GRAVITATIONAL RECOIL OF BINARY BLACK HOLES

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Based on *Gravitational recoil of inspiralling black-hole binaries to second-post-Newtonian order* [Blanchet, Qusailah & Will 2005]

Digest on the history of gravitational recoil

1. General formalisms
   - Near-zone computation of recoil in linearized gravity [Peres 1958]
   - Flux computations of recoil as interaction between quadrupole and octupole moments [Bonnor & Rotenberg 1961, Papapetrou 1971]
   - General multipole expansion ($\forall\,\ell \geq 2$) of linear momentum flux [Thorne 1080]
   - Radiation-reaction computation of recoil and linear momentum balance equation [Blanchet 1996]
2. Core collapse to BH
   - $V_{\text{recoil}} \lesssim 300$km/s (PN calculation) [Bekenstein 1973]
   - Perturbation of Oppenheimer-Snyder collapse to BH [Moncrief 1979]
3. Compact binary systems
   - Recoil for point-mass binaries in Newtonian approximation [Fitchett 1983]
   - Recoil for particle around Kerr BH (perturbation theory) [Fitchett & Detweiler 1984]
   - Particle falling on symmetric axis of Kerr [Nakamura & Haugan 1983]
   - 1PN calculation to the recoil from point-mass binaries [Wiseman 1992]
   - Contributions of spins (PN calculation) [Kidder 1995]

Recent calculations in the case of compact binaries

1. Analytical or semi-analytical
   - Perturbation calculation ($\mu \ll M$) of recoil during final plunge of two BH [Favata, Hughes & Holz 2004]
   - 2PN calculation and estimate of the contribution of the plunge phase [Blanchet, Qusailah & Will 2005] (this work)
   - Application of the effective-one-body (EOB) approach [Damour & Gopakumar 2006]
2. Numerical
   - Perturbation/full numerical (Lazarus code) [Campanelli & Lousto 2004]
   - Binary BH grand challenge [Baker, Centrella, Choi, Koppitz, van Meter & Miller 2006]
   - Binary BH grand challenge [Gonzalez, Sperhake, Bruegmann, Hannam & Husa 2006]
   - Close limit approximation [Sopuerta, Yunes & Laguna 2006]

Flux of linear momentum

1. Use stress-energy tensor of GWs

$$T^{\mathrm{GW}}_{\mu\nu} = \frac{1}{32\pi}\langle\partial_\mu h^{\mathrm{TT}}_{ij}\partial_\nu h^{\mathrm{TT}}_{ij}\rangle$$

2. Derive the linear momentum loss as surface integral at infinity

$$\left(\frac{dP^i}{dt}\right)^{\mathrm{GW}} = -r^2\int d\Omega\, n^i\, T^{\mathrm{GW}}_{00}$$

General expression in terms of radiative moments U_L and V_L [Thorne 1980]

$$\left(\frac{dP^i}{dt}\right)^{\mathrm{GW}} = \sum_{\ell=2}^{+\infty}\frac{1}{c^{2\ell+3}}\left\{\alpha_\ell\, U^{(1)}_{iL}\, U^{(1)}_{L} + \beta_\ell\,\varepsilon_{ijk}\, U^{(1)}_{jL-1}\, V^{(1)}_{kL-1} + \frac{\gamma_\ell}{c^2}\, V^{(1)}_{iL}\, V^{(1)}_{L}\right\}$$

Note that the multipolar order (ℓ) scales with the PN order (c^{-1})

Linear momentum flux at Newtonian order

- The radiative moments U_L, V_L reduce to the source multipole moments

$$
\begin{aligned}
U_L &= I_L^{(\ell)} + \mathcal{O}\left(\frac{1}{c^3}\right) \\
V_L &= J_L^{(\ell)} + \mathcal{O}\left(\frac{1}{c^3}\right)
\end{aligned}
$$

- The source moments I_L, J_L take on their usual Newtonian expressions

$$
\begin{aligned}
I_L &= \int d^3x\, \rho\, \hat{x}_L + \mathcal{O}\left(\frac{1}{c^2}\right) \\
J_L &= \varepsilon_{kl\langle i_\ell} \int d^3x\, \rho\, v_k\, \hat{x}_{L-1\rangle l} + \mathcal{O}\left(\frac{1}{c^2}\right)
\end{aligned}
$$

