newton-Schrödinger equation is obtained, and its solutions are studied. Our theoretical predictions are then successfully checked by a comparison with observational data at various scales, ranging from planetary systems to large scale structures. A possible interpretation of these results is that the underlying fractal geometry of space-time plays the role of a universal structuring "field" that leads to self-organization and morphogenesis of matter in the Universe.

Keywords: relativity, gravitation, fractal space-time, self-organisation

1 Introduction

The theory of scale relativity[1] is founded on the realization that the whole of present physics relies on the implicit assumption of differentiability of the space-time continuum. Giving up the hypothesis of the differentiability of coordinates naturally leads to the proposal that the concept of *fractal space-time*[1-5] is the geometric tool adapted to the research of a new physical description based on non-differentiability.

Since a nondifferentiable, fractal space-time is explicitly resolution-dependent, the same is *a priori* true of all physical quantities that one can define in its framework. We thus need to complete the standard laws of physics by laws of scale. We have suggested[1,4,6] that the principle of relativity, that is known to be a constructive principle for motion laws, can be extended to constrain also these new scale laws.

The domains of application of this theory are typically the asymptotic domains of physics, small length-scales and small time-scales $\Delta x \to 0$, $\Delta t \to 0$ (microphysics), large length-scales $\Delta x \to \infty$ (cosmology), but also the domain of complexity and of structure formation on long time-scales $\Delta t \to \infty$.

A more detailed account of the theory and of its consequences (including the domains of high energy elementary particle physics and cosmology, which are not considered here) can be found in the book[1]and in the contribution of one of us to the 'Fractal 95' conference,[7] and in a more recent review paper.[8]

2 Theoretical framework

2.1 Description of a non-differentiable and fractal space-time

Giving up differentiability of the space-time coordinates has three main consequences: (i) the explicit scale-dependence of physical quantities on space-time resolutions, that implies the construction of new fundamental laws of scale (to which we apply the principle of scale relativity) ; (ii) the multiplication to infinity of the number of geodesics, that suggests jumping to a statistical and probabilistic, fluid-like, description; (iii) the breaking of the time symmetry ($dt \leftrightarrow -dt$) at the level of the space-time geometry, that implies a "two-valuedness" of velocities which we represent in terms of a complex and non-classical new physics. The aim of the present section is to summarize how the giving up of differentiability leads one to introduce such new structures. A more detailed account of the construction of the theory has been given in the 'Fractal 95' volume.[7]

2.2 New scale laws

The basis of our method consists of replacing the classical velocity by a function that depends explicitly on resolution ε, $V = V(\varepsilon)$. Only $V(0)$ is now undefined, while $V(\varepsilon)$ is now defined for any non-zero ε. One particularly efficient application of this method consists of treating the time differential element dt as a variable, and of identifying it with the time resolution, i.e., $dt = \varepsilon_t$.

Now, the simplest possible equation that one can write for the variation of the velocity$V(t,dt)$ in terms of the new scale variable dt is:

$$\frac{\partial V}{\partial \ln dt} = \beta(V) \quad , \tag{1}$$

i.e., a first order, renormalization-group-like differential equation. The β–function here is *a priori* unknown. It can be expanded in terms of a Taylor expansion, $\beta(V) = a + b\ V + ..$, where a and b are "constants" (independent of dt, but possibly dependent on space-time coordinates). Setting $b = (1/D)-1$, we obtain the solution of this equation under the form:

$$V = v + W = v\left[1 + \zeta\left(\frac{\tau}{dt}\right)^{1-1/D}\right] \quad , \tag{2}$$

where v is a mean velocity and W a fractal fluctuation that is explicitly scale-dependent, and where τ and ζ are chosen such that $<\zeta> = 0$ and $<\zeta^2> = 1$. Concerning the value of the fractal dimension, recall that $D = 2$ plays the role of a critical dimension in the whole theory.[1,7,8]

Let us now write the expression for the elementary displacement dX derived from the above value of the velocity. It can be written under the sum of two terms,

$$dX = dx + d\xi \,, \tag{3}$$

with $dx = v \; dt$, and $d\xi = \zeta \; \tau_0^{1-(1/D)} \; dt^{1/D}$, where τ_0 is a constant.

