EXCISION WITHOUT EXCISION

WHY THE PUNCTURE METHOD WORKS!

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STATIONARY SLICE IN SCHWARZSCHIL

$\neq$ UNCHANGING METRIC

ANY SLICE CAN BE DRAGGED ALON

KILLING VECTOR

GIVEN SLICE + EMBEDDING CONDITION

SLICING CONDITION / LAPSE SPECIFICATION

A STATIONARY SLICE IS ONE

WHERE

SPECIFIED LAPSE = KILLING LAPSE

OPTIONAL EXTRA :

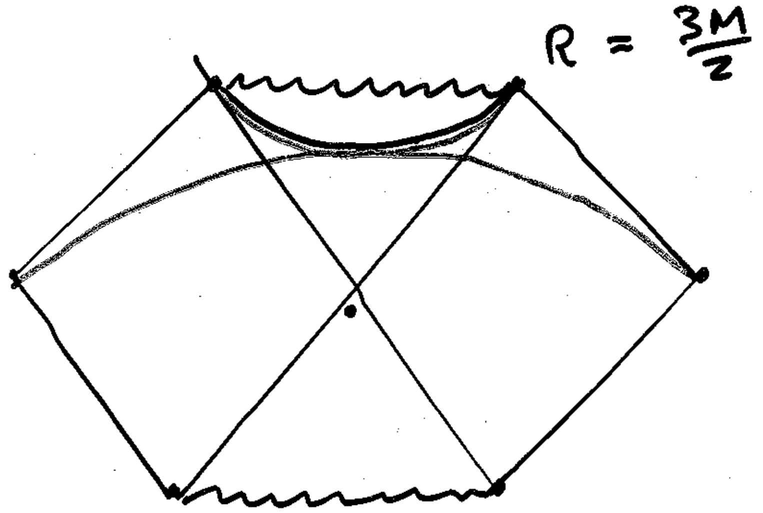
3 - COORDINATE CHOICE / SHIFT SPECIFICATIO

SPECIFIED SHIFT = KILLING SHIFT

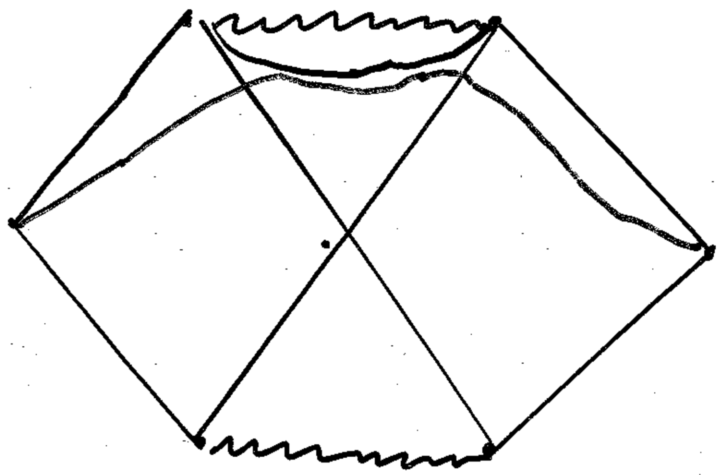
SPHERICAL MAXIMAL SLICES IN SCHWARZSCHILD

