Gravitational waves from accretion onto Schwarzschild black holes: A perturbative approach

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Phys. Rev. D 69 (2004), 124028

The “plunge”: Motivations and overview

Setting: Matter plunging into the black hole in the “test-matter” approximation:

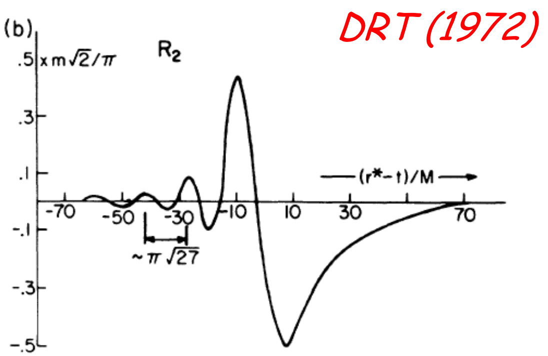
localized source (*δ-like source: a “particle”. Radiation reaction included.*)
extended source (*dust or fluid matter distribution evolved with 2D and 3D (M)GRHydro codes*)

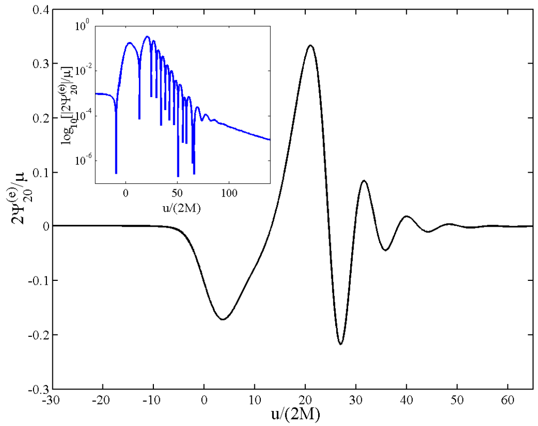
Techniques: black-hole perturbation theory to extract waves as a *complementary* approach to Numerical Relativity simulations. Quick (and approximate) way to gain general ideas about the physics.

Interest: analysis of the features of QNMs excitation (and curvature backscattering in genearl) determined by the “geometrical” size of the matter that is plunging into the black-hole.

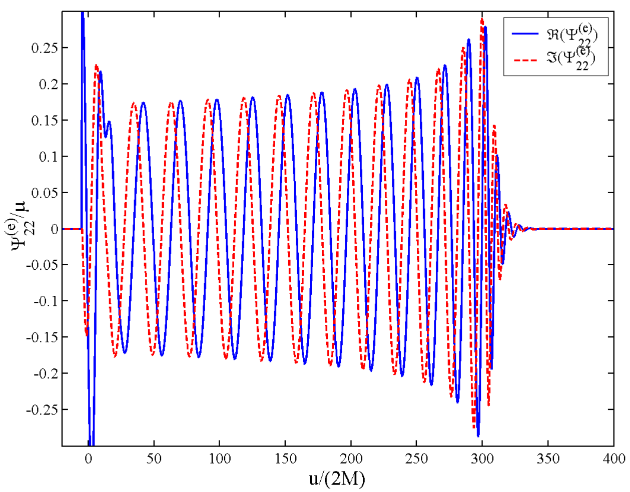
The “plunge” of a particle (BBH in the EMR-limit)

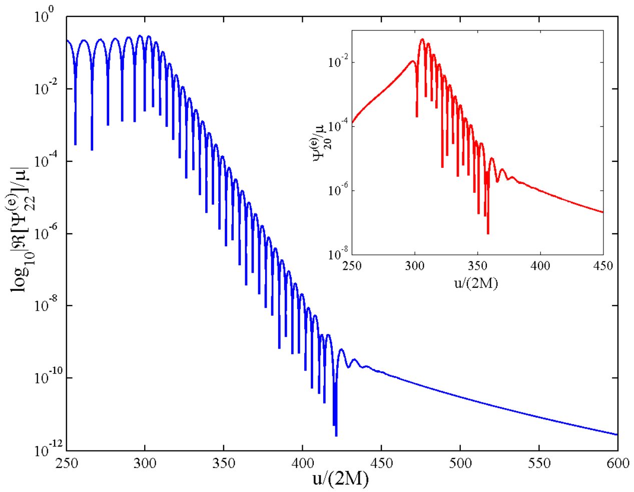
Radial (*axisymmetric: m=0*) plunge of a particle: Waveforms [from DRT (1972) to LP-MP(1997, 2001)]





Transition from quasi-circular inspiral to plunge of a particle (*with 2.5PN radiation-reaction see TD talk*)





Plunge of test fluid (accretion)

Physical setting

Last stages of gravitational collapse (or binary merger): A black hole + accretion flows

Main question: how relevant can be the presence of *black hole quasi-normal modes* in this phase?

Motivations

BHs perturbation theory with general matter source as a complementary approach ***NR*** simulations.

Still technical problems in treating *"excised"* spacetimes in the presence of matter.

Recent progress in gravitational collapse in 3D: *Baiotti et al. (2005)* and *Zink et al. (2005)* .

Previous work

Shapiro&Wasserman (***SW1982***) and Petrich,Shapiro and Wasserman (***PSW1985***): dust accretion using frequency domain computations following DRPP techniques. *Destructive interference effects.*

Papadopoulos and Font (***PF1999***) using Bardeen-Press equation.

Metric perturbations of a Schwarzschild spacetime

Remark: Regge-Wheeler and Zerilli-Moncrief equations from the 10 Einstein equations.
Gauge-invariant and ***coordinate-independent*** formalism

[***Regge&Wheeler1957, Zerilli1970, Moncrief1974, Gerlach&Sengupta1978, Sarbach&Tiglio2001, Martel&Poisson2005, Nagar&Rezzolla 2005***]

$$g_{\mu\nu} = \overset{0}{g}_{\mu\nu} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(h_{\mu\nu}^{\ell m}\right)^{(\mathrm{o})} + \left(h_{\mu\nu}^{\ell m}\right)^{(\mathrm{e})}$$

Regge-Wheeler and Zerilli-Moncrief equations (with sources) in Schwarzschild coordinates

$$\partial_t^2 \Psi_{\ell m}^{(\mathrm{o/e})} - \partial_{r_*}^2 \Psi_{\ell m}^{(\mathrm{o/e})} + V_\ell^{(\mathrm{o/e})} \Psi_{\ell m}^{(\mathrm{o/e})} = S_{\ell m}^{(\mathrm{o/e})}$$

$$r_* = r + 2M \ln[r/(2M) - 1]$$

In the ***wave zone***: GW amplitude and emitted power

$$h_+ - \mathrm{i} h_\times = \frac{1}{r} \sum_{\ell,m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \left(\Psi_{\ell m}^{(\mathrm{e})} + \mathrm{i}\Psi_{\ell m}^{(\mathrm{o})}\right) \, {}_{-2}Y^{\ell m}(\theta,\phi) + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\frac{dE}{dt} = \frac{1}{16\pi} \sum_{\ell,m} \frac{(\ell+2)!}{(\ell-2)!} \left(\left|\frac{d\Psi_{\ell m}^{(\mathrm{e})}}{dt}\right|^2 + \left|\frac{d\Psi_{\ell m}^{(\mathrm{o})}}{dt}\right|^2 \right)$$

The general sources

In Schwarzschild coordinates

$$S^{(e)} = -\frac{8\pi}{\Lambda\left[(\Lambda-2)r+6M\right]}\left\{\frac{\Lambda\left(6r^3-16Mr^2\right)-r^3\Lambda^2-8r^3+68Mr^2-108M^2r}{(\Lambda-2)r+6M}T_{00}^{\ell m}\right.$$
$$+\frac{1}{e^{4b}}\left[2Mr+r^2(\Lambda-4)\right]T_{11}^{\ell m}+2r^3\partial_{r_*}T_{00}^{\ell m}-2\frac{r^3}{e^{2b}}\partial_{r_*}T_{11}^{\ell m}$$
$$\left.+4\frac{\Lambda r}{e^{4b}}T_1^{\ell m}+\frac{1}{e^{2b}}\left[2\Lambda\left(1-\frac{3M}{r}\right)-\Lambda^2\right]T_2^{\ell m}+4\frac{r^2}{e^{4b}}T_3^{\ell m}\right\}. \tag{4}$$

$$S^{(o)} \equiv \frac{16\pi r}{\Lambda-2}e^{2a}\epsilon^{AB}\nabla_B L_A = \frac{16\pi r}{\Lambda-2}\left[\left(1-\frac{2M}{r}\right)\partial_t L_1^{\ell m}-\partial_{r_*}L_0^{\ell m}\right] \qquad \Lambda\equiv\ell(\ell+1)$$