In the present contribution, only the above scale-law with fractal dimension $D = 2$ will be considered. Such a law do come under the principle of scale relativity, since it can be identified with a "galilean" approximation of more general "lorentzian" scale-relativistic laws in which the fractal dimension becomes itself variable with scale.[1,6,8]

2.3 Induced effects on motion laws

One of the geometric consequences that is specific of the nondifferentiability of space (not only of the trajectories), is that there will be an infinity of fractal geodesics that relate any couple of points in a fractal space, as compared with situations where there is only one geodesic (or a finite number) in general relativity.[1,4] This leads one to jump to a statistical description.

The nondifferentiable nature of space-time implies an even more dramatic consequence, namely, a breaking of reversibility at the infinitesimal level ($dt \rightarrow -dt$). Such a discrete symmetry breaking can *not* be derived from only the fractal or nondifferentiable nature of *trajectories*, since it is a consequence of the irreducible nondifferentiable nature of *space-time* itself. Therefore, while the concept of velocity was classically a one-valued concept, we must introduce, if space-time is nondifferentiable, two velocities, v_+ and v_-, instead of one.

Now we have no way to favor v_+ rather than v_-. Both choices are equally qualified for the description of the laws of nature. The only solution to this problem is to consider both the forward ($dt > 0$) and backward ($dt < 0$) processes together. A simple and natural way to account for this two-valuedness consists of using complex numbers and the complex product. The new complex process, *as a whole,* recovers the fundamental property of microscopic reversibility.

Finally, we can describe the elementary displacement dX for both processes as the sum of a mean, $\langle dx_\pm \rangle = v_\pm \; dt$, and a fluctuation about this mean, $d\boldsymbol{\xi}_\pm$, which is then by definition of zero average, $\langle d\boldsymbol{\xi}_\pm \rangle = 0$, i.e.:

$$dX_+(t) \; = \; v_+ \, dt \; + d\boldsymbol{\xi}_+(t) \;\; ; \;\; dX_-(t) \; = \; v_- \, dt \; + d\boldsymbol{\xi}_-(t) \;\; . \tag{4}$$

The fundamental irreversibility of the description is now apparent from the fact that the average backward and forward velocities are in general different. So mean forward and backward derivatives, d_+/dt and d_-/dt, are defined. Once applied to the position vector x, they yield the *forward and backward mean velocities*, $d_+x(t)/dt = v_+$ and $d_-x(t)/dt = v_-$.

Concerning the fluctuations, the fractal behavior writes in three dimensions

$$\langle d\xi_{\pm i} \; d\xi_{\pm j} \rangle = \pm \, 2 \, \mathbb{D} \, \delta_{ij} \; dt \quad , \tag{5}$$

$\mathbb{D}$ standing for a fundamental parameter (equivalent to the above parameter τ_0) that characterizes the new scale law. The $d\boldsymbol{\xi}(t)$'s are of zero mean and mutually independent.

Our main tool now consists of recovering local time reversibility in terms of a new *complex* process: we combine the forward and backward derivatives in terms of a complex derivative operator

$$\frac{\mathcal{d}}{dt} = \frac{(d_+ + d_-) - i(d_+ - d_-)}{2dt} \, , \tag{6}$$

which, when applied to the position vector, yields a complex velocity $\mathcal{V} = \mathcal{d}\boldsymbol{x}(t)/dt$. The real part $\boldsymbol{V}$ of the complex velocity $\mathcal{V}$ generalizes the classical velocity.

Equations (5,6) now allow one to get a general expression for the complex time derivative $\mathcal{d}/dt$. One finds:[1]

$$\frac{\mathcal{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{V} \, . \, \nabla - i \, \mathcal{D} \, \Delta \, . \tag{7}$$

The passage from classical (differentiable) mechanics to the new nondifferentiable mechanics can now be implemented by replacing the standard time derivative d/dt by the new complex operator $\mathcal{d}/dt$ (see Pissondes[9] for recent developments of this method, concerning in particular the Leibniz rule). In other words, this means that $\mathcal{d}/dt$ plays the role of a *scale-covariant derivative* (in analogy with general relativity where the basic tool consists of replacing $\partial_j A^k$ by the covariant derivative $D_j A^k = \partial_j A^k + \Gamma^k_{j\,l} A^l$).

