Negative Komar Mass in regular stationary spacetimes

Marcus Ansorg[1], David Petroff[2]

[1] Max-Planck-Institut für Gravitationsphysik,

Albert-Einstein-Institut, Golm, Germany

[2] Theoretisch-Physikalisches-Institut,

Friedrich-Schiller-Universität, Jena, Germany

Plan of the Talk

1. Introduction
2. The Komar mass
3. Journey into the realm of negative Komar mass
4. Summary

1. Introduction

- We consider a self-gravitating system consisting of a uniformly rotating, homogeneous perfect fluid ring and a central object, being either a black hole or a disk of dust.
- Axisymmetry and stationarity are described by Killing vectors

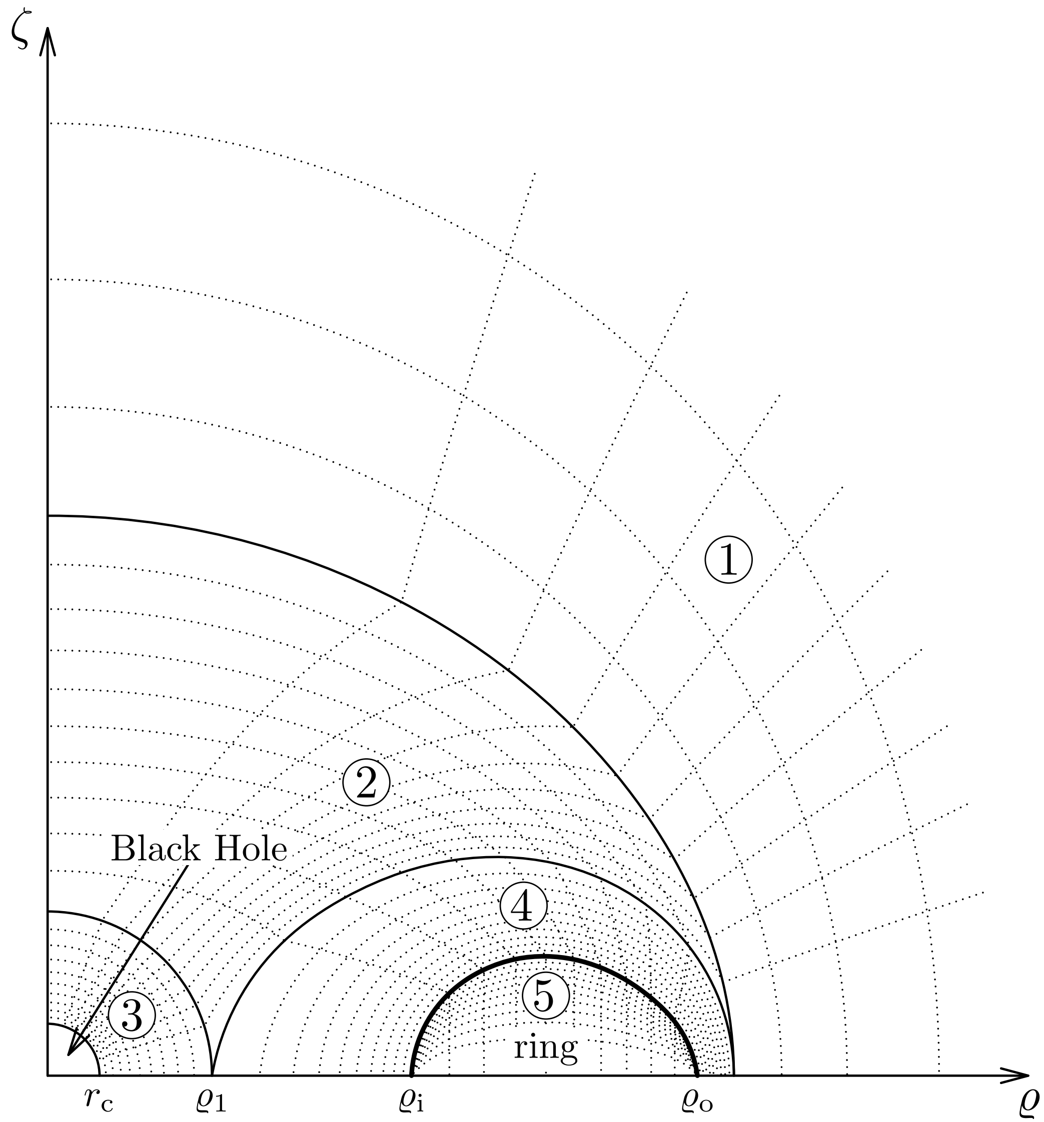
  $$\eta^i \quad \text{and} \quad \xi^i$$

- Line element in Weyl-Lewis-Papapetrou coordinates:

  $$ds^2 = -e^{2\nu}\,dt^2 + \varrho^2 B^2 e^{-2\nu}\,(d\varphi - \omega\,dt)^2 + e^{2\lambda}\left(d\varrho^2 + d\zeta^2\right)$$

- For the metric funtions ν, B, ω, λ we solve the corresponding free boundary value problem with spectral methods.
- The presence of the ring can affect the properties of the central object drastically.
- We illustrate the ring's influence by tracing paths along which the 'Komar' mass of the central object becomes negative.

1. Introduction



The division of the ϱ-ζ plane into the domains used in the spectral methods ($\varrho_i/\varrho_o = 0.56$ and $r_c/\varrho_o = 0.08$).

2. The Komar mass (1)

- Poisson equation in Newtonian gravity

$$\nabla \cdot (\nabla U) = 4\pi\mu$$

- A mass can be assigned to any subregion $V \subseteq \mathbb{R}^3$

$$M(V) = \int_V \mu d^3x = \frac{1}{4\pi} \oint_{\partial V} \nabla U d\vec{f}$$

- Consequences:

$$\begin{aligned} 1) \quad M(V) &= 0 \quad \text{if } V \text{ is a vacuum region with } \mu = 0 \\ 2) \quad M_{\text{total}} &= M(\mathbb{R}^3) = -\lim_{r \longrightarrow \infty} (rU) \end{aligned}$$

2. The Komar mass (2)

- Specific Einstein equation in axisymmetry and stationarity:

$$\nabla \cdot \left(B\nabla\nu - \frac{\omega}{2}\varrho^2 B^3 e^{-4\nu}\nabla\omega \right) = 4\pi\tilde{\mu}(\mu, p; \lambda, \nu, B, \omega)$$

- A 'Komar' mass can be assigned to any subregion $V \subseteq \mathbb{R}^3$

$$M(V) = \int_V \tilde{\mu} d^3x = \frac{1}{4\pi} \oint_{\partial V} \left(B\nabla\nu - \frac{\omega}{2}\varrho^2 B^3 e^{-4\nu}\nabla\omega \right) d\vec{f}$$

- Consequences:

1) $M(V) = 0$ if V is a vacuum region with $\mu = 0$

2) $M_{\text{ADM}} = M(\mathbb{R}^3) = -\lim_{r\to\infty}(r\nu)$

- Define the Komar mass of a black hole as surface integral over an arbitrary boundary ∂V where V contains only the black hole.

2. The Komar mass (3)

- **Question:** Is the black hole's Komar mass always positive ?
- Analysis by means of the 'Smarr' formula (Bardeen, Carter):

$$M_{\mathrm{h}} = \frac{\kappa}{4\pi} A_{\mathrm{h}} + 2\Omega_{\mathrm{h}} J_{\mathrm{h}}$$

- The surface gravity κ and horizon area A_{h} are always positive, but the product can approach zero.
- The ring can cause a 'frame dragging' of the black hole such that its angular velocity Ω_{h} and angular momentum J_{h} assume different signs.
- Requirement: highly relativistic rotating rings, characterized by a large ergosphere (a portion of space in which $\xi^i \xi_i > 0$).
- Can the negative term $2\Omega_{\mathrm{h}} J_{\mathrm{h}}$ dominate over $\kappa A_{\mathrm{h}}/(4\pi)$?
- **Answer:** Yes! The Komar mass of such black holes is negative.

2. The Komar mass (4)

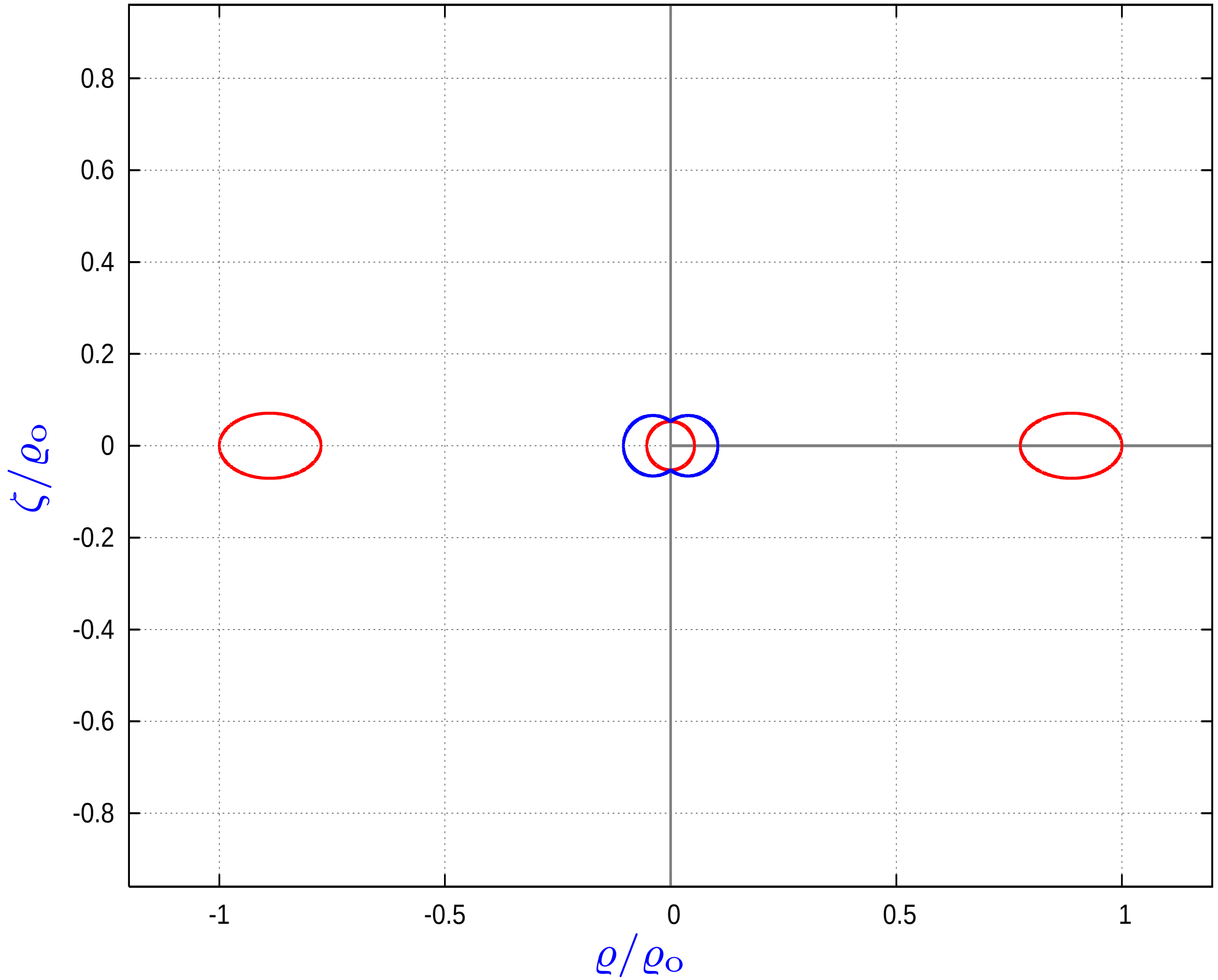
- **Question:** Is the central disk's Komar mass always positive ?
- Analysis by means of the Disk-'Smarr' formula (Bardeen):

$$M_{\rm d} = e^{V_0} M_0 + 2\Omega_{\rm d} J_{\rm d}$$

- The redshift $Z_{\rm d} = e^{-V_0} - 1$ and the *baryonic* mass M_0 are always positive, but the product $e^{V_0} M_0$ can approach zero.
- Again, a 'frame dragging' caused by the ring can lead to different signs of the disk's angular velocity $\Omega_{\rm d}$ and its angular momentum $J_{\rm d}$.
- Requirement: highly relativistic rotating rings, characterized by a large ergosphere (a portion of space in which $\xi^i \xi_i > 0$).
- Can the negative term $2\Omega_{\rm d} J_{\rm d}$ dominate over $e^{V_0} M_0$?
- Answer: Yes! The Komar mass of such dust disks is negative.