A.N & L. Rezzolla, Class. Q. Grav. 22 (2005), R167 (*….but we left some misprints around!*)

K. Martel & E. Poisson, Phys. Rev. D 71 (2005), 104003 (*using a general slicing of Schwarzschild*)

GR (ideal) hydrodynamics in a nutshell

Local conservation laws of the stress energy tensor (Bianchi identities) and of the matter current density (the continuity equation):

$$\nabla_\mu T^{\mu\nu} = 0,$$
$$\nabla_\mu J^\mu = 0.$$

Perfect fluid: *no viscosity*

$$T^{\mu\nu} = \rho h u^\mu u^\nu + p g^{\mu\nu}$$
$$J^\mu = \rho u^\mu$$

$$h = 1 + \varepsilon + \frac{p}{\rho}$$

Equation of state (EoS):

$$p = p(\rho, \varepsilon)$$

Difficulty: the solution can be discontinuous (*simplest example: Burger's equation*)

- ✓ High-Resolution-Shock-Capturing (HSRC) methods based on (approximate) Riemann solvers mediated from Newtonian hydrodynamics.

- ✓Need a formulation of the GR-hydro equations in *flux-conservative* form (which is natural for Euler equations)

GR (ideal) hydrodynamics in a nutshell

Eulerian formulation of the general relativistic hydrodynamics equations as a first-order system of conservation laws (*Banyuls, Font, Ibañez, Martí, Miralles. 1997*).

The metric in the ***ADM 3+1 decomposition***

$$ds^2 = -(\alpha^2 - \beta_i \beta^i) dx^0 dx^0 + 2\beta_i dx^i dx^0 + \gamma_{ij} dx^i dx^j$$

Define the vector $\mathbf{U}(\mathbf{w}) = (D, S_j, \tau)$ of the conserved quantities

$$\mathbf{w} = (\rho, v_i, \epsilon)$$

$D = \rho W,$	Conserved rest mass density
$S_j = \rho h W^2 v_j,$	Conserved velocity
$E = \rho h W^2 - p$	Energy
$\tau \equiv E - D$	Conserved internal energy

$$v^i = \gamma^{ij} v_j$$

$$v^i = \frac{u^i}{\alpha u^0} + \frac{\beta^i}{\alpha}$$

$$W \equiv \alpha u^0 = (1 - v^2)^{-1/2}$$

First order flux-conservative hyperbolic system

$$\frac{1}{\sqrt{-g}} \left(\frac{\partial \sqrt{\gamma} \mathbf{U}(\mathbf{w})}{\partial x^0} + \frac{\partial \sqrt{-g} \mathbf{F}^i(\mathbf{w})}{\partial x^i} \right) = \mathbf{S}(\mathbf{w})$$

GWs from accretion of fluid matter

- ✓ Focus: all the elements to study GWs from accretion flows.

  GWs are expected to come from the time variation of the matter quadrupole moment as well as from pure excitation of the spacetime (i.e., QNMs and curvature backscattering).

- ✓ Test-matter approximation ($\mu \ll M$)
  - *Neglect self-gravity of the accreting layers of fluid*
  - *Neglect radiation reaction effects*.

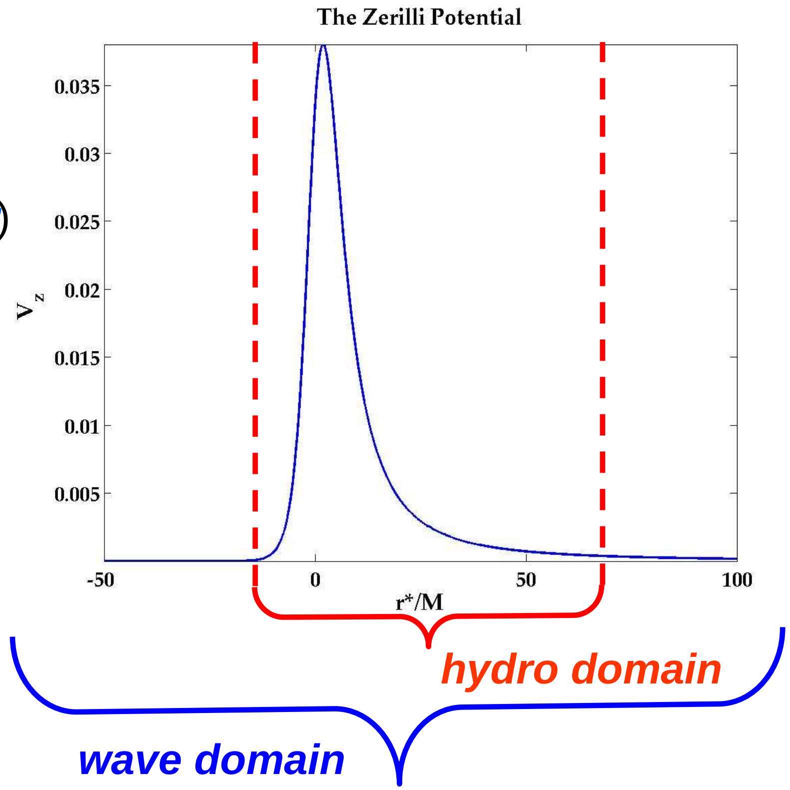
- ✓ Zerilli-Moncrief and Regge-Wheeler equations with a **matter source term**:

  (non-magnetized) dust (e.g quadrupolar shells) or fluid distribution (e.g., thick disks)

Notice: our general relativistic HRSC hydro-code is axisymmetric (2D). We can compute m=0 multipoles only.

Numerical framework

- ✓ Hydro domain much smaller than wave domain
- ✓ Schwarzschild coordinates for hydro code
  (*We are currently implementing EF coordinates*)
- ✓ Update the hydro, then compute the sources
- ✓ Solve perturbations
- ✓ 1D equations: wave-zone observer
- ✓ ID: solve Hamiltonian constraint in the CF approx.
  (unphysical radiation at t=0)

