Equilibrium binary neutron star da
circular orbits

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From geometry to numerics, 20 November, 2006.

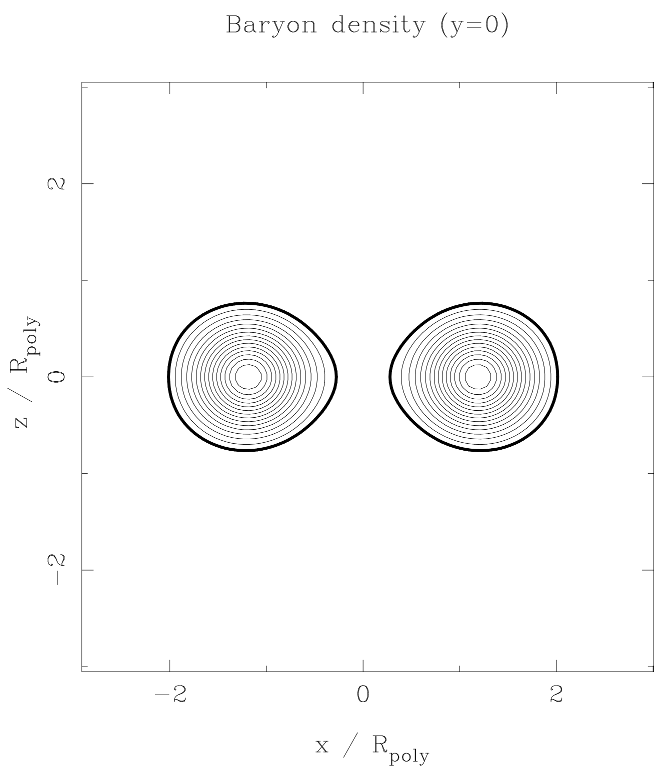
Introduction

∘ Accurate equilibrium solutions in circular orbits are u
investigate the final stage of inspiral.

⋆ Initial data sets of simulations.

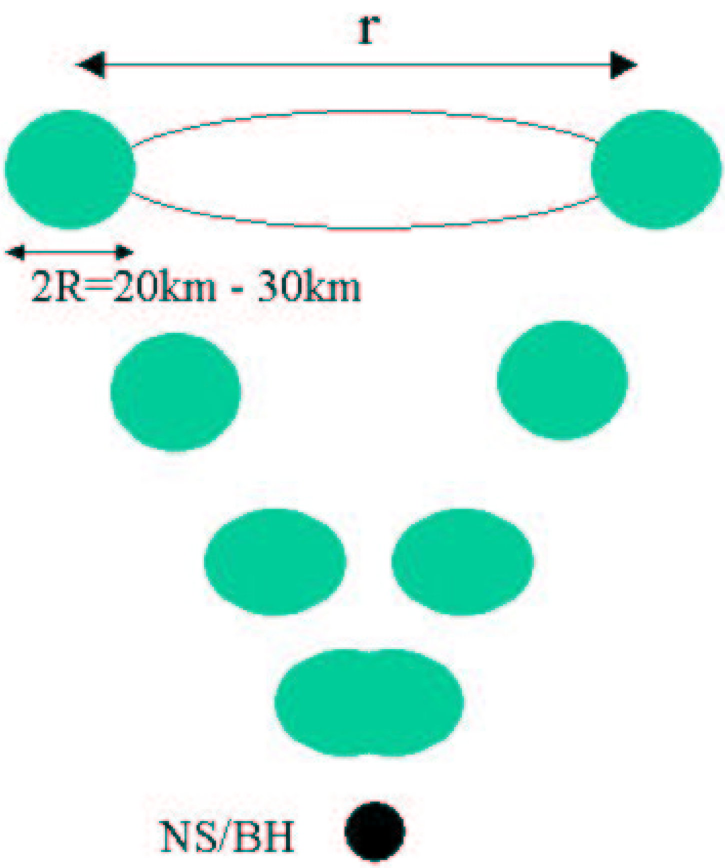
⋆ Reference for the inspiral orbit of simulations; calibrat
circularity of the last couple of orbits

⋆ Determination of the upper limit of the gravitational w
$f_{\rm GW}$ of the inspiraling orbit → constrain nuclear EOS.



Taniguchi and Gourgoulh

○ Th



Inspira

$f \sim 10$

Post-N

Point I

$r \gg R,$

Interm

$f \gtrsim 50$

$r \lesssim 4R$

$t_{\rm GW} \gg$

Finite

Merger

$f > 1$k

Event rate : 1/yr in $\sim 50 - 100$ Mpc (Kalogera et al. 2004)

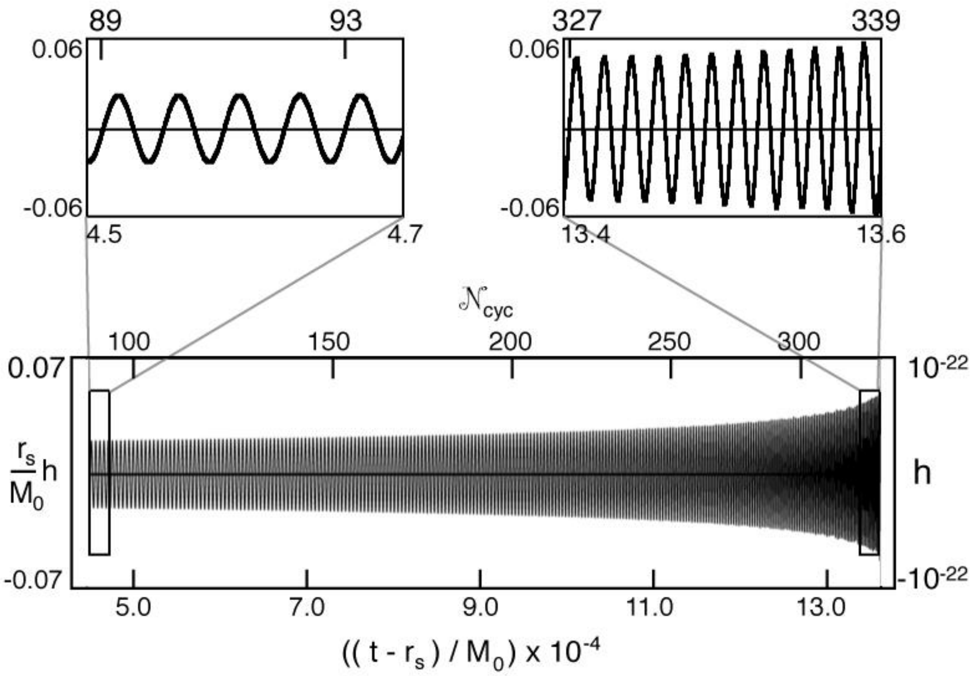
$\sim 40 - 600$ events/yr for Advanced LIGO.

BH-NS merger rate $\gtrsim 10\%$ of the NS-NS m

Preparation for templates of gravitational waveforms.

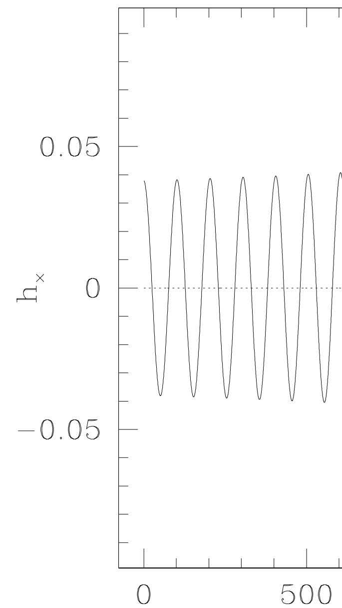
◦ Wavetrain for inspiralling to merger of binary neutron

(Duez, Baumgarte, Shapiro, Shibata and Uryū, PRD 2002.)