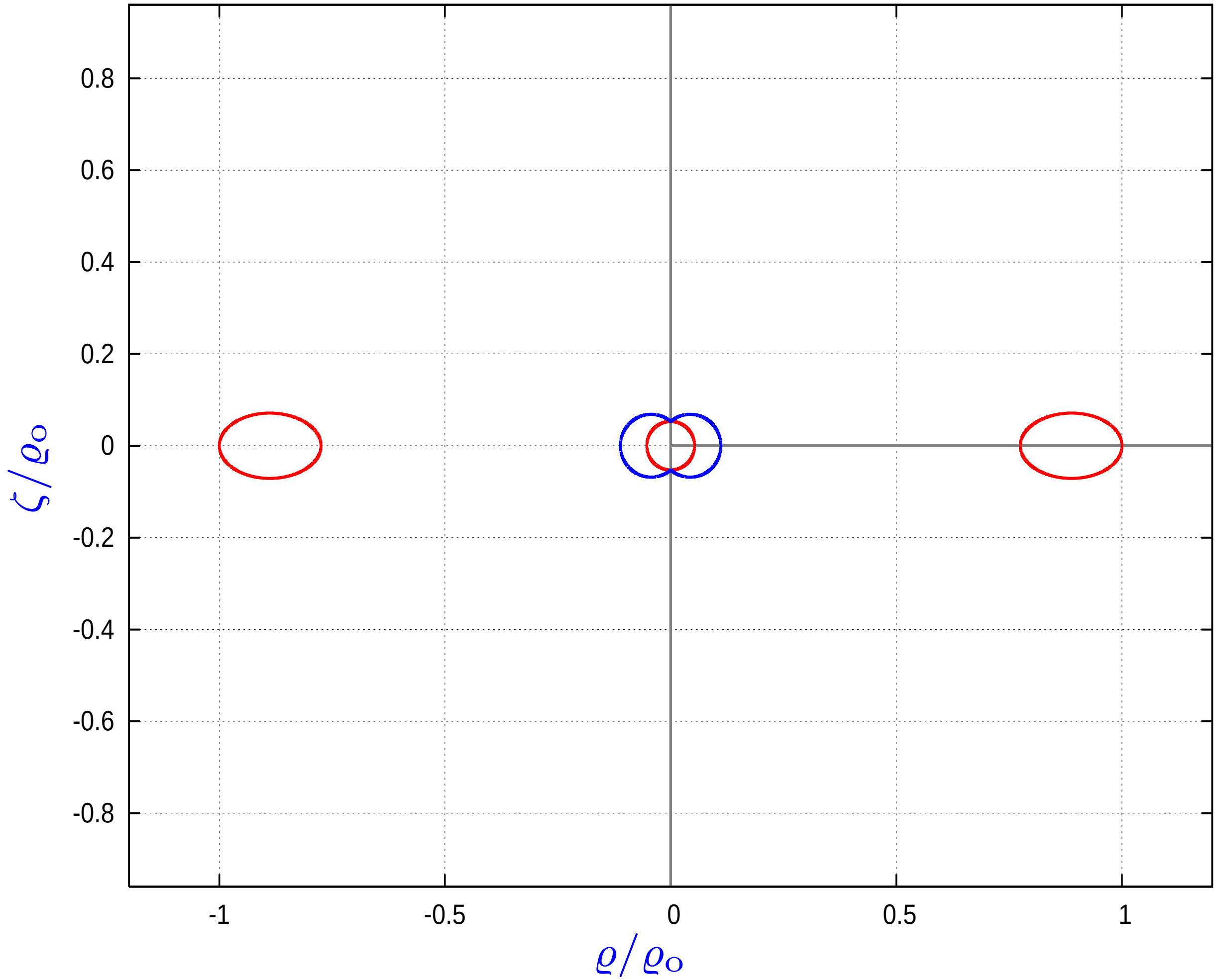
3. Journey

$$M_\mathrm{h}/M_\mathrm{r} = 0.89\,,\ Z_\mathrm{r} = 0.65$$



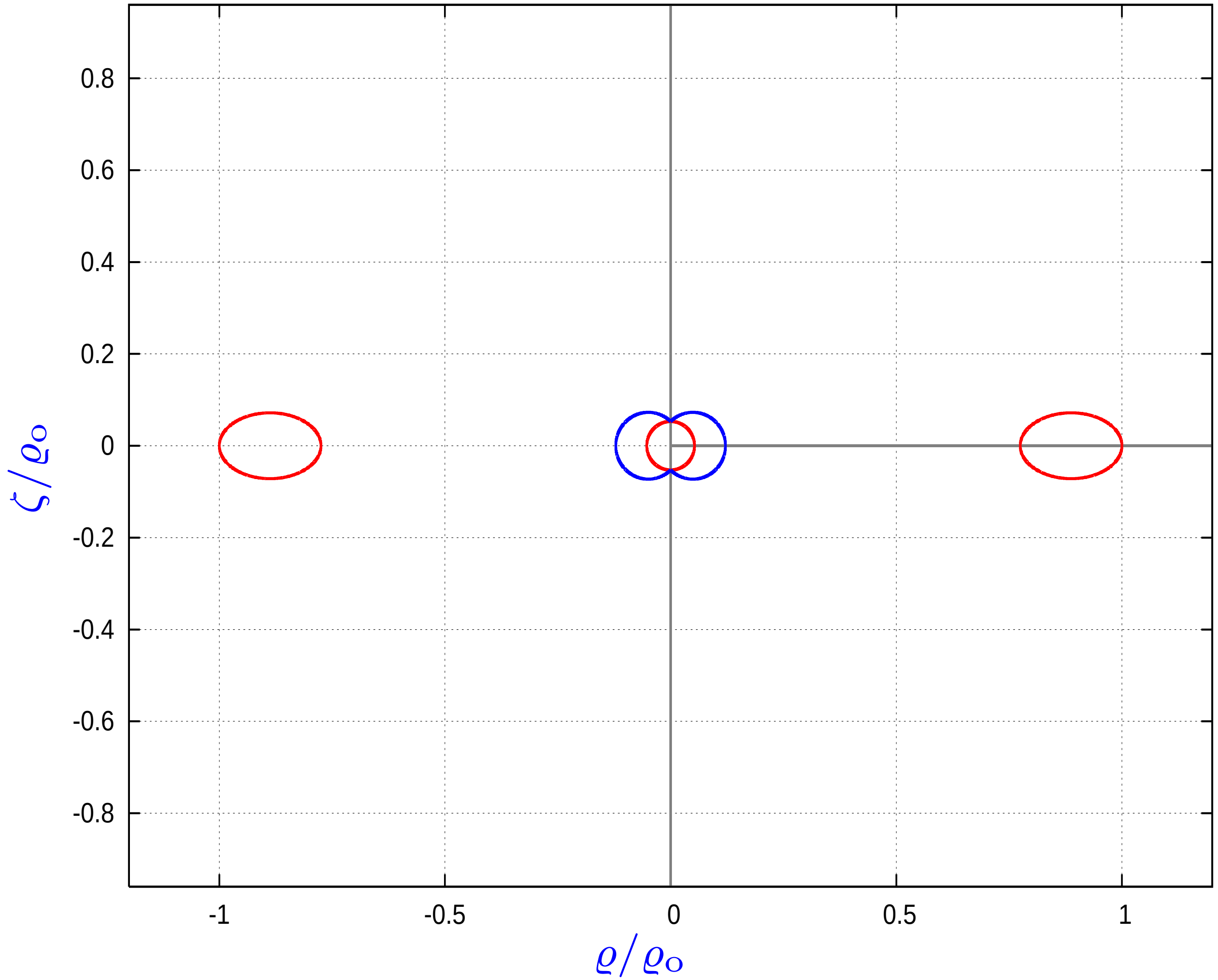
3. Journey

$$M_{\mathrm{h}}/M_{\mathrm{r}} = 0.78\,,\ Z_{\mathrm{r}} = 0.75$$



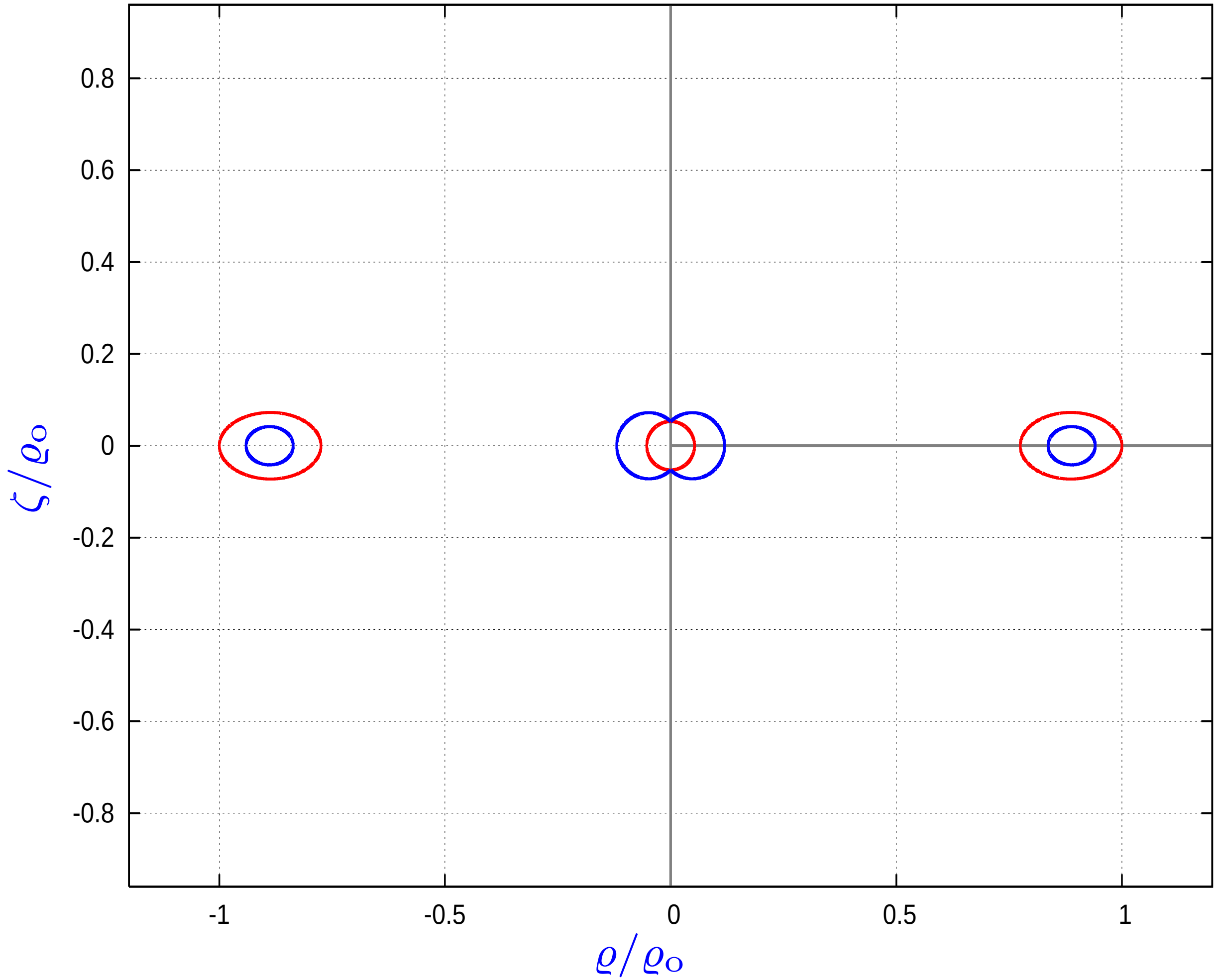
3. Journey

$$M_{\mathrm{h}}/M_{\mathrm{r}} = 0.53\,,\ Z_{\mathrm{r}} = 1.1$$



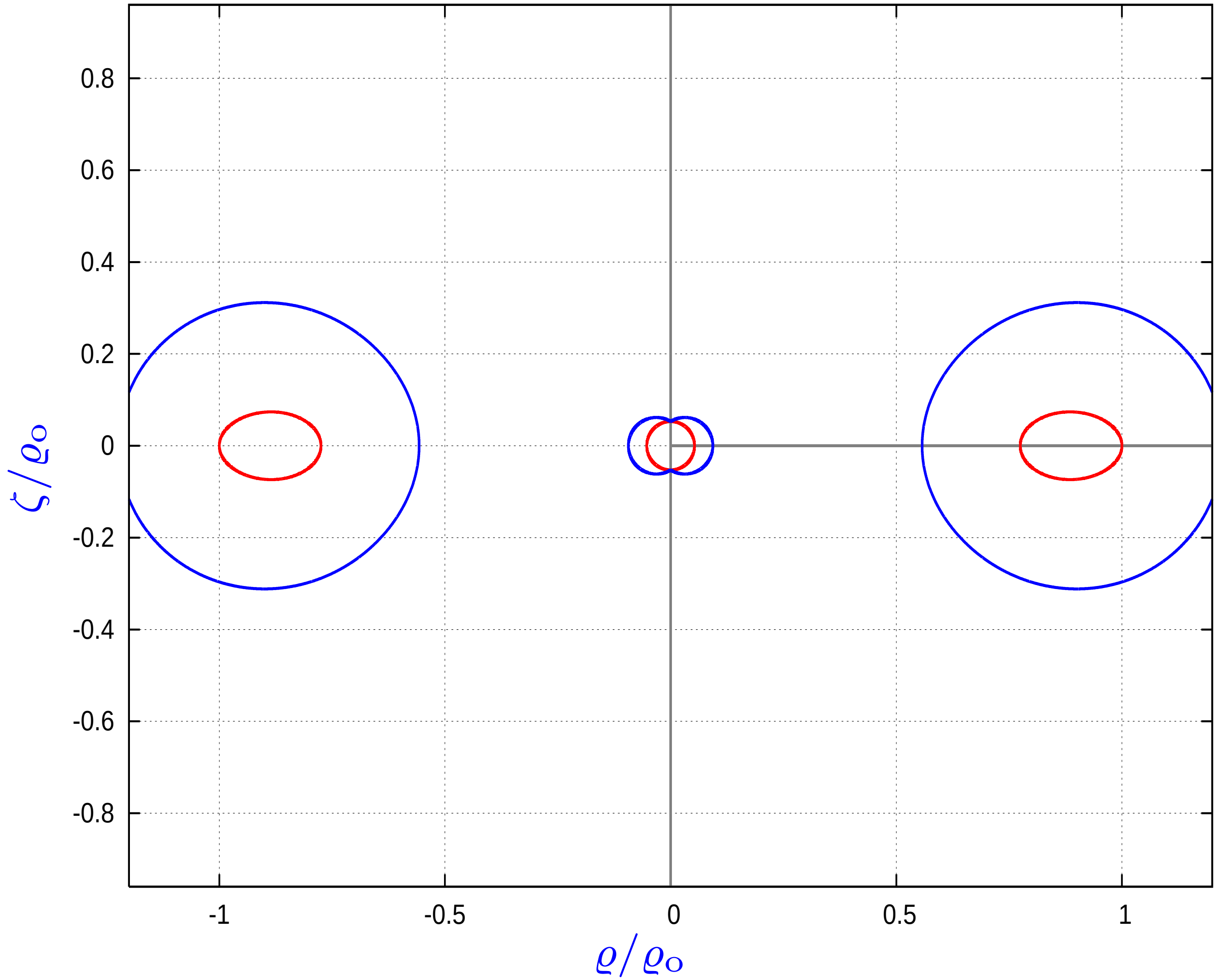
3. Journey

$$M_{\rm h}/M_{\rm r} = 0.33\,,\ Z_{\rm r} = 1.6$$



