Scale-Relativistic Estimate of the Fine Structure Constant

Laurent Nottale

CNRS, DAEC, Observatoire de Paris-Meudon,
92195 Meudon Cedex, France

Abstract. The low energy value of the fine structure constant is theoretically estimated to within 1‰ of its observed value in the framework of the theory of scale relativity, in which the Planck length-scale is reinterpreted as a lowest, limiting, unpassable length, invariant under dilatations. This estimate is performed by using the renormalization group equations to evolve the QED coupling constant from infinite energy, where the bare mean electroweak coupling is assumed to reach its "natural" value $1/4\pi^2$, down to the electron energy. A consequence of this calculation is that the number of Higgs boson doublets is predicted to be $N_H = 1$.

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1. Introduction

Present high energy physics seems to be faced with several unsolved fundamental problems, among which: (i) the problem of divergences of self-energies and charges in QED; (ii) the nature of the Planck scale; (iii) the origin and values of universal scales, such as the Electroweak and GUT scales; (iv) the values of coupling constants. These problems are clearly related with one another.

Indeed, consider in particular the question of the theoretical understanding of the coupling constant values which we address in the present letter. Once it was realized that vacuum polarization by virtual particle-antiparticle pairs resulted in a screening of bare charges, it became clear that the most straighforward programme for theoretically predicting the low energy value of the electromagnetic coupling (i.e., the fine structure constant) was to derive it from its "bare" (infinite energy) value and from its variation with scale. Even though considerable progress has been made thanks to the development of quantum gauge theories and of the renormalization group approach, the hope to implement this programme in the framework of the standard model was finally deceived. The reason for this failure comes precisely from the asymptotic behaviour of present theories: the inverse coupling constants are found to vary logarithmically with scale, so that the charges are found either to become infinite at infinite energy in the abelian group case (more precisely, the divergence occurs at some ultra-high energy given by the "Landau ghost"), or to vanish (asymptotic freedom of non abelian groups). Both situations prevent from defining a bare charge: for example, in the simplest GUT theory based on the SU(5) group, the three fundamental couplings derive from a unique high energy coupling, whose quantization can be theoretically demonstrated, while its bare value is predicted to be zero. One generally attempts to escape from this contradiction by noting that new unknown physics is needed beyond the Planck scale because of the intervening of gravitation...

We have recently proposed a new frame of thought for understanding the asymptotic behavior of the quantum theory at high energy [1], which allows us to reconsider the coupling constant problem. It can be founded on three postulates:

(i) the equations of physics must be written in a scale-covariant way;

(ii) the explicit dependence on scale of the laws of physics at high energy (such as observed in the variation of charges and self-energies in terms of scale and as described in the renormalization group approach [2]) is broken at low energy, i.e., equivalently, at large length-time scale (as shown by the independence of classical laws on resolution); the transition from scale-dependence to scale independence occurs about the Compton length of the electron.

(iii) the Planck scale is invariant under dilatations and contractions.

While postulates (i) and (ii) give back today's physics, our third postulate (iii) is a new interpretation for the Planck scale, which implies profound changes in the asymptotic behavior of quantum field theories. We have called *scale relativity* the resulting theory [1]. Our aim in the present letter is, after having recalled the main results which have already been obtained, to reconsider in its framework the question of the theoretical estimate of the fine structure constant α. This is motivated by the fact that, in scale relativity, the coupling constants are now converging toward *finite* values at infinite energy, so that one may explicitly wonder about the expected value of the quantized bare charge (we shall argue that one can define an “electroweak charge” $(8\alpha/3)^{1/2}$ whose bare value is $1/2\pi$), then compute the corresponding low energy value and finally compare this prediction to the experimental value.