$(M/R)_\infty = 0.05$, N=1.0 polytrope.

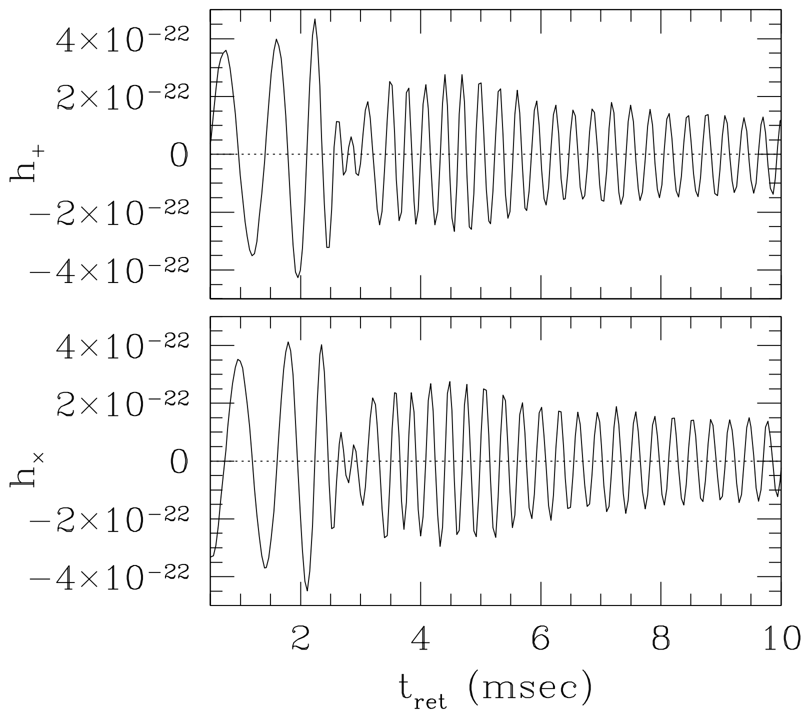
$t_{\text{vis}} > t_{\text{GW}} > P_{\text{orb}}$



$(M/R)_\infty = 0.14$,

ISCO –

- The GW frequency at the final inspiral $\sim$ 1 kHz.
- Merger object (hypermassive neutron star) oscillation
- Target for the narrow band search of Advanced LIGO



Shibata (2005)

ISCO $\rightarrow$ Merger $\rightarrow$ ringdown (HMNS)

Initial data construction

Initial data construction

Data on the initial slice has to satisfy constraints ($G_{\alpha\beta} - 8$

○ Isenberg-Wilson-Mathews (IWM) formulation.

• 4 constraints and the spatial trace of Einstein equati
for spatially conformally flat metric on a maximally emb

$$ds^2 = -\alpha^2 dt^2 + \psi^4 f_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j d$$

f_{ij}: flat metric.

(5 components of the metric coefficients are solved. St
dition in rotating frame is assumed for the fluid equatio

• The balance of the gravity and orbital acceleration is
correctly due to the truncation, which induces an ecce
inspiral orbit.

(IWM formulation agrees with GR in a static and spheric
spacetime, and with the first post-Newtonian approxima

○ Initial data for BBH in IWM formulation.

- For the metric, $ds^2 = -\alpha^2 dt^2 + \psi^4 f_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j$
we solve five equations $G_{\alpha\beta}n^\beta = 0$, and $G_{\alpha\beta}\gamma^{\alpha\beta} = 0$. $(\gamma_a$

$$\mathring{\Delta}\psi + \frac{\psi^5}{8}\left(A_{ab}A^{ab} + \frac{2}{3}K^2\right) = 0,$$

$$\mathring{\Delta}(\alpha\psi) + \psi^5 £_{t-\beta}K - \alpha\psi^5\left(\frac{7}{8}A_{ab}A^{ab} + \frac{5}{12}K^2\right) =$$

$$\mathring{\Delta}\tilde{\beta}_a + \frac{1}{3}\mathring{D}_a\mathring{D}_b\tilde{\beta}^b + 2\alpha A_a{}^b\mathring{D}_b \ln\frac{\psi^6}{\alpha} - \frac{4}{3}\alpha\mathring{D}_a K =$$

- Choosing K=0, and imposing boundary condition at
asymptotics, a sysmtem of the eliptic equations are solv

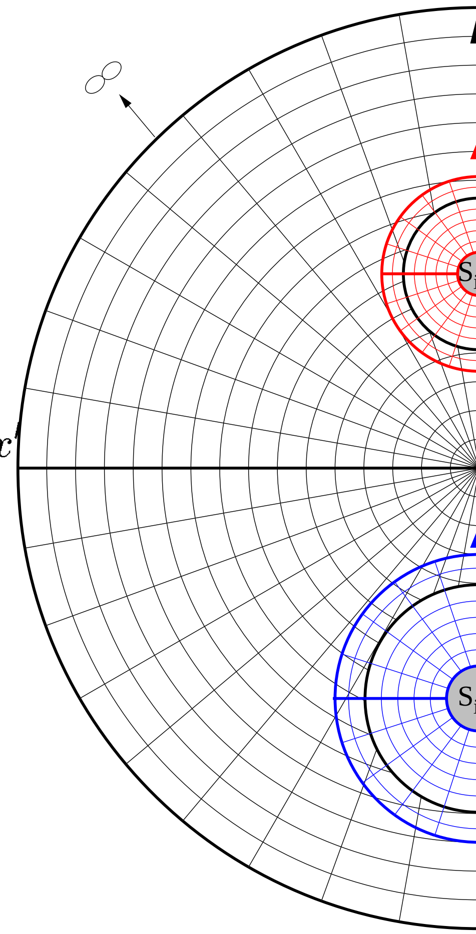
• Coordinate and Poison solver for Binary black hole/neut

◦ Green's integral formula is applied on three overlapped domains of spheircal coordinates. For $\mathcal{L}\phi = \mathcal{S}$,

$$\phi(x) = -\frac{1}{4\pi}\int_V G(x,x')S(x')d^3x'$$
$$+\frac{1}{4\pi}\int_{\partial V}\left[G(x,x')\nabla\phi(x') - \phi(x')\nabla' G(x,x')\right] d^3x'$$

