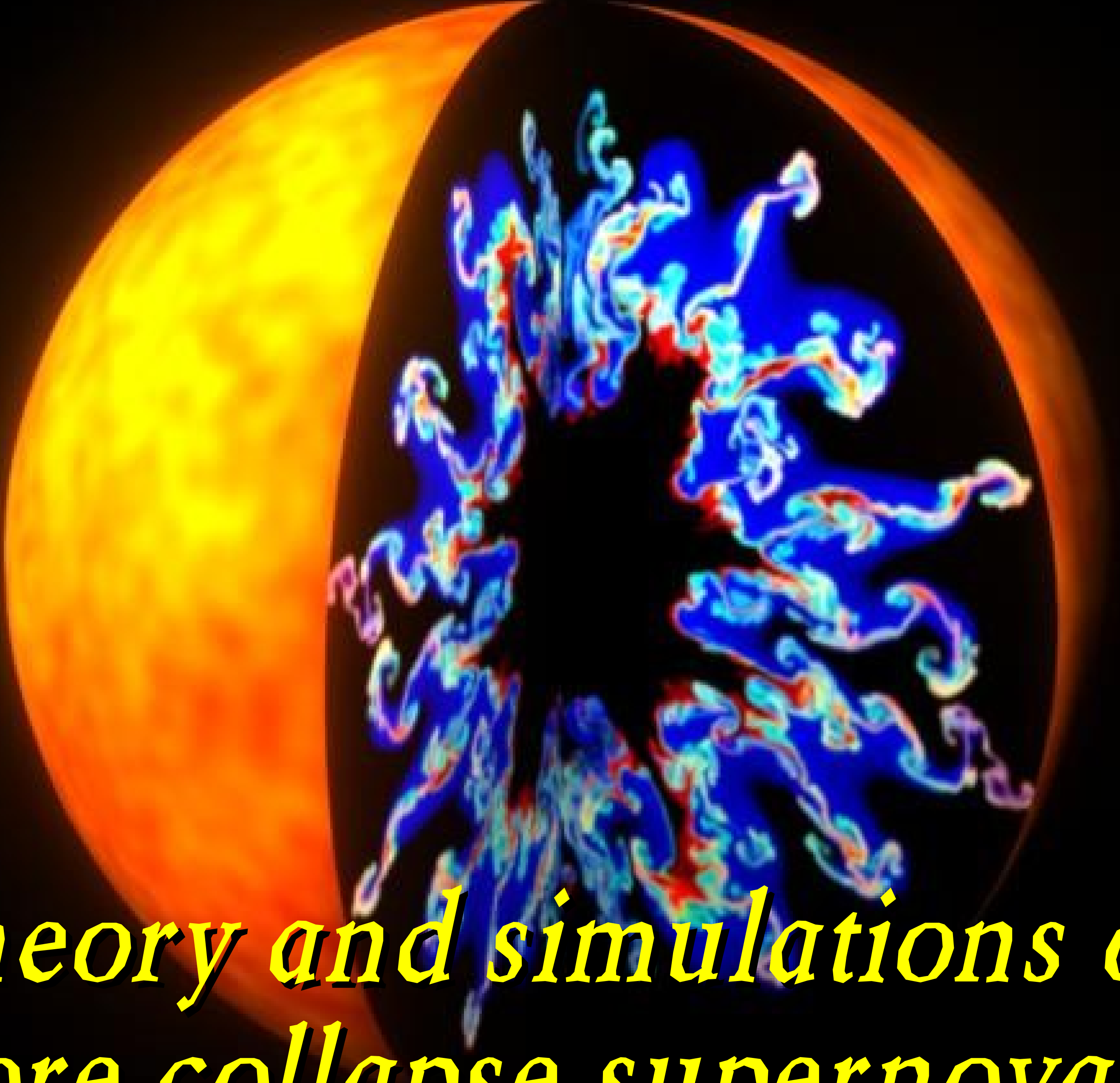
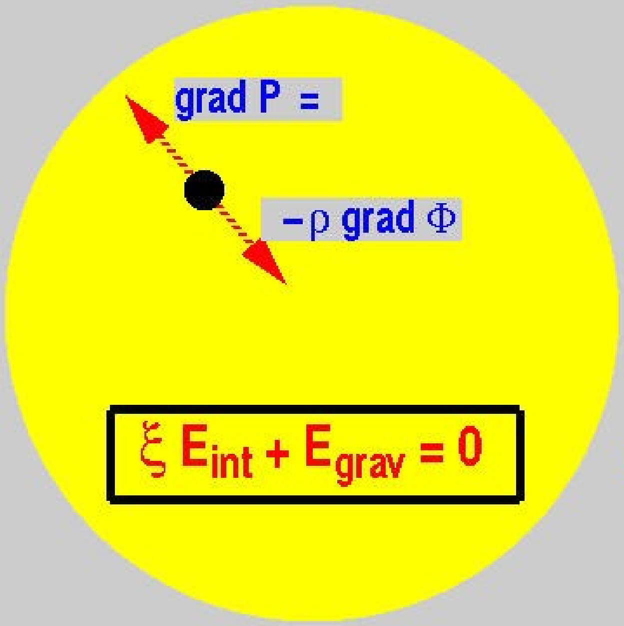


Theory and simulations of core collapse supernovae

Ewald Müller
Max-Planck Institut für Astrophysik

Evolution towards gravitational collapse

stellar evolution mostly hydrostatic, i.e pressure and gravitational forces are in equilibrium



$$\boxed{\xi E_{int} + E_{grav} = 0}$$

virial theorem

$\xi := 3P/\rho u$ ideal gas: $P = (\gamma-1)\,\rho u$ --> $\xi = 3\,(\gamma-1)$

relativ. Fermi gas: $P = 1/3\,\rho u$ --> $\xi = 1$

total energy:

$$\boxed{W := E_{int} + E_{grav} = (1-\xi)\,E_{int} = (\xi-1)/\xi\;E_{grav}}$$

if $\xi = 1$ --> $W = 0$!

Evolution towards gravitational collapse

gas: finite temperature --> star radiates

energy conservation:

$$\boxed{dW/dt + L = 0}$$

luminosity

$$\boxed{L = (\xi-1)\,dE_{int}/dt = -(\xi-1)/\xi\;dE_{grav}/dt}$$

if $L > 0$ --> $dE_{grav}/dt < 0$ <--> contraction --> $dE_{int}/dt > 0$

contraction with $\gamma = 5/3$ ($\xi = 2$) :

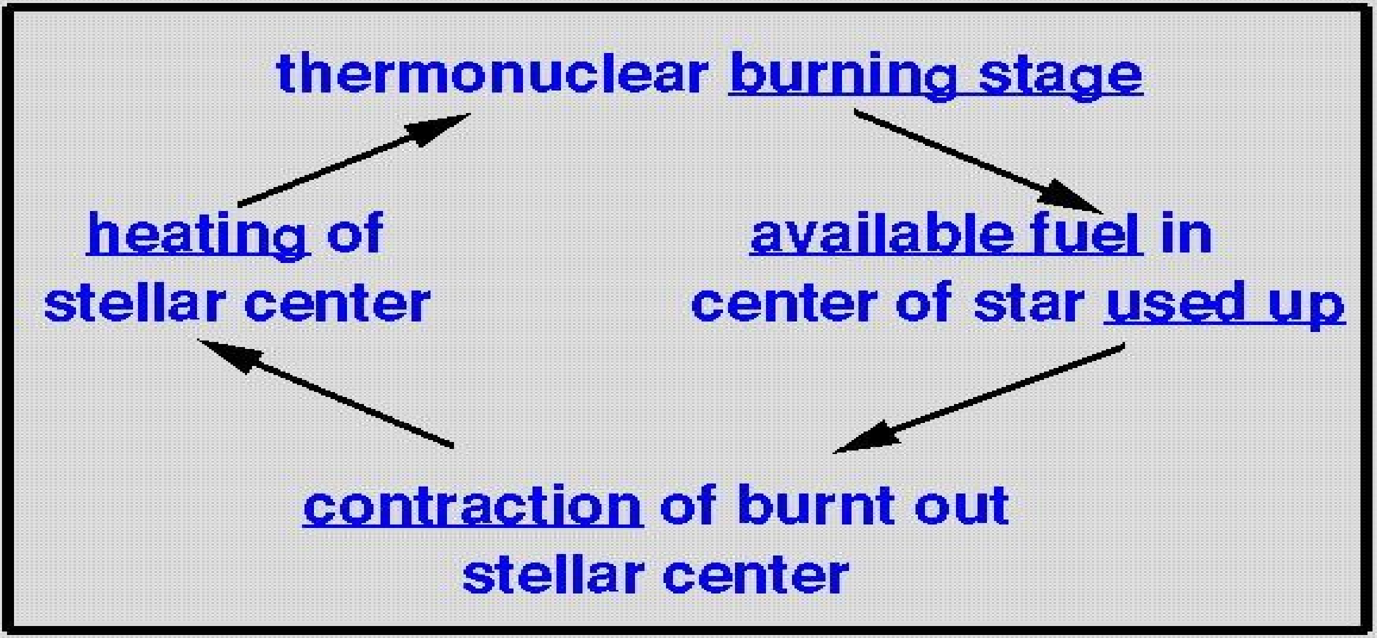
50% of liberated energy are radiated away

50% of liberated energy heat the star

--> star has negative specific heat!

Evolution towards gravitational collapse

- depending on stellar mass:
  number of thermonuclear burning stages



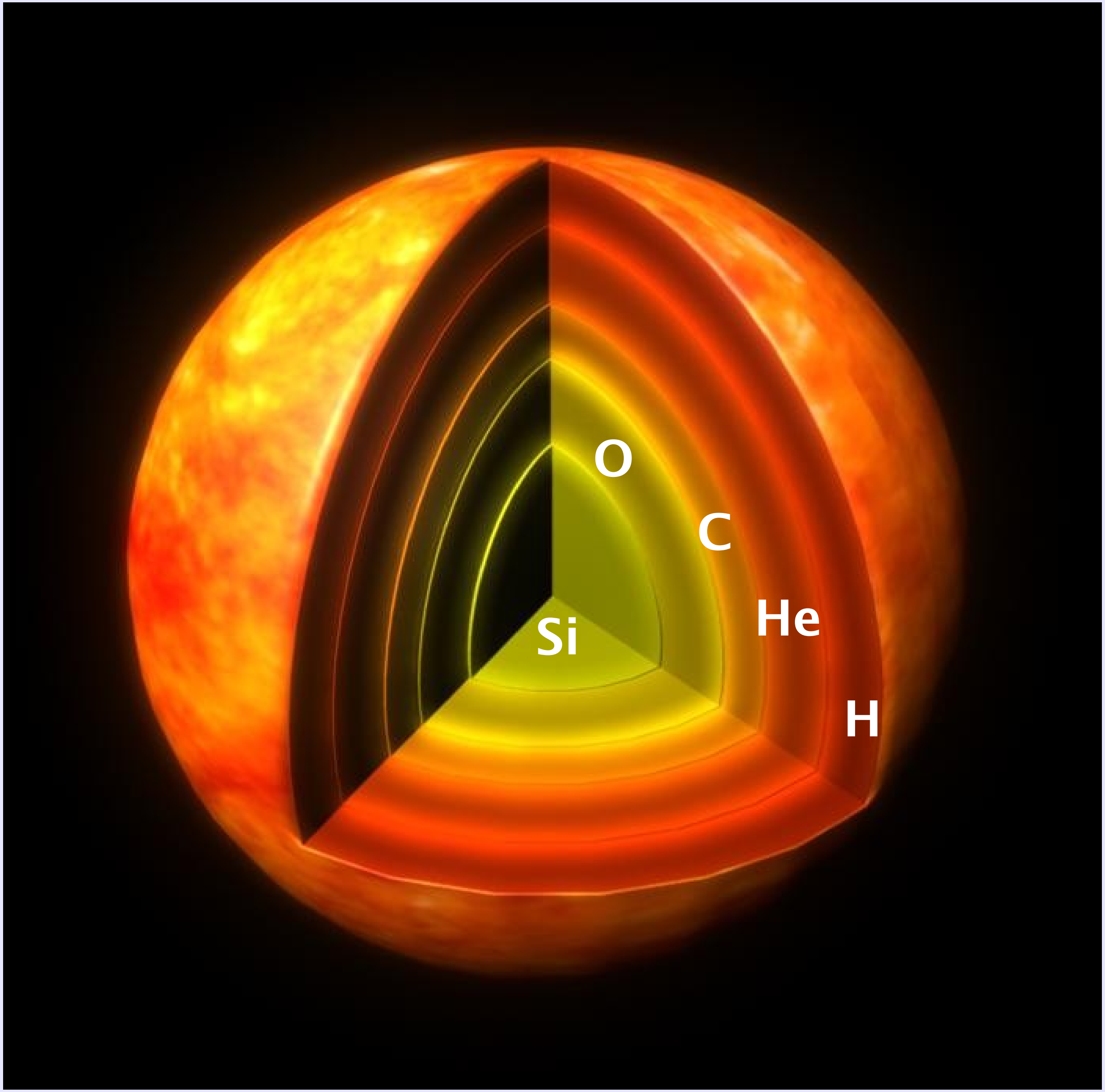
thermonuclear burning stage

available fuel in center of star used up

contraction of burnt out stellar center

heating of stellar center

- every burning stage: central burning + shell burning
- stars with $M > 8 - 10\ M_{Sun}$ experience all physically possible burning stages



Onion-like structure of a presupernova star several million years after its birth:

mass: 10 ... 10^2 M_{sun}

radius: 50 ... 10^3 R_{sun}

- shells of different composition are separated by active thermonuclear burning shells
- core Si-burning leads to formation of central iron core

Note: figure not drawn to scale!

energy sources for a core collapse supernova explosion

gravitational binding energy (SNe II, Ib, Ic)
formation of a compact object of ~1 solar mass
with a radius ~10km

--> $E_b \sim 3 \times 10^{53} (M/M_{sun})^2 (R/10km)^{-1}$ erg

Fe-Ni core: $r \sim 10^{10}$ g/cm^3, $T \sim 10^{10}$ K

--> $P \sim P_e$ (relativistic degenerate Fermi gas)

--> maximum mass (Chandrasekhar)

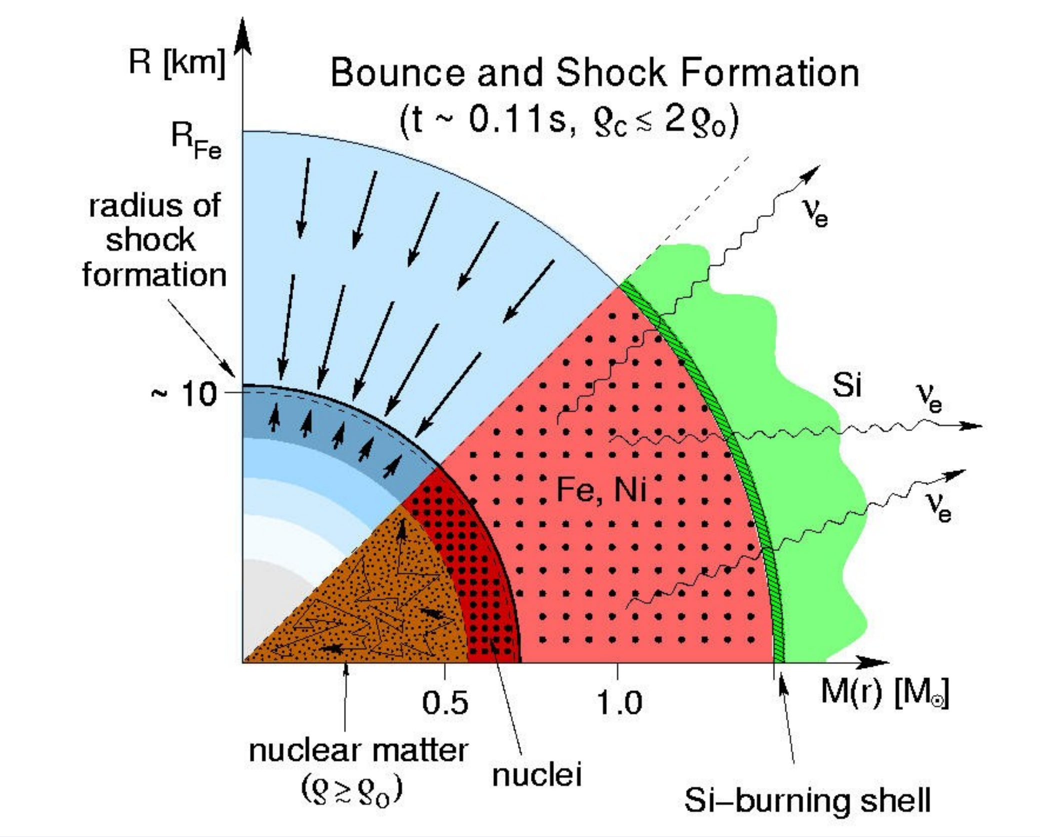
--> core becomes unstable due to:

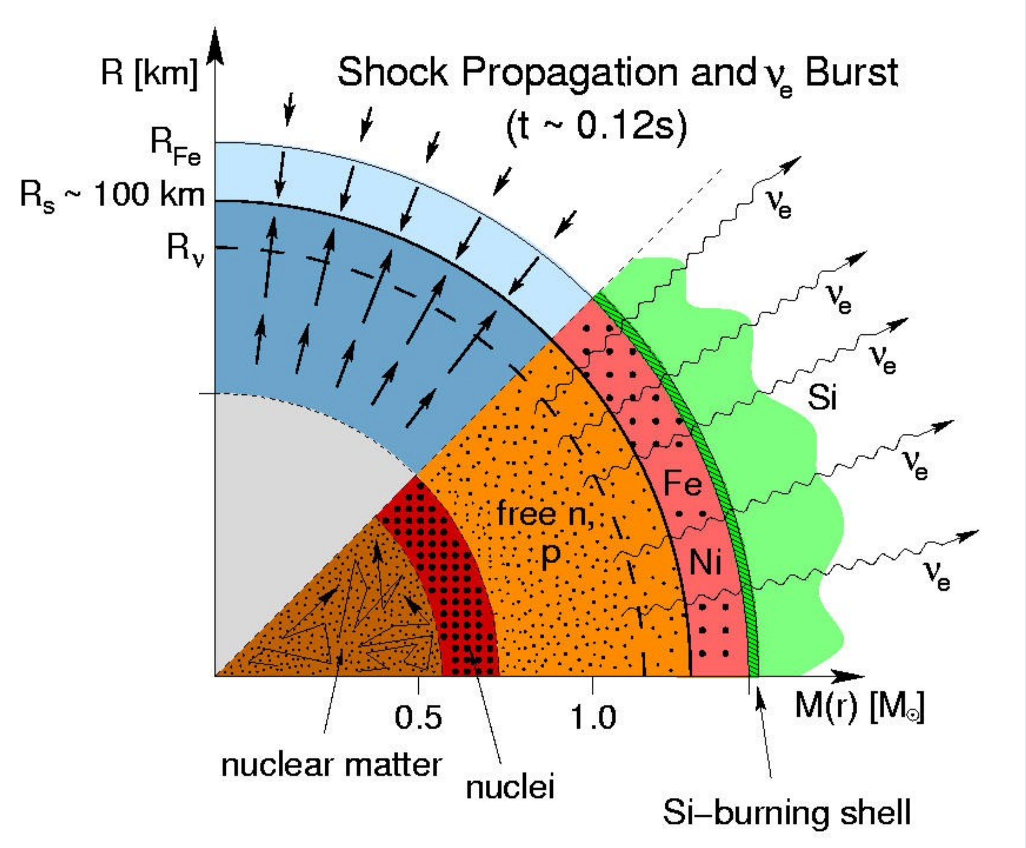
a) electron captures
b) photo-disintegrations

Core collapse supernovae:

- prompt explosion mechanism does not work

(explored during the 1970's and 1980's; commonly accepted early 1990's)



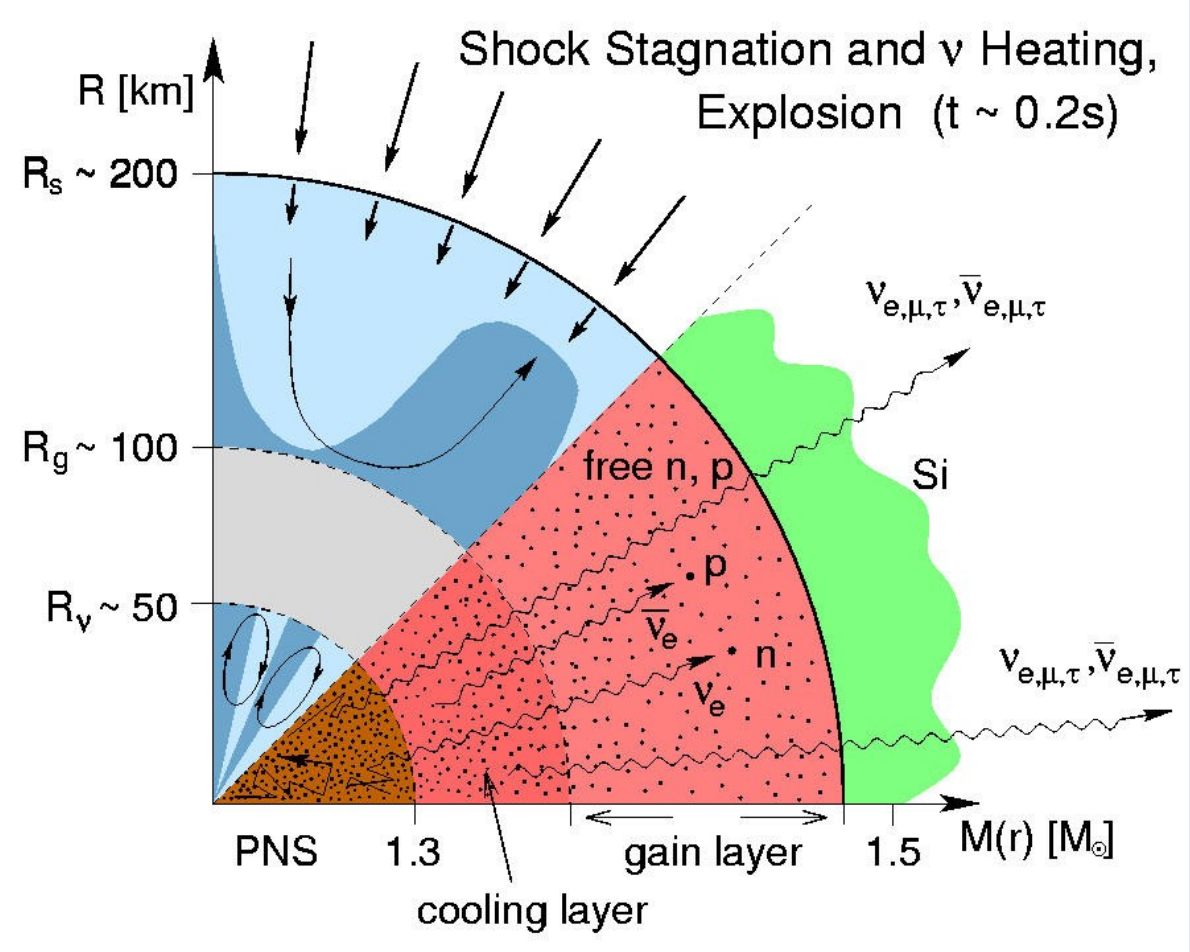


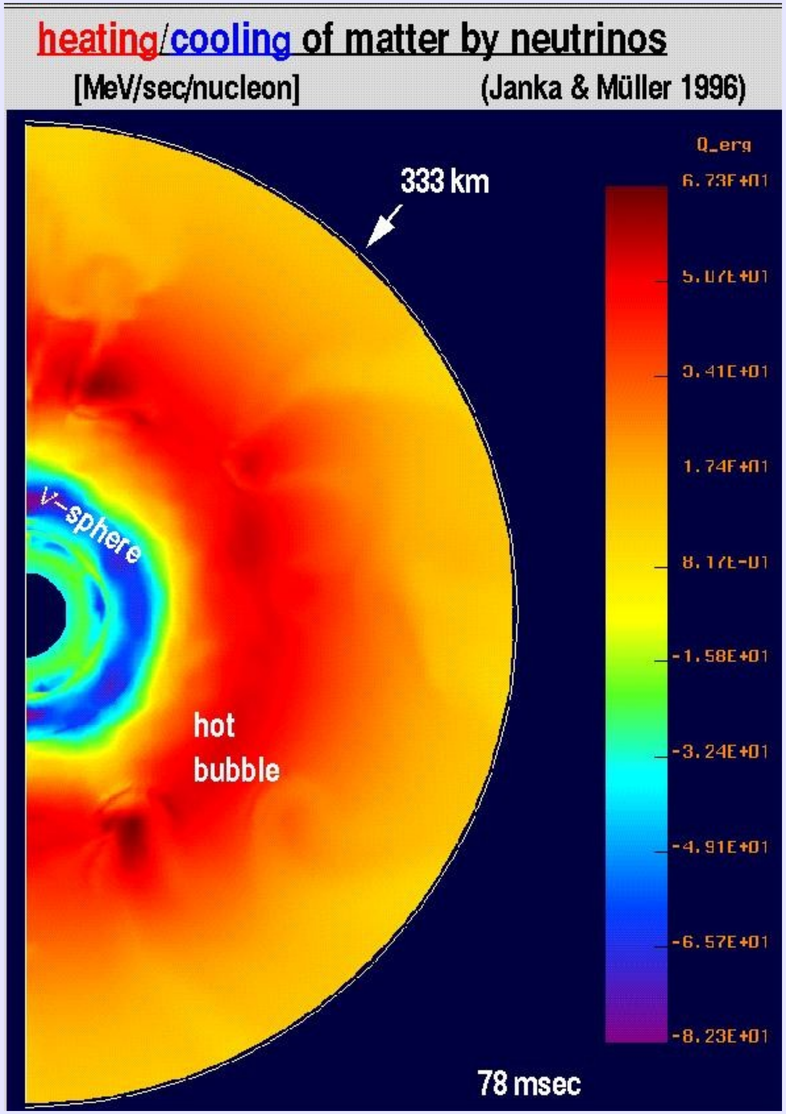
shock wave forms close to sonic point (M ~ 0.7 M_{sun})
initial energy: (5 ... 8) x 10^{51} erg

severe energy losses during shock propagation (8 MeV/nucleon or 1.6 x 10^{51} erg/0.1M_{sun})

current paradigm: neutrino driven delayed explosions

(discovered through computer simulations by Wilson '82, and first analyzed by Wilson & Bethe '95)





Core collapse supernovae: neutrino-driven delayed explosion

(Wilson ´82, Bethe & Wilson '85)

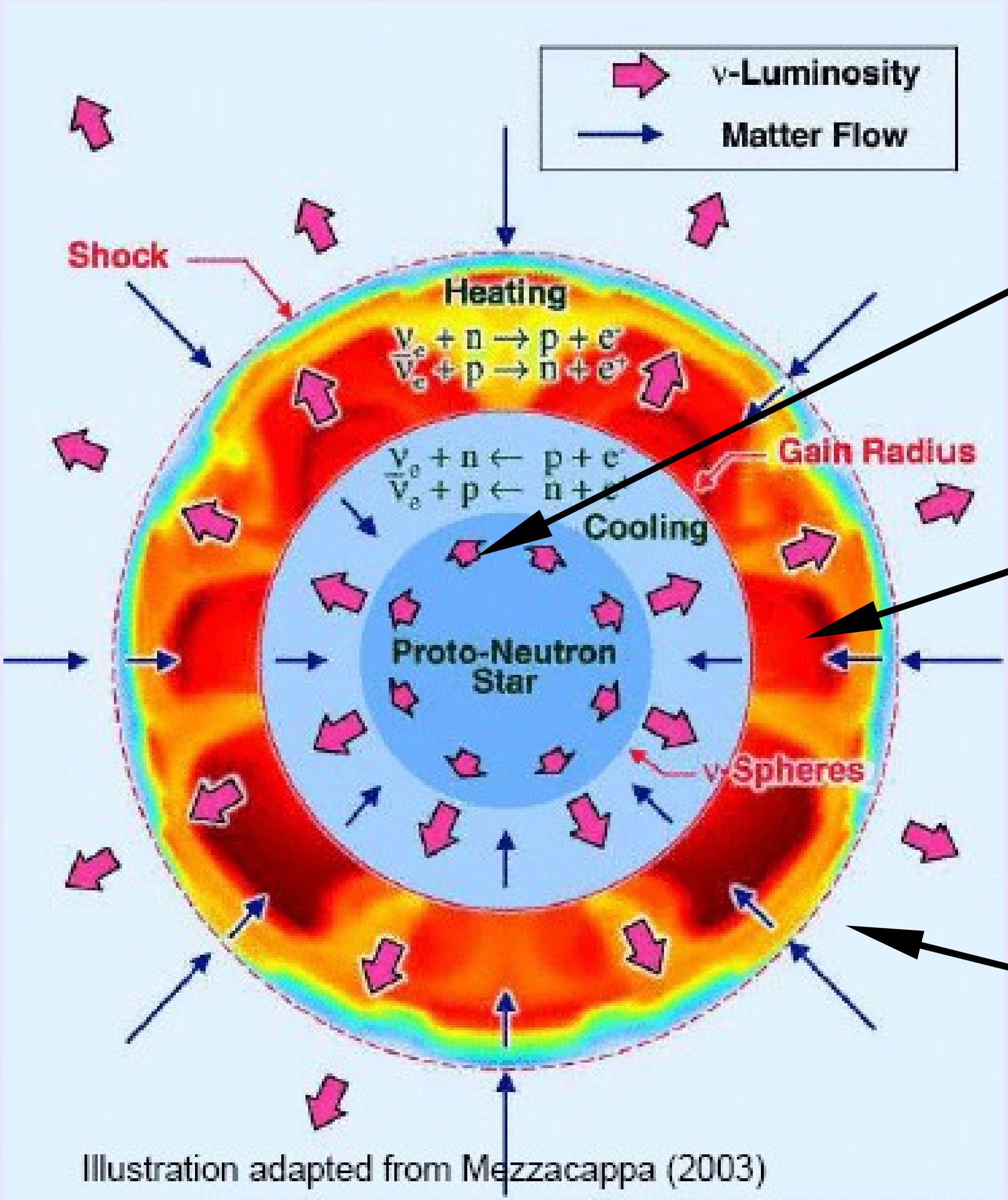


Illustration adapted from Mezzacappa (2003)

neutrinos diffuse out of opaque proto-neutron star ($\tau_\nu \sim 1$)

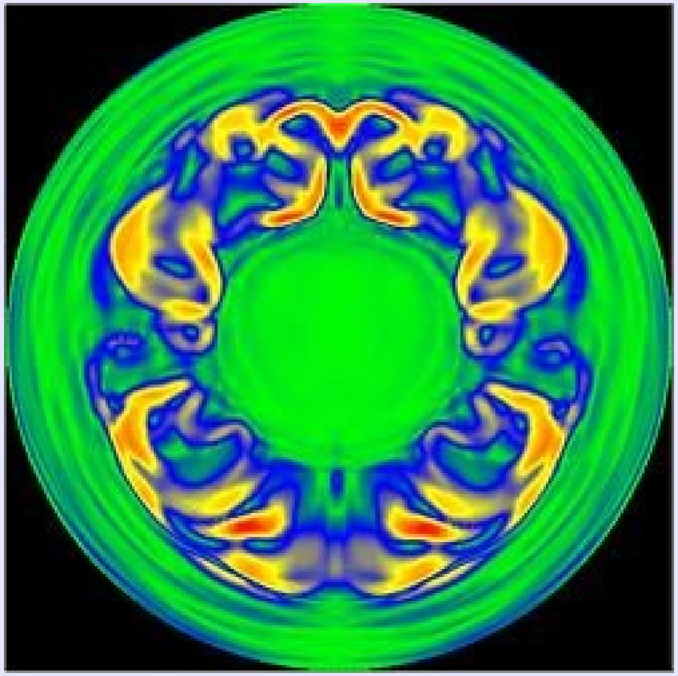
neutrinos heat matter in semi-transparent ($\tau_\nu \sim 1$) post-shock region ---> convection with coexisting downflows and rising hot bubbles sets in

neutrinos stream freely through stellar envelope ($\tau_\nu << 1$)

- observations imply that non-radial flow and mixing are common in core collapse supernovae (see lecture 1)