2. Scale relativity

Let us first consider the current form of renormalization group equations in today's standard theory. For *relevant* fields, the lowest order equation in its simplest form reads, in terms of a *scale dimension* δ

$$\frac{d\varphi}{d\ ln\frac{\lambda_o}{r}} = \delta\ \varphi\ , \tag{1}$$

which is integrated in the scale-invariant power law $\varphi = \varphi_o(\lambda_o/r)^\delta$, i.e., in logarithm form

$$ln\,(\varphi/\varphi_o) = \delta\, ln\frac{\lambda_o}{r}\ . \tag{2}$$

The scale dimension δ is usually assumed to be a *constant*. For marginal fields the lowest order equation reads

$$\frac{d\alpha}{d\ ln\frac{\lambda_o}{r}} = \beta_o\ \alpha^2\ , \tag{3}$$

which is integrated into the well-known logarithmic scale dependence of couplings and self-energies,

$$\bar{\alpha}\,(r) = \bar{\alpha}\,(\lambda_o) - \beta_o\ ln\frac{\lambda_o}{r}\ . \tag{4}$$

Here and in what follows we use the notation $\bar{\alpha} = \alpha^{-1}$. In the above laws, as in the whole of standard physics, the length scale r is allowed to go to zero, so that (4) is asymptotically either divergent or vanishing.

Let us now try to analyse the meaning of the above laws in the light of the scale-relativistic method [1,3]. Our approach consists in (i) redefining the resolution $ln\,(\lambda_o/r)$ as a *state of scale* of the reference system (note its *relative* character); (ii) requiring that the equations of physics be written in a *covariant* way under *scale transformations* of resolutions. Then the general problem of finding the laws of linear transformation of fields in a scale transformation $r \rightarrow r'$ amounts to finding four quantities, $A(\mathbb{V})$, $B(\mathbb{V})$, $C(\mathbb{V})$, and $D(\mathbb{V})$, where $\mathbb{V} = ln\,(r/r')$, such that

$$ln\,\frac{\varphi(r')}{\varphi_o} = A(\mathbb{V})\;\; ln\,\frac{\varphi(r)}{\varphi_o} + B(\mathbb{V})\;\;\delta(r)\quad , \tag{5a}$$

$$\delta(r') = C(\mathbb{V})\;\; ln\,\frac{\varphi(r)}{\varphi_o} + D(\mathbb{V})\;\;\delta(r)\quad . \tag{5b}$$

Set in this way, it immediately appears that the current “scale-invariant” structure of the standard renormalization group (Eq. 2), i.e. $A = 1$, $B = \mathbb{V}$, $C = 0$ and $D = 1$, corresponds to the Galileo group. This is also clear from the law of composition of dilatations, $r \rightarrow r' \rightarrow r''$, which has a simple additive form,

$$\mathbb{V}'' = \mathbb{V} + \mathbb{V}' . \tag{5c}$$

However the general solution to the “special relativity problem” (namely, find A, B, C and D from the principle of relativity) is the Lorentz group [1,4], of which the Galileo group is only a very particular solution. Our proposal [1] is then to implement the above axiom (iii) by replacing the standard law of dilatation, $r \rightarrow r' = \rho r$ by a new relation having Lorentzian form. But this is not the last word to this problem: while the relativistic symmetry is universal in the case of the laws of motion, this is not true for the laws of scale. As experimentally observed and as described by axiom (ii), self-energies and charges are no longer dependent on resolution for scales larger than the electron Compton scale. This implies that the dilatation law must remain Galilean above the Compton scale of the electron. We have shown in Ref. [1] that such a combination of a high energy Lorentzian symmetry and of its breaking to Galilean symmetry at low energy was indeed possible to implement, and that it implied the emergence of a universal, invariant, length-time-*scale*, rather than that of an invariant dilatation as expected from unbroken Lorentzian laws.

For simplicity, we shall consider in what follows only the one-dimensional case. We define the resolution as $r = c\;\delta t$, and define a characteristic Compton scale $\lambda_o = \hbar/m_o c$

(in the calculation of Sec. 4, λ_o will be identified successively with the electron and Z boson scales). The new law of dilatation which implements axioms (i) to (iii) reads, for $r < \lambda_o$ and $r' < \lambda_o$

$$ln \frac{r'}{\lambda_o} = \frac{ln(r/\lambda_o) + ln\rho}{1 + \frac{ln\rho \;\; ln(r/\lambda_o)}{ln^2(\lambda_o/\mathbb{\Lambda})}} \quad , \tag{6}$$

where $\mathbb{\Lambda}$ is the Planck length (currently 1.61605(10) x 10^{-35} m),

$$\mathbb{\Lambda} = (\hbar G/c^3)^{1/2} \quad , \tag{7}$$

and is now interpreted as a limiting lowest length-scale, impassable, invariant under dilatations and contractions. Equation (2) (for $r \leq \lambda_o$) is replaced by

$$ln\,(\varphi/\varphi_o) = \frac{\delta_o\, ln \frac{\lambda_o}{r}}{\sqrt{1 - ln^2(\lambda_o/r) \,/\, ln^2(\lambda_o/\mathbb{\Lambda})}} \quad . \tag{8}$$