- The "Newtonian" linear momentum flux takes the expression

$$
\left(\frac{dP^i}{dt}\right)^{\text{GW}} = \underbrace{\frac{1}{c^7}\left[\frac{2}{63} I^{(4)}_{ijk} I^{(3)}_{jk} + \frac{16}{45} \varepsilon_{ijk}\, I^{(4)}_{jk} J^{(3)}_{kl}\right]}_{\text{corresponds to a 3.5PN radiation reaction effect}} + \mathcal{O}\left(\frac{1}{c^9}\right)
$$

Radiation-reaction calculation of the recoil

- To 3.5PN order the radiation reaction force is electromagnetic-like with both scalar V_reac and vectorial A^i_reac potentials [Blanchet & Damour 1984]
- In a certain gauge the radiation reaction potentials are [Blanchet 1997]

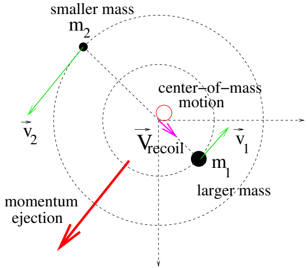
$$\begin{aligned} V_\text{reac} &= -\frac{1}{5c^5} x^{ij} I^{(5)}_{ij} + \frac{1}{c^7}\left[\frac{1}{189} x^{ijk} I^{(7)}_{ijk} + \frac{1}{70} x^2\, x^{ij} I^{(7)}_{ij}\right] + \mathcal{O}\left(\frac{1}{c^9}\right) \\ A^i_\text{reac} &= \frac{1}{21c^7} \hat{x}^{ijk} I^{(6)}_{ijk} + \frac{4}{45c^7} \varepsilon_{ijk} x^{jl} J^{(5)}_{kl} + \mathcal{O}\left(\frac{1}{c^9}\right) \end{aligned}$$

- The total recoil force (integrated over the source) is

$$F^i_\text{reac} = \underbrace{-\frac{1}{c^7}\left[\frac{2}{63} I^{(4)}_{ijk} I^{(3)}_{jk} + \frac{16}{45} \varepsilon_{ijk}\, I^{(4)}_{jk}\, J^{(3)}_{kl}\right]}_{\text{agrees with the Newtonian flux calculation}} + \mathcal{O}\left(\frac{1}{c^9}\right)$$

Gravitational recoil of BH binaries (Newtonian order)

The linear momentum ejection is in the direction of the lighter mass' velocity



In the Newtonian approximation [with $f(\eta) \equiv \eta^2\sqrt{1-4\eta}$]

$$V_{\text{recoil}} = 20\,\text{km/s}\left(\frac{6M}{r}\right)^4 \frac{f(\eta)}{f_{\text{max}}}$$

$$= 1500\,\text{km/s}\left(\frac{2M}{r}\right)^4 \frac{f(\eta)}{f_{\text{max}}} \quad \text{[Fitchett 1983]}$$

Very interesting result which shows the astrophysical relevance of GW recoil but illustrates the fact that the recoil is mainly generated in the strong field region

Linear momentum flux to 2PN order

1. We need to include higher-order radiative moments

$$\left(\frac{dP^i}{dt}\right)^{\text{GW}} \sim \frac{1}{c^7}\left[U^{(1)}_{ijk}\,U^{(1)}_{jk} + \varepsilon_{ijk}\,U^{(1)}_{jl}\,V^{(1)}_{kl}\right] + \frac{1}{c^9}\left[U^{(1)}_{ijkl}\,U^{(1)}_{jkl} + \varepsilon_{ijk}\,U^{(1)}_{jlm}\,V^{(1)}_{klm} + V^{(1)}_{ijk}\,V^{(1)}_{jk}\right] + \frac{1}{c^{11}}\left[U^{(1)}_{ijklm}\,U^{(1)}_{jklm} + \varepsilon_{ijk}\,U^{(1)}_{jlmn}\,V^{(1)}_{klmn} + V^{(1)}_{ijkl}\,V^{(1)}_{jkl}\right]$$