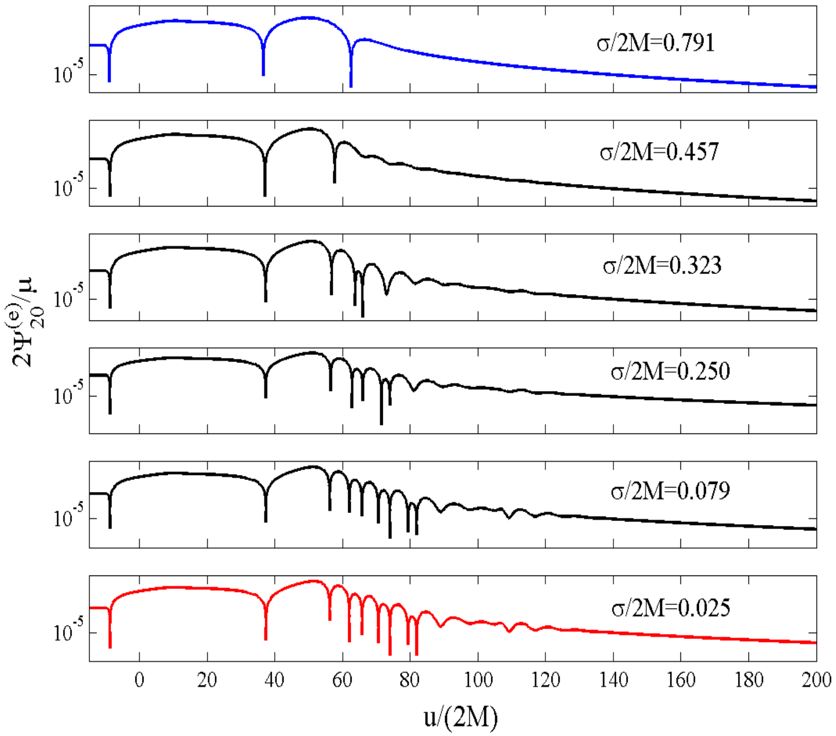


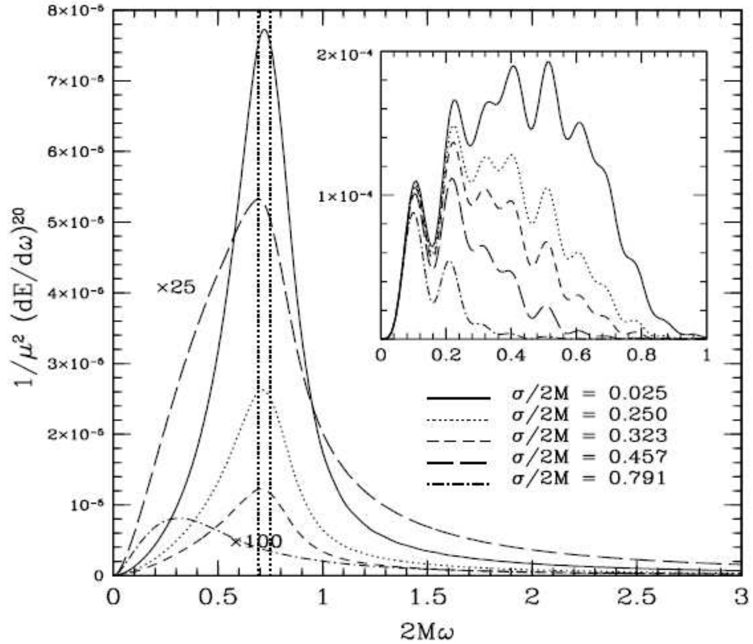
Observer > 200 M

Dust accretion

Quadrupolar dust shells with gaussian radial extent plunging from finite distance with different amount of compactness (width) $\kappa = 1/\sigma^2$ [embedded in a thin spherical atmosphere]

$$\rho = \rho_{\rm bckg} + \rho_0 \exp[-\kappa(r-r_0)^2] \sin^2\vartheta$$

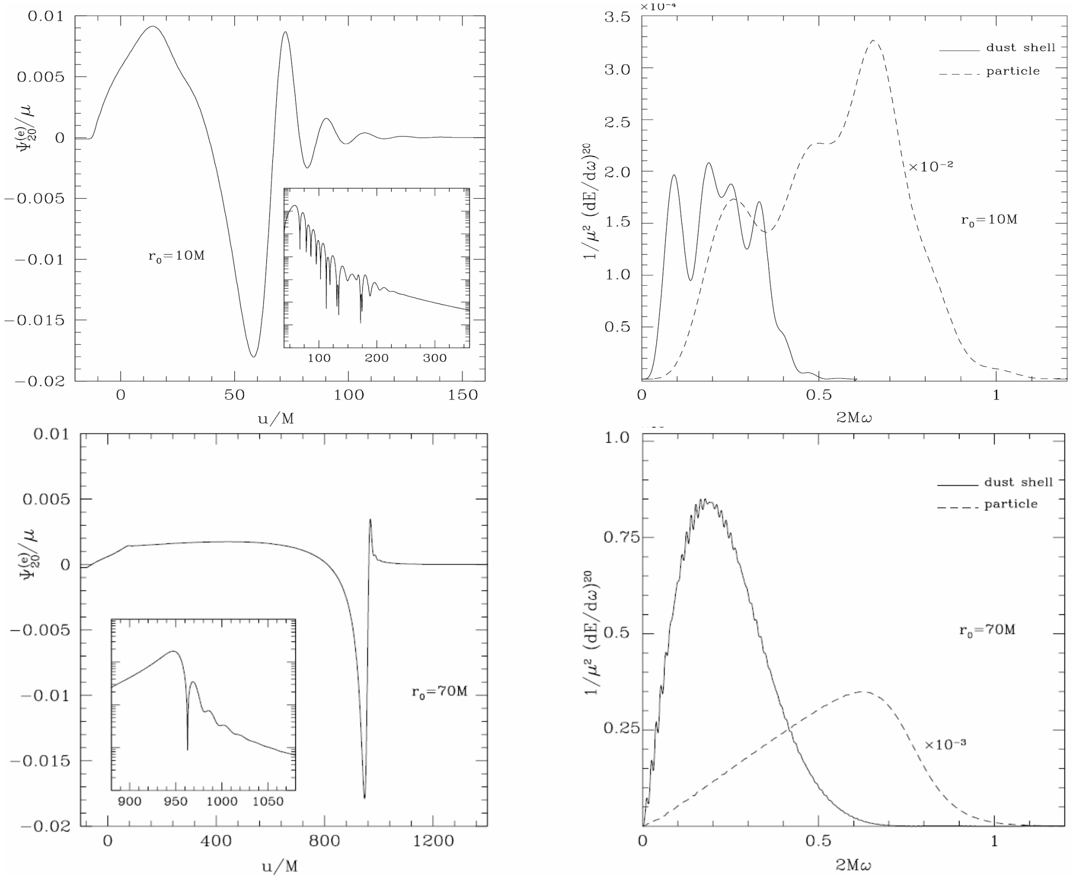




- QNMs in the ringdown phase for narrow shells [but the fit can't be perfect: *Tail effects* (see next slides)]
- Inteference bumps in the (total) energy spectra

Dust accretion

Varying initial position r_0



✓Interference bumps in the energy spectra

✓Tail effects which affect the ringdown

✓The larger the separation, the smaller are the bumps. Tends to a smooth spectrum as expected (***SW 1982 and PSW 1985***).

✓Spacing: $\Delta\omega = 2\pi/T_{\rm infall}$

Emitted energy $\dfrac{2M}{\mu^2}E^{20} \lesssim 10^{-4}$

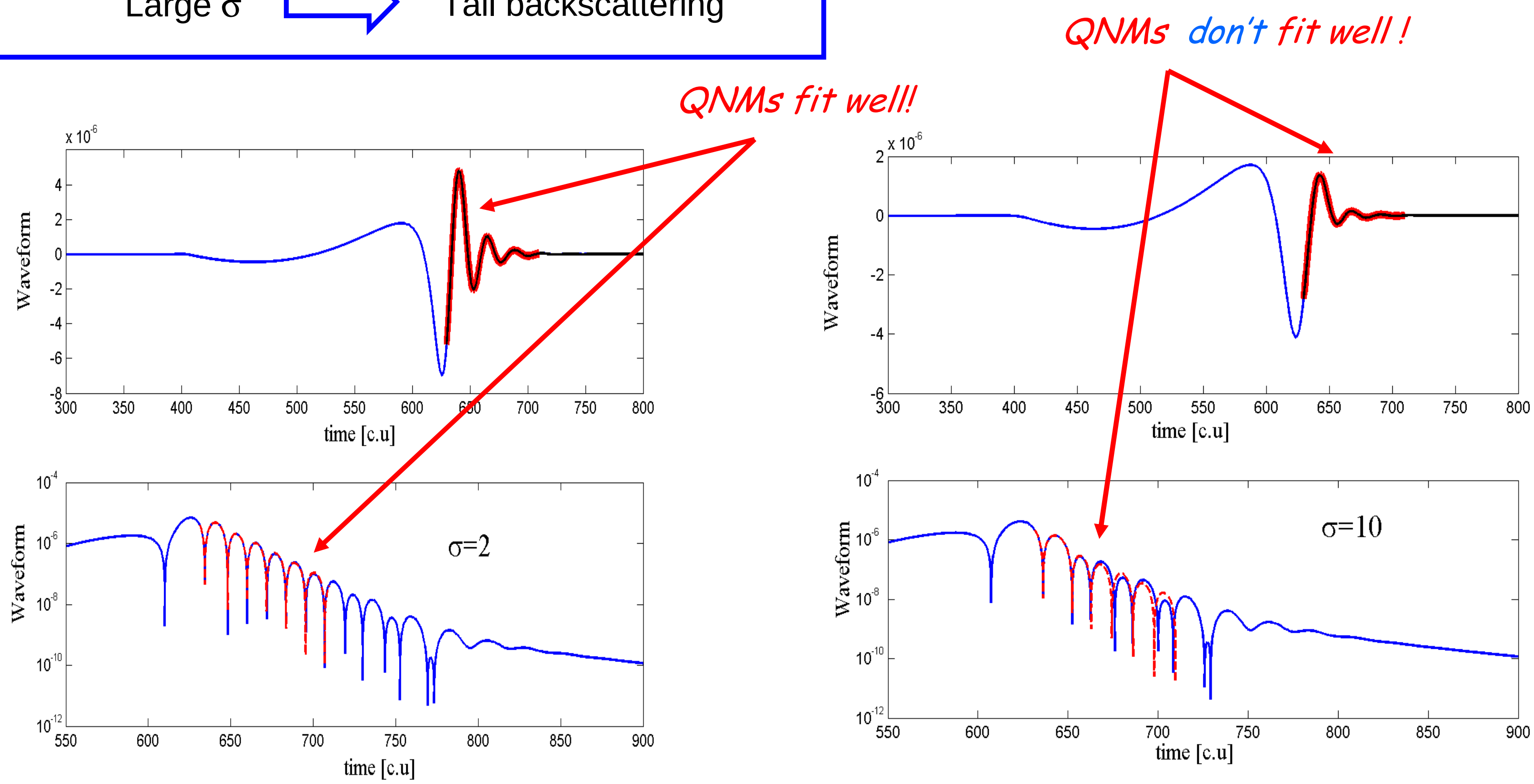
Two order of magnitude (or less) smaller than the ***DRPP*** limit $0.0104mc^2(m/M)$