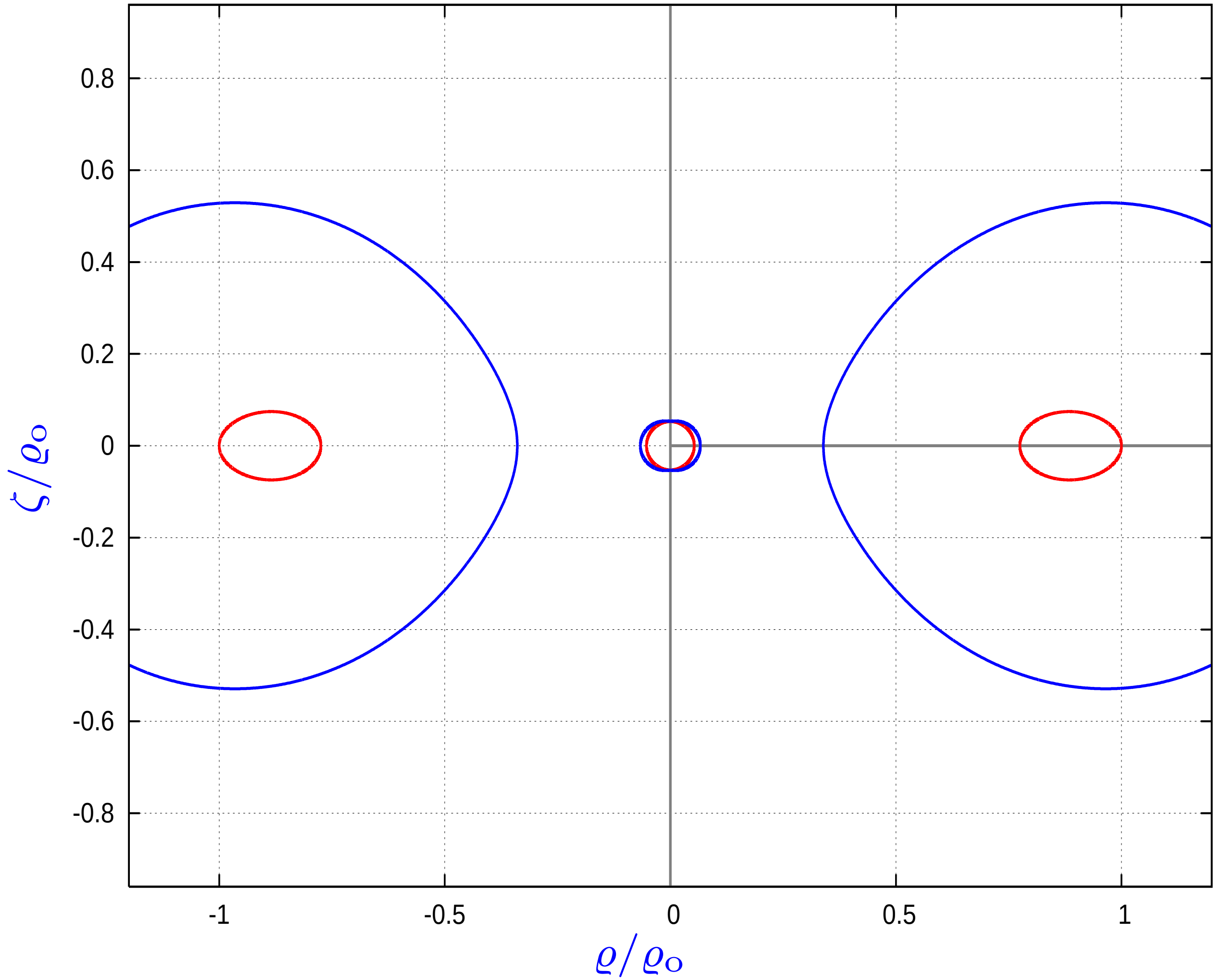
3. Journey

$$M_{\rm h}/M_{\rm r} = 0.16\,,\ Z_{\rm r} = 2.7$$



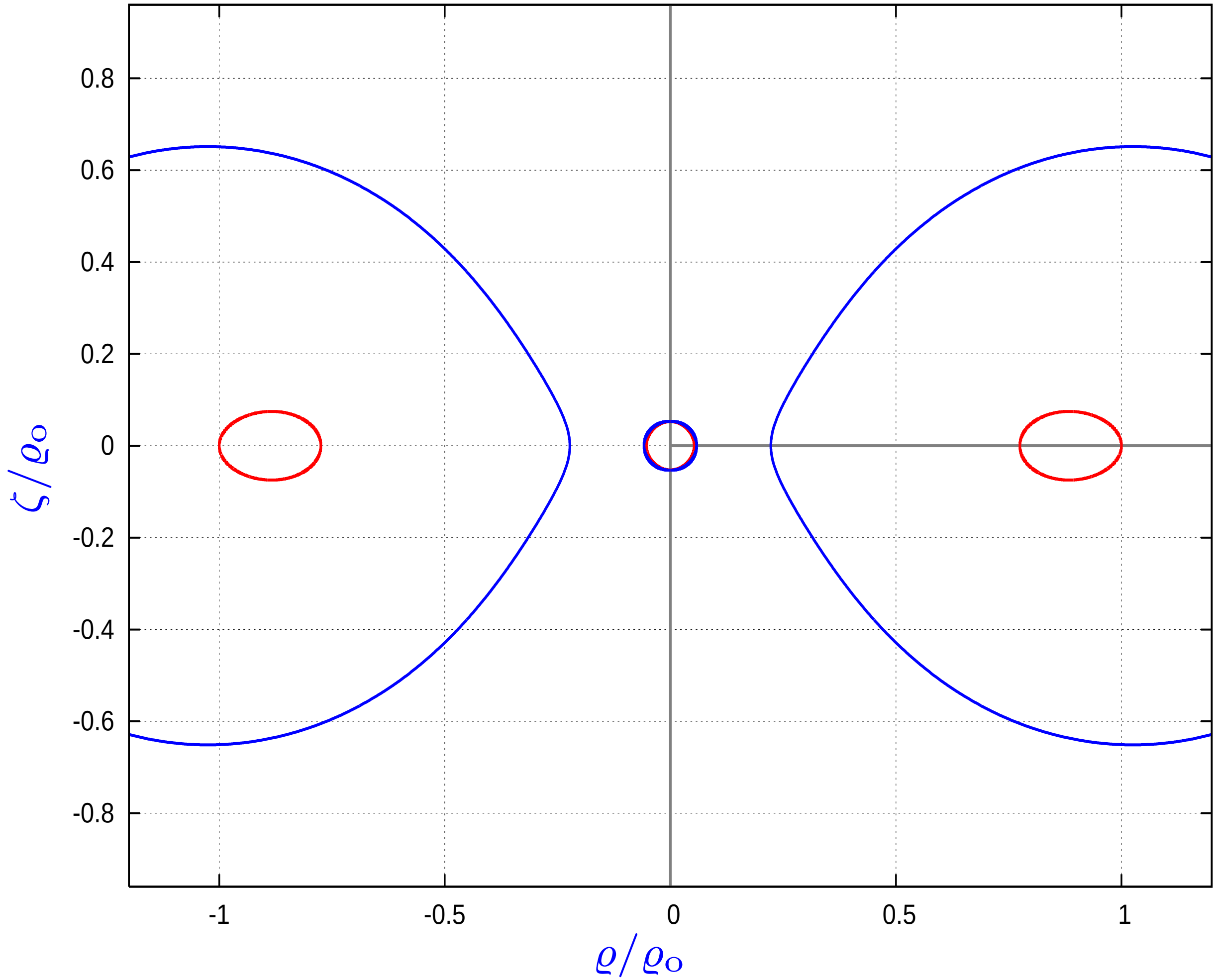
3. Journey

$$M_{\rm h}/M_{\rm r} = 0.094\,,\ Z_{\rm r} = 3.6$$



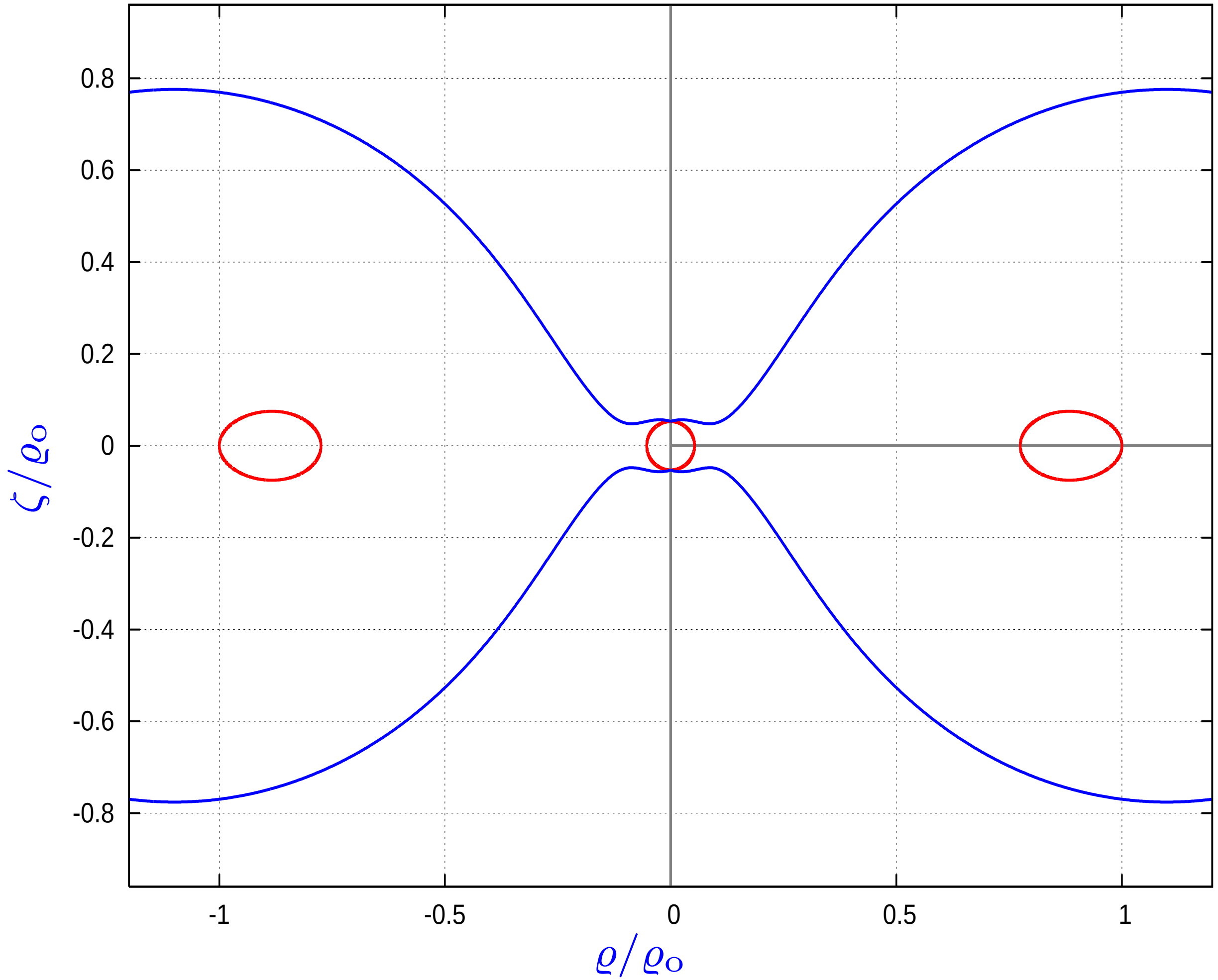
3. Journey

$$M_{\mathrm{h}}/M_{\mathrm{r}} = 0.069\,,\ Z_{\mathrm{r}} = 4.2$$



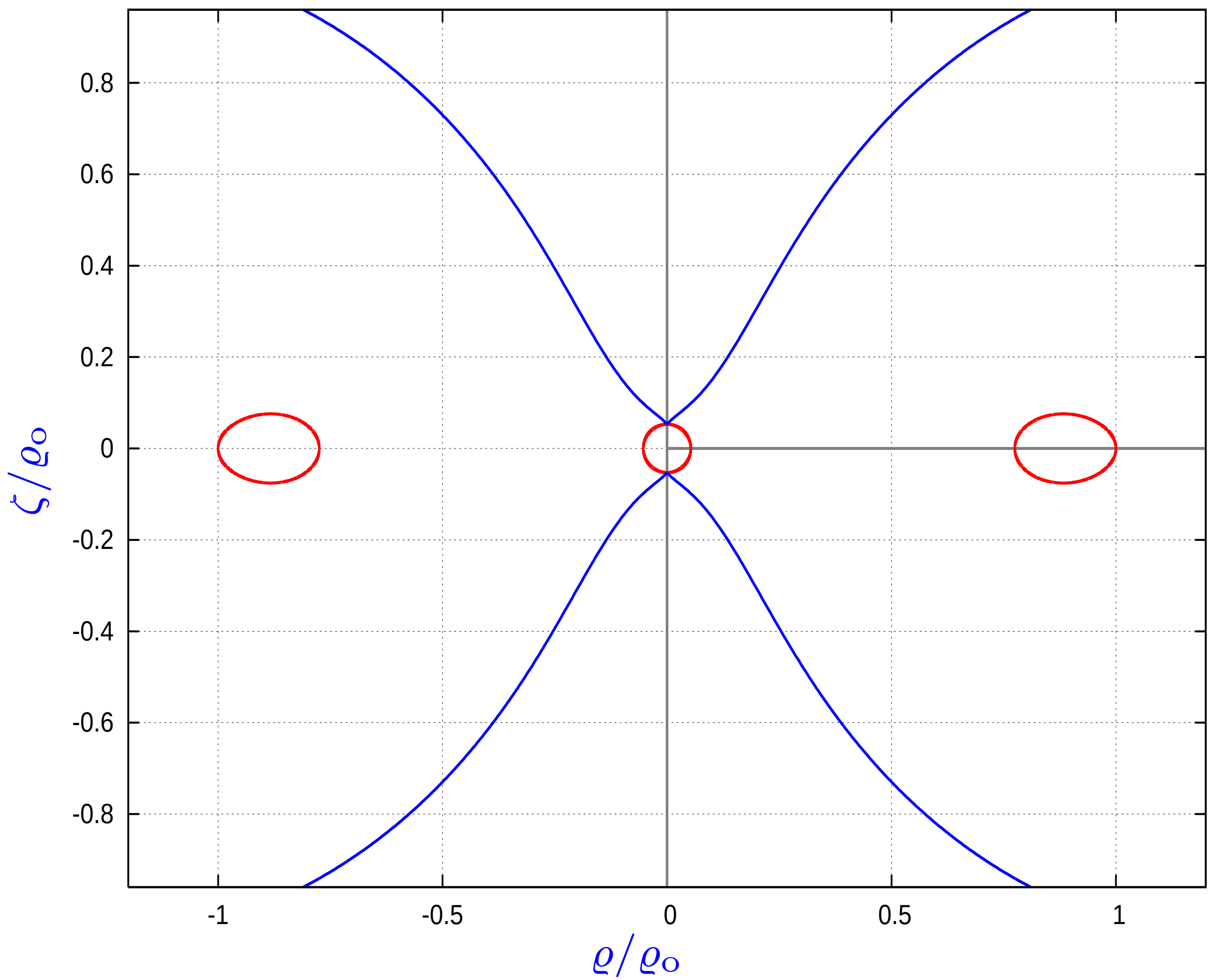
3. Journey

$$M_{\rm h}/M_{\rm r} = 0.048\,,\; Z_{\rm r} = 4.8$$



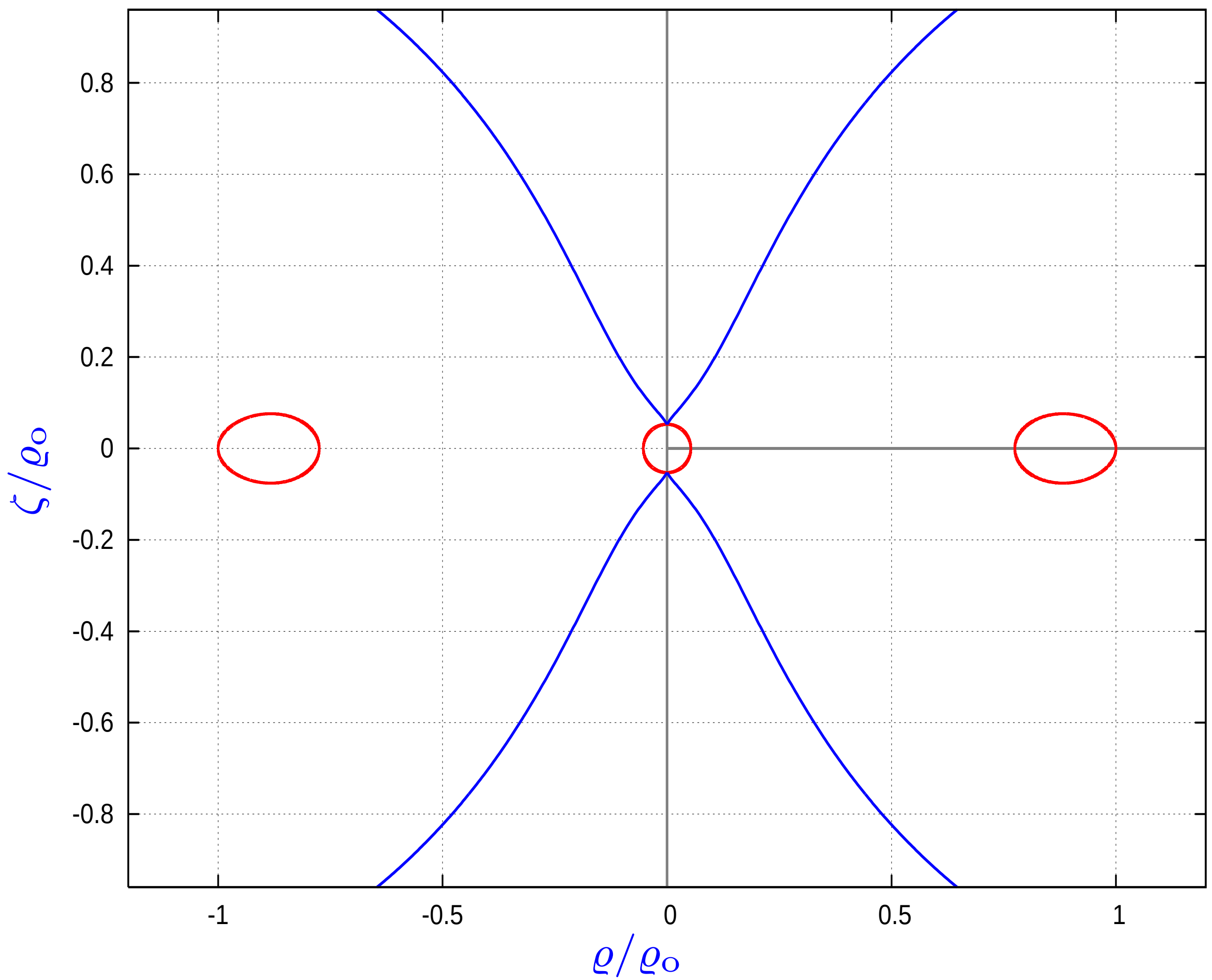