2. To 2PN order the tail contributions are

$$U_{ij} = I^{(2)}_{ij} + \frac{2G\,m}{c^3}\int_{-\infty}^{t} d\tau\, I^{(4)}_{ij}(\tau)\left[\ln\left(\frac{t-\tau}{2}\right) + \frac{11}{12}\right],$$

$$U_{ijk} = I^{(3)}_{ijk} + \frac{2G\,m}{c^3}\int_{-\infty}^{t} d\tau\, I^{(5)}_{ijk}(\tau)\left[\ln\left(\frac{t-\tau}{2}\right) + \frac{97}{60}\right],$$

$$V_{ij} = J^{(2)}_{ij} + \frac{2G\,m}{c^3}\int_{-\infty}^{t} d\tau\, J^{(4)}_{ij}(\tau)\left[\ln\left(\frac{t-\tau}{2}\right) + \frac{7}{6}\right]$$

Application to compact binaries in circular orbits

All the required source multipole moments in the case of compact binaries on circular orbits are known [Blanchet, Iyer & Joguet 2002, Arun, Blanchet, Iyer & Qusailah 2004]

$$
\begin{aligned}
I_{ij} &= \eta\, m \left\{ x^{\langle ij\rangle} \left[1 + \gamma\left(-\frac{1}{42} - \frac{13}{14}\eta\right) + \gamma^2\left(-\frac{461}{1512} - \frac{18395}{1512}\eta - \frac{241}{1512}\eta^2\right)\right] \right.\\
&\quad \left. + r^2 v^{\langle ij\rangle}\left[\frac{11}{21} - \frac{11}{7}\eta + \gamma\left(\frac{1607}{378} - \frac{1681}{378}\eta + \frac{229}{378}\eta^2\right)\right]\right\}, \\
I_{ijk} &= -\eta\,\delta m \left\{ x^{\langle ijk\rangle}\left[1 - \gamma\eta - \gamma^2\left(\frac{139}{330} + \frac{11923}{660}\eta + \frac{29}{110}\eta^2\right)\right]\right.\\
&\quad \left. + r^2\, x^{\langle i} v^{jk\rangle}\left[1 - 2\eta - \gamma\left(-\frac{1066}{165} + \frac{1433}{330}\eta - \frac{21}{55}\eta^2\right)\right]\right\}, \\
J_{ij} &= -\eta\,\delta m \left\{ \varepsilon^{ab\langle i} x^{j\rangle a} v^b \left[1 + \gamma\left(\frac{67}{28} - \frac{2}{7}\eta\right)\right.\right.\\
&\quad \left.\left. + \gamma^2\left(\frac{13}{9} - \frac{4651}{252}\eta - \frac{1}{168}\eta^2\right)\right]\right\}
\end{aligned}
$$

where $\eta \equiv \mu/m$ (mass ratio) and $\gamma \equiv m/r$ (PN parameter)

Result for the 2PN linear momentum [Blanchet, Qusailah & Will 2005]

$$\left(\frac{dP^i}{dt}\right)^{\text{GW}} = -\frac{464}{105} f(\eta)\, x^{11/2} \Bigg[1 + \overbrace{\left(-\frac{452}{87} - \frac{1139}{522}\eta\right) x}^{\text{1PN}} + \overbrace{\frac{309}{58}\pi\, x^{3/2}}^{\text{tail}} + \underbrace{\left(-\frac{71345}{22968} + \frac{36761}{2088}\eta + \frac{147101}{68904}\eta^2\right) x^2}_{\text{2PN}} \Bigg] \hat{\lambda}^i$$

- The recoil of the center-of-mass follows from integrating

$$\frac{dP^i_{\text{recoil}}}{dt} = -\left(\frac{dP^i}{dt}\right)^{\text{GW}}$$

- The recoil velocity V^i_{recoil} can be obtained analytically in the adiabatic approximation (up to the ISCO)

Recoil velocity at the ISCO

Table: Recoil velocity ($\mathrm{km\,s^{-1}}$) at the ISCO defined by $x_{\text{ISCO}} = 1/6$.