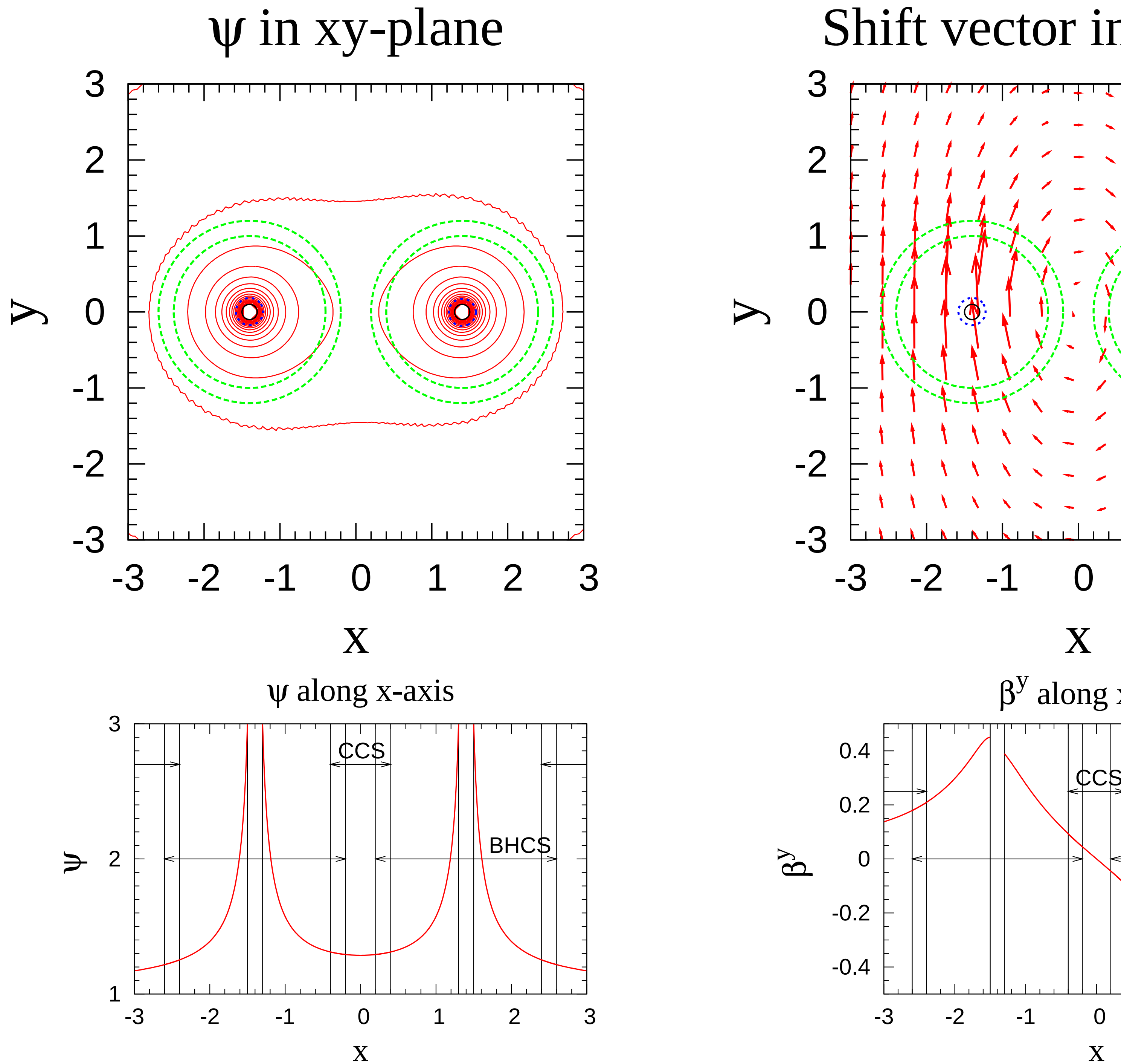
◦ For the central domain,
$G(x,x') = \frac{1}{|x-x'|}$ is chosen.

◦ For the black hole domains,
the Green's function for either Dirichlet or Neumann boundary between two concentric spheres is chosen depending on the condition imposed on the fields.



(Tsokaros, Ury

in preparatio



ψ in xy-plane
Shift vector in
y
x
ψ along x-axis
β^y along x
CCS
BHCS
ψ
β^y
x

New Formulations

○ New formulations for the initial data of binary compa
circular orbits solve all components of Einstein-Euler s
in 3+1 form on a spacelike hypersufrace Σ without a

(1) Waveless Approximation. (Shibata, Uryū, Friedman, 2004

- ⋆ Waveless condition $\partial_t \tilde{\gamma}^{ab} = 0$ is imposed on Σ. (γ_{ab}
- ⋆ All fields have Coulomb type fall off in the asympto
- ⋆ All components of Einstein's equation are written e
- ⋆ For the fluid sources, stationarity in rotating frame

(2) Helically symmetric perfect fluid spacetime.

(Blackburn, Detweiler, 1992; Friedman, Uryū, Shibata, 2002;
Whelan, Beetle, Krivan, Price 2002; Klein 2004)

- ⋆ Helical symmetry $\pounds_k g_{\alpha\beta} = 0$ is imposed.
  (Stationary condition in a rotating frame. $k^\alpha := t^\alpha + \Omega\phi^\alpha$.)
- ⋆ Helically symmetric spacetime is no more asymptoti
- ⋆ Field equations are written mixed elliptic and Helmh
- ⋆ The latter may be solved for half-advanced + half-r

◯ Formulation.

◦ The field equation $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ is projected to Σ and

Hamiltonian constraints : $(G_{\alpha\beta} - 8\pi T_{\alpha\beta})n^\alpha n^\beta = 0$
Momentum constraints : $(G_{\alpha\beta} - 8\pi T_{\alpha\beta})\gamma^\alpha{}_a n^\beta =$
Trace of a projection to Σ : $(G_{\alpha\beta} - 8\pi T_{\alpha\beta})\gamma^{\alpha\beta} = 0.$
Tr free part of a projection to Σ : $(G_{\alpha\beta} - 8\pi T_{\alpha\beta})(\gamma^\alpha{}_a\gamma^\beta{}_b -$

Solved for the metric $\{\psi, \beta^a, \alpha, \tilde{\gamma}_{ab}\}$ on a slice Σ, in a ch

$$ds^2 = -\alpha^2 dt^2 + \psi^4\tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

◦ Gauge conditions:

(1) the maximal slicing $K = 0$.

(2) the spatial gauge $\mathring{D}_a\tilde{\gamma}^{ab} = 0$, Dirac gauge.

◦ A condition to specify the conformal decomposition:

$\det(\tilde{\gamma}_{ab}) = \det(f_{ab})$,

(The confomally rescaled the spatial metric is defined by $\tilde{\gamma}_{ab} = \psi^{-}$

○ Field equations for $\tilde{\gamma}_{ab}$ for the waveless and helical fo

The waveless condition : $\partial_t \tilde{\gamma}^{ab} = 0$.
The helical symmetry : $\pounds_k \tilde{\gamma}_{ab} := \partial_t \tilde{\gamma}_{ab} + \pounds_{\Omega\phi} \tilde{\gamma}_{ab} = 0$.

⋆ Each condition yeilds a different relation for K_{ab}, respe

$$K_{ab} = \frac{1}{2\alpha} \pounds_{\beta} \gamma_{ab} + \frac{1}{3\alpha} \gamma_{ab} D_c(\Omega \phi^c), \qquad \text{w}$$

$$K_{ab} = \frac{1}{2\alpha} \pounds_{\omega} \gamma_{ab} = \frac{1}{2\alpha} \left(\pounds_{\beta} \gamma_{ab} + \pounds_{\Omega\phi} \gamma_{ab} \right), \qquad \text{h}$$

β^a: the shift in non-rotating frame.
ω^a: the shift in rotating frame, $\omega^a = \beta^a + \Omega \phi^a$.

