Generalized Harmonic Evolutions of Binary Black Hole Spacetimes

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From Geometry to Numerics — Institut Henri Poincare
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- Moving Black Holes.

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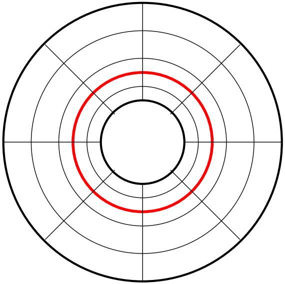

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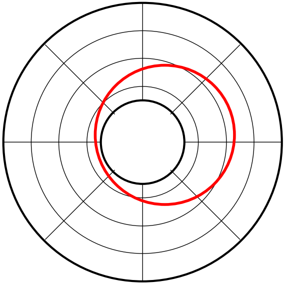

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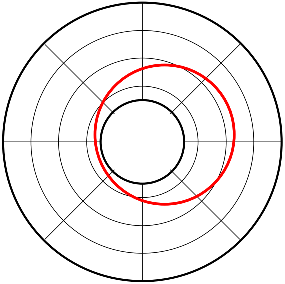

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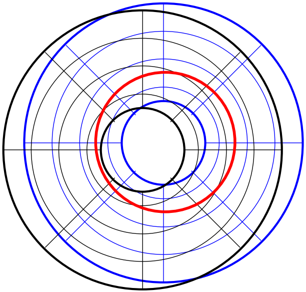

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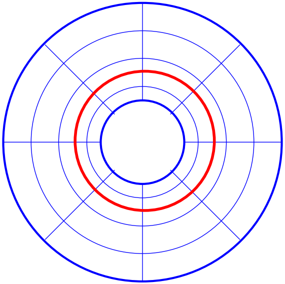

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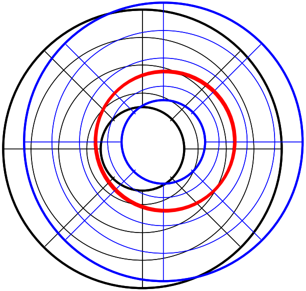

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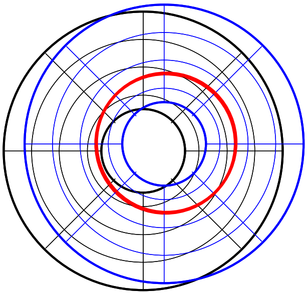

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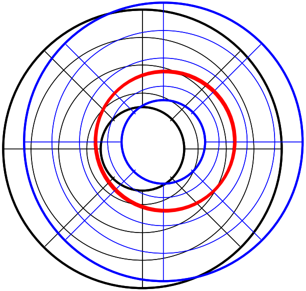

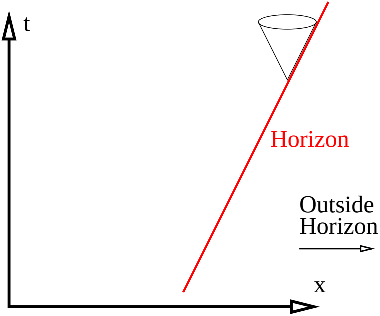

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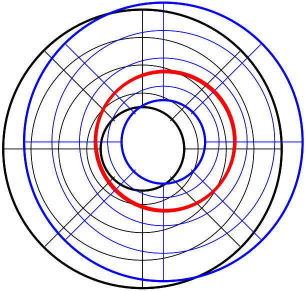

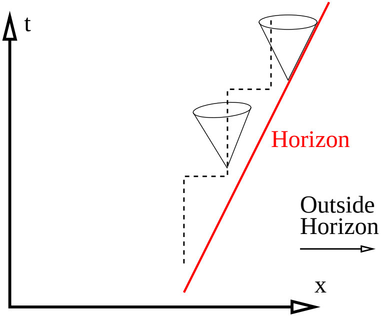

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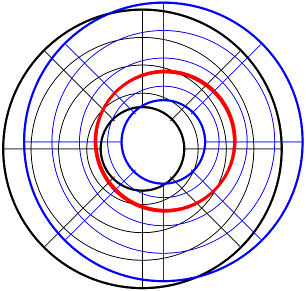