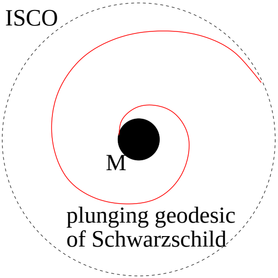
$\eta = \mu/m$	0.05	0.1	0.15	0.2	0.24
Newtonian	2.29	7.92	14.56	18.30	11.78
N + 1PN	0.27	0.77	1.16	1.12	0.55
N + 1PN + 1.5PN (tail)	2.87	9.80	17.74	21.96	13.97
N + 1PN + 1.5PN + 2PN	2.73	9.51	17.57	22.22	14.38

Estimate of the recoil accumulated during the plunge

We make a number of simplifying assumptions

1. The plunge is approximated as that of a test particle of mass μ moving on a geodesic of the Schwarzschild metric of a BH of mass m
2. The 2PN linear momentum flux is integrated on that orbit ($y \equiv m/r$)

$$\Delta V^i_{\text{plunge}} = L \int_{\text{ISCO}}^{\text{horizon}} \left(\frac{1}{m\,\omega} \frac{dP^i}{dt} \right) \frac{dy}{\sqrt{E^2 - (1-2y)(1+L^2\,y^2)}}$$



E and L are the constant energy and angular momentum of the Schwarzschild plunging orbit

Matching to the circular orbit at the ISCO

1. We evolve a circular orbit at the ISCO (where $x = 1/6$) piecewise to a new orbit using energy and angular momentum balance equations

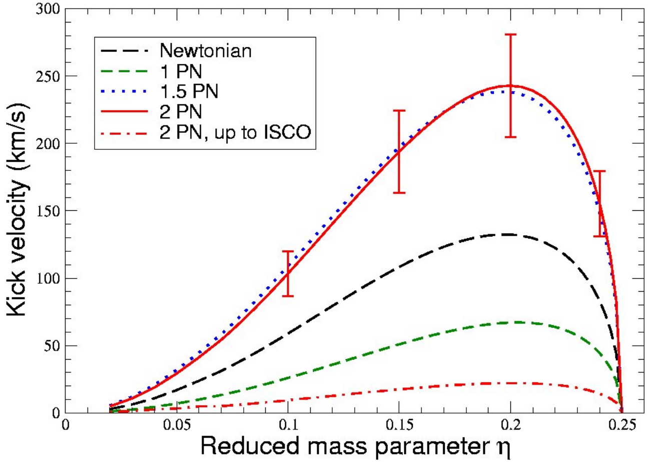
$$\frac{dE}{dt} = -\frac{32}{5}\frac{\eta}{m}x_{\text{ISCO}}^5$$
$$\frac{dL}{dt} = \frac{1}{\omega_{\text{ISCO}}}\frac{dE}{dt}$$

2. We discretize these relations around the ISCO values over a fraction of orbital period $\alpha\, P$ (where $0 < \alpha < 1$)

$$E = E_{\text{ISCO}} - \frac{64\pi}{5}\eta\,\alpha\, x_{\text{ISCO}}^{7/2}$$
$$L = L_{\text{ISCO}} - \frac{64\pi}{5}\eta\,\alpha\, x_{\text{ISCO}}^{2}$$

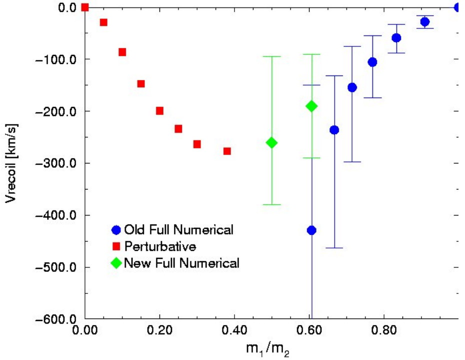
3. We check that the results are insensitive to the value of α below 0.1