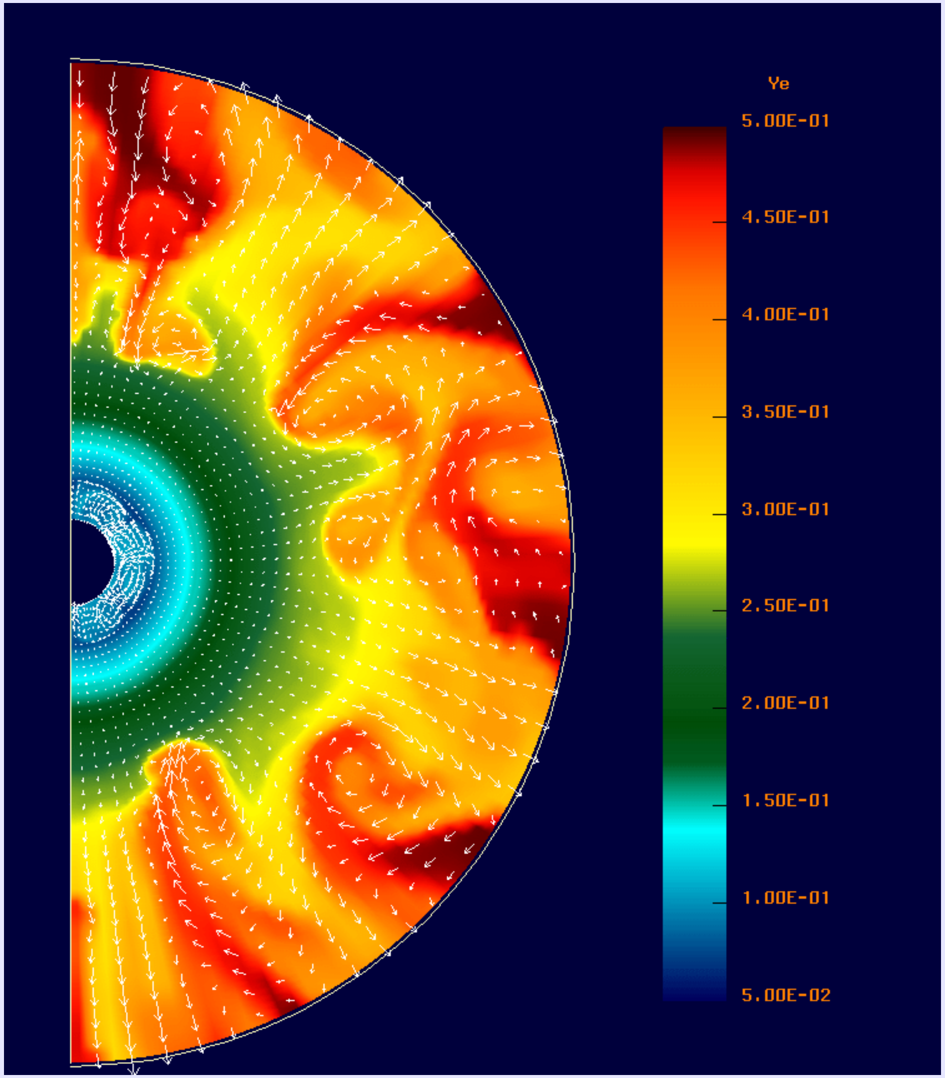
- theoretical models based on delayed explosion mechanism predict non-radial flow and mixing due to
  - Ledoux convection inside the proto-neutron star
    (due to deleptonization and neutrino diffusion)
  - convection inside neutrino heated hot bubble
    (behind shock wave due to neutrino energy deposition)
  - additional flow instabilities (SASI, AAC)
    (between shock and neutrino sphere)
  - Rayleigh-Taylor instabilities in stellar envelope
    (due to non-steady shock propagation; triggered by hot bubble)

Core collapse supernovae need multidimensional modeling !



Ledoux convection inside proto-neutron star due to negative lepton and entropy gradients (Keil, Janka & Müller '96)

- asymmetric ν-emission (few sec) and flow (~100 sec?)



Convection in the surface layers of the proto-neutron star and in the hot bubble 78 msec after core bounce (Janka & Müller '96)

The computational challenge:

a) 6D radiation + 3D hydrodynamics problem

multi-flavor, multi-d transport of neutrinos (fermions!)
coupled to
multi-d multi-fluid self-gravitating hydrodynamic flow

most important SN explosion physics occurs in
semi-transparent region --> Boltzmann solver

b) very different time and length scales

covering up to 10 orders of magnitude in time and space

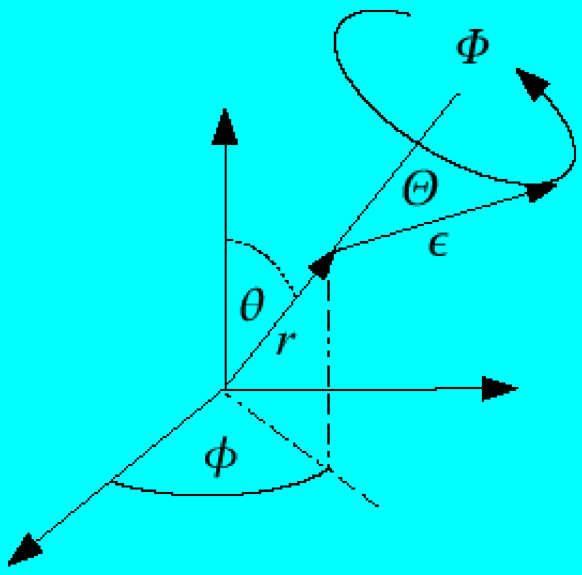
--> implicit treatment of transport equations, symmetry assumptions, adaptive grids (AMR)

Approaches to numerical transport:

- * trapping schemes
- * flux-limited diffusion — reduces dimensionality!
- * variable Eddington factor technique:
  solve Boltzmann transport equation (BTE)
  & moments equations (ME)
- * S-N solver: discretize BTE in all variables — huge matrices!
- * Monte Carlo method: reconstruct phase space distribution (fermions!) by direct sampling — very costly for dynamics!

* <u>Question:</u> choice of reference frame (comoving, mixed, or fixed) and coordinates (Eulerian or Lagrangian)?

The curse of the dimensions



- Boltzmann equation determines neutrino distribution function in phase space

$$f(r,\theta,\phi,\Theta,\Phi,\epsilon,t)$$

- Integration over momentum space yields source terms for hydrodynamics

$$Q(r,\theta,\phi,t),\dot{Y}_e(r,\theta,\phi,t)$$

Solution approach

- **3D** hydro + **6D** direct discretization of Boltzmann Eq. (no serious attempt yet)
- **2D** hydro + **5D** direct discretization of Boltzmann Eq. (planned by DoE's TSI/SSC)
- **2D** hydro + "**ray-by-ray-plus**" variable Eddington factor method (MPA)

Required resources

- >= 1-10 PFlops (sustained!)
- >= 10-100 TFlops/TBytes
- >= 1 TFlops, < 1 TByte

Specialities of neutrino transport in supernovae:

* diffusion or free streaming and stiff matter interactions limit time step ---> implicit schemes advisable

* velocity fields & general relativistic effects

* energy (frequency) bin coupling

* interaction kernels nonlinear (stimulated absorption); transport equation of integro-differential character

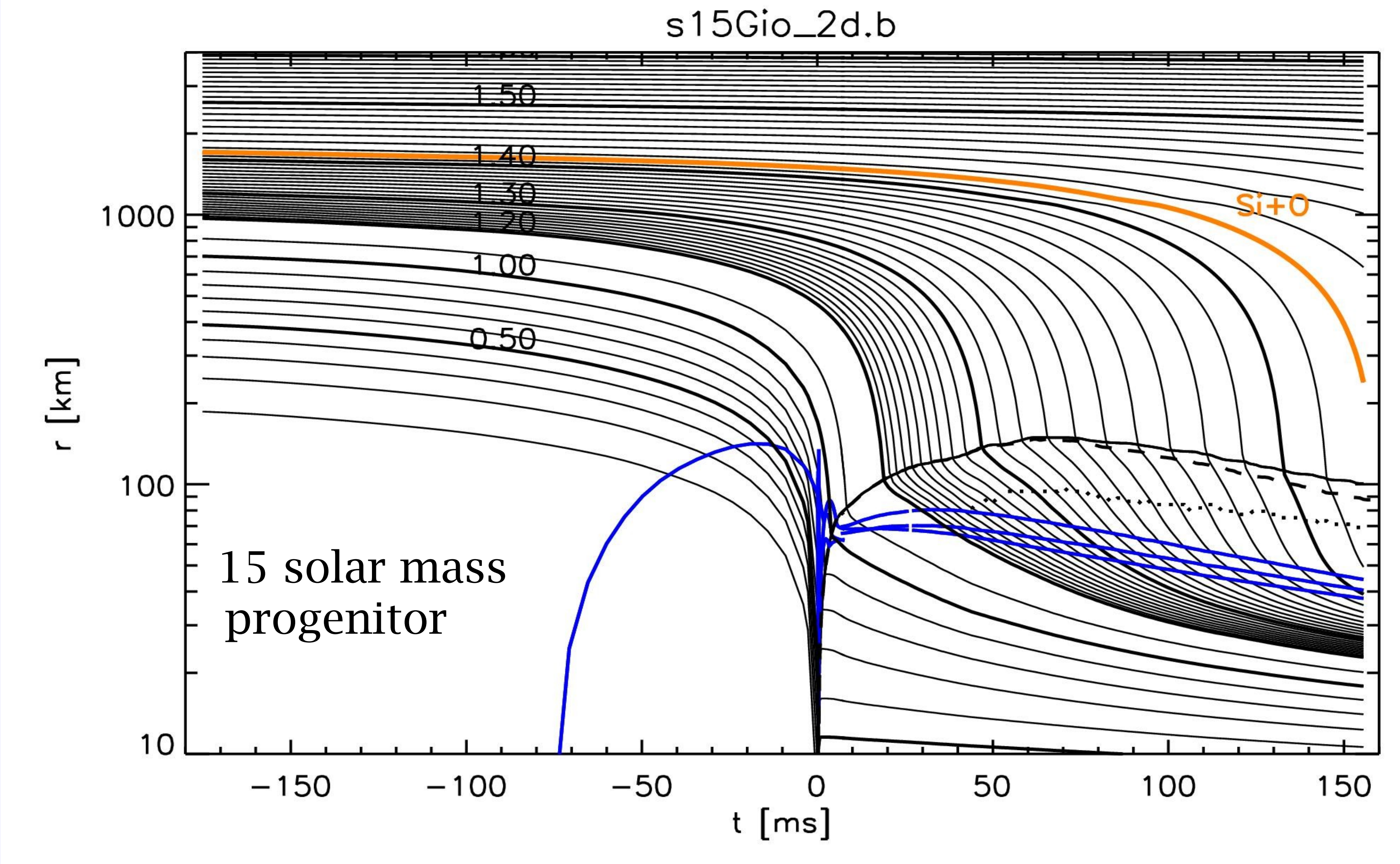
* neutrino-antineutrino coupling

* many time steps necessary ---> conservation form of lepton number, energy & momentum equations advantageous!

* coupling to hydrodynamics: different radial grids and temporal stepping ---> operator split techniques

1D simulation with Boltzmann neutrino transport

(Buras, Rampp & Janka 2002)



15 solar mass progenitor

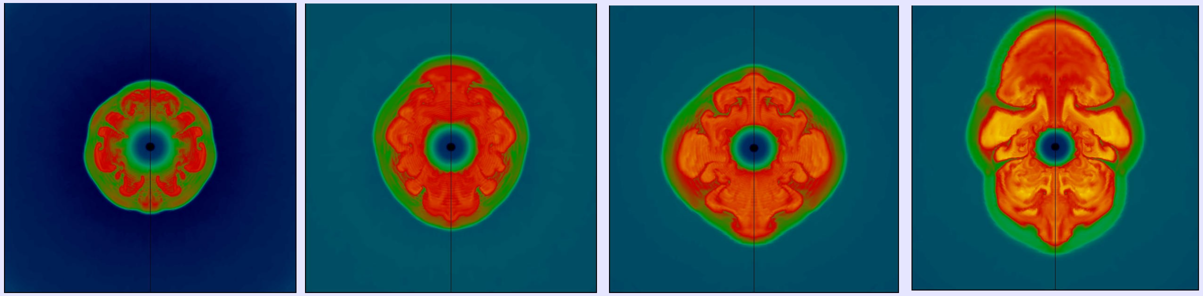
No explosion!

confirmed by Oak Ridge supernova group (Liebendörfer etal 2001)

State-of-the-art hydrodynamic simulations with

Boltzmann n-transport, realistic EOS, relativistic gravity, and realistic progenitors

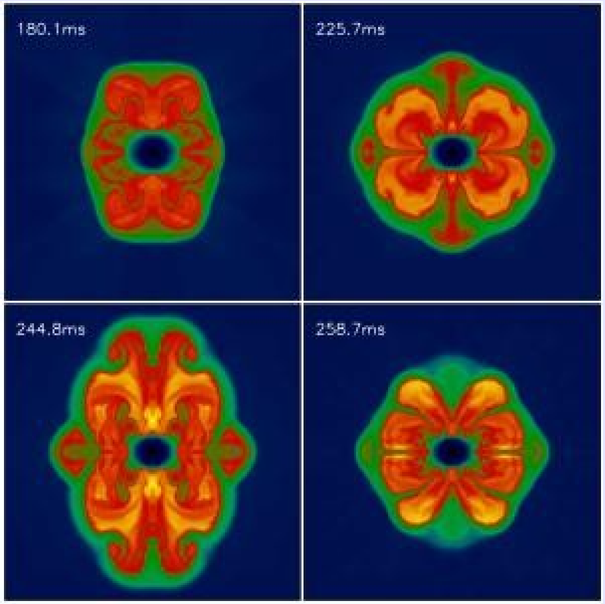
2D HD + (1.5D + 2.5D) NuTrans: $3\ 10^{17}$ ops/simu, i.e. 10^{7}s @ 30Gflops, or
10^{5}s @ 3Tflops



Snapshots from a 2D 180° run of a non-rotating axisymmetric 11.2 M_{sun} progenitor

(Buras, Rampp & Janka 2003)

--> weak explosion (0.3 Bethe)!



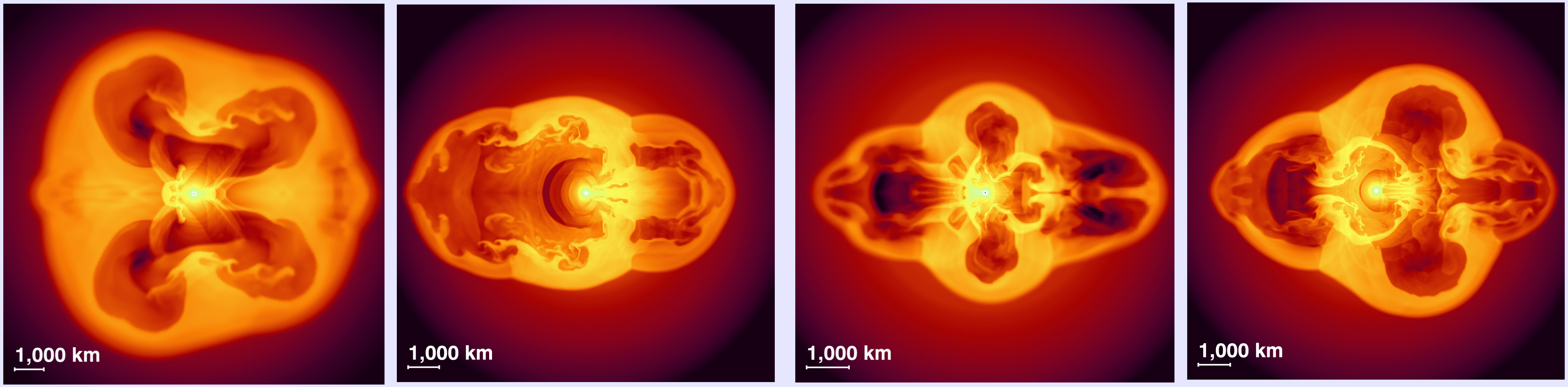
Snapshots from a 2D 90° run of a rotating axisymmetric 15 M_{sun} progenitor

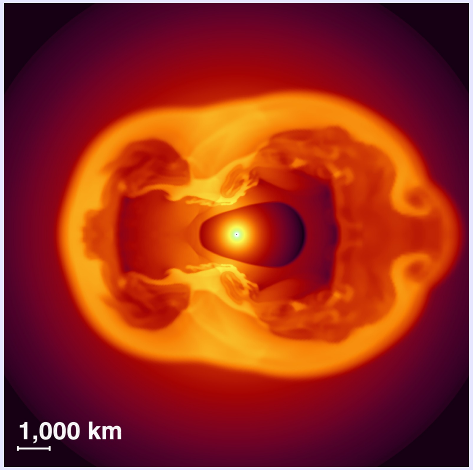
($b_{initial}$ = 0.05% , $\Omega_{i,c}=0.5s^{-1}$; Heger etal 2003)

(Buras, Rampp, Janka & Kifonidis 2003)