⋆ For the time derivative of the extrinsic curvature, we a

$$\pounds_k K_{ab} := \partial_t K_{ab} + \Omega \pounds_{\phi} K_{ab} = 0.$$

⋆ Spatial trace free projection of the Einstein equation i

$$(G_{\alpha\beta} - 8\pi T_{\alpha\beta})\left(\gamma_a{}^{\alpha}\gamma_b{}^{\beta} - \frac{1}{3}\gamma_{ab}\gamma^{\alpha\beta}\right) = \mathcal{E}_{ab} - \frac{1}{3}\gamma_{ab}\gamma^{cd}\mathcal{E}$$

$$\mathcal{E}_{ab} := \frac{1}{\alpha}\pounds_\omega K_{ab} + {}^3R_{ab} - \frac{1}{\alpha}D_aD_b\alpha + KK_{ab} - 2K_{ac}K_b{}^c$$

⋆ A few variations to write the above equation: $h_{ab} := \tilde{\gamma}$

Waveless or Helical

$$\mathring{\Delta} h_{ab} = 2\left(\hat{\mathcal{E}}_{ab} - \frac{1}{3}\tilde{\gamma}_{ab}\tilde{\gamma}^{cd}\hat{\mathcal{E}}_{cd}\right) - \frac{1}{3}\tilde{\gamma}_{ab}\mathring{D}^e h^{cd}\mathring{D}_e h_c$$

$$\hat{\mathcal{E}}_{ab} := \frac{1}{\alpha}\pounds_\omega K_{ab} + R^{\mathrm{NL}}_{ab} + {}^3\tilde{R}^{\psi}_{ab} - \frac{1}{\alpha}D_aD_b\alpha + KK_{ab} - 2K_{ac}$$

Helical

$$\left(\mathring{\Delta} - \Omega^2\partial_\phi^2\right)h_{ab} = 2\left(\bar{\mathcal{E}}_{ab} - \frac{1}{3}\tilde{\gamma}_{ab}\tilde{\gamma}^{cd}\bar{\mathcal{E}}_{cd}\right) - \frac{1}{3}\tilde{\gamma}_{ab}\mathring{D}^e h^{cd}\mathring{D}_e h_{cd} + \frac{1}{3}\tilde{\gamma}_{ab}\Omega$$

$$\bar{\mathcal{E}}_{ab} := \hat{\mathcal{E}}_{ab} - \frac{1}{2}\Omega^2\partial_\phi^2 h_{ab}\,.$$

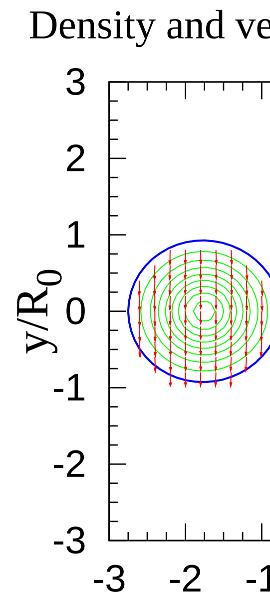
Waveless binary neutron star solu

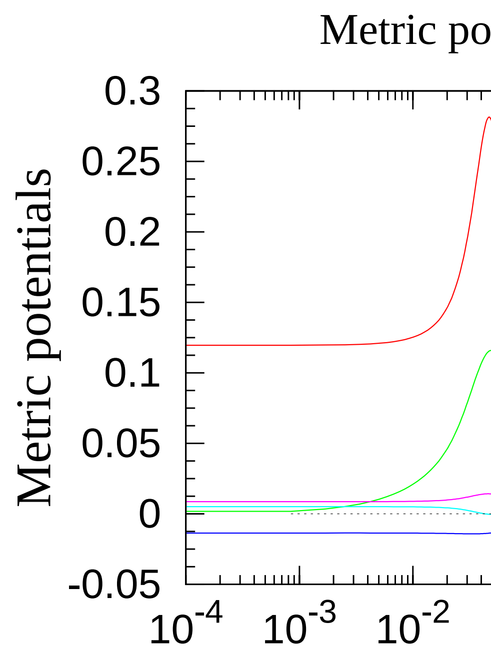
Numerical solution of BNS with non-conformally flat 3-metric.

(Uryū, Limousin, Friedman, Gourgoulhon, Shibata 2006)

Adiabatic EOS: $p = \kappa\rho^\Gamma$, $\Gamma = 2$. $(M/R)_\infty = 0.17$, $d/R_0 = 1.75$.

24 grids along the stellar radius. $(210 \times 48 \times 48)$ for the field.





Non-conformal flat part $h_{ab} = \tilde{\gamma}_{ab} - f_{ab}$ may introduce the following corrections (Shibata and Uryū 2002).

$$\frac{\delta\Omega}{\Omega} = O(h_{ab}), \qquad \frac{\delta\rho}{\rho} = O(v^2 h_{ab}),$$
$$\frac{\delta\psi}{\psi} = O(v^2 h_{ab}), \qquad \frac{\delta M}{M} = O(v^2 h_{ab}),$$
$$\frac{\delta J}{J} = O(h_{ab}).$$

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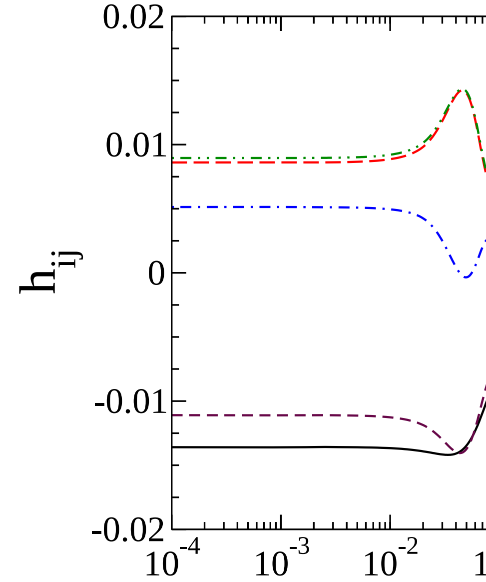
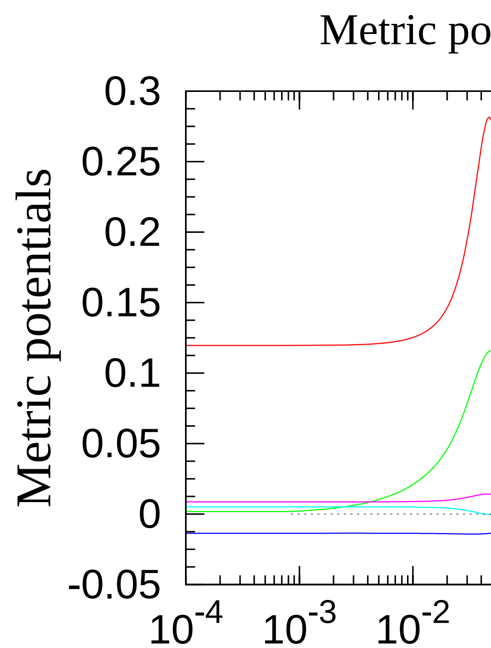
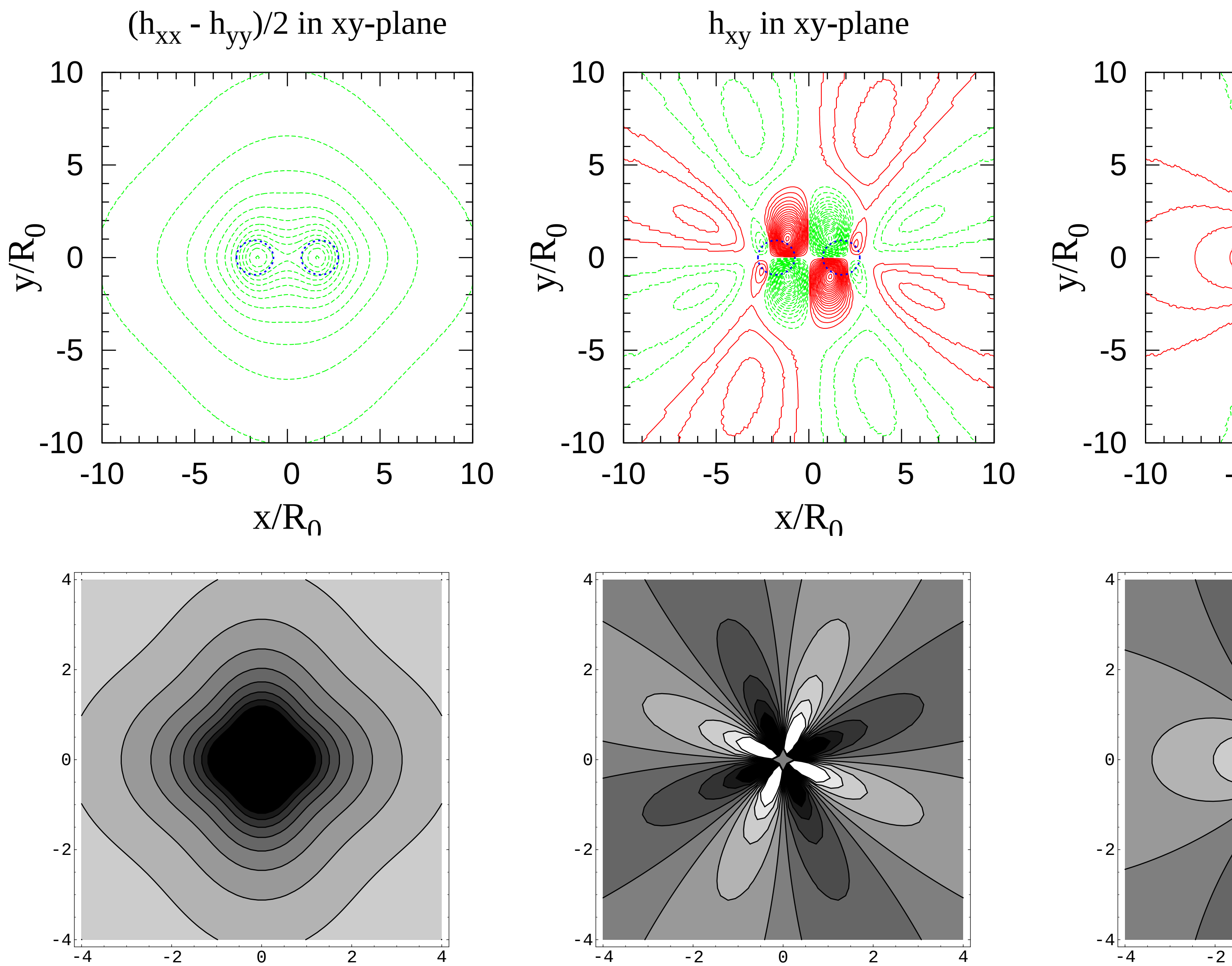