2.4 Scale-covariant mechanics

Let us recall the main steps by which one may generalize classical mechanics using this 'scale-covariance'. We assume that any mechanical system can be characterized by a Lagrange function $\mathcal{L}(\boldsymbol{x}, \mathcal{V}, t)$, from which an action $\mathcal{S}$ is defined. The Lagrange function and the action are now *a priori* complex and are obtained from the classical Lagrange function $L(\boldsymbol{x}, d\boldsymbol{x}/dt, t)$ and classical action S precisely from applying the above prescription $d/dt \rightarrow \mathcal{d}/dt$. The stationary-action principle, applied on this new action with both ends of the action integral fixed, leads to generalized Euler-Lagrange equations

$$\frac{\mathcal{d}}{dt} \frac{\partial \mathcal{L}}{\partial \mathcal{V}_i} = \frac{\partial \mathcal{L}}{\partial x_i} \, . \tag{8}$$

Other fundamental results of classical mechanics are also generalized in the same way. In particular, assuming homogeneity of space in the mean leads to defining a generalized *complex* momentum and a complex energy given by $\mathcal{P} = \partial\mathcal{L} / \partial\mathcal{V}$, $\mathcal{E} = \mathcal{P}\,\mathcal{V} - \mathcal{L}$. If one now considers the action as a functional of the upper limit of integration, the variation of the action from a trajectory to another nearby trajectory yields a generalization of other well-known relations of classical mechanics, namely $\mathcal{P} = \boldsymbol{\nabla} \mathcal{S}$, $\mathcal{E} = -\partial \mathcal{S}/\partial t$.

2.5 Newton-Schrödinger equation

Let us now specialize our study, and consider Newtonian mechanics, i.e., the general case when the structuring field is a scalar field. The Lagrange function of a closed system, $\mathcal{L}$ =

$\frac{1}{2}m\, v^2 - \phi$, is generalized as $L(\boldsymbol{x}, V, t) = \frac{1}{2}\, mV^2 - \phi$, where ϕ denotes a scalar potential. The Euler-Lagrange equations keep the form of Newton's equation of dynamics,

$$m \; \frac{d}{dt} \; V \;=\; -\boldsymbol{\nabla} \phi \quad , \tag{9}$$

which is now written in terms of complex variables and operators. We may now introduce a wave function ψ which *is nothing but another expression for the complex action S*,

$$\psi \;=\; e^{iS/2m\mathbb{D}} \quad . \tag{10}$$

It is related to $V = -\,2i\, \mathbb{D}\, \boldsymbol{\nabla}\, (ln\psi)$. Standard calculations with differential operators finally allow one to transform Eq. (9) into a generalized Schrödinger equation:[1]

$$\mathbb{D}^2\, \Delta\psi \;+\; i\;\mathbb{D}\; \frac{\partial}{\partial t}\, \psi \;-\; \frac{\boldsymbol{\Phi}}{2m}\, \psi \;=\; 0 \quad . \tag{11}$$

The statistical meaning of the wave function (Born postulate) can now be deduced from the very construction of the theory.[10,11] The "particle" is one random geodesic of the infinite family of geodesics, and its probability to be found at a given position must be proportional to the density of the fluid of geodesics (see Hermann[12] for numerical simulations implementing this interpretation). This density can now be easily calculated from our variables, since the imaginary part of Eq. (11) writes $\partial\rho/\partial t + div(\rho \boldsymbol{V}) = 0$, where $\rho = |\psi|^2$. This equation is the equation of continuity, in agreement with ρ representing the fluid density, which is proportional to the density of probability.

3 Application to gravitational structures

3.1 Theoretical predictions

A preferential domain of application of our new framework is gravitation. Indeed, gravitation is already understood, in Einstein's theory, as the various manifestations of the geometry of space-time at classical scales. Now our proposal may be summarized by the statement that space-time is not only Riemannian but becomes also fractal at very large scales. The various manifestation of the fractal geometry of space-time could therefore be attributed to new effects of gravitation (this becomes a matter of definition).

Let us consider the special case of an "external" gravitational field that is unaffected by the evolution of the structure which is forming. Typical examples of such a case are the Kepler two-body problem, and cosmology (particles embedded into a background with uniform density). For this type of problem, the equations of evolution are:

$$\Delta\phi \;=\; -4\,\pi\, G\, \rho \tag{12a}$$

$$\mathbb{D}^2\, \Delta\psi \;+ i\;\mathbb{D}\; \frac{\partial\psi}{\partial t} \;=\; \frac{\phi}{2m}\; \psi \tag{12b}$$

Solving for these equations will yield a probability density $P = |\psi|^2(\boldsymbol{x},t)$ for the test particles subjected to the potential ϕ.