Understanding curvature backscattering

Scattering of Gaussian pulses of different widths (*with S. Bernuzzi*)

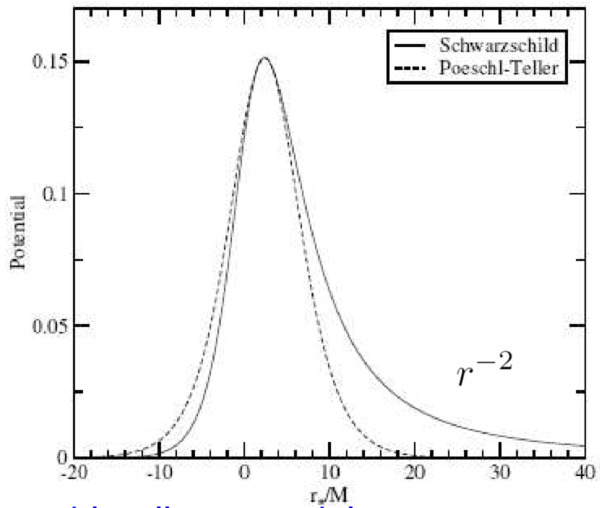
$$\psi = \exp\left[-(r - r_c)^2/\sigma^2\right]$$

Large σ ⇒ Tail backscattering



Understanding curvature backscattering

Isolate the contribution of the peak !



Poeschl-Teller potential

$$U = \frac{V_{\max}}{\cosh\left[\alpha(r_* - r_*^{\max})^2\right]}$$

- Exponential decay versus $1/r^2$ decay

- The first frequencies can be computed with an error of few percents with respect to the real values (*Ferrari&Mashhoon 1984*)

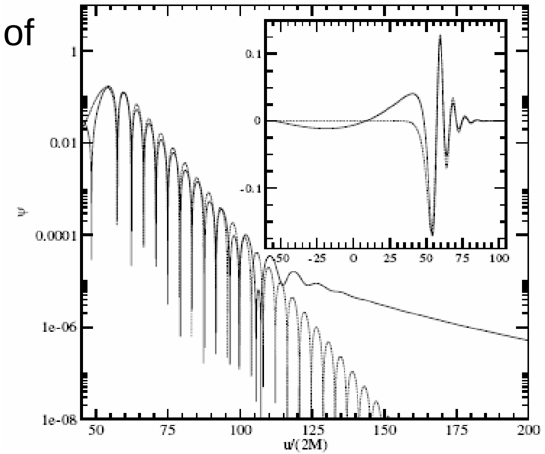
Scattering of Gaussian pulses of different widths:

$$\psi = \exp\left[-(r-r_{\rm c})^2/\sigma^2\right]$$

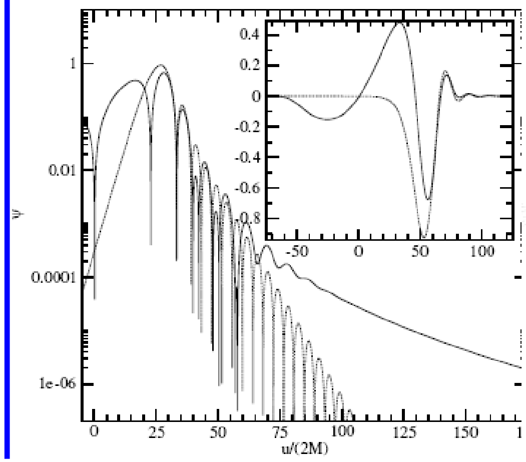
- No tail in the case of PT

- Always the same ringdown

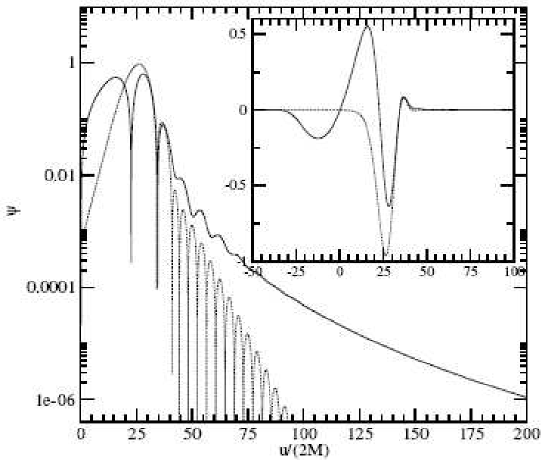
$\sigma = M$



$\sigma = 9.5M$



$\sigma = 11M$



Improving the physical setting: Thick accretion disks

Relativistic tori (i.e. geometrically thick disks) orbiting around black holes are expected to form in at least two different scenarios:

- ✓ *after the gravitational collapse of the core of a rotating massive star (M>25Msun)*
- ✓ *after a neutron star binary merger*

Numerical simulations both in Newtonian physics (*Ruffert&Janka 2001*) as well as in the relativistic framework (*Shibata et al. 2003*) of these scenarios have shown that, under certain conditions a massive disk can be formed

Why can these object be astrophysically interesting?

- ✓ Barotropic fluid configurations with angular momentum: non-Keplerian objects with a cusp ($<6M$)
- ✓ Can be hydrodynamically unstable: the runaway instability
  [but stabilazable without self-gravity and magnetic fields (*Daigne&Font 2004*)].
- ✓ Proposed model for HFQPOs oscillations in X-ray light curves in BH binaries (*Rezzolla et al. 2003*)
- ✓ If high densities are considered, the variations of the quadrupole moment due to oscillation make them GWs sources which could be detectable (within the Galaxy) by ground based interferometers.
- ✓ GWs emission computed via quadrupole fomula only (*Zanotti et al. 2003*)

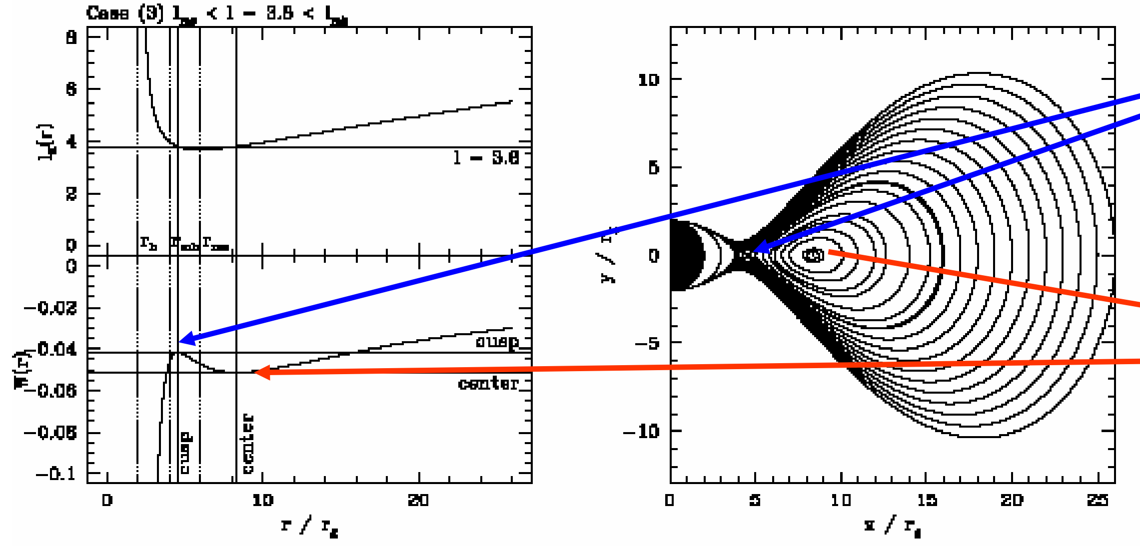
Thick accretion disks

Barotropic matter (polytropic EOS) around a Schwarzschild (or Kerr) BH with a certain angular momentum.