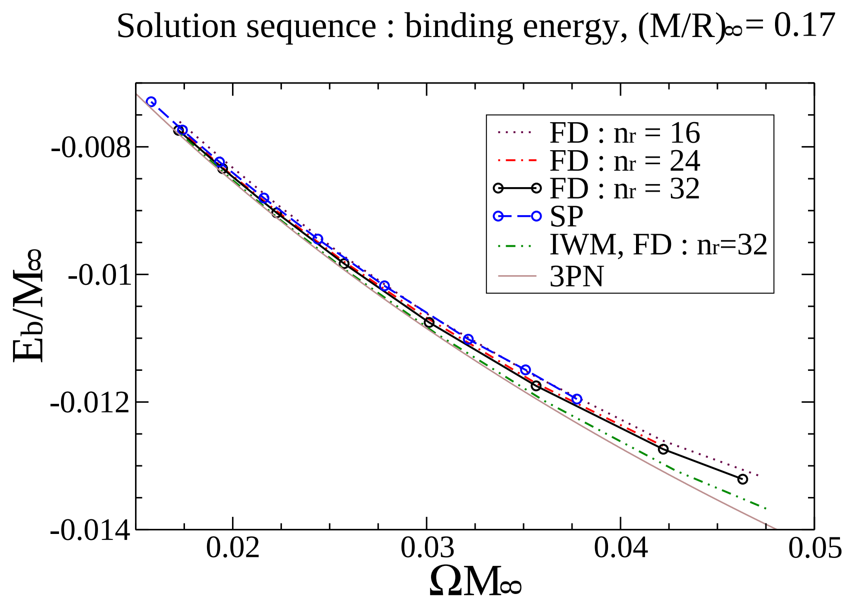


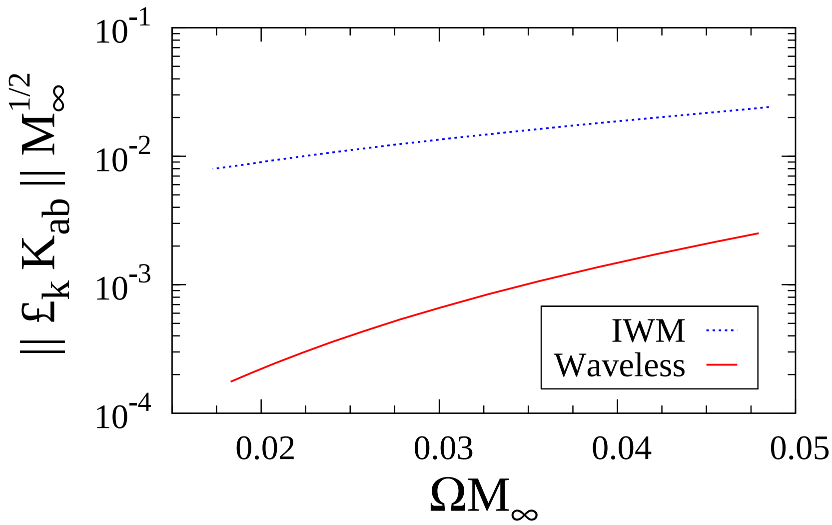
○ Contours for h_{ij}: Waveless BNS vs. 2PN point mass

(2PN in K=0, and transverse gauge: Asada, Shibata, Futama



◯ Solution sequence for $\Gamma = 2$ adiabatic EOS with $(M/$





- ⋆ $E_b(\Omega)$ of the wav deviates from 3PN quence at $\Omega M_\infty \gtrsim 0.$
- ⋆ $dE_b/d\Omega$ of the wav is $\sim 10-15\%$ larger; ror in GW may accu during the last ~ 2 c to the 3PN and IWM
- ⋆ Deviation from the is estimated by com defined by

$$||\, \pounds_k K_{ab} \,|| := \Big[\int_V \gamma^{ac} \gamma^{bd} \, \pounds_k$$

where $K_{ab} = \frac{1}{2\alpha} \pounds_\omega \gamma_{ab}$

Helically symmetric models

○ Helically symmetric scalar field; a toy model.

(Yoshida, Bromley, Read, Uryū, Friedman, (2006))

⋆ A scalar wave equation with source terms that models
of the Einstein equation is solved under the helical sy

$$\Box\psi - \lambda\mathcal{N}[\psi] = s,$$

$$s(t,r,\theta,\phi) = \sum_{\pm} \frac{q}{\sqrt{(2\pi)^3}} \ \exp\left[-\frac{\left(\vec{r} \pm \vec{R}(t)\right)^2}{\sigma^2} \right], \quad \mathbf{R}(t) = a\,[\cos(\Omega t)$$

⋆ Three different nonlinear terms, $\mathcal{N}[\psi] = \psi^3$, $\mathcal{N}[\psi] = |\nabla$
spatial gradient), and $\mathcal{N}[\psi] = \psi\Box\psi$ are used, whose st
adjusted by a coefficient λ.