Let us specialize our study to the Kepler two-body problem. We assume that the central potential $\phi = GmM/r$ dominates all other contributions, and look for stationary solutions. In this case, Eq. (12b) becomes:

$$2\,\mathbb{D}^2\,\Delta\psi + \left[\frac{E}{m} + \frac{GM}{r}\right]\psi = 0 \quad . \qquad (13)$$

One of the main differences of our macroscopic theory compared with standard, microphysical quantum mechanics, lies in the physical meaning of the parameter $\mathbb{D}$. In the microphysical situation, it is totally constrained and related[1,8] to the Planck constant as $\mathbb{D} = h/4\pi m$. In the gravitational case, the equivalence principle implies that it must be independent of the test particle mass m and proportional to the active gravitational mass M.[8,13,14] Therefore we write

$$\mathbb{D} = \frac{GM}{2w} \,, \qquad (14)$$

where w has the dimension of a velocity. Solving now Eq. (13), we find that the probability for the particle to orbit at a given distance r from the central body is given by functions similar to the hydrogen atom orbitals, i.e. $P_n(r) \propto r^{2n}\, e^{-2r/n\alpha}$ for circular orbits, with $\alpha = GM/w^2$. The peaks of probability lie at distances $a_n(\text{peak}) = GMn^2/w^2$, and the mean at $a_n(\text{mean}) = GM(n^2+n/2)/w^2$. From Kepler's third law, the velocity of a particle orbiting at peak is $v_n = w / n$.

Our interpretation of this result is that the gravitational system will have a tendency to make structures according to the above distribution. Moreover, our equations naturally yield a hierarchy of structures.[13,14] Let us summarize the argument. Consider a system of test-particles (e.g., planetesimals) in the dominant potential of the Sun. Their evolution on (relatively) long time-scales is governed by Eq. (13), in terms of a constant w_j. The particles then form a disk whose density distribution is given in the inner region by $P_{n_j}(r)$**.** This distribution can then be fragmented in sub-structures still satisfying Eq. (13), since the central potential remains dominant, but with a different constant w_{j+1}. We can iterate the reasoning on several hierarchy levels. The matching condition between the orbitals implies $w_{j+1} = k_j\, w_j$, with k_j integer**.** Our own Solar System is organized following such a hierarchy on at least 3 levels, from the Sun radius to the outer planets (see below).[13,14]

3.2 Results

Our theory accounts for the observed distributions of position[1,14], angular momentum[1,14] and mass[14] of planets in the Solar System (see Fig. 1). The same is true for the satellites of giant planets,[15] and for the asteroid belt.[16] The theory allows one to make new predictions, as e.g. structures in the distribution of comet perihelion,[17] and the possibility of a new intramercurial small planet.[14] Such an object is currently searched for at Observatoire de la Côte d'Azur using data from the coronagraphic instrument LASCO of the SOHO satellite.

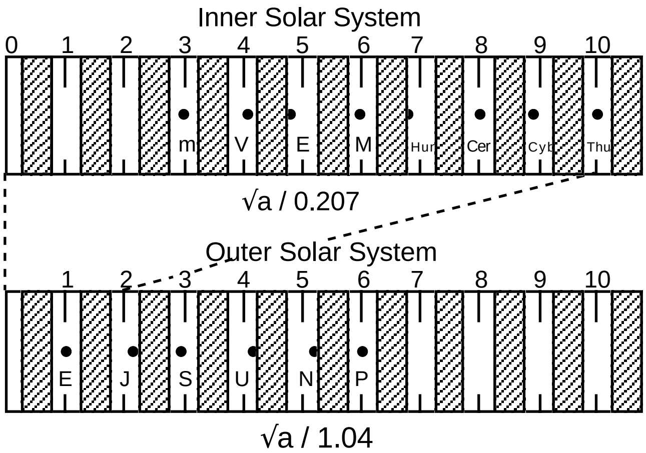

Figure 1. Comparison of the observed semi-major axes of planet orbits in the Solar System with our *a priori* prediction, $a_n = 0.043\ n^2$ AU for the inner system (from $w_0 = 144$ km/s) and $a_n = 1.08\ n^2$ AU for the outer system. The whole inner Solar System corresponds to the "orbital" $n = 1$ of the outer system, which allows one to match the two constants. We have divided the space in two domains, high probability density (white) and low probability density (grey). The probability to obtain such a result by chance is $\approx 2^{-13} \approx 10^{-4}$.