$$\nabla^2 N - K \cdot K N = 0$$

SEVERAL INTERESTING STATIONARY SPHERICAL MAXIMAL SLICES



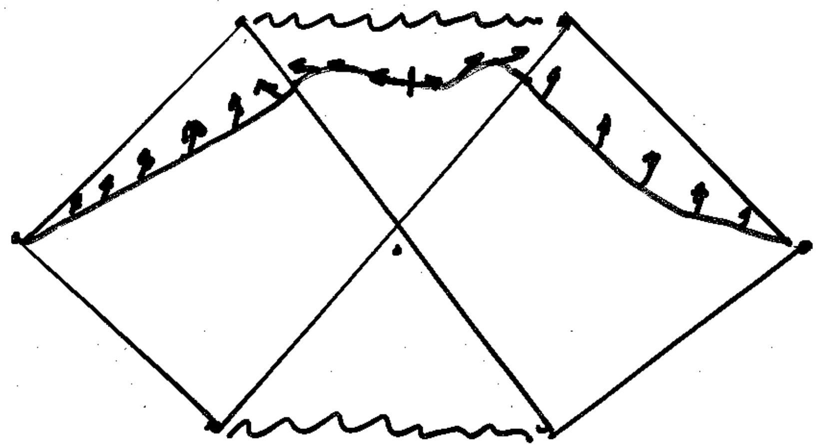
LATE TIME BEHAVIOUR OF MAXIMAL FOLIATION OF SCHWARZSCHILD



$$C \to \left(\frac{3}{2}\right)^{3/2}$$

EXPONENTIAL DECAY

METRIC + EXTRINSIC CURVATURE ASYMPTOTE TO CONSTANT VALUE BUT NOT STATIONARY



EACH HALF APPROXIMATES TRUE STATIONARY SLICE AND IS DRAGGED BY KILLING VECTOR.

GAP OPENS IN CENTER AND IS FILLED BY CYLINDER

$$\frac{dL}{dt} = \frac{2}{\sqrt{3}}$$

MAJOR SUCCESS:

1. BROWNSVILLE
2. NASA

INDEPENDENT BUT SIMILAR CODES

1. BSSN
2. '1 + log' SLICING
$$(\partial_t - \beta^i \nabla_i)\alpha = -2\alpha\, \mathrm{tr} K$$
3. PUNCTURE METHOD
4. COORDINATE CHOICE: 'GAMMA FREEZING'
$$\partial_t^2 \beta^i = 3/4\, \partial_t \tilde{\Gamma}^i$$
(SMARR - YORK 'MINIMUM SHEAR' WRITTEN IN BSSN LANGUAGE)

CONFORMAL FACTOR DIVERGES AT THE PUNCTURE(S)
EXPECTED TO BE A MAJOR SOURCE OF NUMERICAL DIFFICULTY

ONLY PRECAUTION WAS TO ENSURE GRID POINTS NEVER COINCIDED WITH PUNCTURES

JENA GROUP

(BERND BRÜGMANN, MARK HAMMAN, SASCHA HUSA + DENIS POLNEY (POTSDAM))