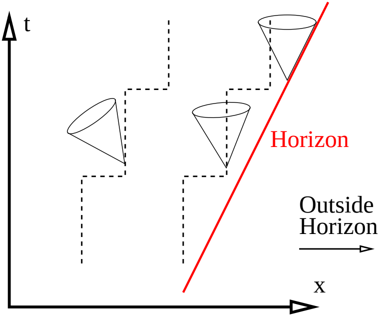

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- Problems:
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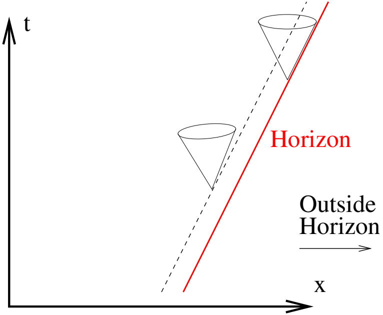
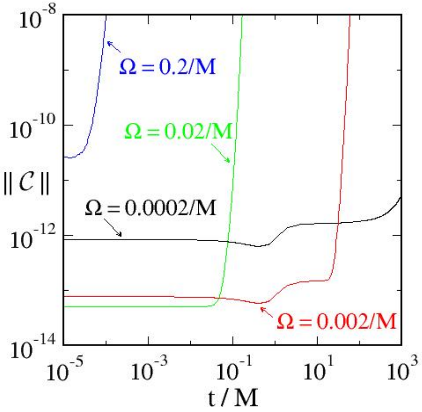
- Solution:
  Choose coordinates that smoothly track the location of the black hole.



Evolving Black Holes in Rotating Frames

- Coordinates that rotate with respect to the inertial frames at infinity are needed to track the horizons of orbiting black holes.
- Evolutions of Schwarzschild in rotating coordinates are unstable.



- Evolutions shown use a computational domain that extends to $r = 1000M$.
- Angular velocity needed to track the horizons of an equal mass binary at merger is about $\Omega \approx 0.2/M$.
- Problem caused by asymptotic behavior of metric in rotating coordinates: $\psi_{tt} \sim \rho^2\Omega^2$, $\psi_{ti} \sim \rho\Omega$, $\psi_{ij} \sim 1$.

Dual-Coordinate-Frame Evolution Method

- Single-coordinate frame method uses the one set of coordinates, $x^{\bar{a}} = \{\bar{t}, x^{\bar{\imath}}\}$, to define field components, $u^{\bar{\alpha}} = \{\psi_{\bar{a}\bar{b}}, \Pi_{\bar{a}\bar{b}}, \Phi_{\bar{\imath}\bar{a}\bar{b}}\}$, and the same coordinates to determine these components by solving Einstein's equation for $u^{\bar{\alpha}} = u^{\bar{\alpha}}(x^{\bar{a}})$:

$$\partial_{\bar{t}} u^{\bar{\alpha}} + A^{\bar{k}\,\bar{\alpha}}{}_{\bar{\beta}} \partial_{\bar{k}} u^{\bar{\beta}} = F^{\bar{\alpha}}.$$

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$$\partial_{\bar{t}} u^{\bar{\alpha}} + A^{\bar{k}\,\bar{\alpha}}{}_{\bar{\beta}} \partial_{\bar{k}} u^{\bar{\beta}} = F^{\bar{\alpha}}.$$

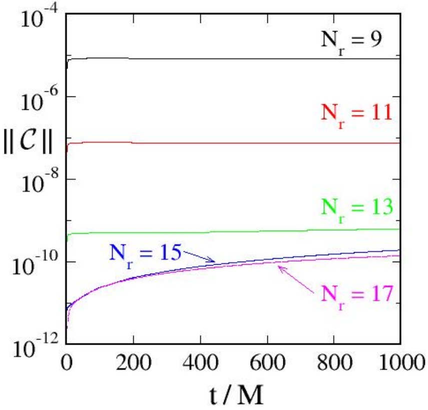
- Dual-coordinate frame method uses a second set of coordinates, $x^a = \{t, x^i\} = x^a(x^{\bar{a}})$, to determine the original representation of the dynamical fields, $u^{\bar{\alpha}} = u^{\bar{\alpha}}(x^a)$, by solving the transformed Einstein equation:

$$\partial_t u^{\bar{\alpha}} + \left[\frac{\partial x^i}{\partial \bar{t}} \delta^{\bar{\alpha}}{}_{\bar{\beta}} + \frac{\partial x^i}{\partial x^{\bar{k}}} A^{\bar{k}\,\bar{\alpha}}{}_{\bar{\beta}}\right] \partial_i u^{\bar{\beta}} = F^{\bar{\alpha}}.$$

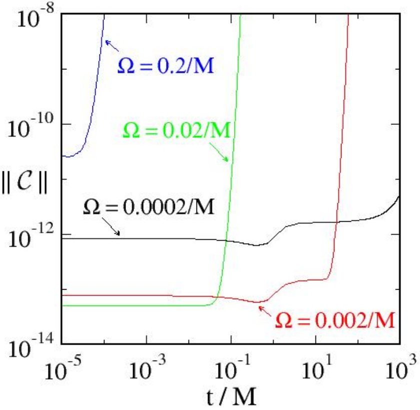
Testing Dual-Coordinate-Frame Evolutions

- Single-frame evolutions of Schwarzschild in rotating coordinates are unstable, while dual-frame evolutions are stable:

Dual Frame Evolution



Single Frame Evolution



- Dual-frame evolution shown here uses a comoving frame with $\Omega = 0.2/M$ on a domain with outer radius $r = 1000M$.