Consider just constant *l* disks (but don't worry of the runaway instability. Fixed background spacetime.)

$$\frac{\nabla_i p}{\rho h} = -\nabla_i W + \cancel{\frac{\Omega \nabla_i l}{1 - \Omega l}}$$

$$W = \frac{1}{2} \ln \frac{r^2 (r - 2M) \sin^2 \theta}{r^3 \sin^2 \theta - l(r - 2M)}$$



cusp < *6M*

Keplerian points

center

➢Torus surrounded by a thin spherical (***Michel 1972***) atmosphere.

➢ Mass of the torus << Mass of the black hole

GWs from disk oscillations

Characteristic GWs amplitude (from *Zanotti et al. 2004*).
Disks (around Kerr BHs) with average density in the range $10^{12} \div 10^{14} \mathrm{g/cm}^3$

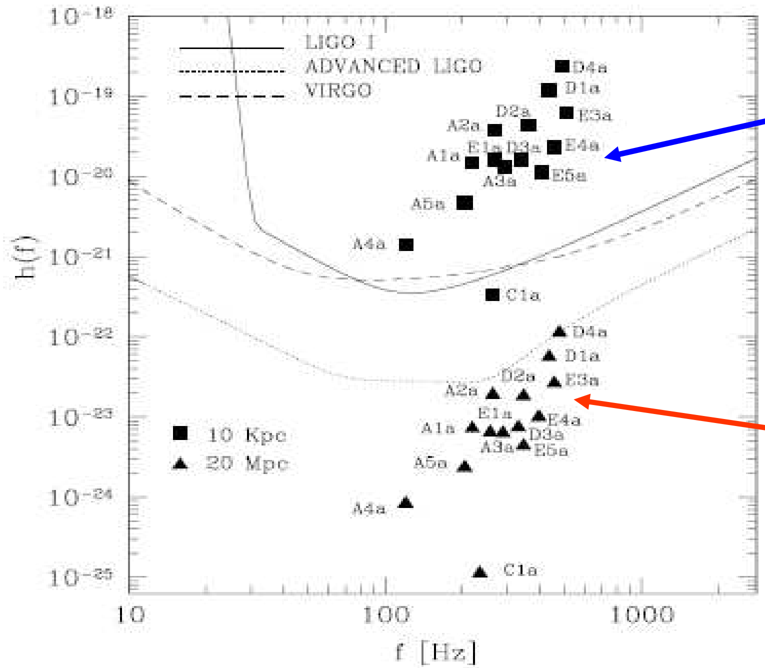


Figure 7. Characteristic wave amplitudes for the tori models of Table 1 with respect to the strain noise of LIGO I and the planned sensitivity of Advanced LIGO, respectively. The amplitudes are computed at both a galactic distance of 10Kpc and at an extragalactic distance of 20Mpc. The planned strain noise of VIRGO is also reported for comparison.

10Kpc

20Mpc

Mass of the torus at most 10% of the mass of the BH

$$p = \kappa \rho^{\gamma}$$

$$\gamma = 4/3$$

$$M = 2.5 M_{\odot}$$

Notice these are inferior limits due to the semplifications of the model (no self-gravity, small mass)

Disk oscillations

✓GWs extracted using the quadrupole formula (***no spacetime** reaction calculable*)

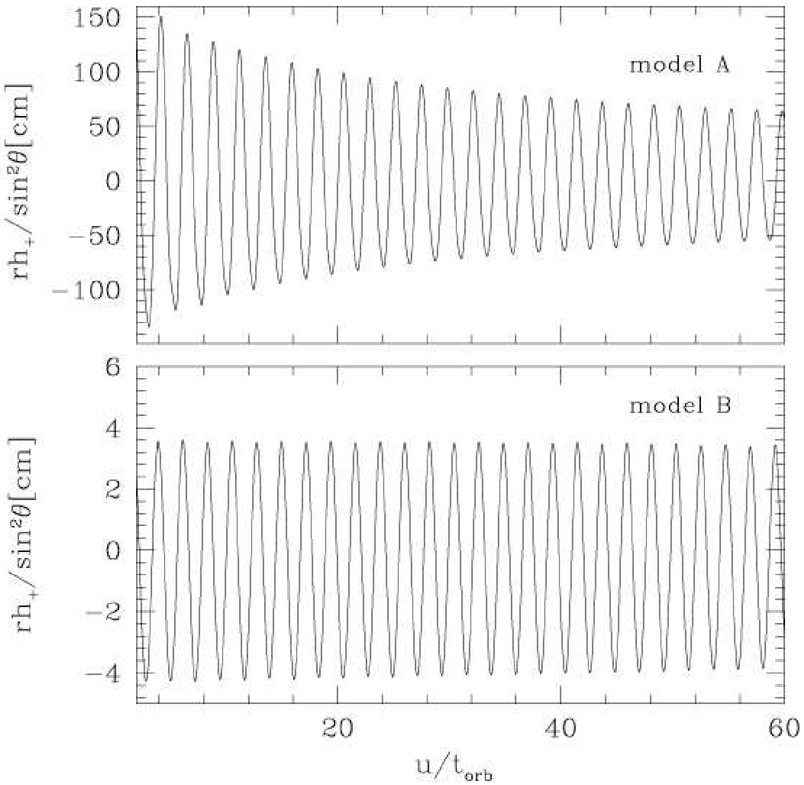
✓The torus oscillates in the potential well due to a (*small*) radial velocity perturbation

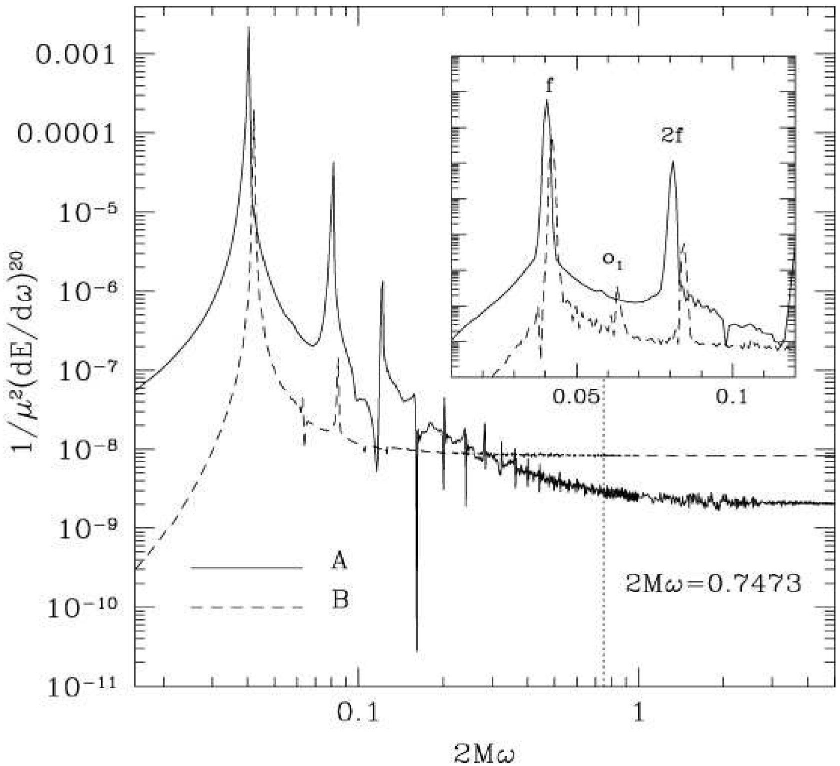
✓Perturbation expressed in terms of the radial velocity of the Michel solution: $v_r = \eta (v_r)_{\rm Michel}$