RAN SCHWARZSCHILD INITIAL DATA USING

BSSN, PUNCTURE, 1+ log, GAMMA FREEZIN

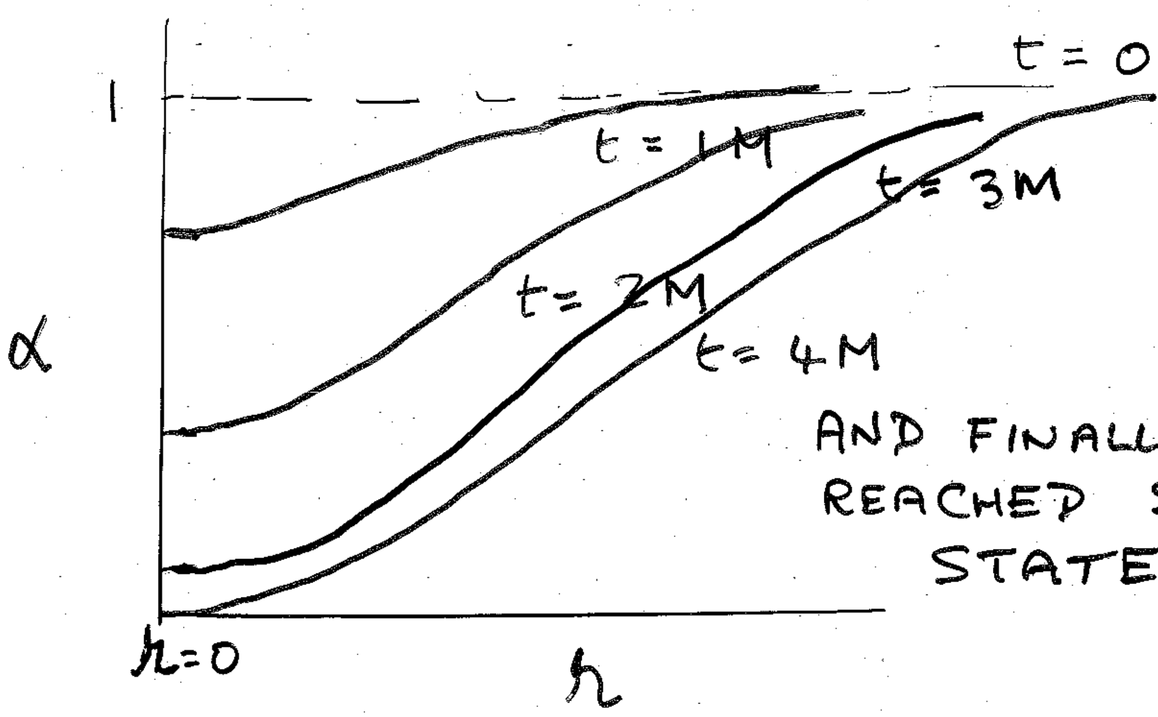
MANIFORD IS FLAT SPACE

STARTED WITH

$$g_{ij} = \left(1 + \frac{M}{2r}\right)^4 \delta_{ij}, \qquad K^{ij} = 0$$

$$N = 1 \quad , \quad \beta^i = 0$$

STRANGE BEHAVIOUR



AND FINALLY REACHED STEADY STATE

WHY DOES CODE CAUSE TOPOLOGY CHANG

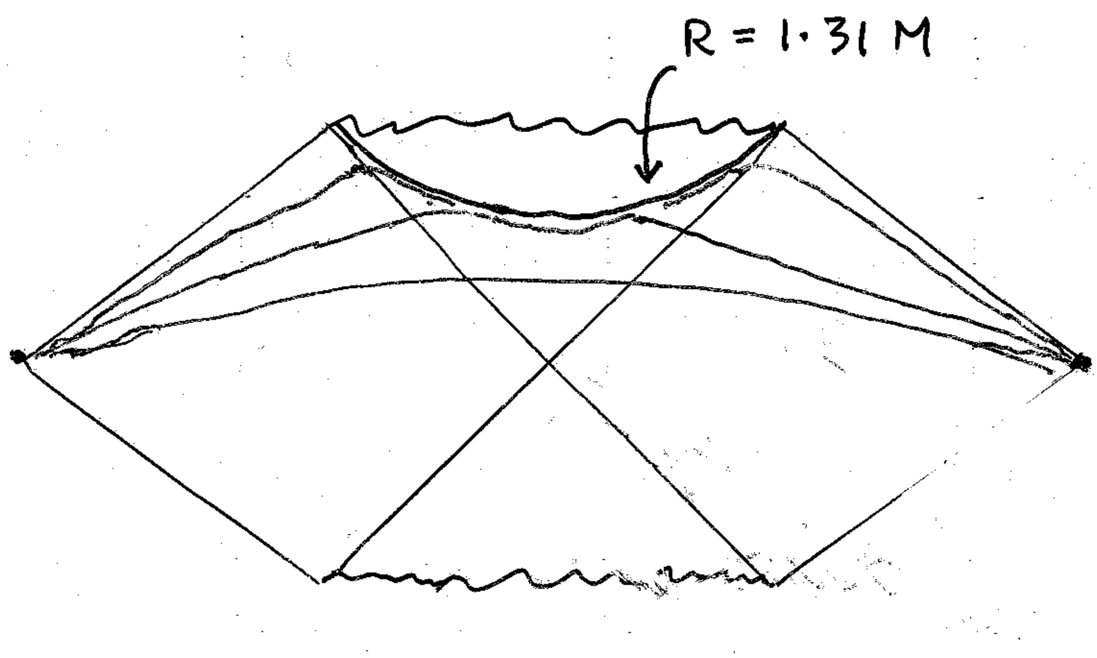
BACK TO FUNDEMENTALS

TAKE INITIAL DATA FOR EXTENDED SCHWARZSCHILD

PROPOGATE IN '1 + log' GAUGE WITH $N \equiv 1$ AS INITIAL CONDITION

THE SLICINGS, AS EMBEDDED HYPERSURFACE IN PENROSE DIAGRAM, SHOULD NOT DEPEND ON EITHER EVOLUTION SCHEME OR SHIFT CHOICE.