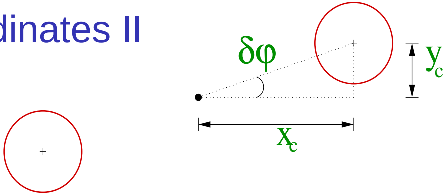
Horizon Tracking Coordinates

- Coordinates must be used that track the motions of the holes.
- For equal mass non-spinning binaries, the centers of the holes move only in the $z = 0$ orbital plane.
- The coordinate transformation from inertial coordinates, $(\bar{x}, \bar{y}, \bar{z})$, to co-moving coordinates (x, y, z),

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{a(\bar{t})} \begin{pmatrix} \cos\varphi(\bar{t}) & -\sin\varphi(\bar{t}) & 0 \\ \sin\varphi(\bar{t}) & \cos\varphi(\bar{t}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix},$$

  with $t = \bar{t}$, is general enough to keep the holes fixed in co-moving coordinates for suitably chosen functions $a(\bar{t})$ and $\varphi(\bar{t})$.
- Since the motions of the holes are not known *a priori*, the functions $a(\bar{t})$ and $\varphi(\bar{t})$ must be chosen dynamically and adaptively as the system evolves.

Horizon Tracking Coordinates II



- Measure the comoving centers of the holes: $x_c(t)$ and $y_c(t)$, or equivalently $Q^x(t) = [x_c(t) - x_c(0)]/x_c(0)$ and $Q^y(t) = y_c(t)/x_c(t)$.
- Choose the map parameters $a(t)$ and $\varphi(t)$ to keep $Q^x(t)$ and $Q^y(t)$ small.

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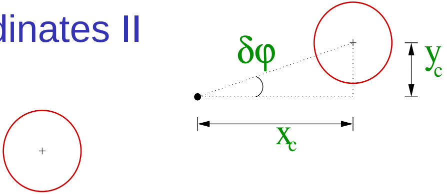

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- Choose the map parameters $a(t)$ and $\varphi(t)$ to keep $Q^x(t)$ and $Q^y(t)$ small.
- Changing the map parameters by the small amounts, δa and $\delta\varphi$, results in associated small changes in δQ^x and δQ^y:

$$\delta Q^x = -\delta a, \qquad\qquad \delta Q^y = -\delta\varphi.$$

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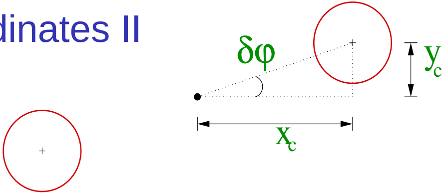

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- Measure the quantities $Q^y(t)$, $dQ^y(t)/dt$, $d^2Q^y(t)/dt^2$, and set

$$\frac{d^3\varphi}{dt^3} = \lambda^3 Q^y + 3\lambda^2\frac{dQ^y}{dt} + 3\lambda\frac{d^2Q^y}{dt^2} = -\frac{d^3Q^y}{dt^3}.$$

The solutions to this "closed-loop" equation for Q^y have the form $Q^y(t) = (At^2 + Bt + C)e^{-\lambda t}$, so Q^y always decreases.

Horizon Tracking Coordinates III

- In practice the coordinate maps are adjusted only at a prescribed set of adjustment times $t = t_i$.

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- In the time interval $t_i < t < t_{i+1}$ we set:

$$\begin{aligned}\varphi(t) &= \varphi_i + (t - t_i)\frac{d\varphi_i}{dt} + \frac{(t-t_i)^2}{2}\frac{d^2\varphi_i}{dt^2} \\ &+ \frac{(t-t_i)^3}{2}\left(\lambda\frac{d^2Q_i^y}{dt^2} + \lambda^2\frac{dQ_i^y}{dt} + \lambda^3\frac{Q_i^y}{3}\right),\end{aligned}$$