The motion of isolated binary galaxies also comes under the two-body problem. This allowed us[8] to explain the effect, discovered by Tifft,[18] of redshift quantization in galaxy pairs in terms of ≈72 and ≈144 km/s (see Fig. 5).

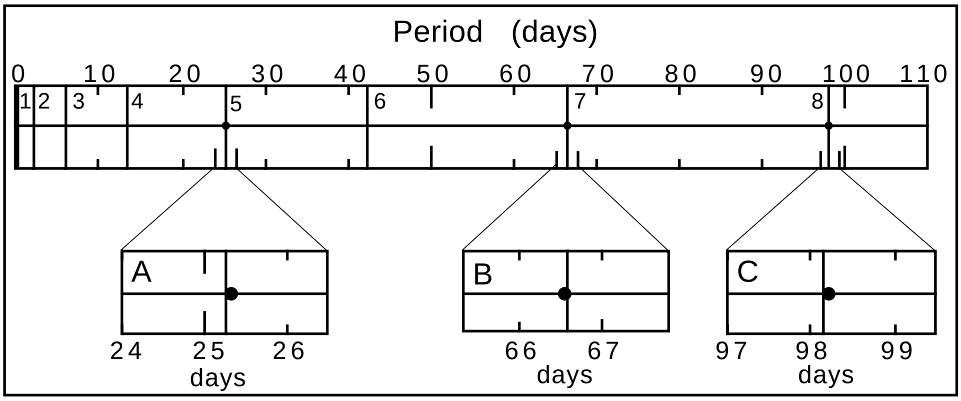

Figure 2. Comparison of our prediction,[13] $T_n = T_0\ (n^2+n/2)^{3/2}$, with the observed periods[19] for the planetary companions of the pulsar PSR B1257+12. The agreement is far better than the resolution of the diagram, so that we have included three insets enlarged by a factor of ≈10 to show the small residual differences. The probability to obtain such an agreement by chance is $P < 10^{-4}$. The periods of possible new planets can be predicted (unoccupied sites, see text).

Moreover, we find by inserting in Eq. (12) the potential of a uniform density that matter in the Universe must structure locally according to the various modes of the isotropic 3-dimensional harmonic oscillator, i.e., to the symmetries described by the SU(3) group.[8] This result yields a new general framework for structure formation,[8,10] and contributes to explain the Tifft[20] effect of "global" redshift quantization (in units of ≈36 km/s), recently confirmed by Guthrie & Napier.[21]

The various "quantizations" effects recalled above can all be expressed in terms of "Tifft's constant", $w_0 \approx 144$ km/s. Our theory predicts that our own Solar System, as well as extra-solar planetary systems, are structured in terms of the same constant, or one of its multiple or sub-multiple. Let us review some of these predicted effects and their observational corroboration in more detail.

(i) Solar System. The Solar System is structured according to the hierarchical law described above. As can be seen in Figs. 1,3, the velocities of planets in the *inner Solar System*, including the mass peaks of the asteroid belt, are given by $v_{n_2} = w_0 / n_2$, with $w_0 = 144.3 \pm 1.2$ km/s. The whole inner system achieves the fundamental orbital of the outer system, whose peak is the Earth ($n_2 = 5$).

According to our hierarchical mechanism, we then expect the *outer system*, including the Earth at $n_1 = 1$, to have velocities given by $v_{n_1} = w_0/5n_1$. We find $w_0 = 140 \pm 3$ km/s. The 'mean' formula gives a better fit for the outer system, as expected for the dominant planets. When accounting for second order terms, we obtain $w_0 = 144.8 \pm 2.6$ km/s, in good agreement with the Tifft extragalactic value and with the inner solar system value.