THE EMBEDDING MUST BE LEFT-RIGHT SYMMETRIC



AT LATE TIMES $(t > 50M)$
EACH SLICE CONSISTS OF 3 PATCHES

TWO OUTER ONES
COMING FROM EACH I_0 TO BECOME
CYLINDERS AT $R = 1.31M$
EACH SATISFIES THE STATIONARY
'1 + log' EQUATION
THESE ARE JOINED BY A CYLINDER
OF RADIUS $R = 1.31M$

AS TIME ELAPSES, THE CYLINDER
EXTENDS AND 'PUSHES' THE 'SHOLDERS'
OUT, IN SUCH A WAY THAT THE 'SHOLDERS
ARE KILLING TRANSPORTED, SO THAT
THE GEOMETRY + EXTRINSIC CURVATURE
DOES NOT CHANGE.

THE '1 + log' CONDITION IS 'SYMMETRY
SEEKING'; IT CANNOT FIND A SLICE
WHICH HAS A SINGLE SYMMETRY.
RATHER IT FINDS A SLICE MADE
UP OF SYMMETRIC PATCHES.

NEXT TO CONSIDER IS COORDINATE CHOICE !

γ - FREEZING BREAKS LEFT - RIGHT SYMMETRY

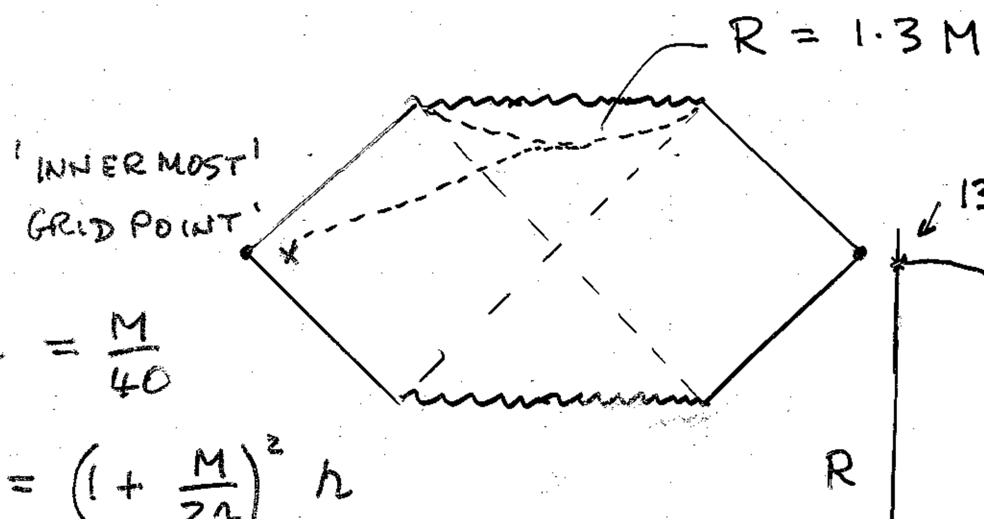
$$\partial_t^2 \beta^i = \tfrac{3}{4} \partial_t \tilde{\Gamma}^i$$