The scale dimension is now varying with scale (for $r \leq \lambda_o$) as

$$\delta(r) = \frac{\delta_o}{\sqrt{1 - ln^2(\lambda_o/r) \,/\, ln^2(\lambda_o/\mathbb{\Lambda})}} \quad , \tag{9}$$

so that Eq.(8) may be given the explicitly scale-covariant form $\varphi = \varphi_o\,(\lambda_o/r)^{\delta(r)}$. As a consequence the mass-energy scale and length scale are no longer inverse, but related by the scale-relativistic generalized Compton formula

$$ln\,\frac{m}{m_o} = \frac{ln\;(\lambda_o/\lambda)}{\sqrt{1 - \frac{ln^2(\lambda_o/\lambda)}{ln^2(\lambda_o/\mathbb{\Lambda})}}} \quad , \tag{10}$$

i.e., $m/m_o=(\lambda_o/\lambda)^{\delta(\lambda)}$, with $\delta(\lambda_o) = 1$. A similar generalization holds for the Heisenberg relations [1]. Concerning coupling constants and self-energies, the fact that the lowest order terms of their β-functions are quadratic implies that their variation with scale is unaffected by scale-relativistic corrections [1], *provided it is written in terms of length scale*. The passage to mass-energy scale is now performed by using Eq.(10).

A detailed justification of the above formulas can be found in Refs.[1]. Let us simply note here that: (i) these new laws are valid only below the electron scale, i.e. when the reference Compton length λ_o is equal to or smaller than the Compton length of the electron $\lambda_e = \hbar/m_e c$ (ii) in scale relativity, one should be cautious that all scales are *relative*, i.e. only scale *ratios* keep a physical meaning; (iii) while Eq. (10) has observational meaning, as we shall see hereafter, Eq. (6) should be understood as having only a *virtual* meaning. Indeed, assume that a system is prepared in such a way that its de Broglie length is λ_o. Then if a measurement is performed on this system at resolution $r << \lambda_o$, the Heisenberg relation implies that it will get a new impulsion $p = p_o + \Delta p \approx \Delta p \approx \hbar/r$, so that its de Broglie length itself is changed and becomes of the order of r.

3. Emergence of the GUT Scale

Before coming to the theoretical estimate of the electromagnetic coupling, let us recall the results which have already been obtained in this new framework [1].

(i) The problem of the divergence of charge and self-energy is solved: they now have finite non-zero values at infinite energy.

(ii) A new fundamental scale emerges, which is given by the length scale corresponding to the Planck energy. Let us set $\mathbb{C}_Z = ln(\lambda_Z/\mathbb{\Lambda}) \approx ln(m_{\mathbb{P}}/m_Z)$: this new scale is given to lowest order by $ln(\lambda_Z/\lambda) = \mathbb{C}_Z/\sqrt{2}$ (see Eq. 10), and is thus $\approx 10^{-12}$ times smaller than the *W/Z* length scale. In other words, this is but the GUT scale (10^{14} GeV in the standard theory).

(iii) As a consequence, the *four* fundamental couplings, U(1), SU(2), SU(3) *and gravitational* converge in the new framework toward about the same scale, which now corresponds to the Planck energy.

(iv) The GUT energy now being of the order of the Planck one ($\approx 10^{19}$GeV), the predicted lifetime of the proton ($\propto m_{\mathrm{GUT}}^4/m_{\mathrm{p}}^5 >> 10^{38}$ yrs) becomes compatible with experimental results [1] ($> 5.5 \times 10^{32}$ yrs, [16]).

4. Theoretical Estimate of the Low Energy QED Coupling

We shall now attempt to perform a completely scale-relativistic calculation relating the low energy value of the fine structure constant ($\bar{\alpha} =$ 137.0359914(11), [15]) to its (formal) infinite energy value. For this purpose we define from the U(1) and SU(2) couplings an averaged electroweak inverse coupling $\bar{\alpha}_0 = \frac{3}{8}\bar{\alpha}_2 + \frac{5}{8}\bar{\alpha}_1$, which is simply related to the (running) fine structure constant α by $\bar{\alpha} = \frac{8}{3}\bar{\alpha}_0$. Let us first demonstrate that in the new framework the infinite energy coupling is finite.