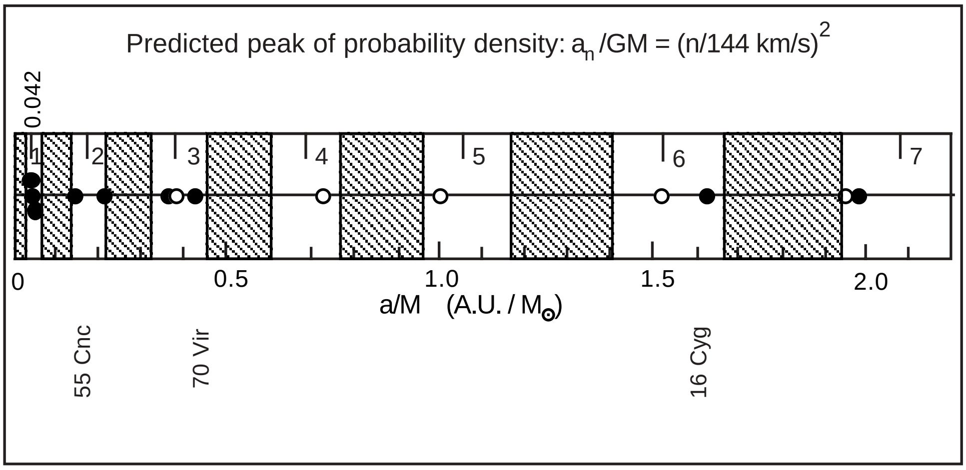

Figure 3. Comparison[13] of the observed values of the ratio *a/M* (where *a* is the semi-major axis of a planet and *M* the mass of its star) for inner solar system planets and extra-solar companions of solar-type stars, with our theoretical prediction $a_n /M = 0.043\, n^2$ (*a* in AU and *M* in M⊙. The grey zones stand for the domains of predicted low probability density. This is a highly significant result, since the probability to obtain such a result by chance is $P \approx 2^{-14} \approx 6\ 10^{-5}$, and $P \approx 2^{-9} \approx 2\ 10^{-3}$ for the subsample of new exoplanets.

(ii) Planetary system around PSR B1257+12. It has been recently shown[13,22] that the system of 3 planets discoverd by Wolszczan[19] around the pulsar PSR B1257+12 does follow the 'mean' law, $T_n = 2\pi GM\, w^{-3} n^3\, (1 + 2/n)^{3/2,}$ with such a precision that second and third order terms can be successfully checked (see Fig. 2).

The planets rank n = 5, 7 and 8. Taking $M_{PSR} = 1.4 \pm 0.1\ M_\odot$, we find $w = 426 \pm 10$ km/s = 3 x (142 ± 3.3) km/s. Conversely, using our determination $w_0 = 144.7 \pm 0.6$ km/s (see below), we derive a pulsar mass of $M_{PSR} = 1.48 \pm 0.02\ M_\odot$. We can predict[13,22] with high precision the periods of possible additional planets, e.g., T_1 = 0.322 days, T_2 = 1.958 days, T_3 = 5.960 days, T_4 = 13.38 days, T_6 = 42.66 days and T_9 = 138.5 days.

(iii) Extra-Solar planets around solar-type stars. Mercury does not rank n = 1 in the inner solar system, but n = 3. This means that, in addition to the preferential values 0.39, 0.69, 1.05, 1.5 AU of the semi-major axes (that agree with Mercury, Venus, Earth and Mars, see Figs. 1,3), we also expect two orbitals closer than Mercury to a solar-type star, at ≈ 0.18 AU (n = 2) and ≈ 0.05 AU (n = 1).[1,14] It is then quite remarkable that the first extra-solar planet, discovered by Mayor & Queloz,[23] around a solar-type star (51 Peg), lies precisely at 0.05 AU from its star, i.e., at the 'fundamental' level of our quantization law.

As can be seen in Fig. 3, the other recently discovered planets (see refs. in Nottale[13]) also agree with our prediction. The 8 new planets yield $w_0 = 143.9 \pm 3.1$ km/s, in agreement with other independent determinations of the Tifft constant. When combined with the inner Solar System planets, we find $w_0 = 144.1 \pm 1.8$ km/s. Moreover, one can demonstrate in a highly significant way that this law is a genuine quantization law, not only an average one: the probability to obtain such a distribution by chance is $P \approx 2\ 10^{-5}$.