Redo the analysis using perturbation theory: solution of ZM and RW equations

Model	N_r	N_θ	μ/M	κ (cgs)	l	$r_{\rm cusp}$	$r_{\rm center}$	ρ_c (cgs)	$r_{\rm in}$	$r_{\rm out}$	L (km)	$t_{\rm orb}$ (ms)	ΔW	η
A	320	150	0.10	3.01×10^{13}	3.75	4.835	7.720	3.58×10^{13}	4.83	11.57	24.87	1.66	0	0.1
B	300	150	0.02	3.58×10^{13}	3.75	4.835	7.720	1.39×10^{13}	5.81	10.32	16.65	1.66	-0.002	0.01
C	300	150	0.10	9.60×10^{13}	3.80	4.576	8.352	1.15×10^{13}	4.57	15.89	41.76	1.86	0	0.01
D	300	150	0.01	6.19×10^{14}	3.95	4.107	9.971	2.83×10^{11}	5.49	29.08	87.09	2.43	-0.015	0.02
O_2	300	150	0.066	6.19×10^{14}	3.95	4.107	9.971	7.69×10^{11}	4.93	42.52	138.75	2.43	-0.008	0.02
O_3	300	150	0.02	1.59×10^{15}	3.95	4.107	9.971	1.05×10^{11}	4.107	76.39	266.75	2.43	0	0.008

Disk oscillations



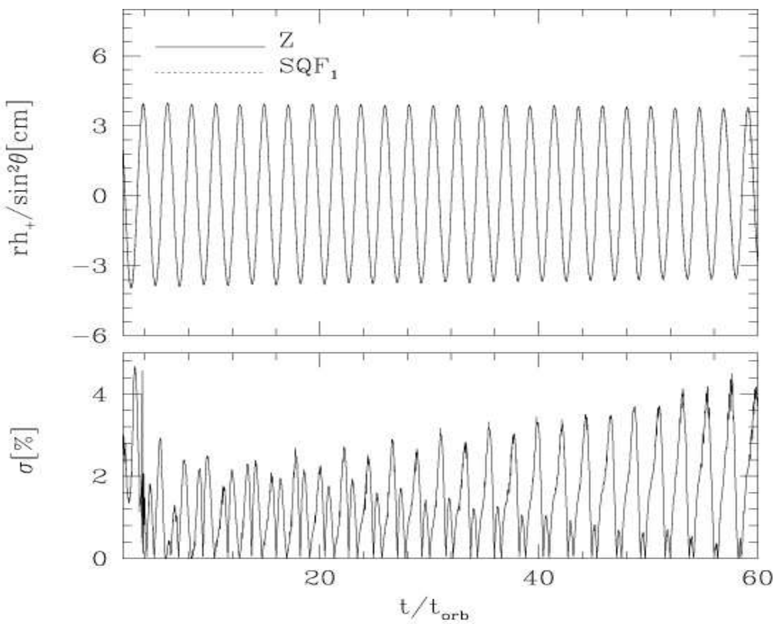


Main result: apparently, ***no QNMs*** of the black hole are present, although some amount of matter plunges into the hole at every oscillation.

Notice: Different statement from Ferrari et al. in frequency domain (PRD 73, 124028 (2006))

Disk oscillations

Analysis of model B



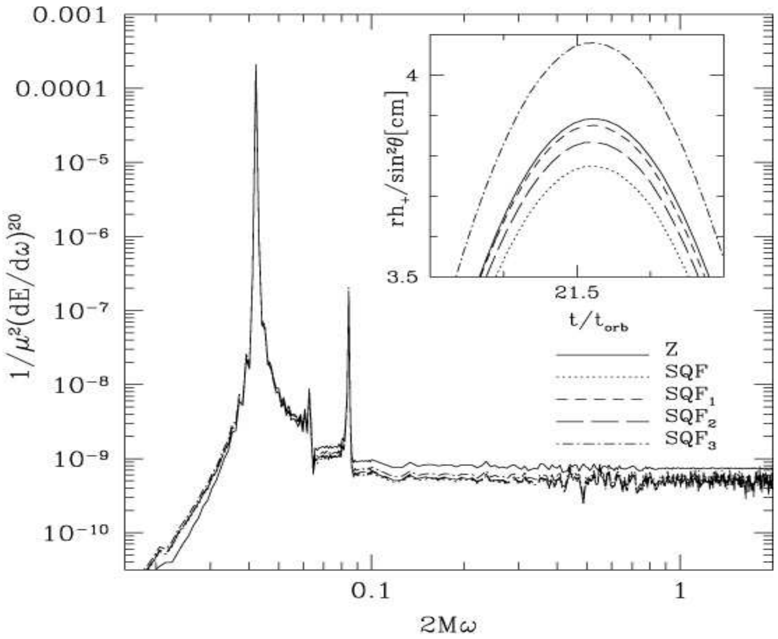


TABLE II: Relative differences $\Delta E^{20} = |E^{20} - E|/E^{20}$ among the emitted energies computed using different versions of the quadrupole formula (E) and the gauge-invariant approach (E^{20}) for disk models with different position of the center.

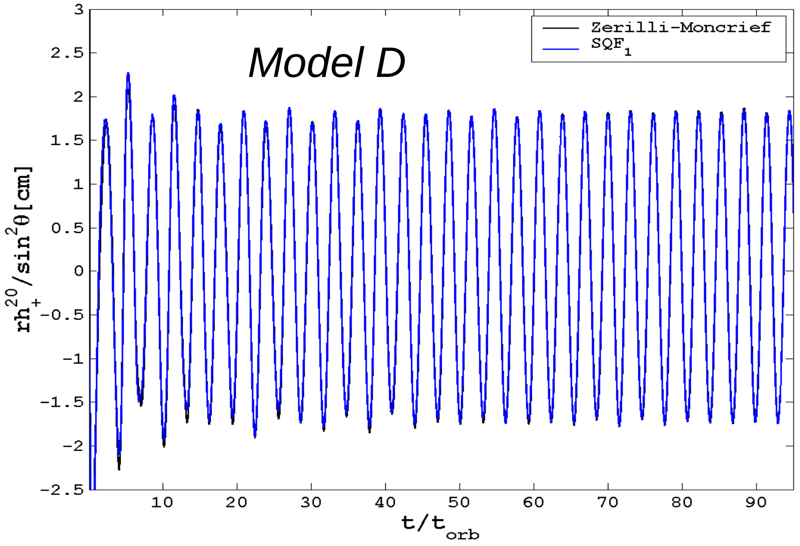
Model	$r_{\rm center}$	$\Delta E^{20}_{\rm SQF}$	$\Delta E^{20}_{\rm SQF_1}$	$\Delta E^{20}_{\rm SQF_2}$	$\Delta E^{20}_{\rm SQF_3}$
A	7.720	2.9%	0.7%	0.8%	11.0%
B	7.720	5.9%	0.3%	2.6%	10.6%
C	8.352	4.9%	1.8%	3.2%	6.7%
D	9.971	2.7%	1.7%	2.2%	3.8%

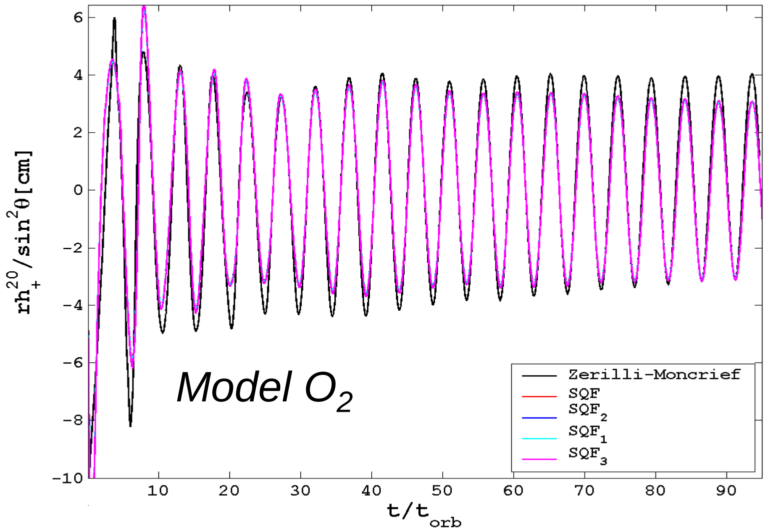
✓Smallest difference with SQF$_1$

✓Quadrupole formula seems reliable !