where Q^x, Q^y, and their derivatives are measured at $t = t_i$, so these maps satisfy the closed loop
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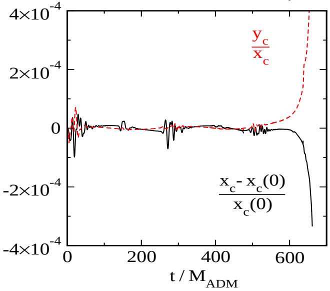
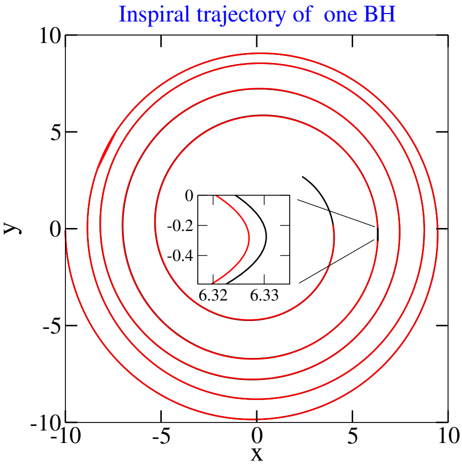
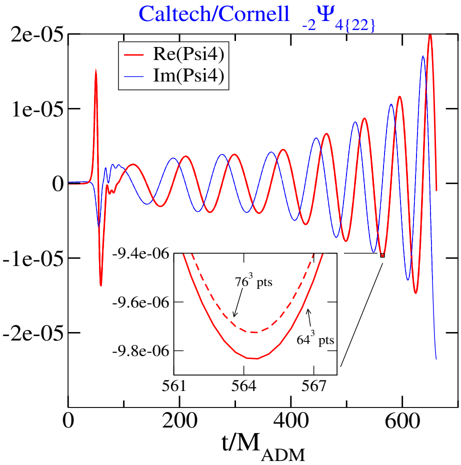


- **This works!** We are now able to evolve binary black holes using horizon tracking coordinates until just before merger.

Evolving Binary Black Hole Spacetimes

- We can now evolve binary black hole spacetimes with excellent accuracy and computational efficiency through many orbits.

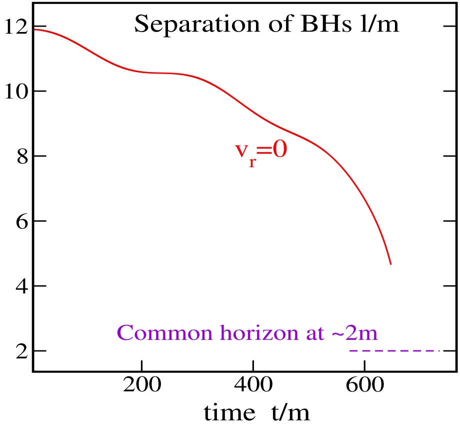




Inspiral Movie   Merger Movie Ψ_4 Movie

Improved Circular Orbit Initial Data

- The conformal thin sandwich initial data has initial radial velocity $\dot{R} = 0$. True circular orbit initial data should have $\dot{R} < 0$, but small.
- Conformal thin sandwich initial data therefore results in slightly elliptical orbits:



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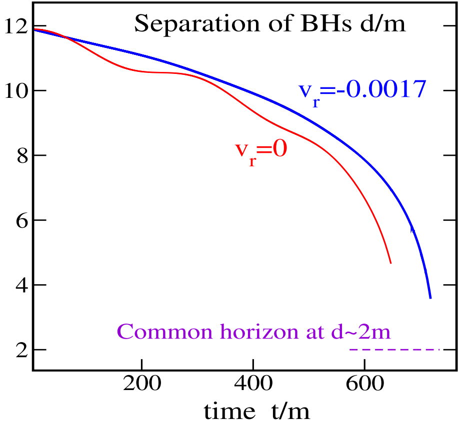

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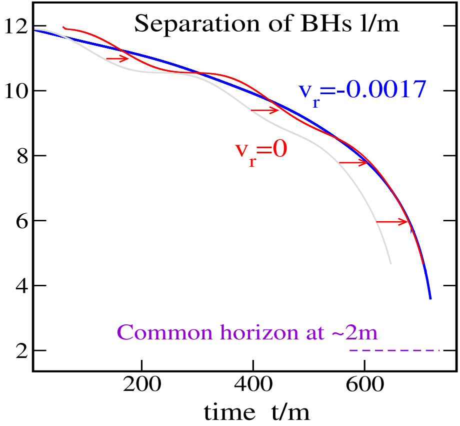

- Small changes in the initial $\dot{R}$ and Ω gives a more circular orbit.
- Orbit point with $\dot{R} = 0$ is near apcenter, with larger R than corresponding point in circular orbit.
- Orbits at time shifted points have nearly identical gravitational waveforms.