IN BSSN SPLIT METRIC

$$g_{ij} = \phi^4 \tilde{g}_{ij}$$

WITH $\det \tilde{g} = 1$

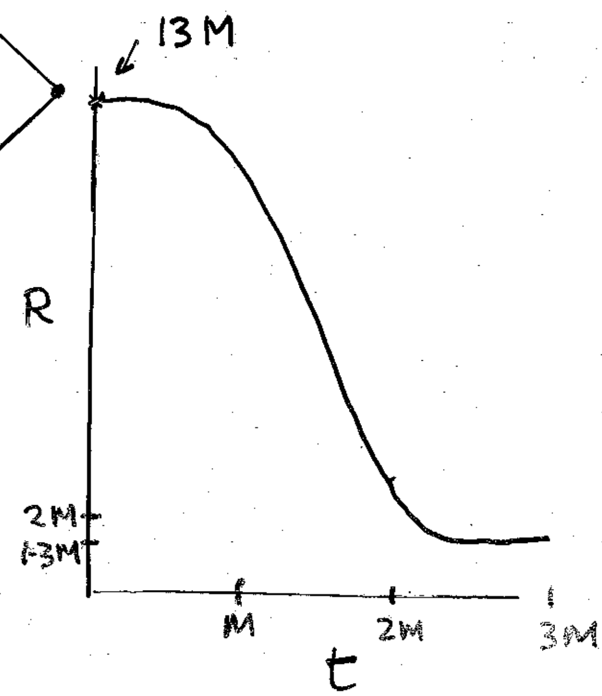
DEPENDS ON ORIGINAL CHOICE OF COORDINATES



$$h = \frac{M}{40}$$

$$R = \left(1 + \frac{M}{2h}\right)^2 h$$

$$= 13\,M$$



'EXPLANATION': FOLIATION SHOWS LOT OF SLICE STRETCHING BETWEEN HORIZONS.

Γ - FREEZING WISHES TO MAINTAIN METRIC REGULAR. ONLY WAY TO ACHIEVE THIS IS TO MOVE LEFT-MOST GRID-POINTS TO RIGHT.

'MAGIC': '1 + log' IS SYMMETRY-SEEKING

Γ- FREEZING IS ALSO SYMMETRY-SEEKING

IT FINDS THE KILLING-SHIFT SO THAT

$$\partial_t \tilde{\Gamma} = 0$$

$$\partial_t \beta^i = 0$$

WE WILL NOTICE SYMMETRIES IN COMPUTER RUNS IF BOTH THE GEOMETRY STABILIZES AND COORDINATES LOCK IN.

CONJECTURE: THIS BEHAVIOUR IS STABLE!

ONLY ONE TEST SO FAR

JENA RAN (BSSN, 1+ log, Γ - FREEZING)

$$g_{ij} = \left(1 + \frac{M}{2r}\right)^4 \delta_{ij}$$

$$K^{ij} = 0$$

$$\alpha = \left(1 + M/2r\right)^{-2}$$ (PRE-COLLAPSED LAPSE)

$$\beta = 0$$

INITIALLY BEHAVED DIFFERENTLY BUT SETTLED TO SAME FINAL STATE

TESTS: OTHER INITIAL LAPSES

(SASCHA) LUMP OF SPHERICAL MASSLESS SCALAR FIELD ON SCHWARZSCHILD (TO MIMIC RADIATION)

KERR

ASK SUCCESSFUL 4-ORBIT CODERS NHETHER THE INNER HORIZONS 'FALL INTO THE PUNCTURES' AT EARLY TIMES

TWO REQUESTS FOR
BROWNSVILLE + GODDARD

① BY $t = 3M$, HAS INNER HORIZON (FALLEN INTO PUNCTURE) i.e. VANISHED FROM GRID ?

② BY $t = 6M$, IS ${}^{(3)}R$ 'NEAR PUNCTURE' REACHING CONSTANT VALUE OF

$${}^{(3)}R = \frac{2}{R^2} = \frac{2}{\left(\frac{3}{2}M\right)^2} \quad \text{(BROWNSVILLE)}$$

$$= \frac{2}{(1.31\ M)^2} \quad \text{GODDARD}$$

DOES REGION WITH ${}^{(3)}R$ = CONSTANT INCREASE WITH TIME ?