Disk oscillations

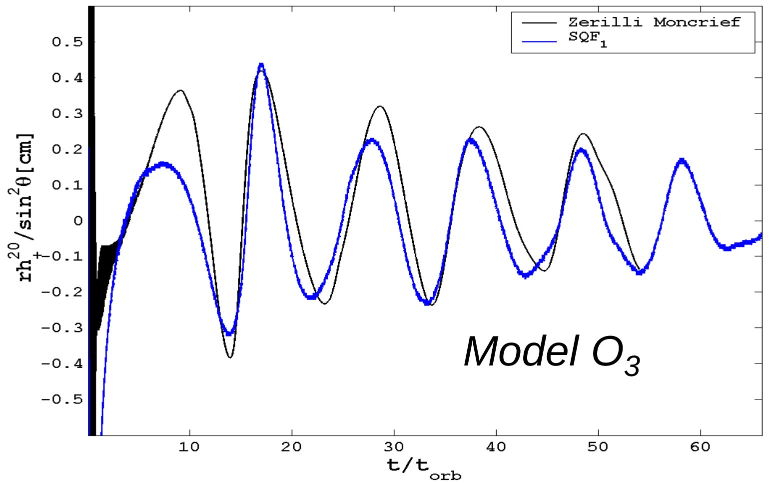
But large differences changing the model !





The width of the torus is also a crucial parameter !

- ✓Differences in the emitted energy > 20%.
- ✓Effect of *curvature backscattering*
- ✓Compare with Tanaka et al., *PTP 90, p.65 (2003)*



Disk plunge

TABLE II: Stable (D_0 and D_1) and marginally stable (D_2) constant angular momentum thick disks orbiting around a Schwarzschild black hole of mass $M = 2.5M_\odot$. From left to right, the columns report the name of the model, the number of radial and polar gridzones used in the simulations, the disk-to-hole mass ratio, the polytropic constant κ of the isoentropic EOS $p = k\rho^\gamma$ with $\gamma = 4/3$, the value of the specific angular momentum l, the position of the cusp $r_{\rm cusp}$ and of the center $r_{\rm center}$ of the disk, the rest–mass density at the center ρ_c, the location of the inner ($r_{\rm in}$) and outer ($r_{\rm out}$) disk boundaries, the value of the potential barrier ΔW and the orbital period at the center $t_{\rm orb}$.

Model	N_r	N_θ	μ/M	κ (cgs)	l	$r_{\rm cusp}$	$r_{\rm center}$	ρ_c (cgs)	$r_{\rm in}$	$r_{\rm out}$	ΔW	$t_{\rm orb}$ (ms)
D_0	300	150	0.0077	2.25×10^{13}	3.72	5.06	7.27	7.31×10^{12}	5.26	9.50	-1×10^{-4}	1.51
D_1	300	150	0.0463	9.00×10^{13}	3.80	4.57	8.35	6.86×10^{12}	5.21	14.54	-0.002	1.87
D_2	300	150	0.0779	1.05×10^{14}	3.80	4.57	8.35	8.74×10^{12}	4.57	15.89	0	1.87

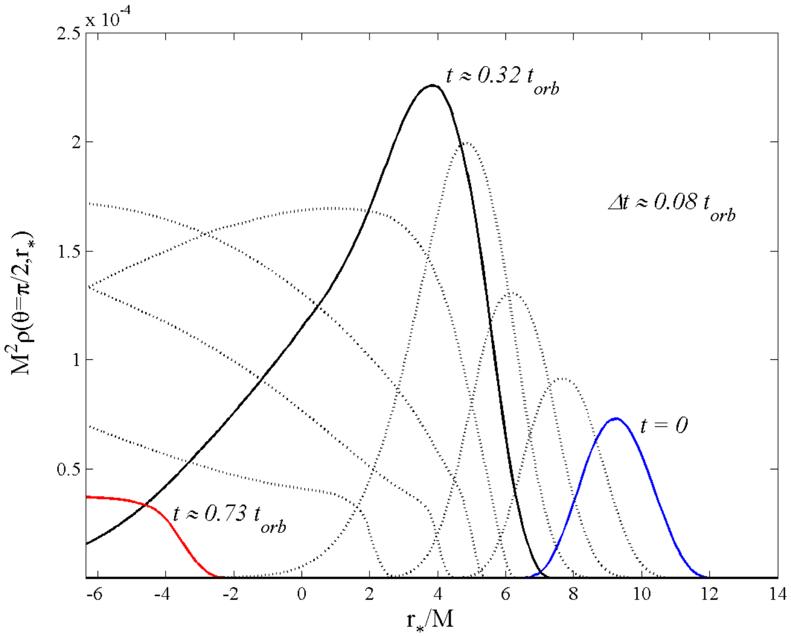
Violent accretion

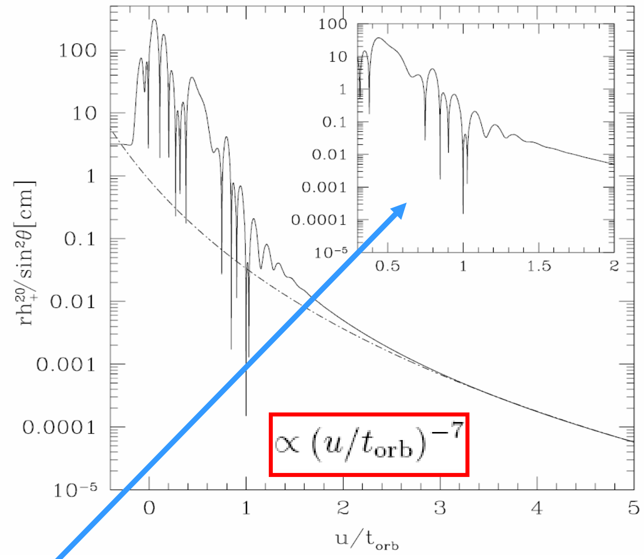
Model $\mathbf{D_0}$: high radial velocity perturbation: the torus completely plunges on the BH as a whole.

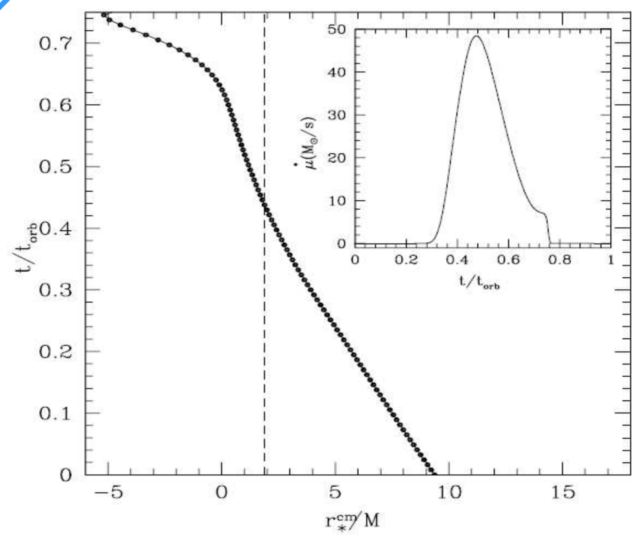
Disk plunge (D_0)

Violent accretion (η=0.2):