3. Journey

$$M_h/M_r = 0.013\,,\ Z_r = 6.4$$



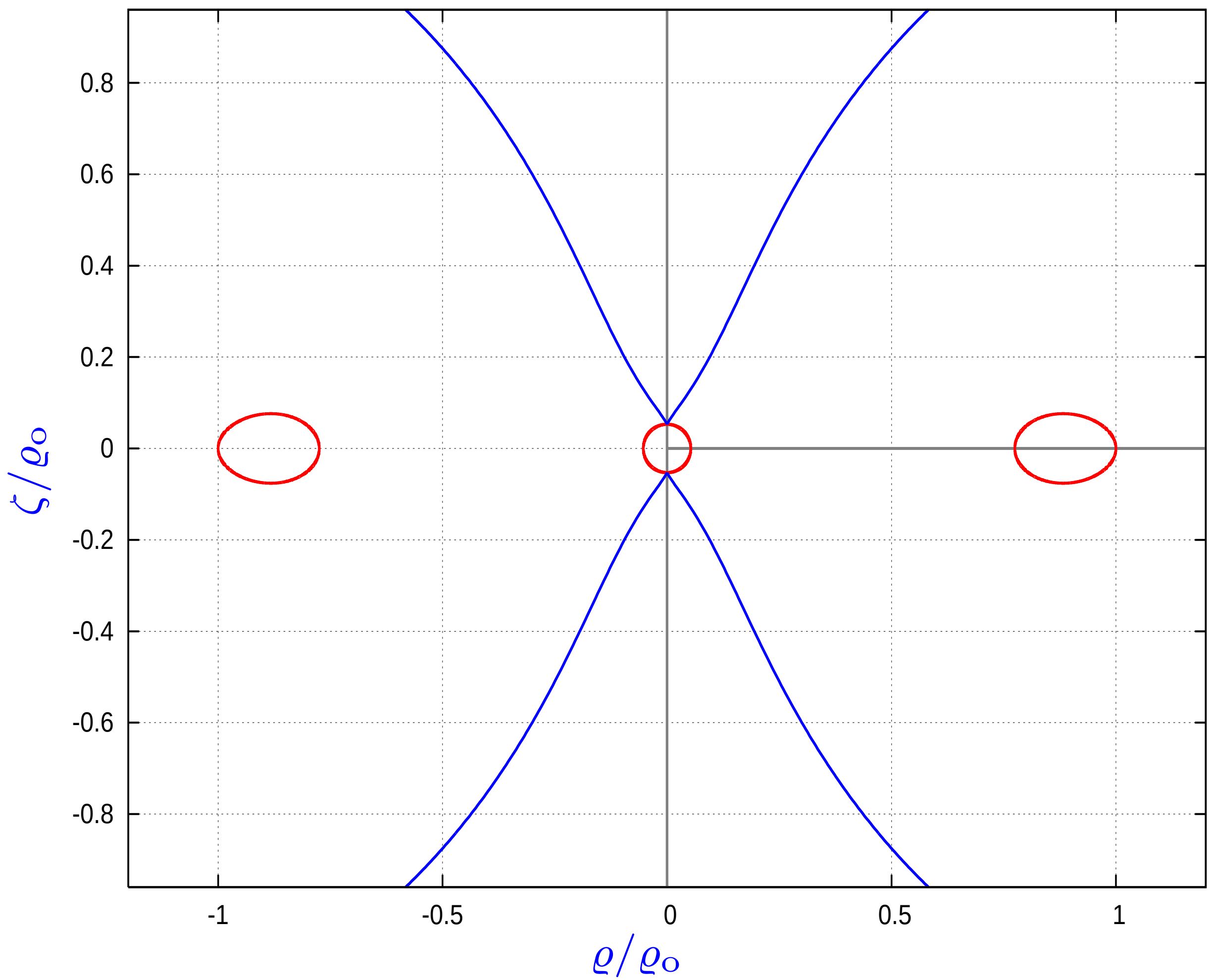
3. Journey

$$M_{\rm h}/M_{\rm r} = 0.00070\,,\; Z_{\rm r} = 7.3$$



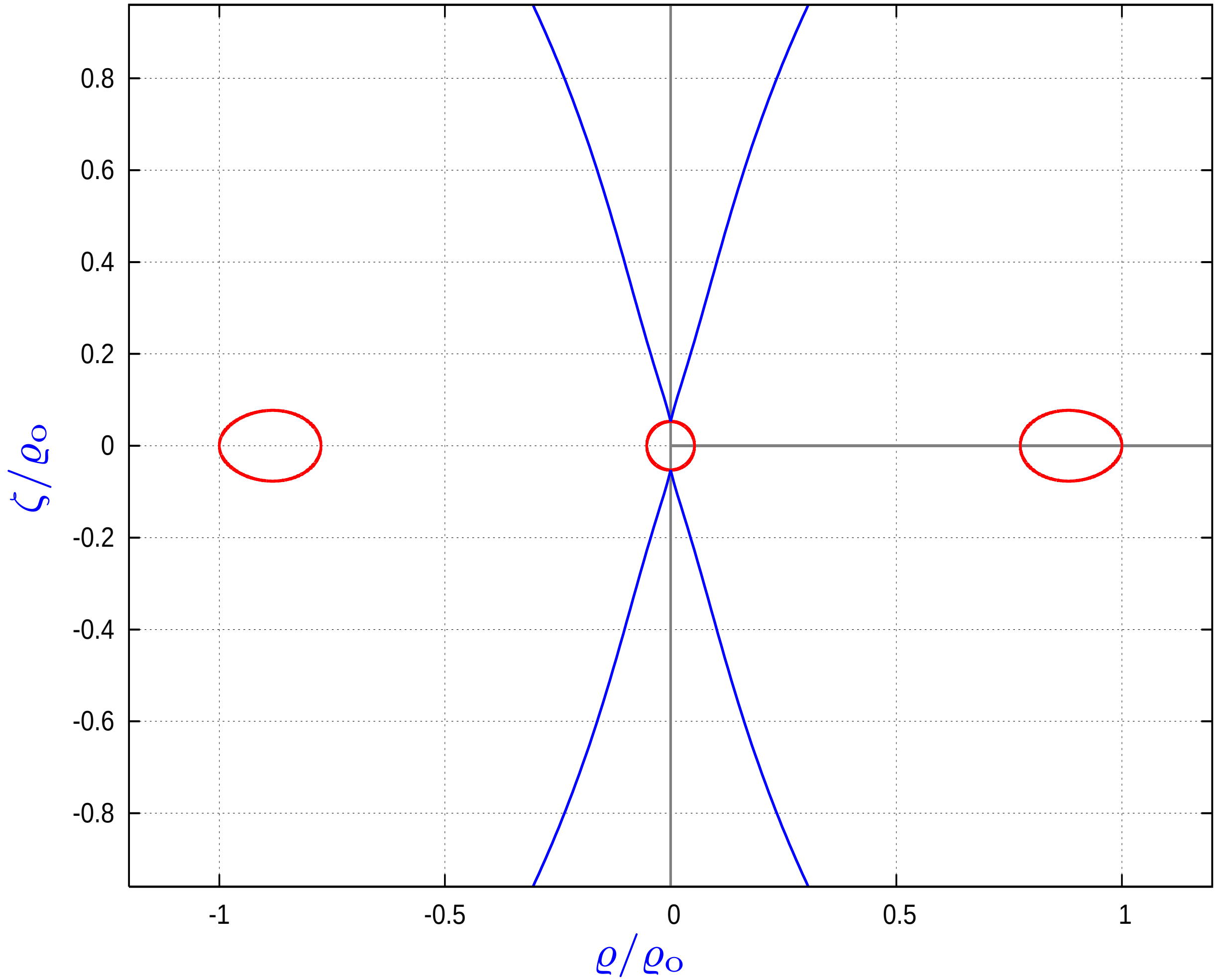
3. Journey

$$M_{\rm h}/M_{\rm r} = -0.0055\,,\; Z_{\rm r} = 7.8$$



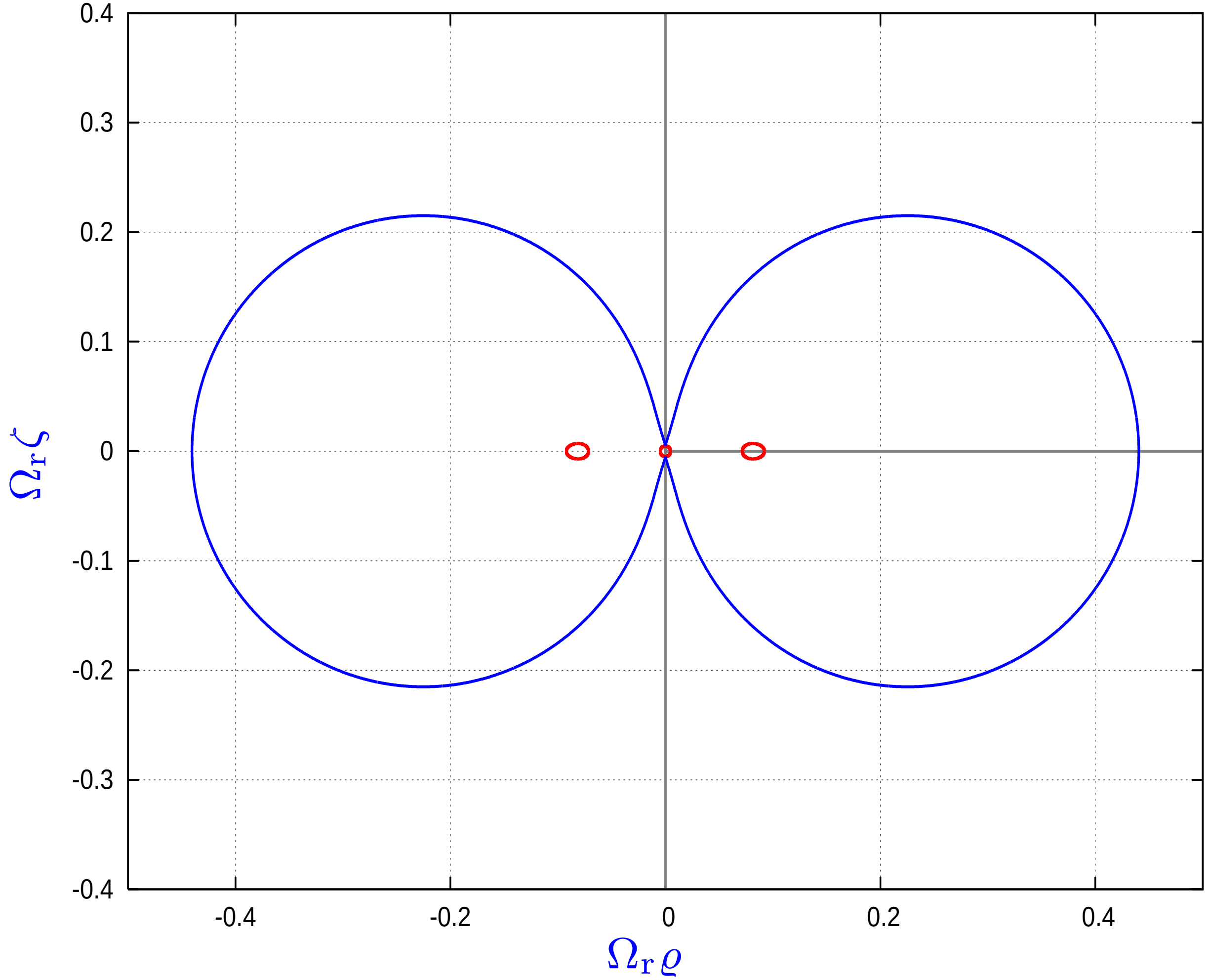
3. Journey

$$M_{\rm h}/M_{\rm r} = -0.04\,,\ Z_{\rm r} = 13$$



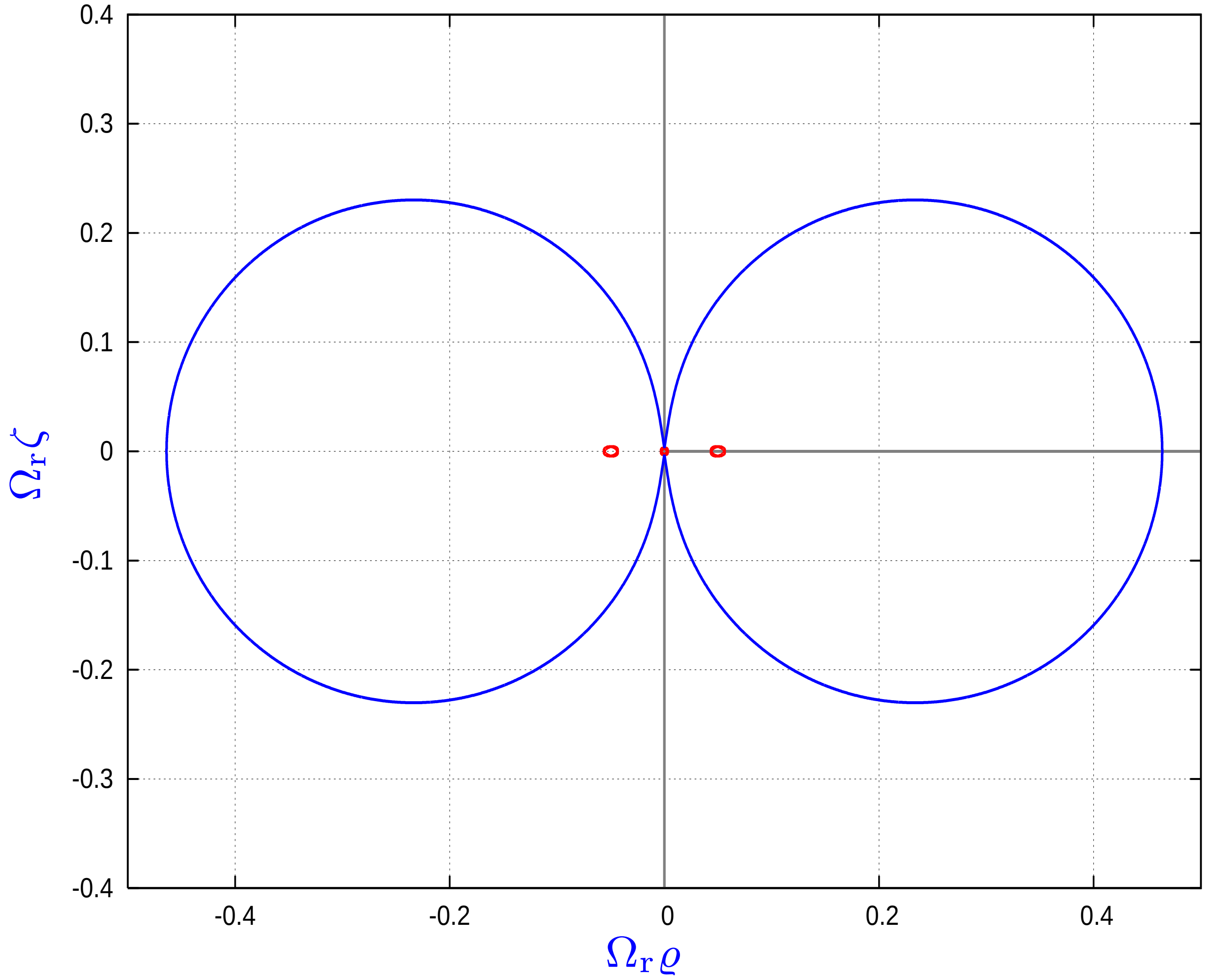
3. Journey

$$M_\mathrm{h}/M_\mathrm{r} = -0.04\,,\ Z_\mathrm{r} = 13$$