Between the Planck and Z boson length-scale, the variation of $\bar{\alpha}$ is given to leading order by the solution to its renormalization group equation [5,7] ,

$$\bar{\alpha}(r) = \bar{\alpha}(\lambda_Z) - \frac{10+N_H}{6\pi} \, ln\frac{\lambda_Z}{r} \quad , \tag{11}$$

where N_H is the number of Higgs doublets. So, while in the standard theory the QED coupling is divergent at the "Landau ghost" $ln\frac{\lambda_Z}{r} \approx 3\pi\ \bar{\alpha}_Z / 5$, in scale relativity the length-scale r is limited by the lowest scale $\mathbb{\Lambda}$. The new convergence of charge at infinite energy is better understood by writing Eq.(11) in terms of a running mass-energy scale m:

$$\bar{\alpha}\,(m) \;=\; \bar{\alpha}(m_Z) \;-\; \frac{10+N_H}{6\pi}\;\frac{ln(m/m_Z)}{\sqrt{1 + ln^2(m/m_Z)/\mathbb{C}_Z{}^2}} \quad . \tag{12}$$

Making m tend to infinity yields the finite result:

$$\bar{\alpha}\,(\infty) \;=\; \bar{\alpha}(m_Z) \;-\; \frac{10+N_H}{6\pi}\;\mathbb{C}_Z \quad . \tag{13}$$

This finiteness of the bare charge sets in completely renewed terms the problem of the theoretical prediction of the low energy charge. Knowing that the bare charge is neither null nor infinite, we can now wonder about its expected value. We shall only give here a simple dimensional argument. Consider a Coulomb-like force between two "charges". It is given by $F = e^2 / 4\pi\varepsilon_0 r^2 = \tilde{\alpha}\,\hbar c / r^2$. Hence the dimensional equation of $\tilde{\alpha}\,\hbar c = F\, r^2$ is $[\mathrm{ML}^3\mathrm{T}^{-2}]$. The natural possible values for the length scale in this dimensional equation are the reduced Compton length $\mathrm{L} = \hbar/\mathrm{M}c$ or Compton wavelength $h/\mathrm{M}c$, and similarly for the time scale $\mathrm{T} = \hbar/\mathrm{M}c^2$ or $h/\mathrm{M}c^2$. Combining these possibilities yields $\tilde{\alpha} = 1/4\pi^2$, 1, 2π or $8\pi^3$. The only value *smaller than* 1 (i.e., compatible with the smallness of the observed electric and weak charges) is $\tilde{\alpha} = 1/4\pi^2$. Knowing that the low energy electric charge results from a mixing of the high energy U(1) and SU(2) charges in the electroweak theory, we suggest to identify this coupling $\tilde{\alpha}$ with the electroweak coupling α_0 defined above, $\bar{\alpha}_0 = \frac{3}{8}\bar{\alpha}_2 + \frac{5}{8}\bar{\alpha}_1$, which is such that $\alpha_0 = \alpha_1 = \alpha_2$ at unification scale. This corresponds to a bare inverse fine structure constant of $\bar{\alpha}(\infty) = 32\pi^2/3$.

In order to deduce the low energy fine structure constant from this bare value, we shall now run the inverse coupling, thanks to its renormalization group equation [2,5,6], from the Planck length scale (i.e. infinite mass-energy scale) to the Z boson scale, then to the electron scale.

If we disregard for the moment the purely scale-relativistic correction which arises from the fact that the Z mass-scale and Z length-scale are no longer directly inverse (see Eq.10), we have $\mathbb{C}_Z \approx ln(m_{\mathbb{P}}/m_Z) = 39.436$ (where $m_{\mathbb{P}}$ is the Planck mass), and we find numerically for the first order variation of the inverse QED coupling (Eq. 12):

$$\Delta\bar{\alpha}^{(1)}_{\Lambda Z} \;=\; 23.01 \;+\; 2.1\,(N_H - 1) \quad . \tag{14}$$