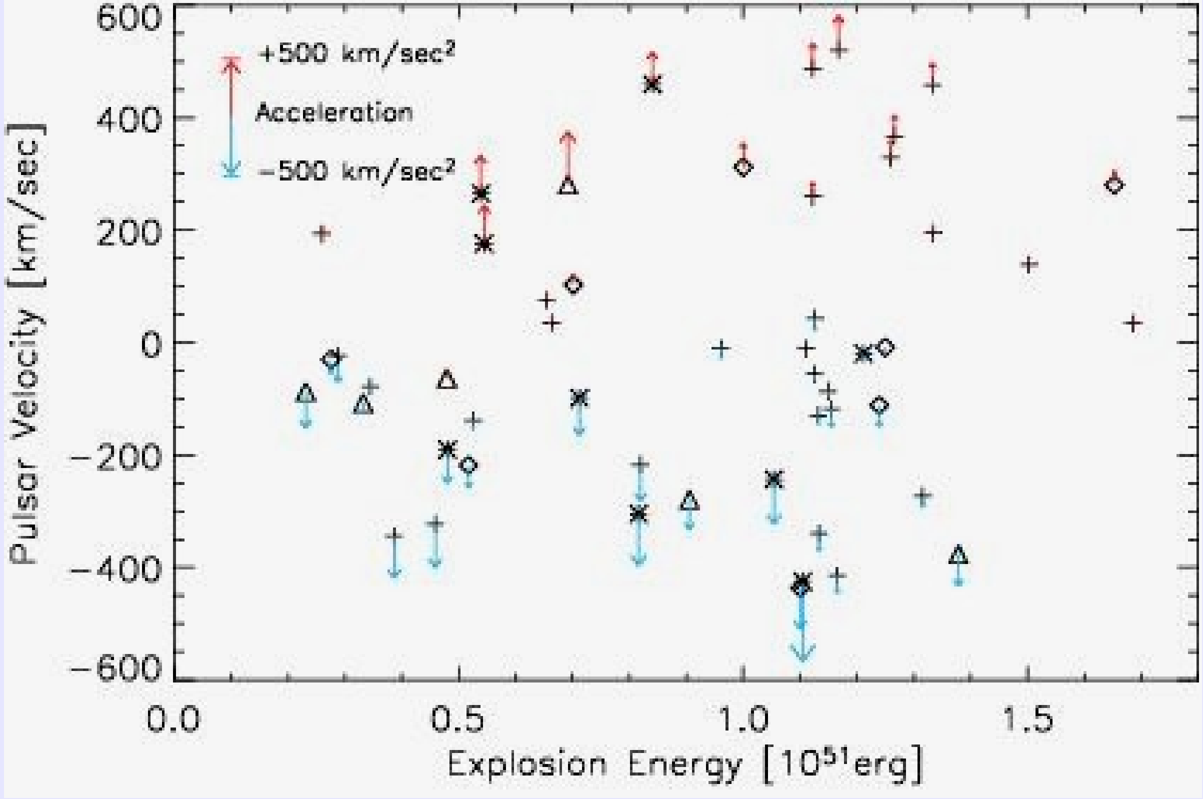
Large scale asymmetries & neutron star kicks

Growth of dominant low order (l=1,2) modes in post-shock layer
---> neutron star kicks (Scheck etal '03)

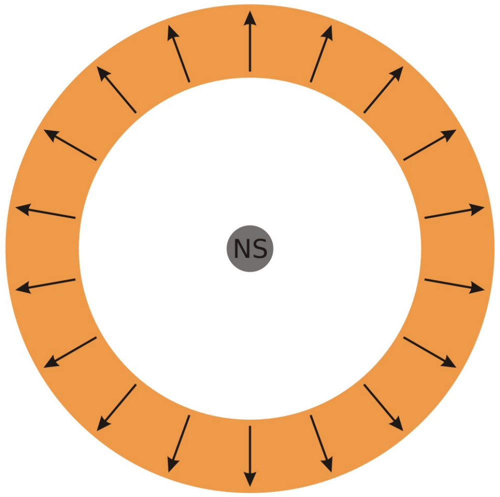




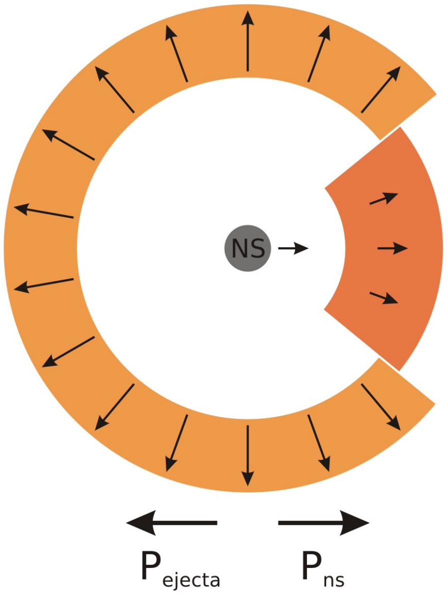
density distribution
1 sec after
core bounce



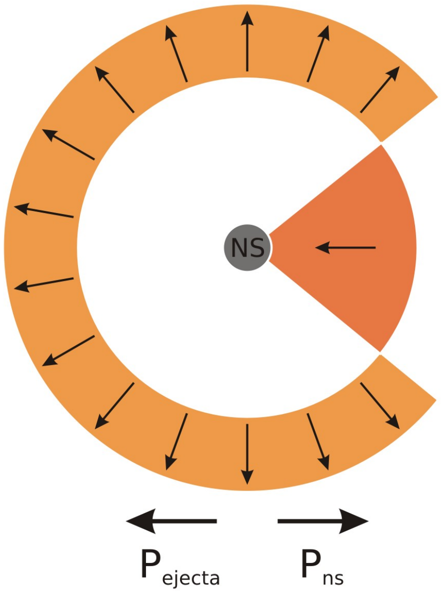
kick velocity
uncorrelated
with explosion
energy



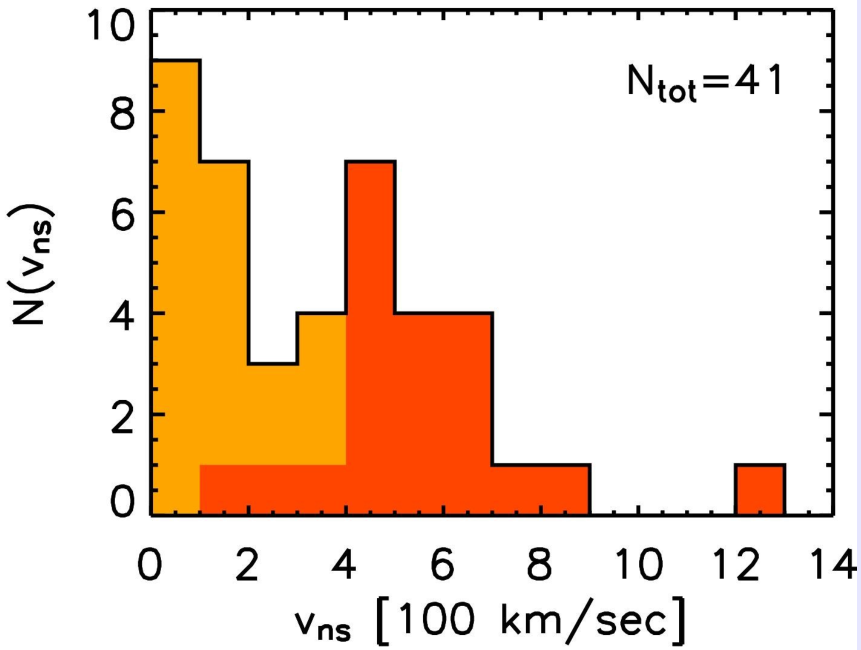
spherical explosion:
no kick



anisotropic explosion: kick due
to gravitational acceleration

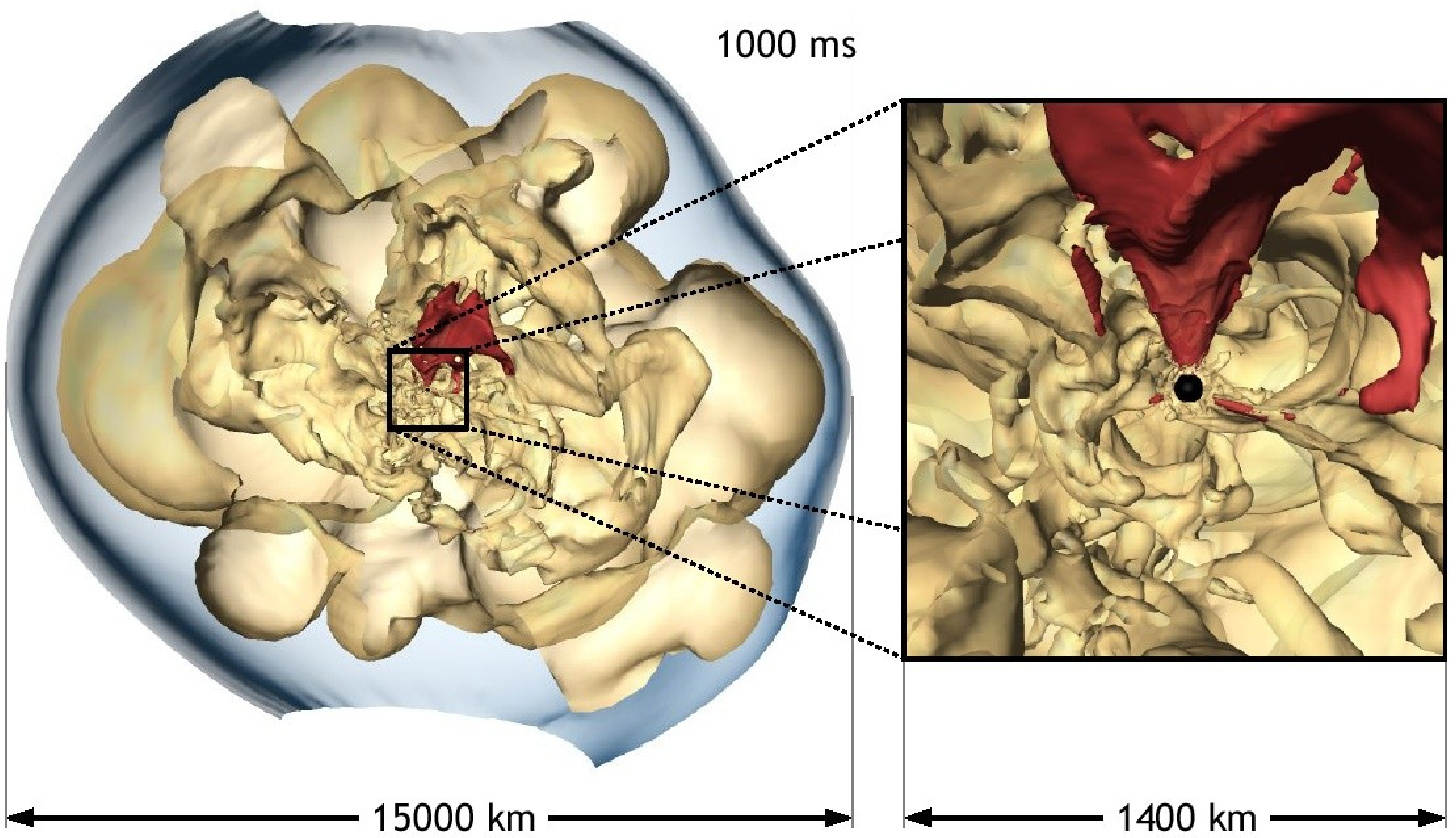


anisotropic explosion: kick due
to anisotropic accretion



large set of simulations shows
bi-modal kick velocity distribution
(Scheck 2005)

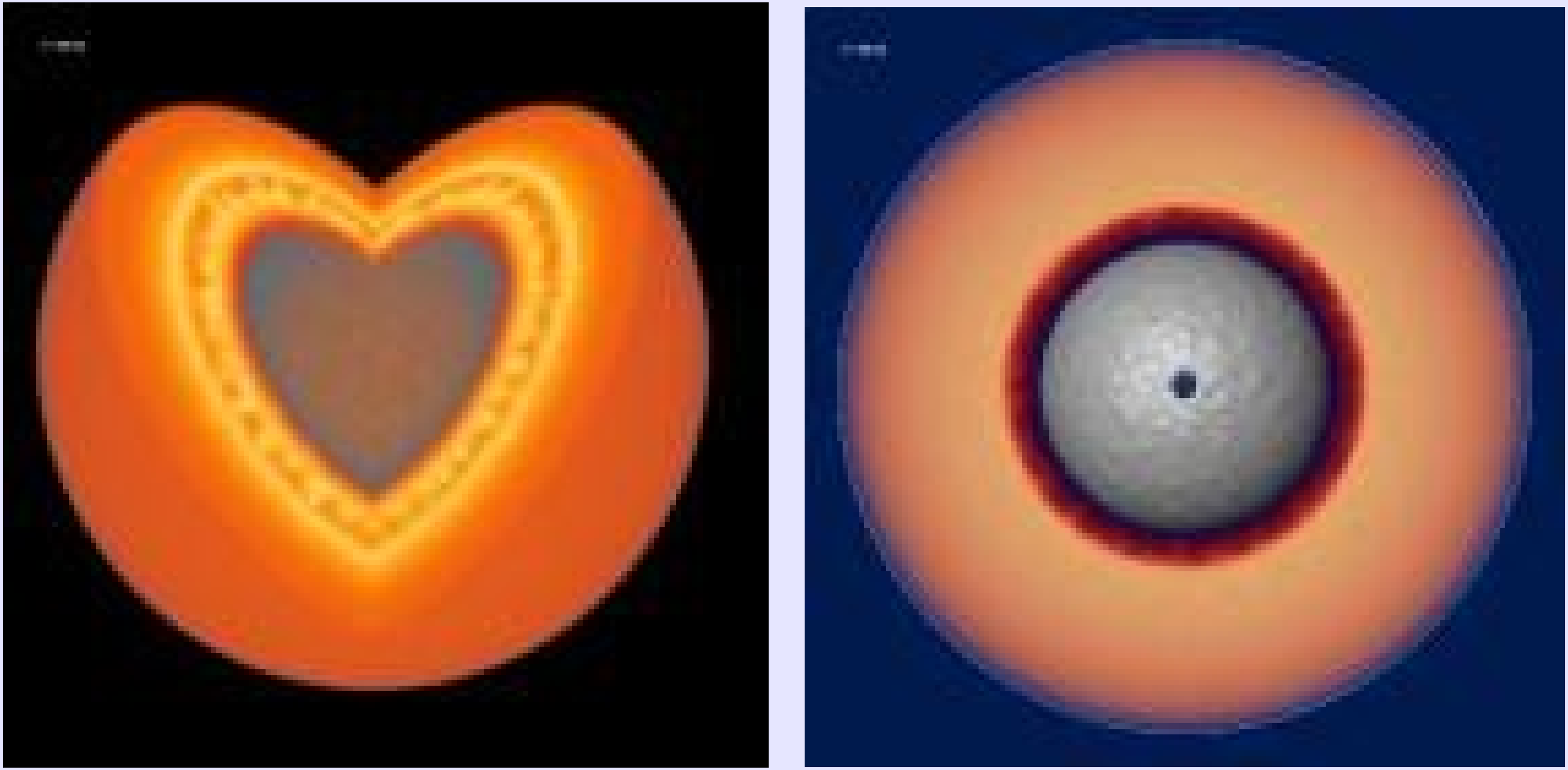
Global dipolar oscillations of the post-shock layer also seen in recent 3D simulations neglecting (Blondin etal '03) or simplifying (Scheck etal '04) the treatment of ν-transport



3D core collapse simulation: shock, Y_e=const & downflow to NS (Scheck 2004)

Growth of dominant post-shock low-order (l=1,2) modes: 3D simulations

(Scheck et al. 2006)



Provide a look into the heart of a core collapse supernova!

Without special ingredients that are not commonly accepted
(e.g. strong magnetic fields, exotic neutrino physics, fast rotation)
one gets (Scheck et al. 2003, 2005, 2006)

- a pronounced global anisotropy, even “one-sided” explosions

- high neutron star kick velocities (record: 1200 km/s)

- large-scale mixing of the ejecta as required by observations of SN1987A (Kifonidis et al. 2006)

Is convection the only cause of anisotropies?