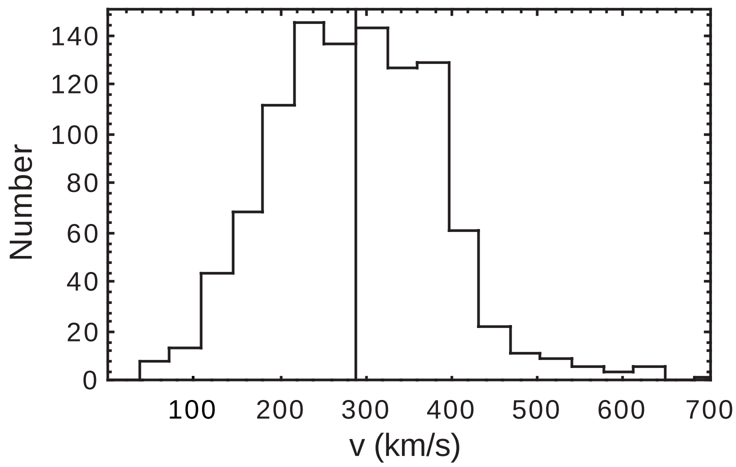

Figure 4. Distribution of the mean orbital velocities of binary stars in the Brancewicz and Dvorak catalog[24] of 1048 eclipsing binaries. The global average is 289.4 ± 3.0 km/s (vertical line), i.e. two times the Tifft constant w_0 = 144.7 km/s.[25]

(iv) Binary stars. A crucial test of the theory is to verify that it applies to pure two-body systems. We have shown that the theory can be successfully checked with isolated binary galaxies and single planets such as (presumably) the 51 Peg system. Similar results are obtained for double stars. Hence, the average velocity of eclipsing binaries in the Brancewicz and Dvorak catalog[24] of eclipsing binaries (N = 1048 objects) is $w = 289.4 \pm 3.0$ = 2 x (144.7 ± 1.5) km/s (see Fig. 4), and the distribution of velocities of binary systems including a pulsar show a periodicity of (145.8 ± 3.0) ÷ 3 km/s.[25]

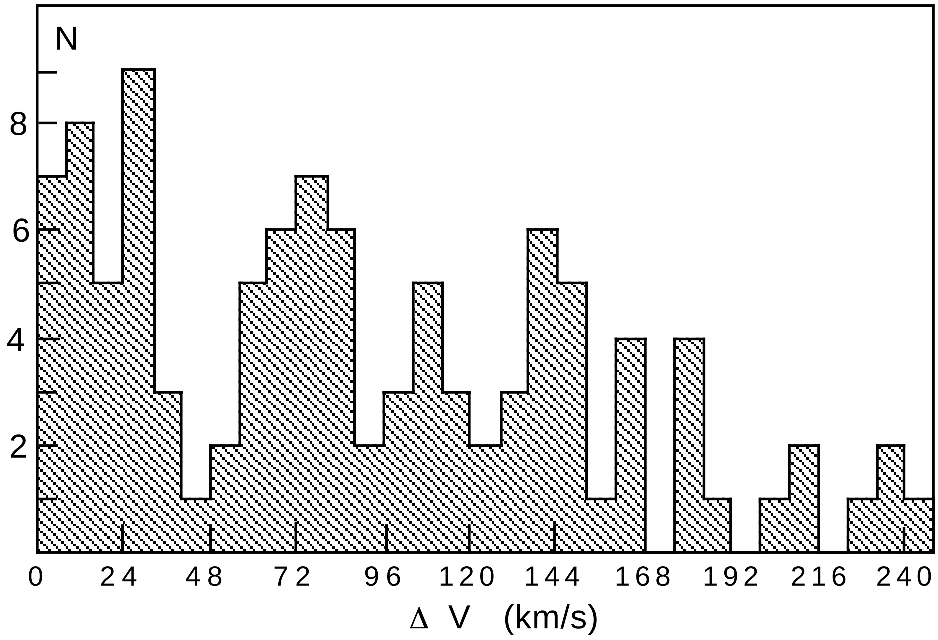

Figure 5. Histogram of velocity differences for the Schneider-Salpeter[26] sample of 107 isolated binary galaxies with high precision 21 cm redshifts, from their Table 1. The higher probability of the values 72 km/s and 144 km/s (and possibly 24 km/s) is clearly apparent and can be shown to be statistically significant.[27]

(v) Binary galaxies. Tifft[18] has discovered that the velocity differences in galaxy pairs are quantized in terms of submultiples of ≈144 km/s (see Fig. 5). These are exactly the values theoretically predicted and seen in our own solar system and in extrasolar planetary system around solar-type stars. One can also demonstrate that the binary galaxy velocity peaks are in correspondence through Kepler's third law with equivalent peaks observed in their relative position distribution:[27] this is a strong indication that they are probability density peaks of real velocities (i.e. that these redshift differences come from the Doppler effect, rather than being "anomalous redshifts" of unknown origin).