To next-to-leading order, its variation between infinite energy and the Z mass is given by [6,7,1]

$$\Delta\bar{\alpha}^{(2)}_{\mathbb{A}Z} \;=\; -\frac{104+9N_H}{6\pi(40+N_H)}\, ln\{1-\frac{40+N_H}{20\pi}\,\alpha_1(\lambda_Z) ln\frac{\lambda_Z}{\mathbb{A}}\}$$

$$+\frac{20+11N_H}{2\pi(20-N_H)}\, ln\{1+\frac{20-N_H}{12\pi}\,\alpha_2(\lambda_Z)\, ln\frac{\lambda_Z}{\mathbb{A}}\} \;+\frac{20}{21\pi}\, ln\{1+\frac{7}{2\pi}\,\alpha_3(\lambda_Z) ln\frac{\lambda_Z}{\mathbb{A}}\} \qquad (15)$$

This expression depends on the values of the U(1), SU(2) and SU(3) couplings at the Z scale. One could use their known values, but they are themselves derived in part from the observed value of the low energy fine structure constant. Fortunately, our prediction of the GUT scale provides us with a possibility to make a self-consistent calculation (see Fig.1). Since this is a second order correction, we do not need to have very precise estimates of the couplings as input in Eq. (15). (More precise theoretical estimates of the 3 fundamental couplings will be given in a forthcoming work). So we first compute the GUT scale:

$$ln(\lambda_Z/\lambda_{\mathrm{GUT}}) \approx \mathbb{C}_Z/\sqrt{2} \;\;\Rightarrow\;\; \mathbb{C}_{\mathrm{GUT}} = ln(\lambda_{\mathrm{GUT}}/\mathbb{A}) \approx 11.5 \;\;, \qquad (16)$$

then run the inverse coupling $\bar{\alpha}_0$ from infinite mass scale to GUT scale and assume an exact convergence of the three couplings at the GUT scale:

$$\bar{\alpha}_1(\lambda_{\mathrm{GUT}}) \approx \bar{\alpha}_2(\lambda_{\mathrm{GUT}}) \approx \bar{\alpha}_3(\lambda_{\mathrm{GUT}}) \approx \bar{\alpha}_0(\lambda_{\mathrm{GUT}}) \approx 4\pi^2 + \frac{11}{16\pi}\,\mathbb{C}_{\mathrm{GUT}} \approx 42.0, \qquad (17)$$

and finally use the lowest order solutions to the renormalization group equations [5] to obtain the couplings at Z scale:

$$\bar{\alpha}_1\,(\lambda_Z) \;=\; \bar{\alpha}_1\,(\lambda_{\mathrm{GUT}}) \;+\; \frac{40+N_H}{20\pi}\;ln(\lambda_Z/\lambda_{\mathrm{GUT}}) \;\;, \qquad (18a)$$

$$\bar{\alpha}_2\,(\lambda_Z) \;=\; \bar{\alpha}_2\,(\lambda_{\mathrm{GUT}}) \;-\; \frac{20-N_H}{12\pi}\;ln(\lambda_Z/\lambda_{\mathrm{GUT}}) \;\;, \qquad (18b)$$

$$\bar{\alpha}_3\,(\lambda_Z) \;=\; \bar{\alpha}_3\,(\lambda_{\mathrm{GUT}}) \;-\; \frac{7}{2\pi}\;ln(\lambda_Z/\lambda_{\mathrm{GUT}}) \;\;. \qquad (18c)$$

The precise value of N_H is unimportant at the precision searched. We find $\bar{\alpha}_1(\lambda_Z) \approx 60$, $\bar{\alpha}_2(\lambda_Z) \approx 28$ and $\bar{\alpha}_3(\lambda_Z) \approx 10.6$ (the current values are 59.22(14), 30.10(23) [7] and 8.93(23) [8]). Inputing these estimates in Eq.(15), we find the second order correction to be:

$$\Delta\bar{\alpha}^{(2)}_{\mathbb{A}Z} \;=\; 0.73 \pm 0.03 \;\;, \qquad (19)$$

(inputing the experimental values would have given $\Delta\bar{\alpha}^{(2)}_{\mathbb{A}Z} = 0.76$).