- indications for a second low-mode, oscillatory instability

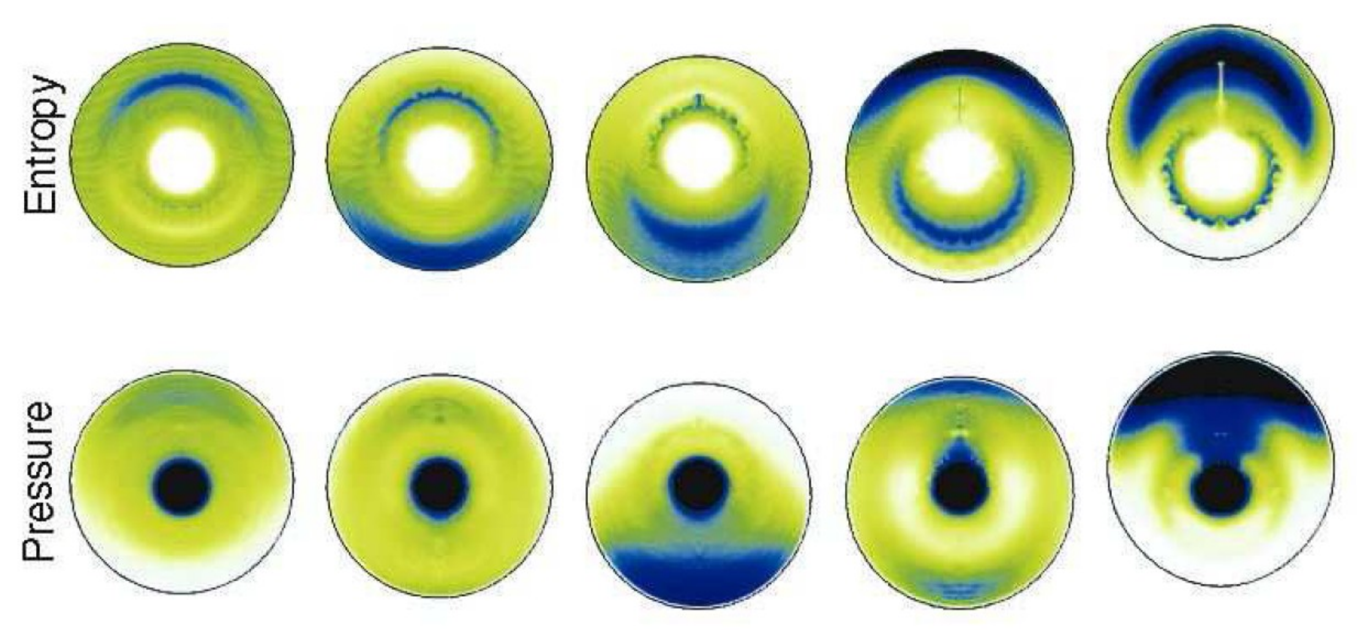
- nature and growth rate of the instability?

Standing accretion shock instability

Blondin et al. (2003)

hydrodynamic simulations of flow behind standing accretion shock

---> low-mode oscillatory instability (“sloshing”)
redistribution of energy unbinds matter (interpreted as an explosion)



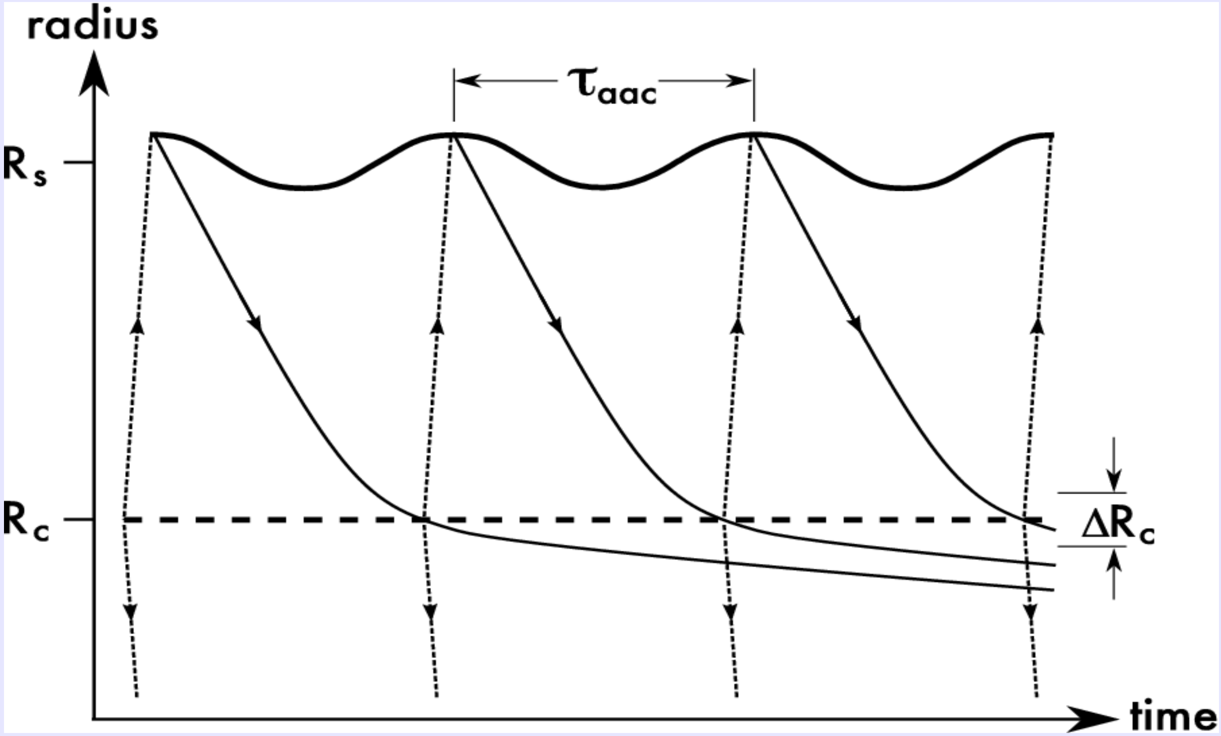
same behaviour is found if neutrino cooling and a microphysical EOS are included
(Blondin et al. 2005, Ohnishi et al. 2005)

Advective-Acoustic Cycle (Foglizzo 2002; accretion disks)

interaction of two kinds of perturbations:

- advected perturbations (entropy, vorticity) propagating with flow velocity v
- acoustic perturbations (pressure waves) propagating with v ± c

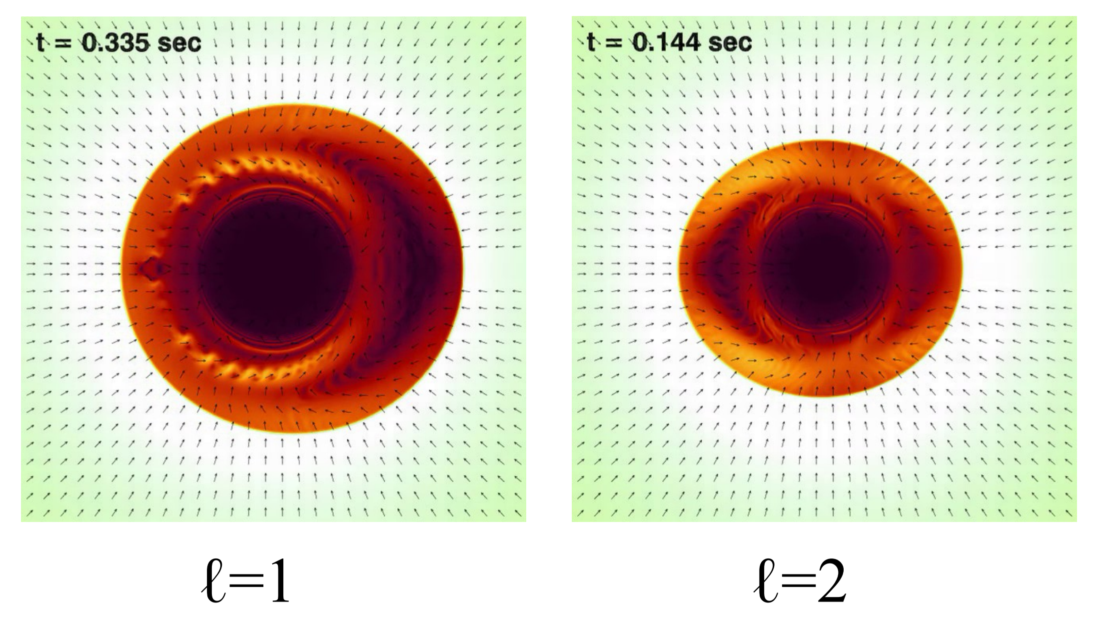
shock --->



Scheck et al. 2006

- neutrino heating is boosted (by a factor ~2) by AAC and convection
- AAC is a non-radial, low-mode oscillatory instability that can grow (and trigger explosions) under conditions which do not allow for the growth of convection

(short advection time scale, small entropy gradient, small initial perturbations)



ℓ=1 ℓ=2

AAC is likely responsible for the excitation of low-ℓ modes, which cause large neutron star kicks

Core collapse supernovae & the equation of state

EoS currently applied in simulations

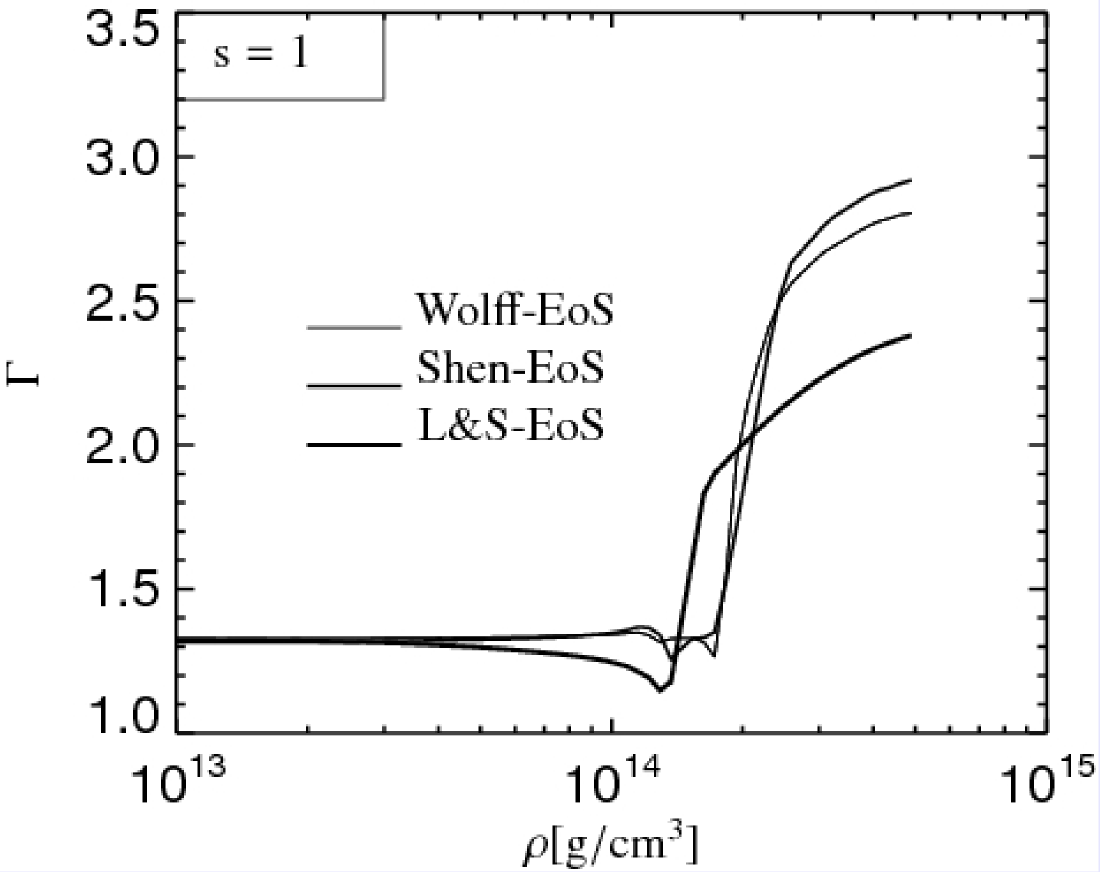
Lattimer & Swesty '91 (compressible liquid drop; Skyrme interaction; K=180 MeV, 29.3 MeV)

Shen et al. '98 (relativistic mean field; K=281 MeV, 36.9 MeV)

Wolff & Hillebrandt '84 (Hartree-Fock, Skyrme interaction; K=263 MeV, 32.9 MeV)

- extrapolated to supra-nuclear densities

- differ in the value of the adiabatic index around and above the phase transition to homogeneous nuclear matter

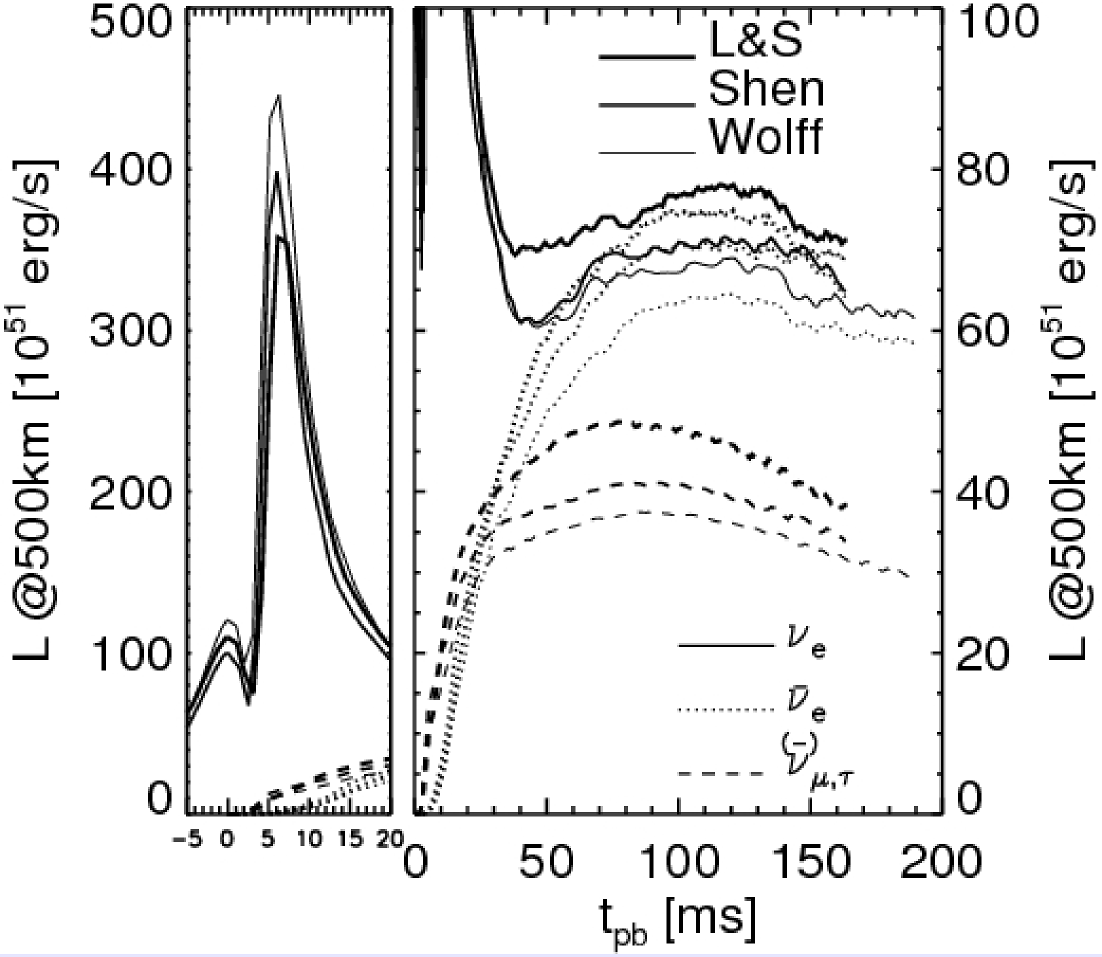


Marek '03

EoS dependence of simulation results (Marek '03)