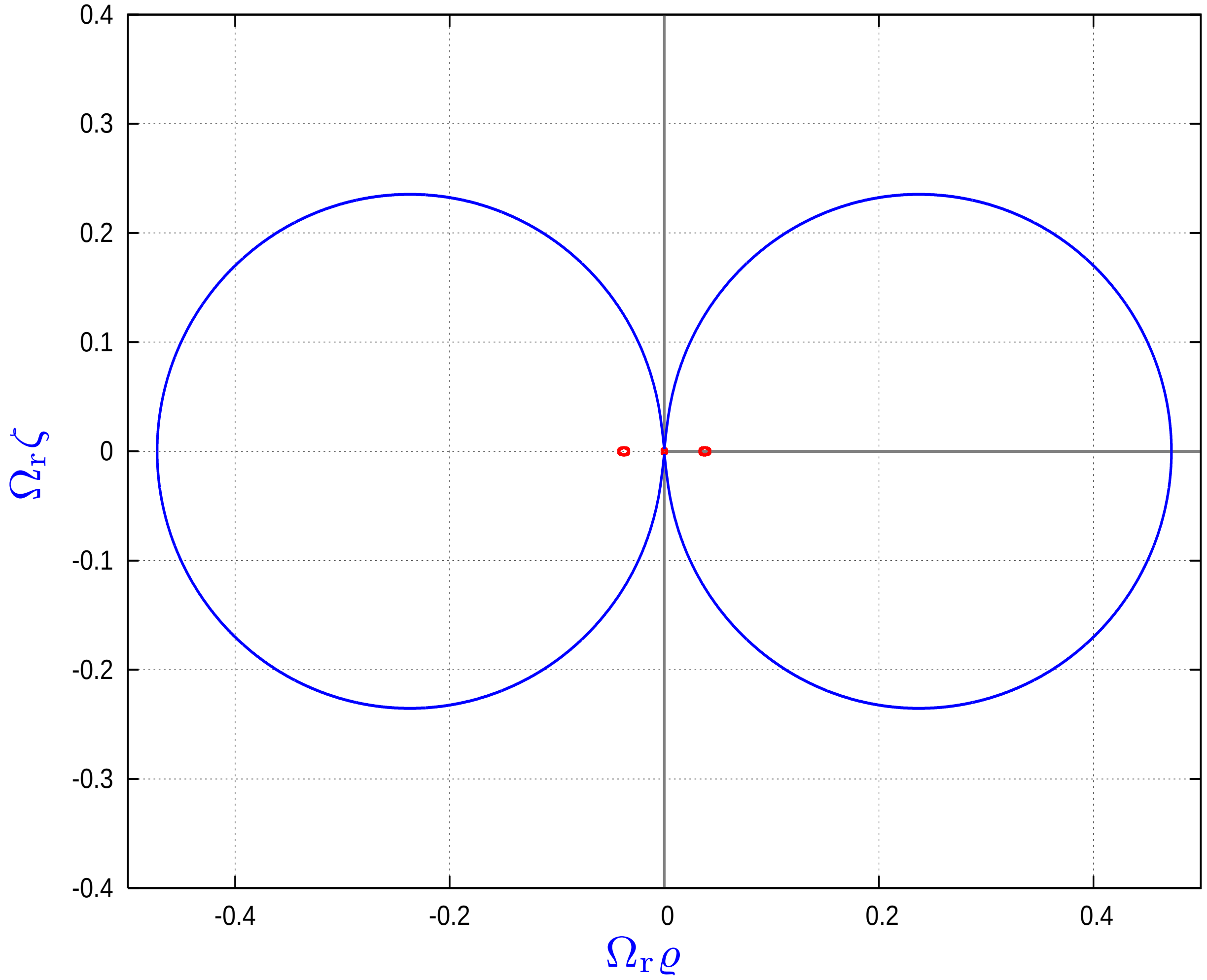
3. Journey

$$M_{\rm h}/M_{\rm r} = -0.060\,,\; Z_{\rm r} = 24$$



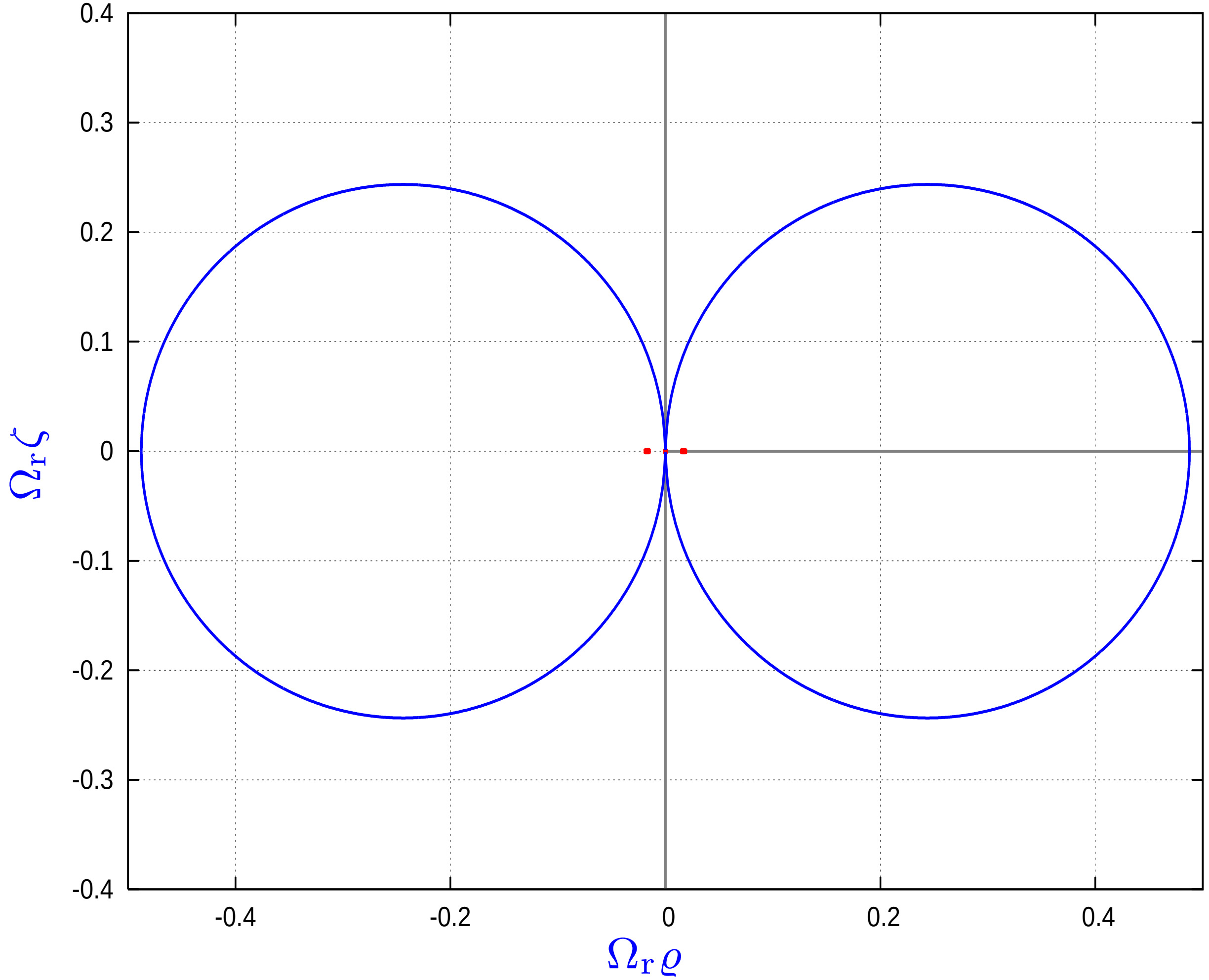
3. Journey

$$M_{\rm h}/M_{\rm r} = -0.067\,,\; Z_{\rm r} = 32$$



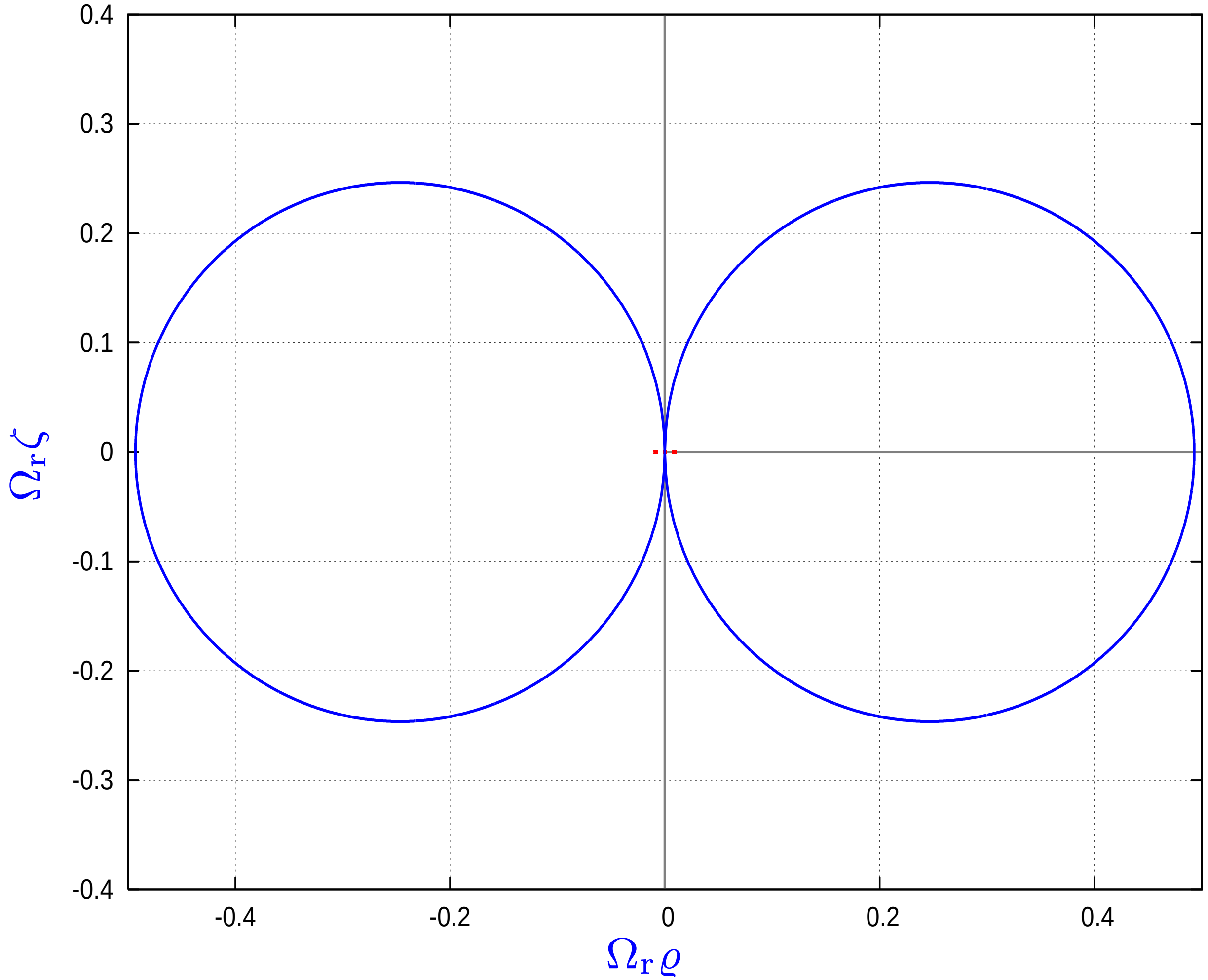
3. Journey

$$M_{\rm h}/M_{\rm r} = -0.077\,,\ Z_{\rm r} = 75$$

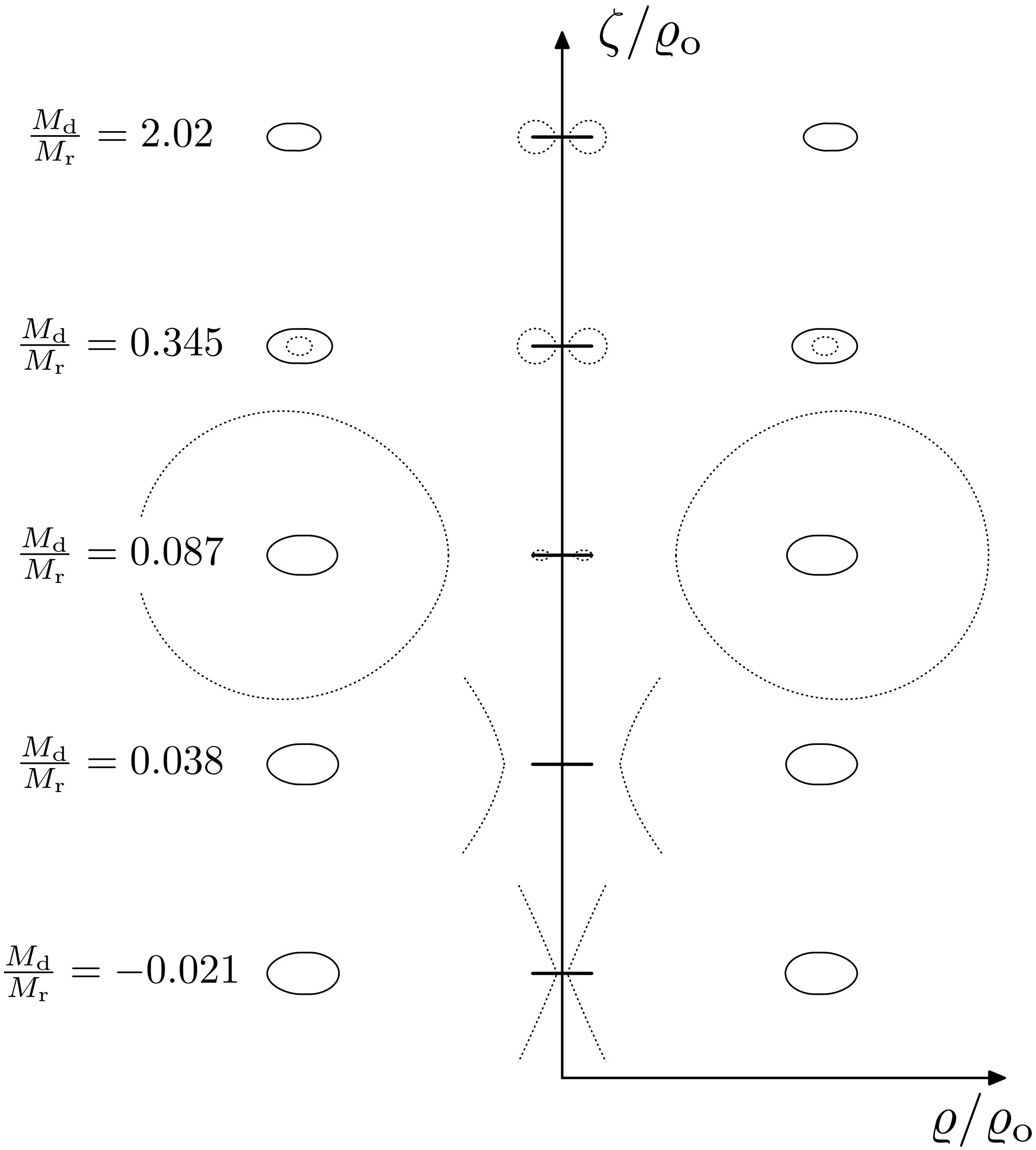


3. Journey

$$M_{\mathrm{h}}/M_{\mathrm{r}} = -0.080\,,\ Z_{\mathrm{r}} = 150$$