(vi) Local Supercluster of galaxies. In the present contribution, we have mainly developed the theory and its comparison with observational data for the case of a Kepler two-body potential. Recall however that, by inserting in Eq. (12) the potential of a uniform density, it is found that matter in the Universe must structure locally according to the various modes of the isotropic 3-dimensional harmonic oscillator, i.e., to the symmetries described by the SU(3) group.[8] This allows one to explain the Tifft[20] effect of global quantization of galaxy redshifts in units of ≈36 km/s, recently confirmed by Guthrie & Napier.[21] A re-analysis of their sample in the framework of our theory yields w_0 = 144.7 ± 2.2 km/s (see Fig. 6).

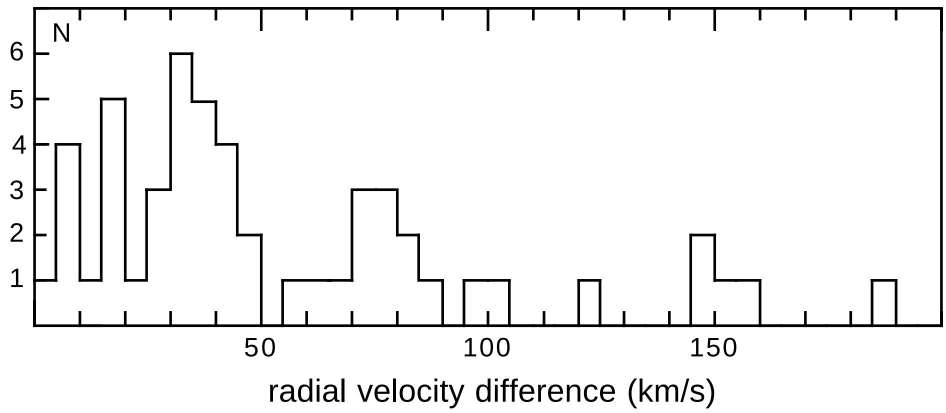

Figure 6. Histogram of velocity differences between pairs in the subsample of galaxies belonging to groups, from the Guthrie & Napier[21] sample (galaxies with high precision 21 cm redshifts in the local supercluster). Peaks of probability density are apparent for the values ≈144 km/s, 72 km/s and mainly for 36 km/s, in agreement with the highly significant peak obtained at ≈37 km/s by Guthrie and Napier in their power spectrum analysis.

In the present limited contribution, we have mainly considered the problem of velocity structures, but the theory also predicts structures in angular momentum, obliquities and inclinations,[28] and other orbital elements,[17] which have been successfully checked. Moreover, it provides us with a generalized framework (which includes and extend the standard one) to study the formation and evolution of gravitational structures.[8,10]

5 Conclusion

One of the most important consequences of the quantum-like nature of the world at long time and/or large length-scales suggested in scale-relativity is the profound unity of structures that it implies between very different scales. This unity is a consequence:
(i) of the universality of the structuring "force", (i.e. of the fractal geometry of space-time),
(ii) of that of physics, as manifested in the Lagrangian / Hamiltonian formalism, i.e., in the underlying symplectic structure of physics,
(iii) and of the recovered prevalence of the space-time description.

This was already apparent in standard quantum mechanics, in the fact that several features of the solutions to a given Schrödinger equation can be detailed before the precise form of the potential is specified. The structures come in large part from the quantum terms themselves (which we have interpreted as a manifestation of the fractal space-time geometry) and from the matching and limiting conditions for the wave function (then for the probability density distribution).

Now, as can be seen on the various generalized Schrödinger equations one can write for different fields,[10] these quantum terms are in common to all of them. Moreover, these equations describe different systems that must be matched together at different scales in the real world: the matching conditions will then imply a unity and a continuity of the structures observed at these different scales. This prediction has already been fairly well verified for various gravitational potentials from the scale of star radii ($<10^6$ km) to extragalactic scales (≈100 Mpc). We shall in future work investigate whether it also applies to systems whose structure do not depend on the gravitational field only, but also on magnetic fields, pressure and dissipative terms (e.g., stellar interiors, stellar atmospheres...).

Acknowledgments

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