We must now run the QED inverse coupling from the Z scale to the electron scale. This problem has been considered by many authors [5,7,10]. The main difficulty is to estimate the QCD contribution, but recent progress have been made on this problem. The running QED coupling variation is directly given by the variation in the QED vacuum polarization, which can itself be deduced from the well-known ratio *R* of the cross sections $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ [9,10]. A detailed computation based on the available experiment results by Burkhardt *et al* [10] have yielded:

$$\Delta\bar{\alpha}_{Ze}^{\mathrm{h}} = 3.94 \pm 0.12 \quad . \tag{20}$$

Holdom and Lewis [9] have recently shown that a constituent quark model including the momentum dependence of the dynamical quark mass is able to reproduce this result theoretically, but with a much larger uncertainty (they find $\Delta\bar{\alpha}_{Ze}^{\mathrm{h}} = 3.85 \pm 0.3$ if $\Lambda_s^{(4)} =$ 100 MeV and 4.1 ± 0.6 if $\Lambda_s^{(4)} = 200$ MeV).

The leptonic contribution is given by (see e.g. ref. [11], p. 635):

$$\Delta\bar{\alpha}_{Ze}^{\mathrm{L}} = \frac{2}{3\pi}\left\{ln\left(\frac{m_Z}{m_e}\right) + ln\left(\frac{m_Z}{m_\mu}\right) + ln\left(\frac{m_Z}{m_\tau}\right) - \frac{5}{2}\right\} = 4.30 \pm 0.05 \; . \tag{21}$$

The full variation between electron and *Z* boson is thus 8.24±0.13. From the observed value $\bar{\alpha}_e = 137.04$, this yields $\bar{\alpha}_Z = 128.80$, in agreement with Amaldi et al. [7] or Holdom and Lewis [9].

Let us finally consider the scale-relativistic corrections to $\Delta\bar{\alpha}$. Between *Z* and Planck scale, one should account for the fact that the *Z* length-scale is now slightly different from its mass scale. Inverting Eq.(10) yields

$$ln\,\frac{\lambda_e}{\lambda_Z} = \frac{ln(m_Z/m_e)}{\sqrt{\left(1 + \frac{ln^2(m_Z/m_e)}{\mathbb{C}_e^2}\right)}} \tag{22}$$

where $\mathbb{C}_e = ln(m_{\mathbb{P}}/m_e) = 51.52797(7)$ [12]. From $m_Z = 91.18(2)$ [13], we find $ln(\lambda_e/\lambda_Z) = 11.772$, so that $\mathbb{C}_Z = 39.756$. With this more precise value of $\mathbb{C}_Z$, $\Delta\bar{\alpha}_{\Lambda Z}$ is increased by +0.19. But the same effect holds between electron and *WZ* scales. Let us compare the mass-scales and Compton length-scales of elementary particles in scale relativity, when taking the electron scale as reference scale:

$ln(\lambda_e/\lambda_\mu) = 5.30$; $ln(m_\mu/m_e) = 5.33$,
$ln(\lambda_e/\lambda_{uds}) = 6.37$; $ln(m_{uds}/m_e) = 6.42$,
$ln(\lambda_e/\lambda_c) = 7.73$; $ln(m_c/m_e) = 7.82$,
$ln(\lambda_e/\lambda_\tau) = 8.06$; $ln(m_\tau/m_e) = 8.16$,
$ln(\lambda_e/\lambda_b) = 8.89$; $ln(m_b/m_e) = 9.03$.
$ln(\lambda_e/\lambda_t) = 12.05$; $ln(m_t/m_e) = 12.39$.

(for *u*, *d*, *s* quarks the quoted mass is an effective mass ≈ $m_{proton}/3$; for the *t*-quark, we have taken a value recently deduced from a fit of precise electroweak data, $m_t = 122\pm22$ GeV [18]). Including these corrections decreases $\Delta\bar{\alpha}_{Ze}$ by 0.37. The net scale-relativistic correction is then finally:

$$\Delta\bar{\alpha}^{\text{Sc-rel}} = -0.18 \quad . \tag{23}$$

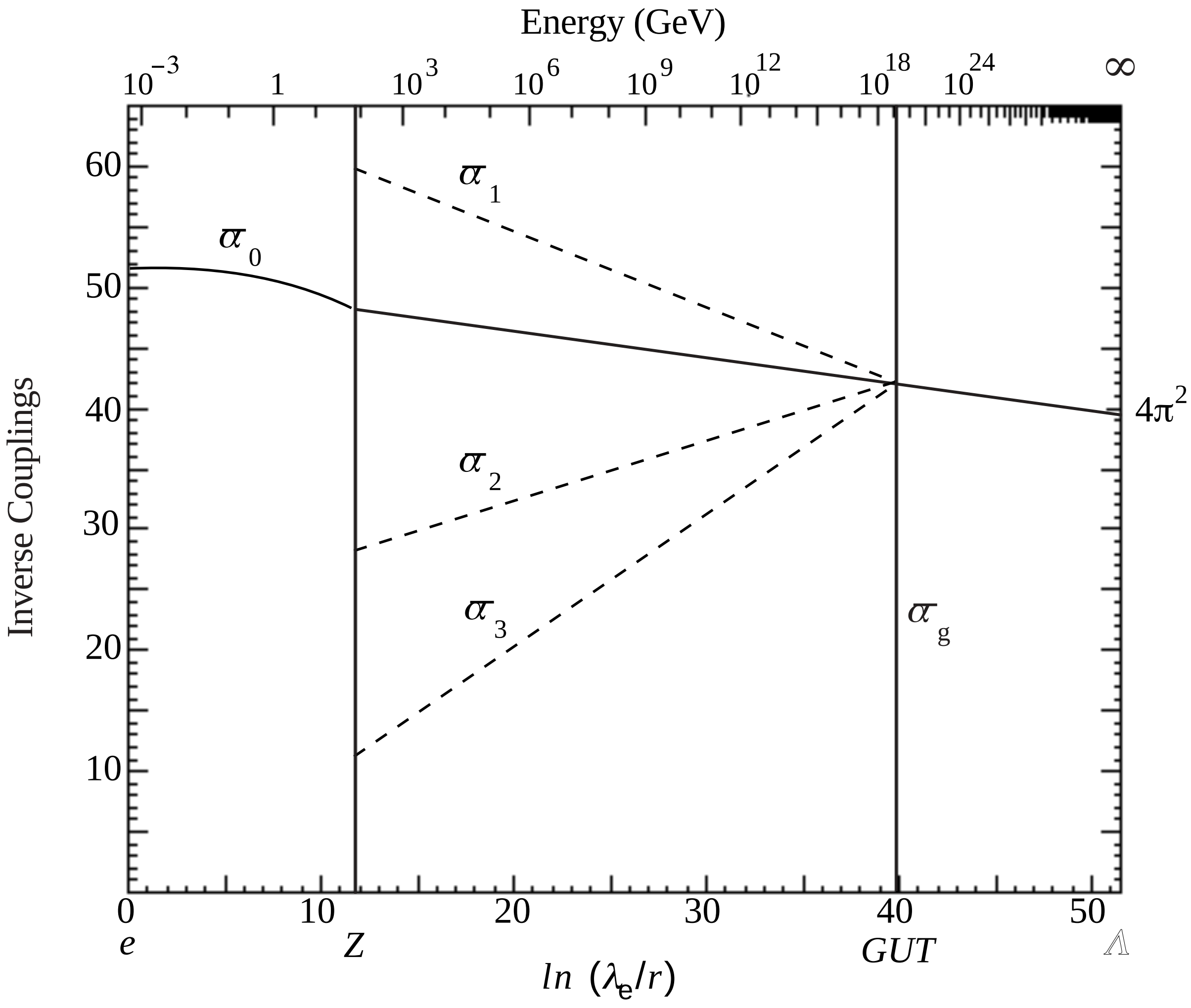

Fig. 1. Evolution of the inverse coupling constant $\bar{\alpha}_0 = \frac{3}{8}\bar{\alpha}$ between the Planck length-scale (which corresponds to infinite energy in scale relativity, see upper graduation) and the electron scale, beyond which it keeps its constant low energy value [experimentally ≈ 137.036 x (3/8)]. The conjecture that the bare coupling at infinite energy is $4\pi^2$ (i.e. that the corresponding "charge" is $1/2\pi$) allows us to estimate its low energy value to better than 1‰ by using the renormalization group equations for the running coupling. An approximate estimate of the U(1), SU(2) and SU(3) inverse couplings (broken lines) is used for computing the next-to-leading order effects (see text).