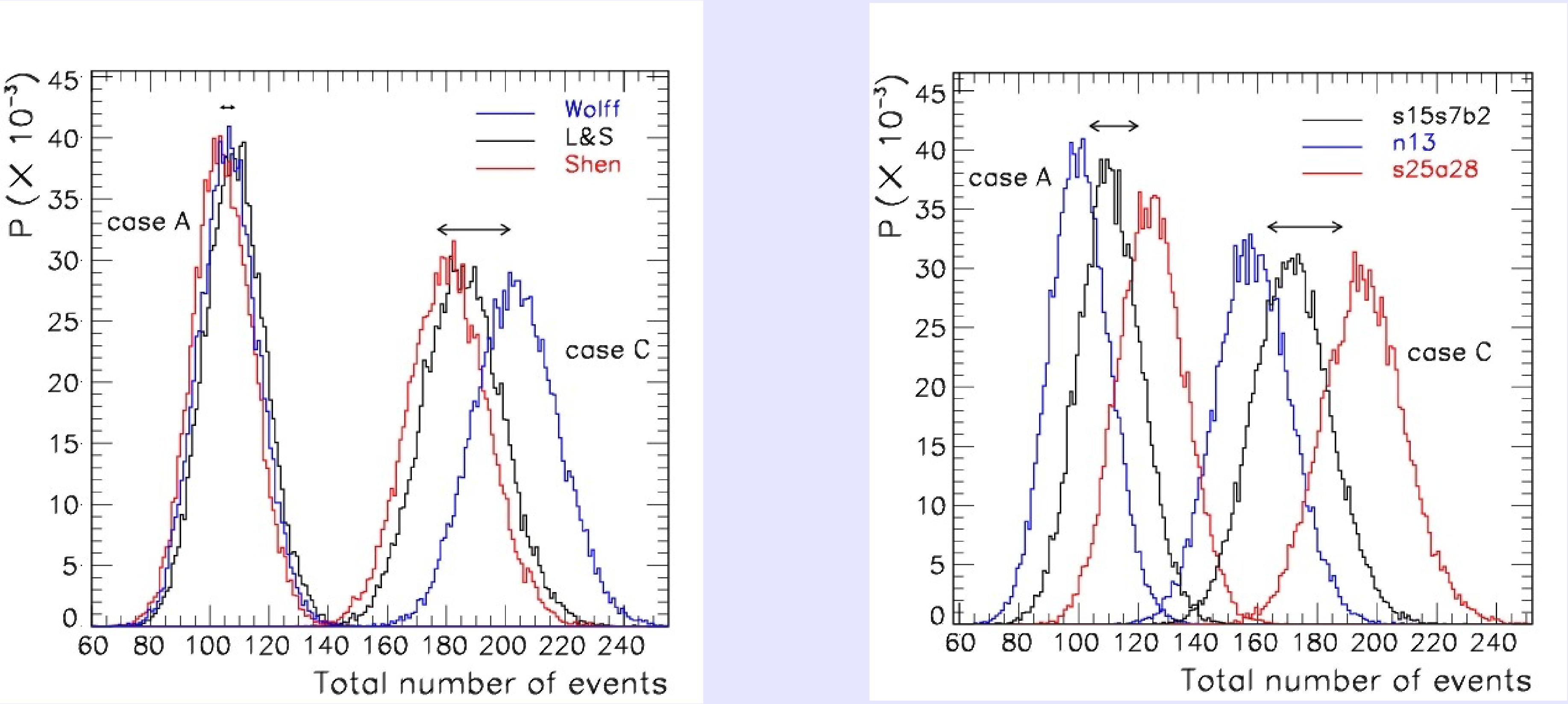
- maximum density at bounce
- density of post-bounce quasi-hydrostatic equilibrium state
- shock formation radius (~0.05M_{sun} further outward for stiffer EoS)
- shock stagnation radius (~10km further outward for stiffer EOS)
- maximum shock expansion
- contraction of proto-neutron star

- peak luminosity during prompt ν-burst & evolution of post-bounce ν-luminosity



Marek '03

EoS effects are hard to measure, i.e. (supra-nuclear) EoS is hard to constrain by observations of core collapse SNe (Kachelrieß et al. '05)



distribution of the observed total number of neutrino events for 20000 SNe at d=10kpc

for different EoS

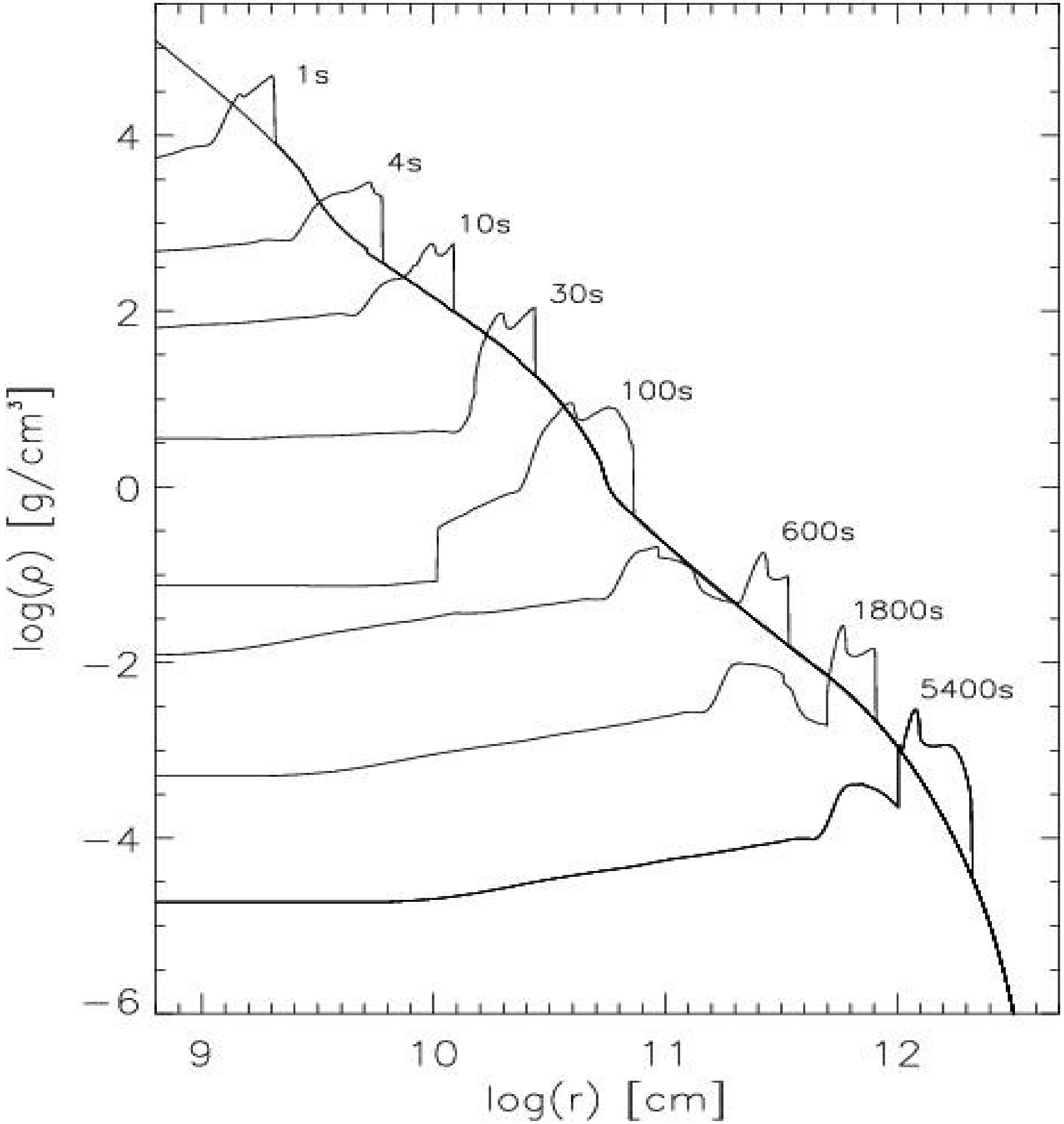
for different progenitors

Neutrino mixing: (A) normal mass hierarchy, large mixing angle,
(C) any hierarchy, small mixing angle

More promising approach: observations of neutron stars!

Rayleigh-Taylor instabilities & mixing in supernova envelopes

Shock propagation through envelope of progenitor star (Müller et al., 1990)



shock propagation is non steady

--->

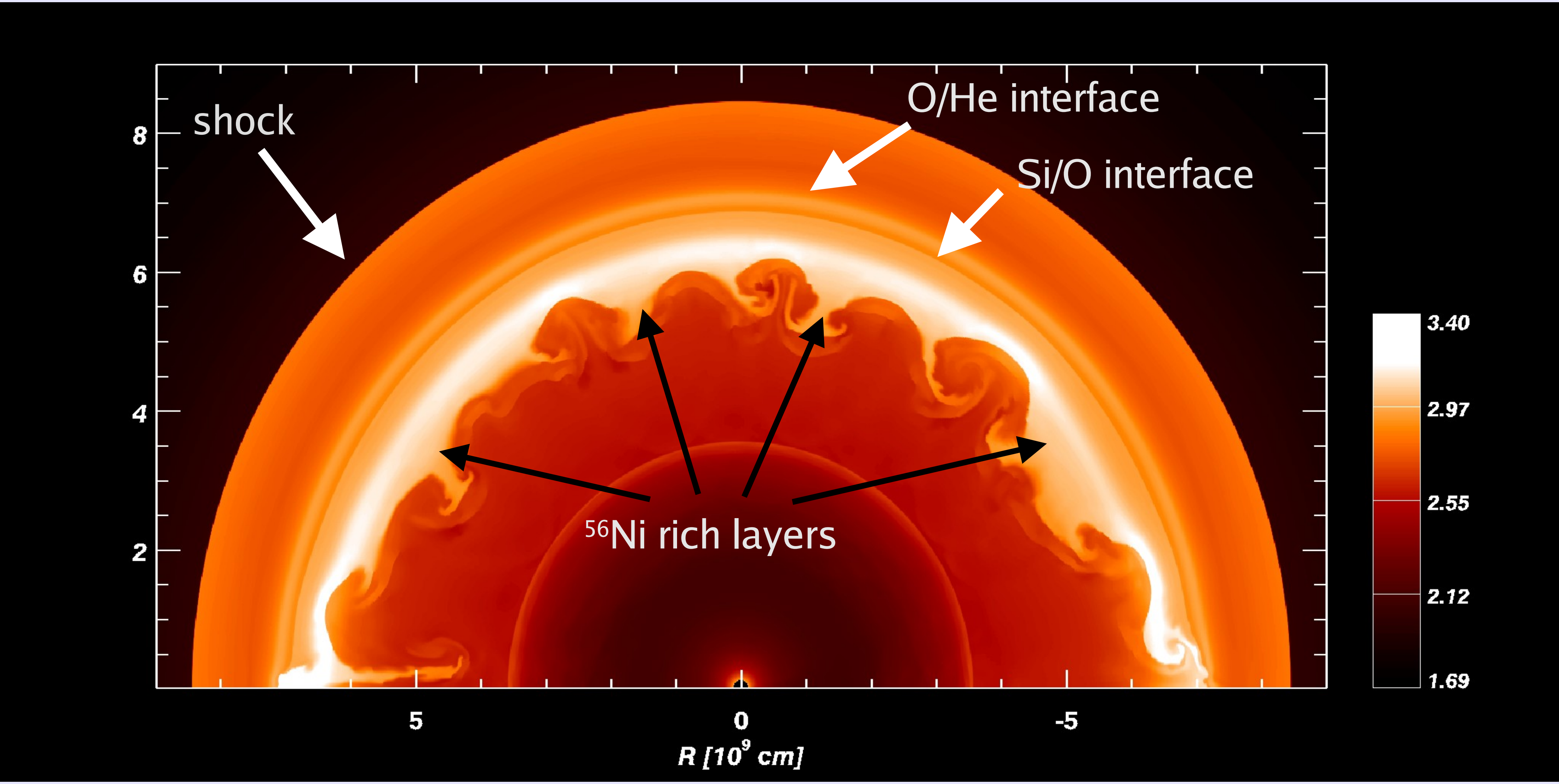
density inversions

--->

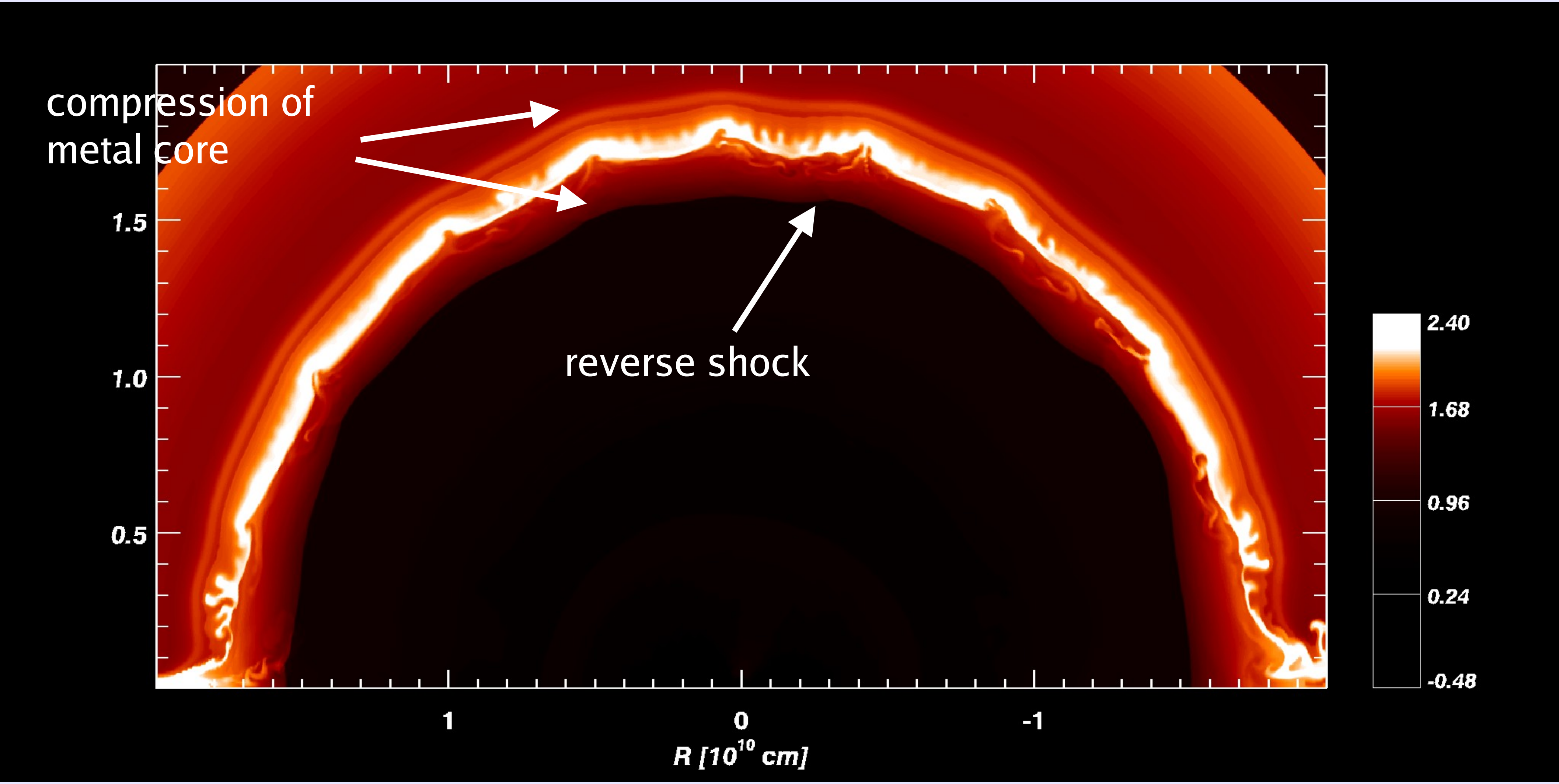
Rayleigh-Taylor instabilities

Results of a 2D AMR simulation of a globally almost spherical, neutrino-driven, “fast” explosion model