Snapshots of the matter density in the equatorial plane



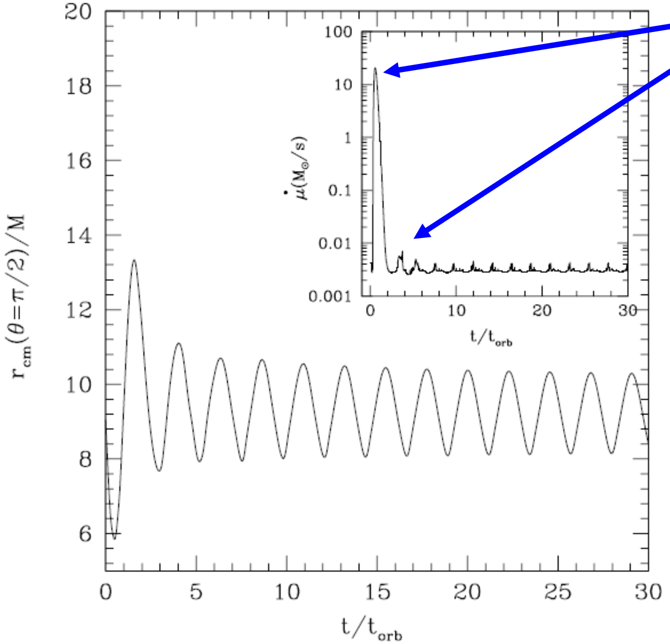


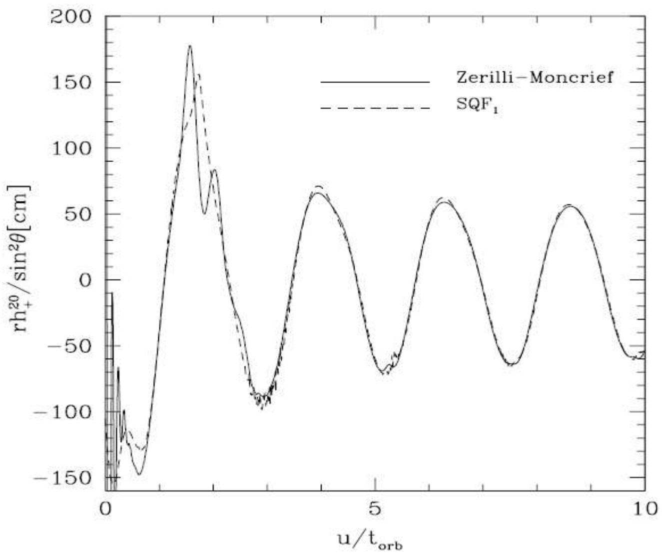


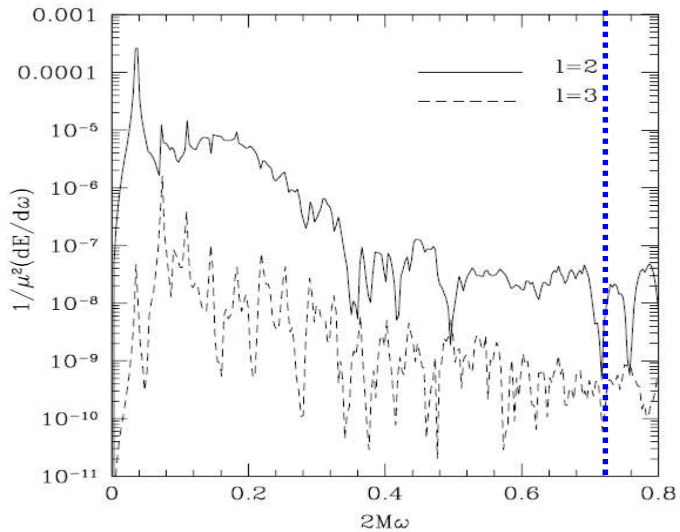
✓Very weak ringdown phase (*due to loss of compactness*)

✓Energy: $\dfrac{2M}{\mu^2}E^{20} = 9.22 \times 10^{-6}$

Other kind of backscattering effects







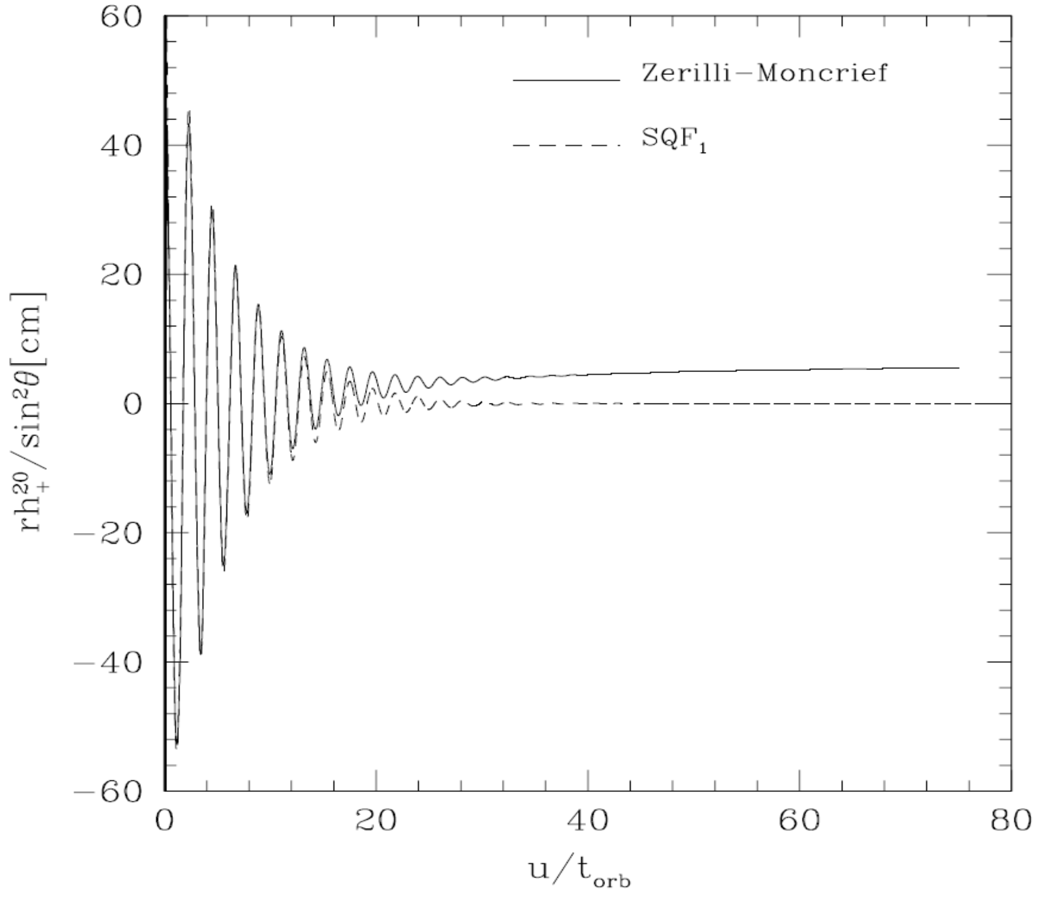
Perturbation is not high enough to have a complete plunge

The remnant is pushed back by the centrifugal barrier

Damped (spacetime) oscillations at low frequencies.
But *there are no QNMs!*

Other kind of backscattering effects

Reduce “by hand” angular momentum.
Secular "drift" due to the tail of the curvature potential



In progress work: Eddington-Filkenstein coordinates

Recent work in collaboration with *P. Montero* (Valencia)

- ✓"New" 2D hydro code in EF coordinates (aiming at having one 3D soon)
- ✓ Successful tests: implementattion of Michel accretion and of stationary tori
- ✓ Currently implementing GWs extraction with the STMP formalism

Advantages:

✓less resolution needed (good for working in 3D one day)
✓Excision of the inner boundary (no waves from inner boundary).
✓*Simpler and more elegant to do simulations in this coordinates…*

Conclusions

Hybrid procedure: BH linear perturbation theory + nonlinear GR-hydro evolution to account for the dynamics of complex matter flows as a complementary approach to full Numerical Relativity simulations.

Dust shells

✓QNMs excitation + tail (backscattering) effects.
✓Interference effects (bumps & reduction of the energy respect to the DRPP limit)

Thick disks

✓Disk oscillations: fluid modes (***in principle detectable***)
✓Comparison between SQF and perturbation theory
✓Backscattering effects + QNMs ringdown (just if complete accretion occurs)

Present and future work

Implementation of GR-Hydro and perturbations equations in horizon-penetrating coordinates.
The aim is to have, on one side a *GR(M)Hydro 3D code* and, on the other, a *2D one for rotating black holes* (including a solver for the penetrating Teukolsky equation).