• Using the symmetry relation $\pounds_k\psi = (\partial_t + \Omega\partial_\phi)\psi = 0$, t
equation is rewritten

$$(\nabla^2 - \Omega^2\partial_\phi^2)\Psi - \lambda\mathcal{N}[\Psi] = S.$$

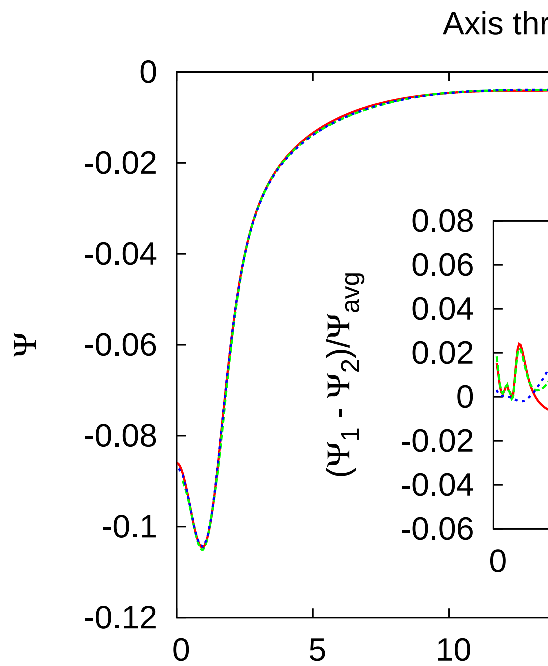
The half retarded+half advanced solution to this equat
with each source is solved using iteration.

○ Divergence of iteration for unfavorable sign of λ.

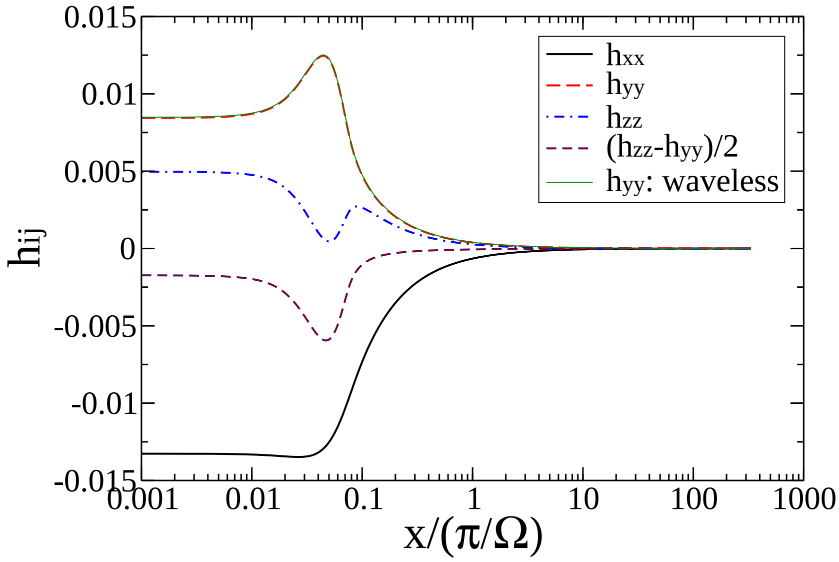
⋆ For each nonlinear term, strikingly different behaviou opposite signs of λ.

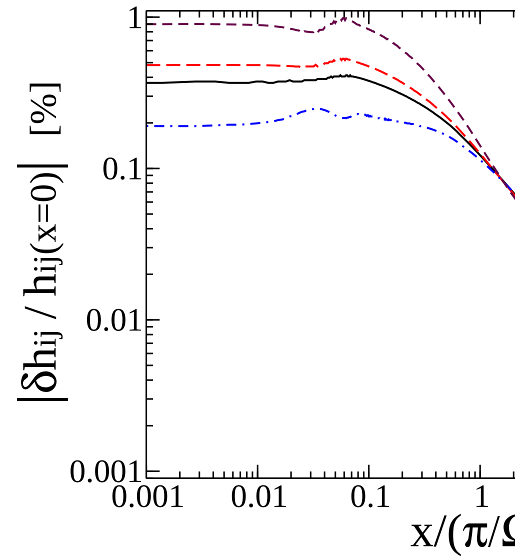
⋆ In each case, the sign of λ favor for the convergence the sign of the source term.

$\mathcal{N}[\Psi]$	Code	λ_{min}	λ_{max}
Ψ^3	KEH	-2.3	>10000
	FD	-2.5	>1000
	ES	-2.4	>1000
$\|\nabla\Psi\|^2$	KEH	<-1000	7.7
	FD	-2.0	5.6
	ES	<-1000	7.3
$\Psi\Box\Psi$	KEH	-0.96	10
	FD	-1.8	>1000
	ES	-1.7	>1000



○ Comparison of the full GR helical BNS and waveless





⋆ Near zone helical solution is calculated: Helical symme
in the near zone $r < \pi/\Omega$, and waveless condition is use

⋆ The near zone helical solution agrees well with the wav
Inaccuracy arises from neglecting gravitational wave m
the near zone where Coulomb field dominates.

⋆ When the boundary of helically symmetric domain
$r \sim 1 - 1.5\pi/\Omega$ or larger, the helically symmetric BNS
not converge. A solution behaves to be unbound aroun

Summary and Discussion

Two new formulations, (1) the waveless formulation ar
lically symmetric formulation, for the binary neutron st
initial data have been developed and implemented in ne
cessfully:

⋆ For (1) : BNS solution sequence has been computed.
⋆ For (2) : ◦ Hybrid solution for BNS (WAT in the asyn
◦ Numerical toy model have been calculated.

- These are more reliable initial data for the binary insp

⋆ Only at distances larger than about 10^4 M is the ener
radiation field comparable to the mass of the binary sy
However, we haven't made a success for calculating a fu
in a few wavezone where it is asymptotically decresein
field+standing wave. (Nuemrical problem?)

⋆ It is important to investigate an improvment for these
data to induced less eccentricity in the inspiral orbit. A
slowly turning on of the radiation reaction may furthe
eccentricity.

○ M_{ADM}-M_{K} relation for binary systems.

⋆ We derived asymptotic conditions for an equality M_{AD}
satisfied for non-axisymmetric systems. (Shibata, Uryū, Fri

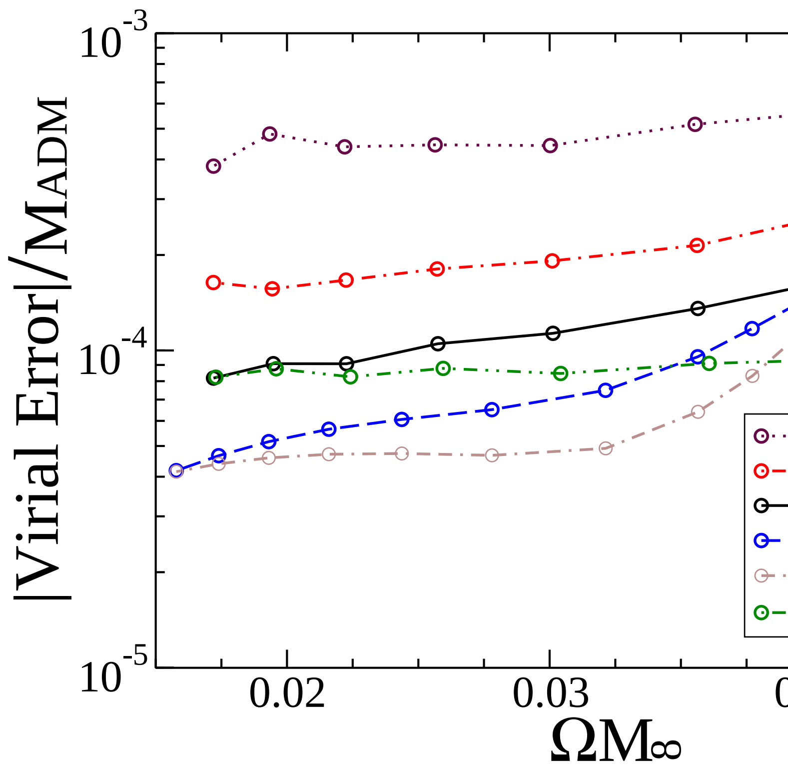
⋆ The condition for $M_{\mathrm{ADM}} = M_{\mathrm{K}}$ is satisfied in the prese