Kifonidis et.al 2003



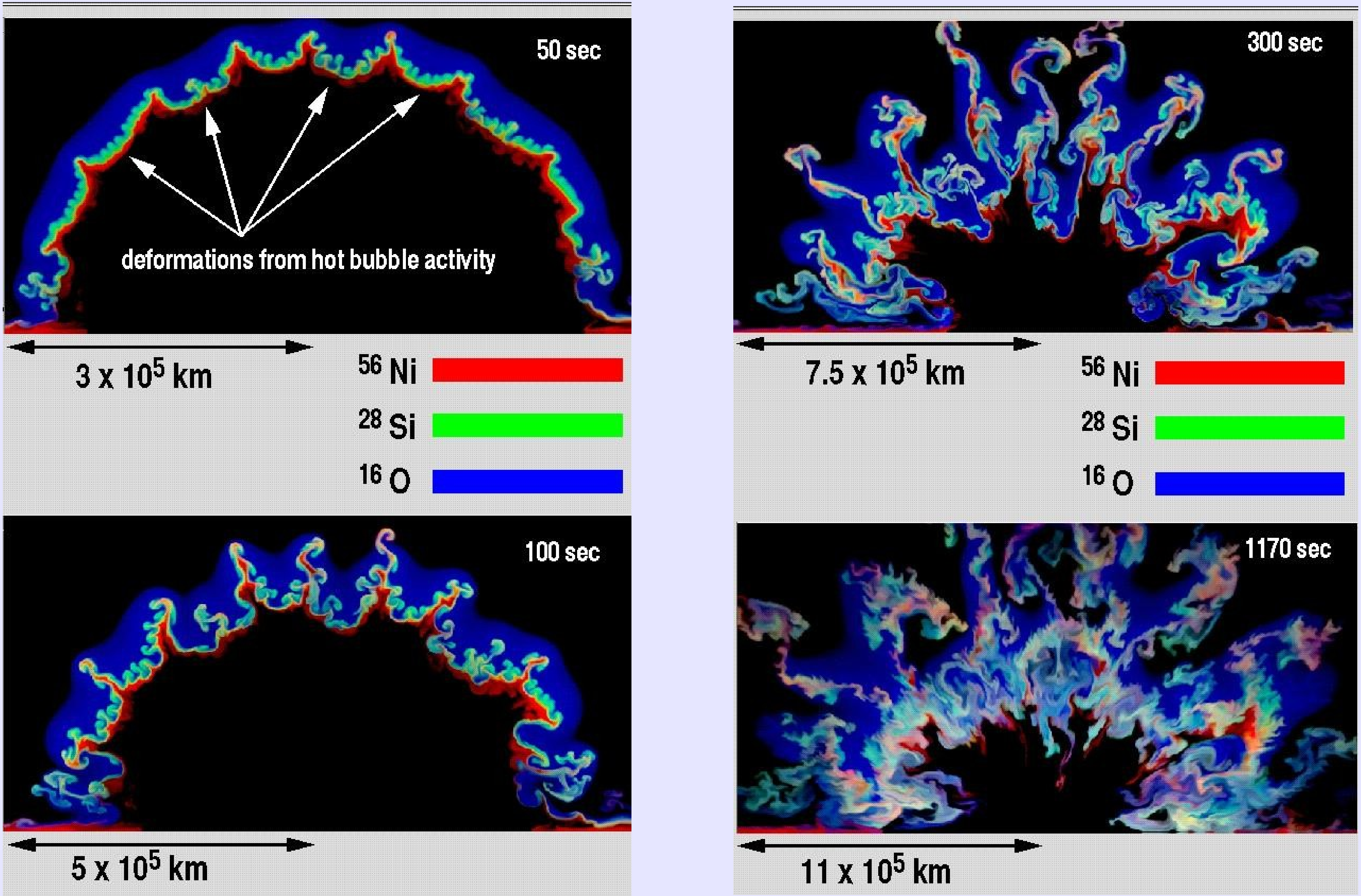
log (density), 4 s post-bounce


compression of metal core

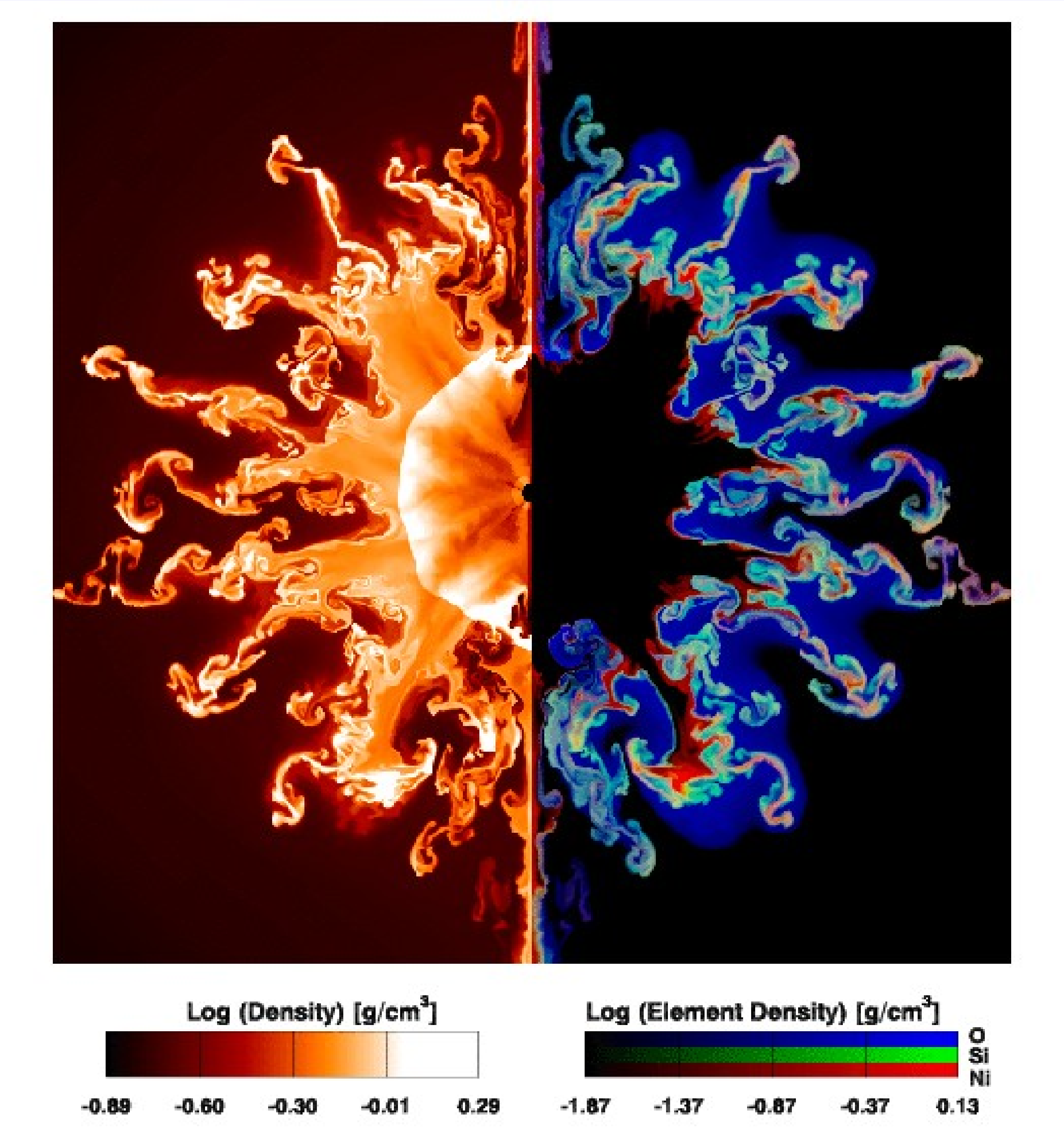
reverse shock

log (density), 20 s post-bounce

Rayleigh-Taylor instabilities & mixing in stellar envelope

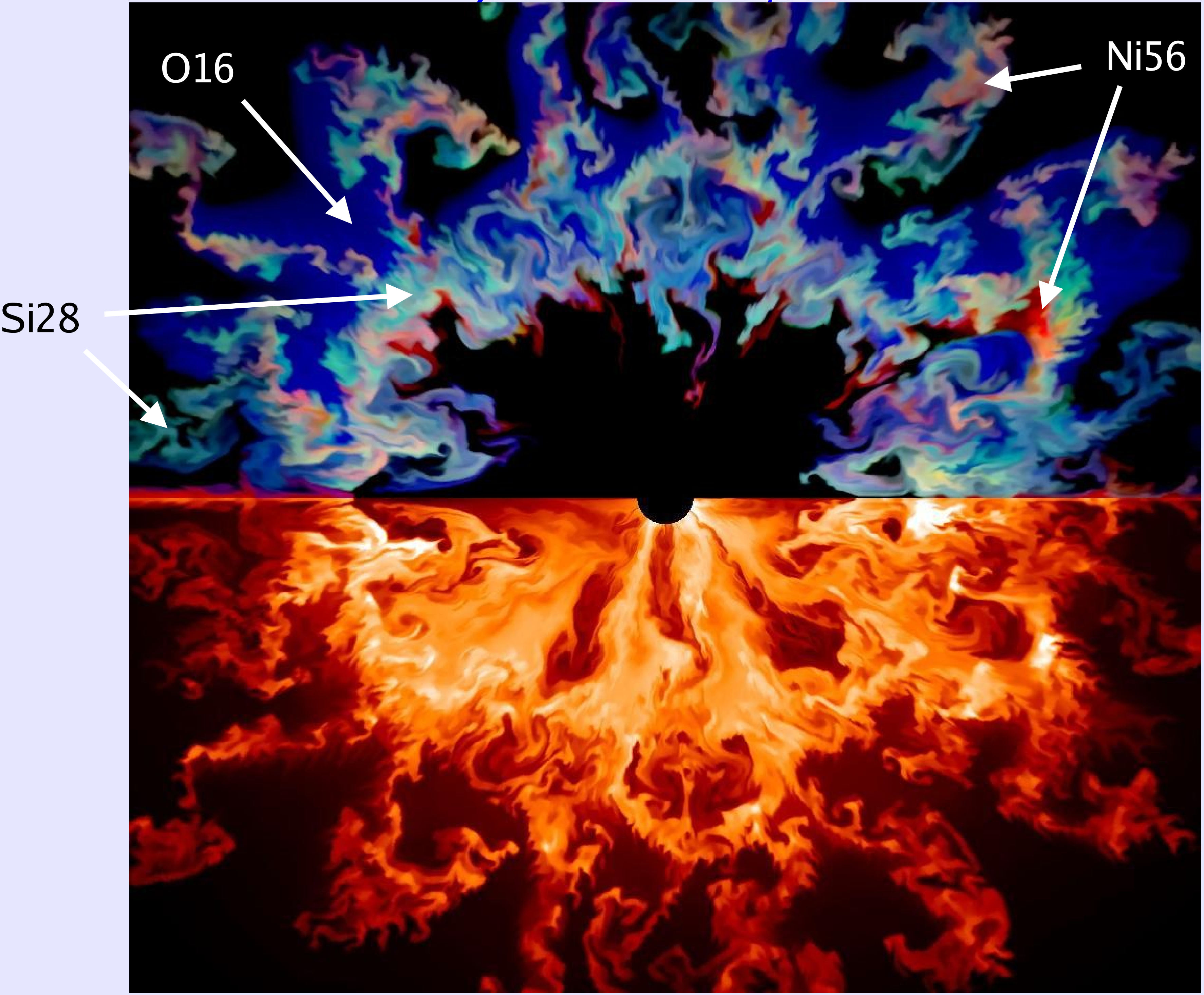


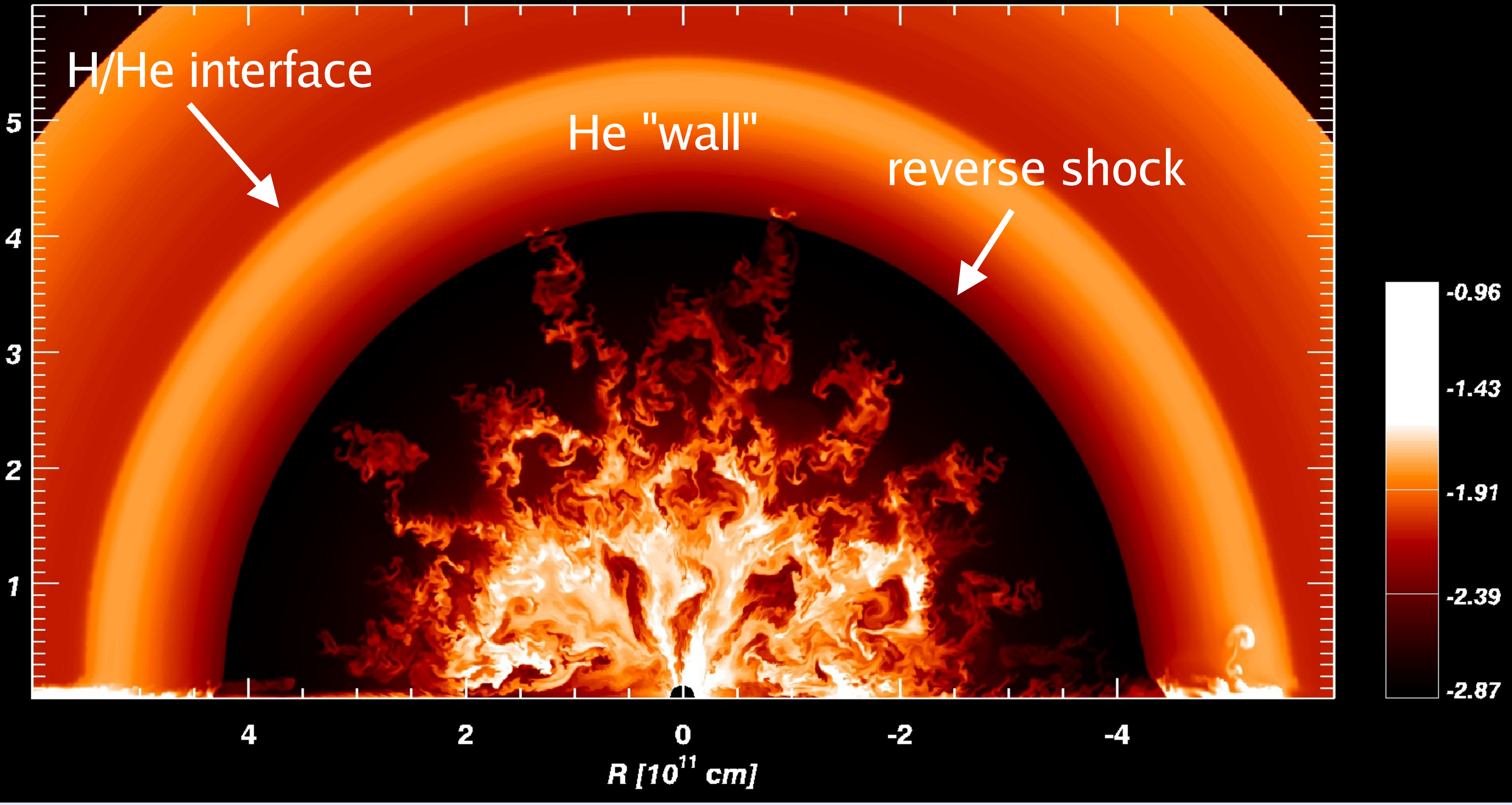
AMR simulation of shock propagation through stellar envelope (Kifonidis, Plewa, Janka & Müller 2003



density &
elements, 300 s

density & elements, 1170 s





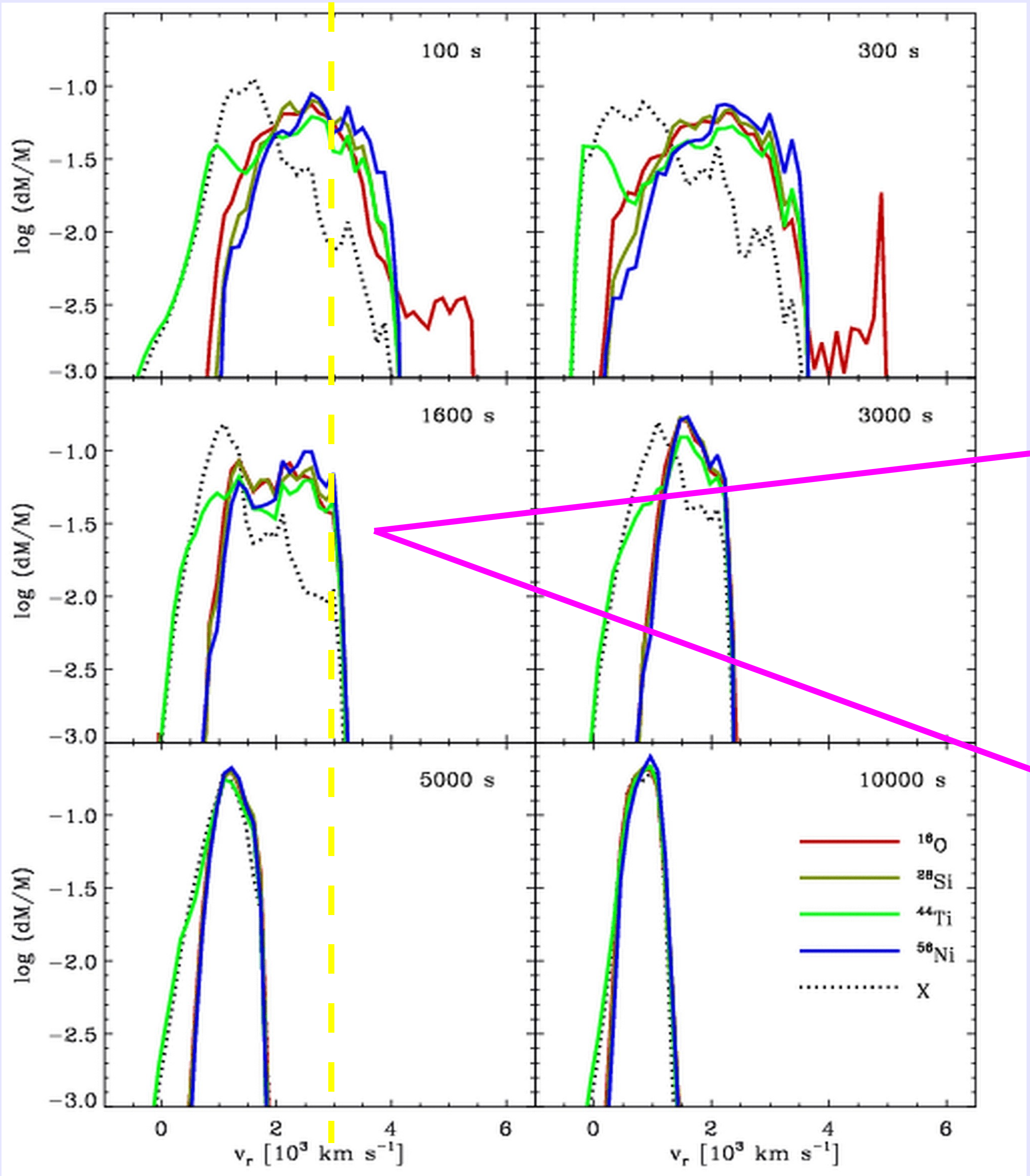
H/He interface

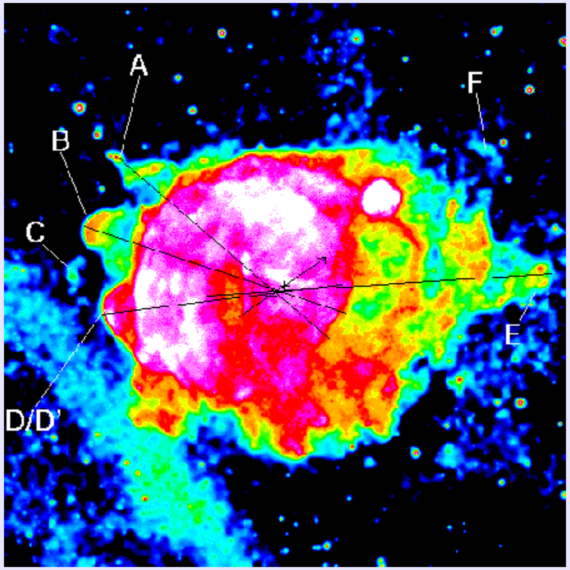
He "wall"