Estimation of the recoil up to coalescence at $r = 2m$



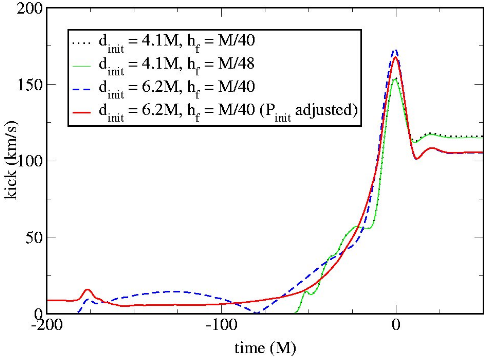
[Blanchet, Qusailah & Will 2005]

Brownsville group [Campanelli & Lousto 2005]



For the mass ratio $\eta = 0.24$ corresponding to $m_2/m_1 = 0.66$ the final kick is around $\sim 200\,\mathrm{km/s}$ but with large error bars

Goddard group [Baker, Centrella, Choi, Koppitz, van Meter & Miller 2006]



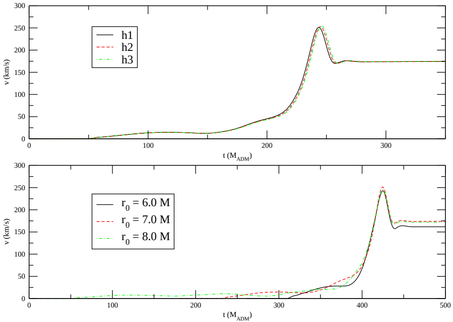
$\Longleftarrow$ 2PN peak

For the mass ratio $\eta = 0.24$ (corresponding to $m_2/m_1 = 0.66$)

- Kick at the maximum is $\sim 170\,\text{km/s}$
- Final kick is $\sim 105\,\text{km/s}$

We note that the kick at the maximum is in rather good agreement with the 2PN calculation for this mass ratio (namely $\sim 160\,\text{km/s}$)

Jena group [Gonzalez, Sperhake, Bruegmann, Hannam & Husa 2006]



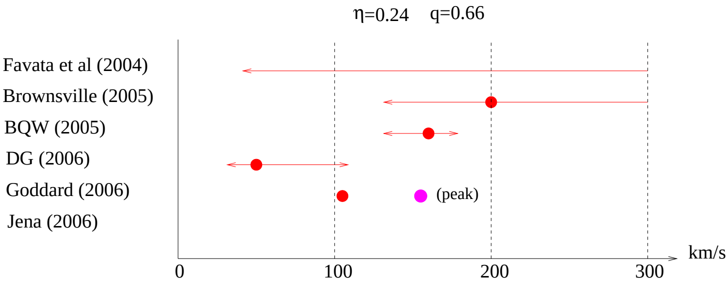
⟸ 2PN peak

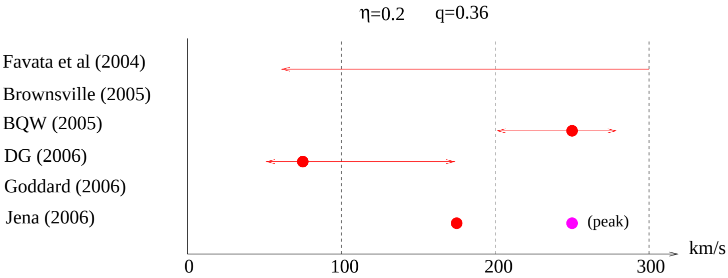
For the mass ratio $\eta = 0.195$ ($m_2/m_1 = 0.36$)

- Kick at the maximum is $\sim 250\,\text{km/s}$
- Final kick is $\sim 175\,\text{km/s}$

Again the kick at the maximum is in good agreement with the 2PN calculation for this mass ratio (namely $\sim 250\,\text{km/s}$)

Summary of comparisons





Conclusions

- The gravitational recoil is likely to have important astrophysical consequences in models for massive BH formation involving successive mergers from smaller BH seeds
- The computation of the recoil at 2PN order gives a maximal contribution of 22 km/s up to the ISCO (probably very accurate)
- For a mass ratio of 0.36 the recoil up to the BH coalescence at $r = 2m$ is estimated at ~ 250 km/s using some approximation in the plunge phase
- Recent progresses in numerical relativity confirm this estimate but show a subsequent decrease of the recoil presumably due to the ring-down phase to the value ~ 175 km/s
- The braking of the recoil velocity in the ring-down phase should be better understood theoretically