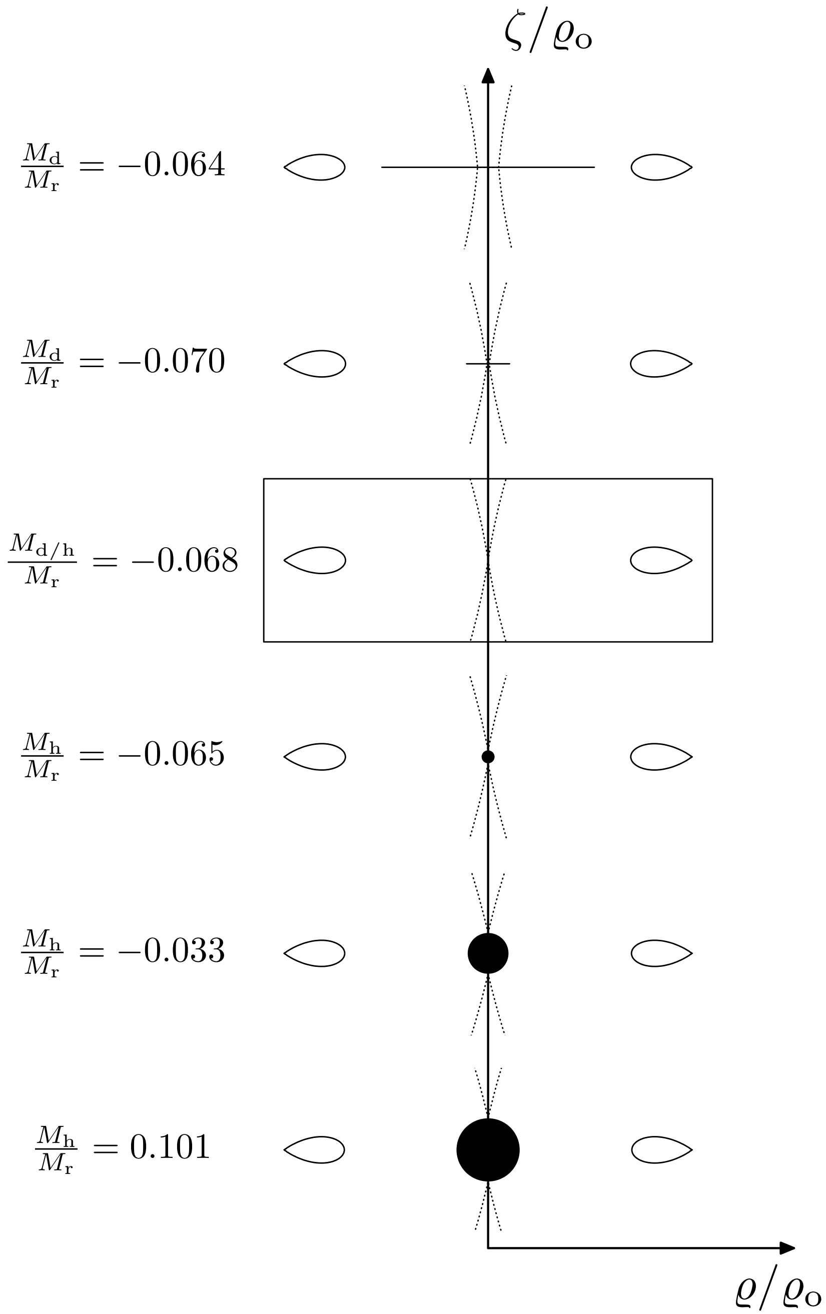


3. Journey

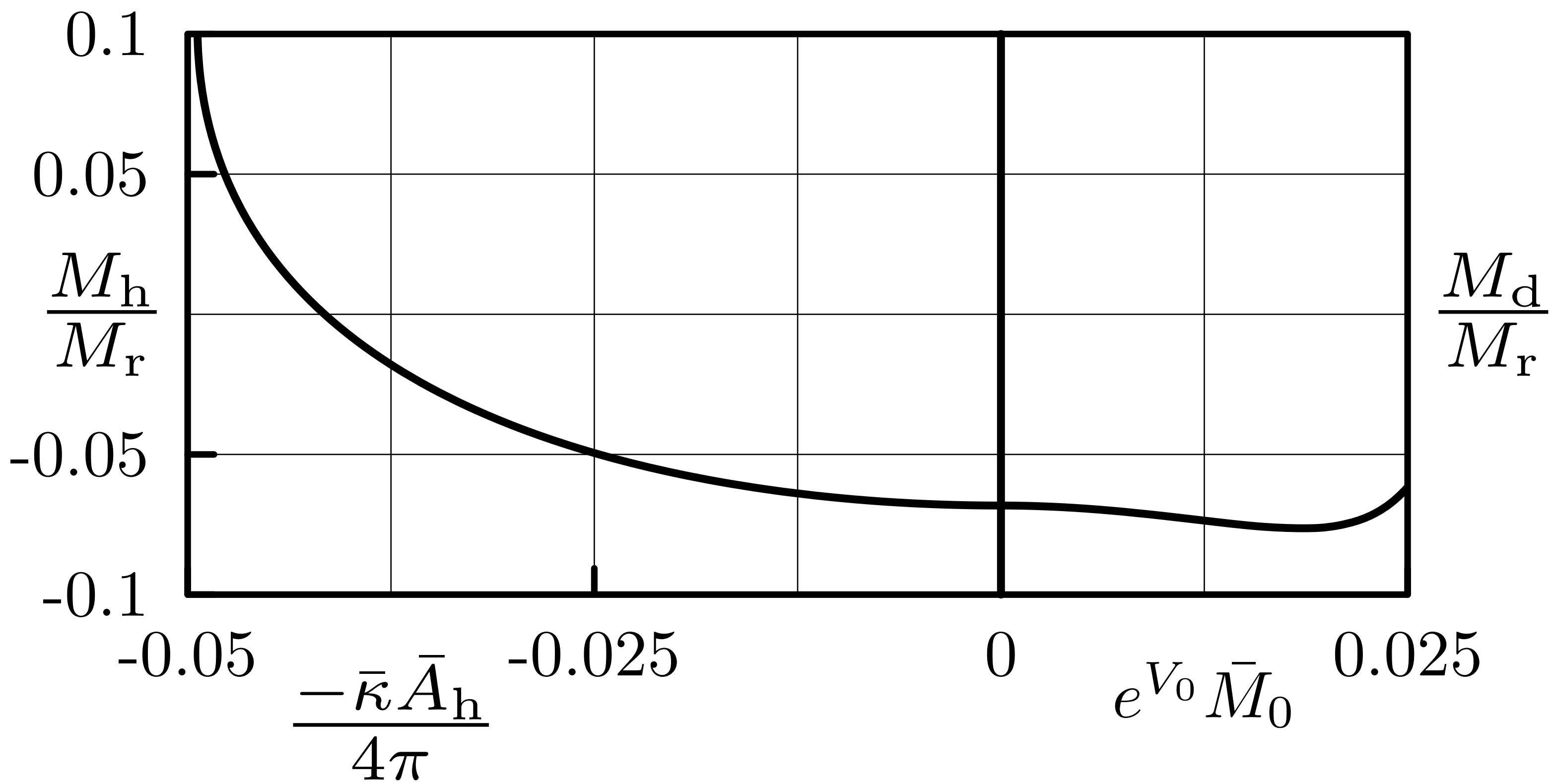


ζ/ϱ_o
$\frac{M_d}{M_r} = 2.02$
$\frac{M_d}{M_r} = 0.345$
$\frac{M_d}{M_r} = 0.087$
$\frac{M_d}{M_r} = 0.038$
$\frac{M_d}{M_r} = -0.021$
ϱ/ϱ_o

3. Journey


ζ/ϱ_o
$\frac{M_d}{M_r} = -0.064$
$\frac{M_d}{M_r} = -0.070$
$\frac{M_{d/h}}{M_r} = -0.068$
$\frac{M_h}{M_r} = -0.065$
$\frac{M_h}{M_r} = -0.033$
$\frac{M_h}{M_r} = 0.101$
ϱ/ϱ_o

3. Journey



The ratio of the Komar Mass of the central object to that of the ring

versus a measure of the distance to the degenerate black hole solution

4. Summary (1)

- The Komar mass of an object in axisymmetry and stationarity can be used on either side of the parametric transition from matter to a black hole.
- It can become negative if
  - (i) The object is placed within the strong gravitational field of a source with greater positive Komar mass.
  - (ii) This source is rapidly rotating so as to produce a large ergosphere encompassing the object.
  - (iii) The object is counter-rotating at a limited rate.
  - (iv) The object exerts a finite influence on the source (it is not close to a ‘test’–object).

4. Summary (2)

- The Komar mass is not an intrinsic property of an object. It is a feature of an object within a specific highly relativistic spacetime geometry.
- Question: What is the maximally attainable ratio

$$-M^{\mathrm{negative}}/M^{\mathrm{positive}} \quad ?$$