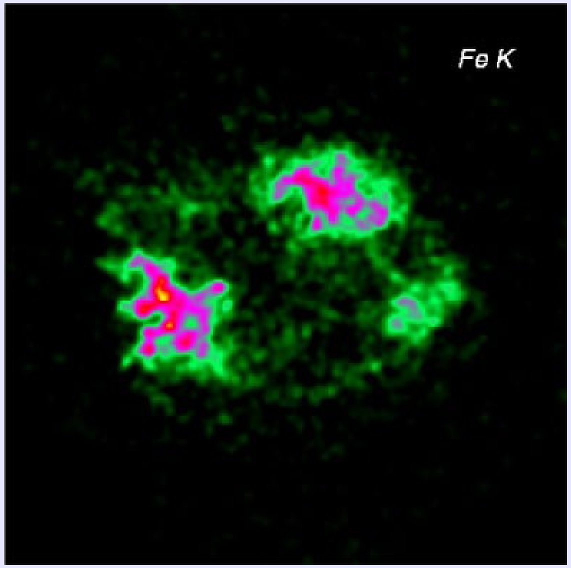
reverse shock

log (density), 1620 s post-bounce

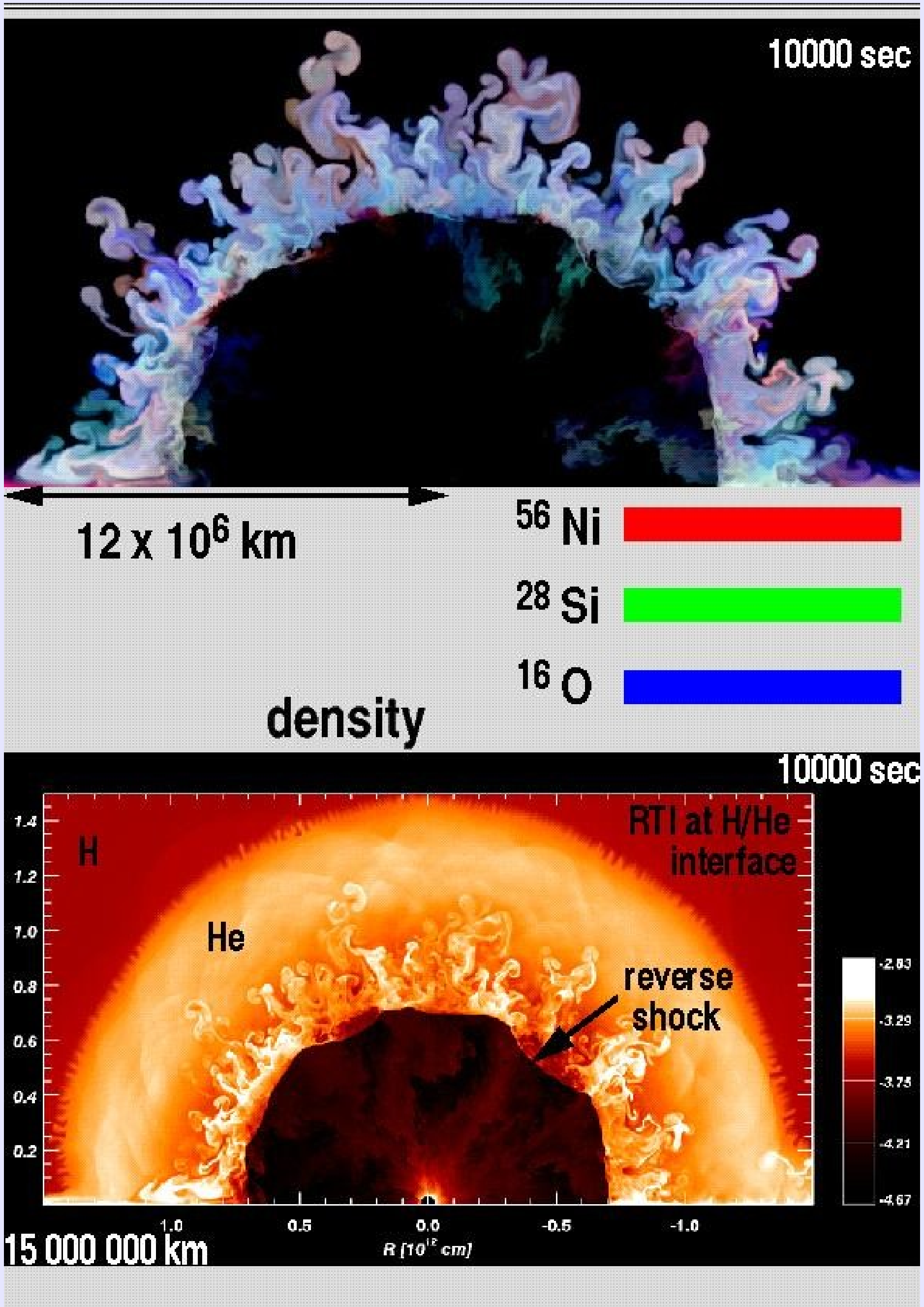
Evolution of the velocity distribution



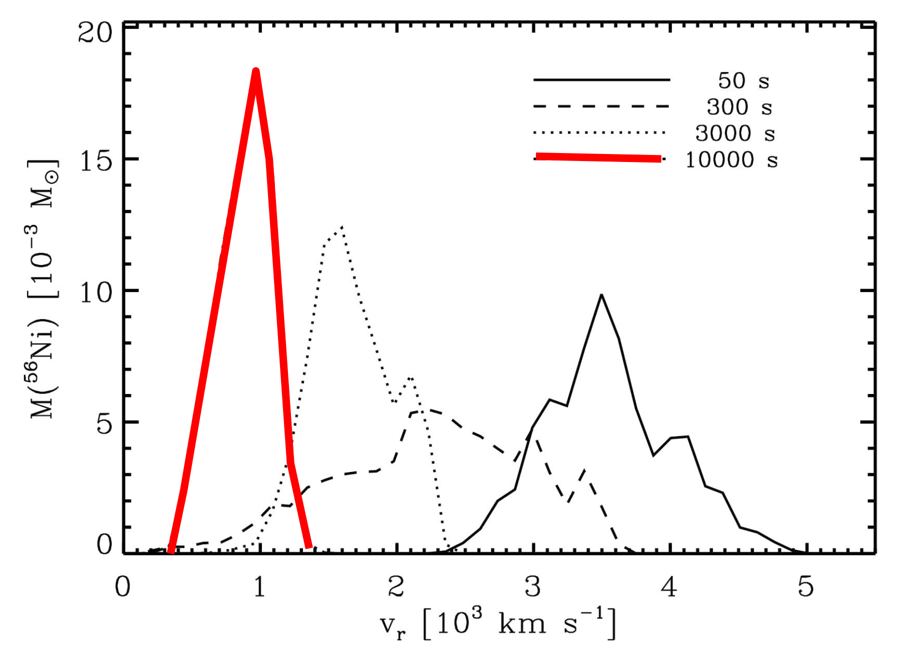




Instabilities, mixing and nucleosynthesis in enevelope



AMR simulation of shock propagation through stellar envelope
(Kifonidis, Plewa, Janka & Müller 2003



- results of simulations <u>in accordance</u> with observations of <u>SNe Ib/Ic</u>

- simulations <u>do not reproduce</u> large velocities of Fe/Ni observed in <u>SN 1987A</u>

Bochum event data matched?

"new" model (with low mode neutrino-driven convection) shows 40% higher initial metal clump velocities than "old" (high mode) model

--->

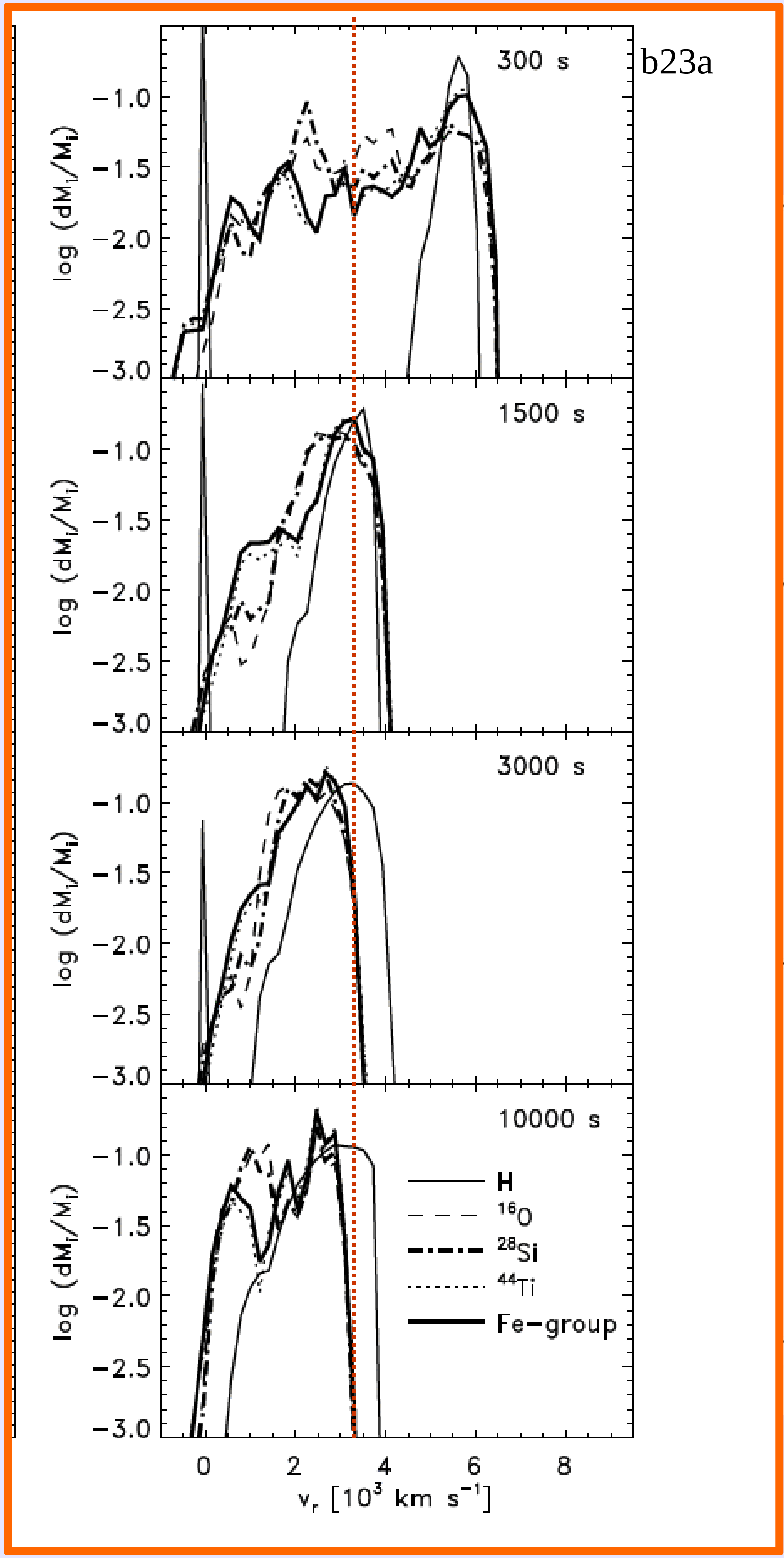
timescale for clump propagation through He-core shorter than timescale for reverse shock formation

--->

fastest clumps do not interact with reverse shock

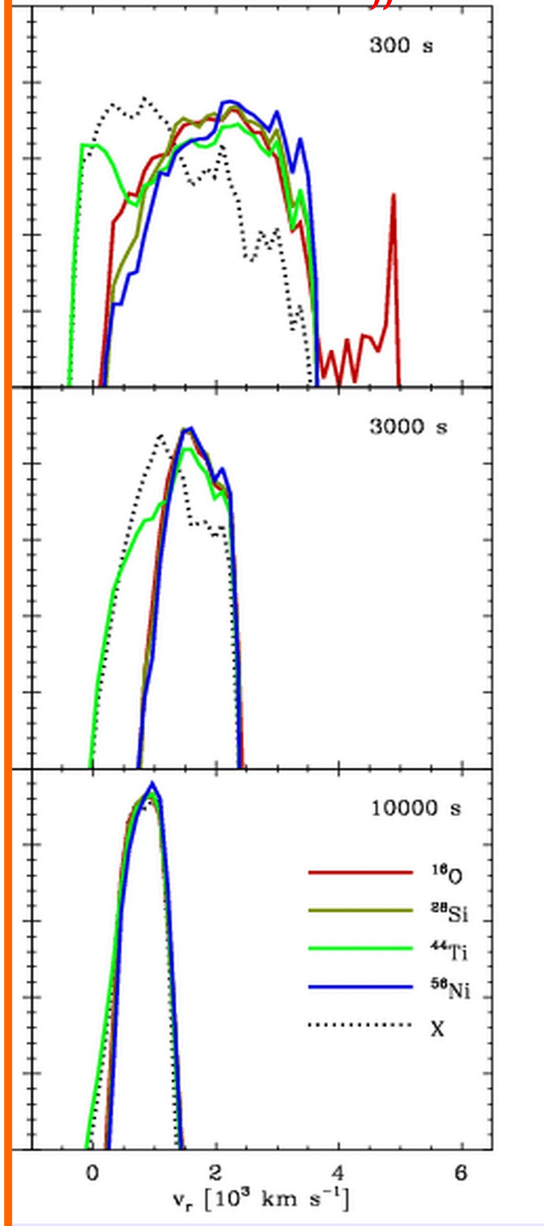
--->

no strong slow-down of clumps!



"new"

„old"



Collapse to black hole:

Fryer '99:

- fate of progenitor star (Scalo IMF)

$8\,M_\odot \leq M \leq 25\,M_\odot$ --> NS

$25\,M_\odot \leq M \leq 40\,M_\odot$ --> BH delayed (1.2%)

fall back (~min ... ~ hr ;
He shell τ_{hyd})

$40\,M_\odot \leq M$ --> BH directly (no SN ; 0.3%)

Collapse to black hole:

Baumgarte, Shapiro & Shibata '00:

supramassive NS: $M_{\text{rigid\_rot}} > M_{\text{non\_rot}}$

hypermassive NS: $M_{\text{diff\_rot}} > M_{\text{rigid\_rot}}$ (magn. braking: $\tau \sim 100$ s)

small B, fast rotator --> bar instability --> quasi-periodic GW signal

large B, slow rotator --> magnetic braking --> collapse to BH
--> quasi-normal modes

$\nu \sim 4$ kHz $(3M_\odot / M_{BH})$