$$M_{\mathrm{ADM}} = -\frac{1}{2\pi}\int_{\infty} D^a\psi\, dS_a$$

$$M_{\mathrm{K}} = \frac{1}{4\pi}\int_{\infty} D^a\alpha\, dS_a$$

$(M_{\mathrm{ADM}} - M_{\mathrm{K}})/M_{\mathrm{ADM}} < 0.02\%$

$M_{\mathrm{ADM}} = M_{\mathrm{K}}$ relates to the virial relation that measure the accuracy of numerical solutions.



Gravitational two body problem with helical killing vector. Friedman, Uryu

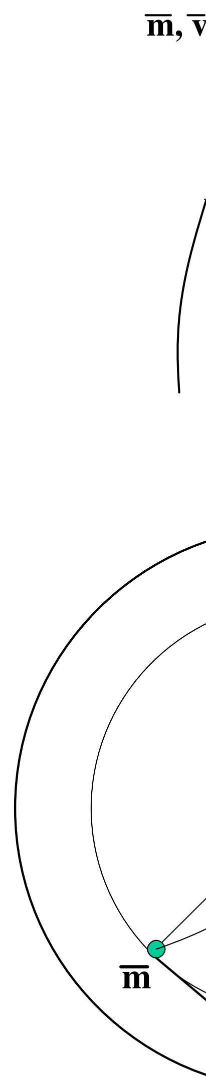
(cf. Schild, 1953; EM two body.)

◯ A toy model in post-Minkowski gravity with radiation and helical symmetry.

∘ Fokker action of two point mass with an interaction that models post-Minkowski gravity is derived.

∘ A particle feels the force created by the half retarded+half advanced field of the other particle, whose Green's function is written $G(x,\bar{x}) = \delta[(x-\bar{x})^2]$, $\Box G(x,\bar{x}) = -4\pi\delta(x-\bar{x})$.

∘ Solutions for the two body problem in circular orbits and its conserved energy and angular momentum are calculated.



○ Fokker action for post-Minkowski gravity.

∘ Parameter-invariant action : (Ramond 1973)

$$I(\tau_1,\tau_2,\bar\tau_1,\bar\tau_2) = -m\int_{\tau_1}^{\tau_2} d\tau\,(-\dot x_\alpha \dot x^\alpha)^{1/2} - \bar m\int_{\bar\tau_1}^{\bar\tau_2} d\bar\tau\,(-\dot{\bar x}_\alpha\,\dot{\bar x}^\alpha)^{1/2} + \int_{\tau_1}^{\tau_2} d\tau$$

$$\Lambda(w,\dot x,\dot{\bar x}) = 2m\bar m\,\delta(w)\frac{(\dot x_\alpha \dot{\bar x}^\alpha)^2 - \frac{1}{2}\dot x_\alpha \dot x^\alpha\,\dot{\bar x}_\beta \dot{\bar x}^\beta}{(-\dot x_\gamma \dot x^\gamma)^{\frac{1}{2}}(-\dot{\bar x}_\delta \dot{\bar x}^\delta)^{\frac{1}{2}}}, \qquad w := (x$$

• Fokker action yeilds correct equations of motion, and fo
conserved quantities by taking a limit after the variation

$$\delta I(\tau_1,\tau_2,\bar\tau_1,\bar\tau_2) = 0, \text{ then let } \begin{cases} \tau_1 \to -\infty,\ \tau_2 \to \\ \bar\tau_1 \to -\infty,\ \bar\tau_2 \to \end{cases}$$

cf) An action for a geodesic motion in the post-Minkow

$$-m\int_{\tau_1}^{\tau_2} d\tau\,(-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta)^{1/2} = -m\int_{\tau_1}^{\tau_2} d\tau\,\left[-(\eta_{\alpha\beta}+h_{\alpha\beta})\dot{x}^\alpha\right.$$

$$\simeq\quad -m\int_{\tau_1}^{\tau_2} d\tau\,(-\dot{x}_\alpha\dot{x}^\alpha)^{1/2} + \frac{1}{2}m\int_{\tau_1}^{\tau_2} d\tau\, h_{\alpha\beta}\frac{\dot{x}^\alpha\dot{x}^\beta}{(-\dot{x}_\gamma\dot{x}^\gamma)^{1/2}} =: I$$

where

$$h^{\alpha\beta}(x) = 4\bar{m}\int_{-\infty}^{\infty} d\bar{\tau}\,\delta\,(w)\,\frac{\dot{\bar{x}}^\alpha\dot{\bar{x}}^\beta - \frac{1}{2}\bar{\eta}^{\alpha\beta}\dot{\bar{x}}^\alpha\dot{\bar{x}}^\beta}{(-\dot{x}_\gamma\dot{x}^\gamma)^{1/2}}.$$

$$\Box\,\tilde{h}^{\alpha\beta} = -16\pi\bar{T}^{\alpha\beta}, \quad \bar{T}^{\alpha\beta} = \bar{m}\int_{-\infty}^{\infty} d\bar{\tau}\,\delta(x-\bar{x})\frac{\dot{\bar{x}}^\alpha\dot{\bar{x}}^\beta}{(\dot{\bar{x}}_\gamma\dot{\bar{x}}^\gamma)^{1/2}}$$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad \tilde{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}h.$$

○ Circular solutions.

For a paticle $\{m, x^\alpha(\tau)\}$, $x^\alpha = t\,t^\alpha + a\varpi^\alpha$, $\dot{x}^\alpha = \gamma k^\alpha$, a
for a particle $\{\bar{m}, \bar{x}^\alpha(\bar{\tau})\}$, $x^\alpha = \bar{t}\,t^\alpha + \bar{a}\bar{\varpi}^\alpha$, $\dot{\bar{x}}^\alpha = \bar{\gamma}\bar{k}^\alpha$.

$$k^\alpha = t^\alpha + \Omega\phi^\alpha : \text{helical killing vector.}$$

⋆ The equations of motion, the conserved 4-momentum
lar momentums are integrated to be a set of algebraic rel
in terms of $\{\varphi, \Omega, v, \bar{v}, \gamma, \bar{\gamma}, m, \bar{m}\}$. ($\varphi$: the retarded angle

$$\varphi^2 = v^2 + \bar{v}^2 + 2v\bar{v}\cos\varphi, \qquad v = a\Omega, \qquad \bar{v} = \bar{a}\Omega,$$
$$\eta_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = -\gamma^2(1 - v^2) = -1, \qquad \eta_{\alpha\beta}\dot{\bar{x}}^\alpha\dot{\bar{x}}^\beta = -\bar{\gamma}^2(1 - \bar{v}^2)$$

⋆ The energy turned out to be the same for the scalar
and tensor (PM) interactions.