We have also considered other possible contributions, which happen to be negligible at the precision considered. This includes the second order terms between electron and *Z* scales $\{\Delta\bar{\alpha}^{(2)} = (3\alpha/4\pi)\Delta\bar{\alpha}^{(1)} \approx 0.015\}$, the third order terms between

Planck scale and Z scale ($\Delta\bar{\alpha}^{(3)}_{\Lambda Z} \approx \pm 0.02$), the effects of Yukawa couplings and Higgs mass ($\lesssim 0.01$).

Combining all the relevant contributions, we obtain a theoretical estimate of the fine structure constant:

$$\bar{\alpha}_e = \bar{\alpha}(\infty) + \Delta\bar{\alpha}^{(1)}_{\Lambda Z} + \Delta\bar{\alpha}^{(2)}_{\Lambda Z} + \Delta\bar{\alpha}^{\mathrm{h}}_{Ze} + \Delta\bar{\alpha}^{\mathrm{L}}_{Ze} + \Delta\bar{\alpha}^{\text{Sc-rel}}$$

$$= 137.08 + 2.11\,(N_H - 1) \pm 0.13 \; . \qquad (24)$$

This leads for 1 Higgs doublet to $\bar{\alpha}_e$ = 137.08 ± 0.13, where we have added in quadrature the estimated errors. Though such an estimate cannot compete with the precision of the experimental one (currently 137.0359914(11) [15] at zero energy, which implies $\bar{\alpha}_e$ = 137.028 at electron energy), they nevertheless agree to better than 1‰, which is a very encouraging result.

We can also use our calculation the reverse way: start from the observed low energy fine structure constant and compute the bare inverse coupling $\bar{\alpha}_0$. We find $\bar{\alpha}_0(\infty)$ = 39.465 ± 0.049, which agrees within error bars with our expectation $4\pi^2$ = 39.478.

Finally our results can be utilized to make predictions on other quantities by setting the low and high energy values of the coupling. As a consequence of the fact that each Higgs doublet contributes by +2.11 in the final result, we can predict with a high level of confidence that the number of Higgs doublets is 1:

$$N_H = 1.02 \pm 0.06 \; . \qquad (25)$$

Another spinoff of our calculation is a prediction of the hadron contribution to the inverse coupling variation $\Delta\bar{\alpha}^{\mathrm{h}}_{Ze}$ = 3.86 ± 0.05, improved with respect to the present experimental value [Eq. 20].

5. Summary and Conclusion

We have performed in the present letter a calculation of the variation of the inverse QED coupling (“fine structure constant”) from infinite energy to electron energy, by using its renormalization group equations in the framework of a scale-relativistic extension of the standard model assuming three families of elementary particles and the ‘great desert’ hypothesis. We find $\Delta\bar{\alpha}_{\infty e} = 31.80 + 2.11(N_H - 1) \pm 0.13$. This calculation allows us to get an estimate to better than 1‰ of the low energy fine structure constant, based on the conjecture that the bare dimensionless “charge” $\alpha_0^{1/2}$ is $1/2\pi$. Such a result is made possible by the new structure of space-time postulated in scale relativity: in this theory, the Planck length plays the new role of a lowest, unpassable scale, invariant under dilatations, which may be reached only at infinite energy and replaces the zero point of standard physics. A direct consequence of this calculation is that the number of Higgs doublets can be only one, in agreement with the minimal version of the electroweak theory with an explicit Higgs mechanism.

One can finally note that, though it is only in the framework of scale relativity that our result is fully understandable (since the Planck length-scale corresponds there to infinite energy-scale), a similar though less significant result may be obtained in the framework of the standard model. Assuming that the inverse coupling $\bar{\alpha}_0$ is $4\pi^2$ at the Planck scale, the same calculation would yield $\bar{\alpha}_e = 137.26 \pm 0.13$, which is marginally consistent (1.8σ) with the observed value. Note also that, even though we have based our calculation on the (conjectured) value of $\bar{\alpha}_0$ at the Planck scale, one expects this value not to be actually achieved in nature, since physics drastically changes at the GUT scale [17]. So this limit must be understood as purely virtual, and defined in the framework of a formal high energy electroweak theory.

We shall in forthcoming works present other theoretical estimates of fundamental parameters of the standard model, depending more tightly on the scale-relativistic structure, and shall also attempt to justify in more detail the $1/2\pi$ conjecture.

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