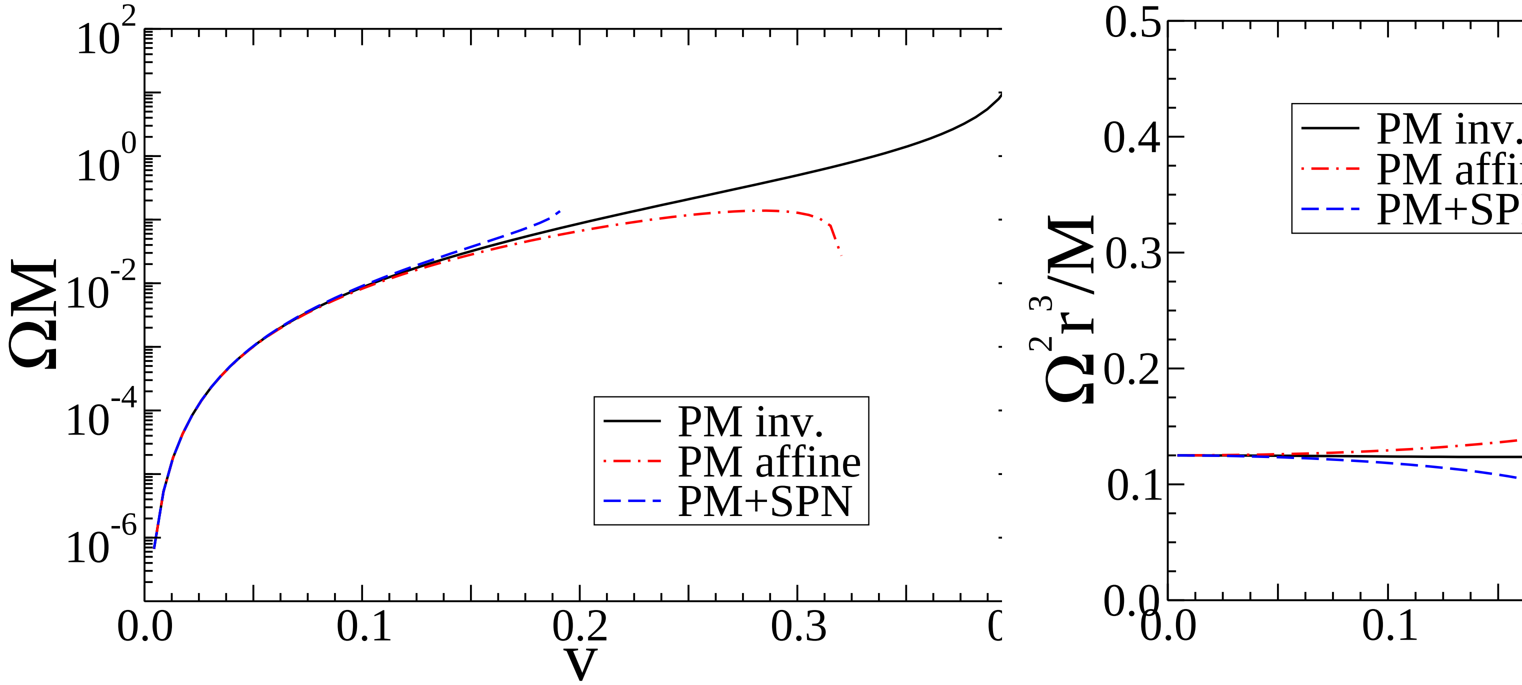
$$E = -P_0 = \frac{m}{\gamma} + \frac{\bar{m}}{\bar{\gamma}}.$$

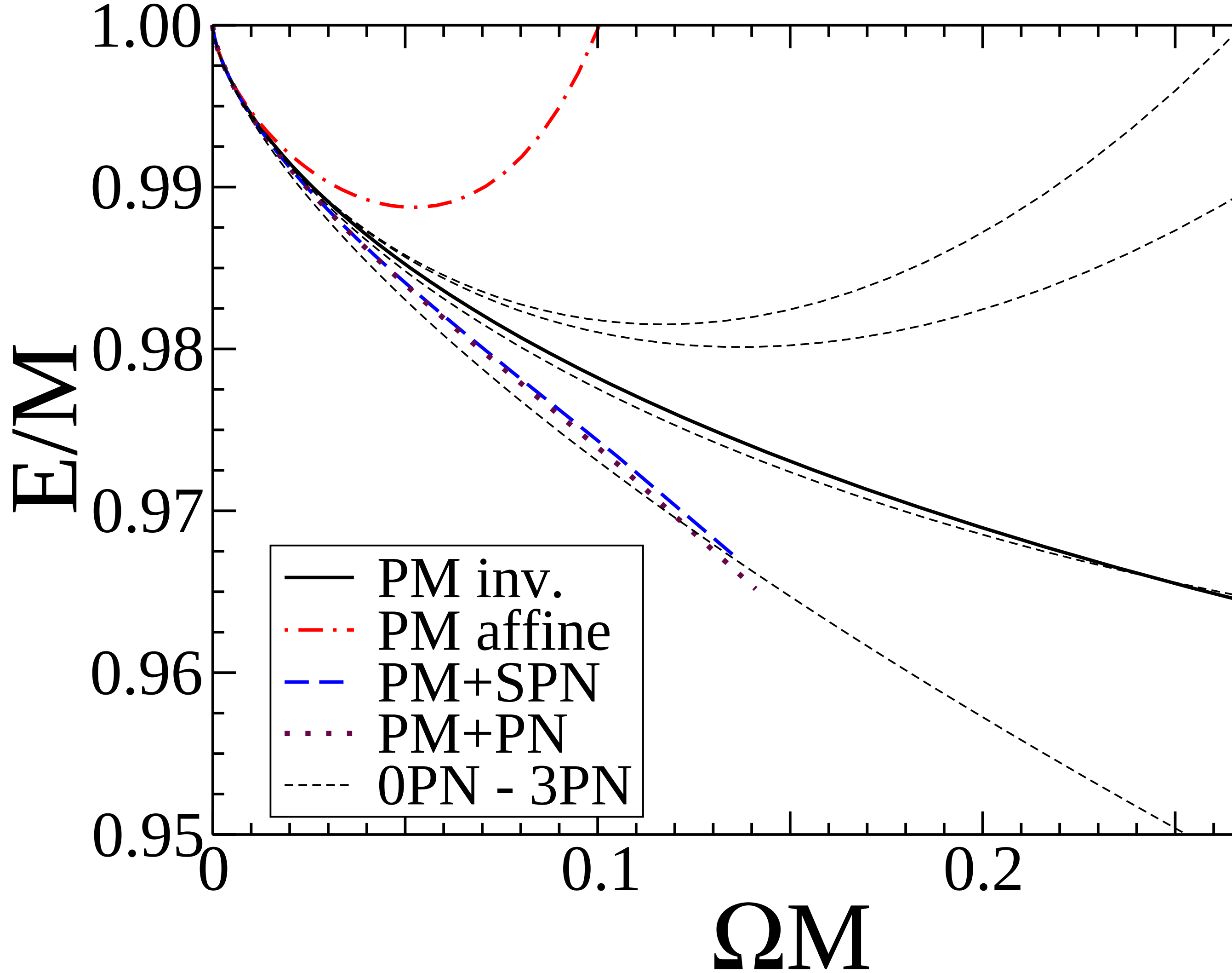
⋆ Angular momentum is proportional to the potential do

$$L = L_{12} = \frac{m\gamma}{2\Omega} h_{\alpha\beta} k^\alpha k^\beta.$$

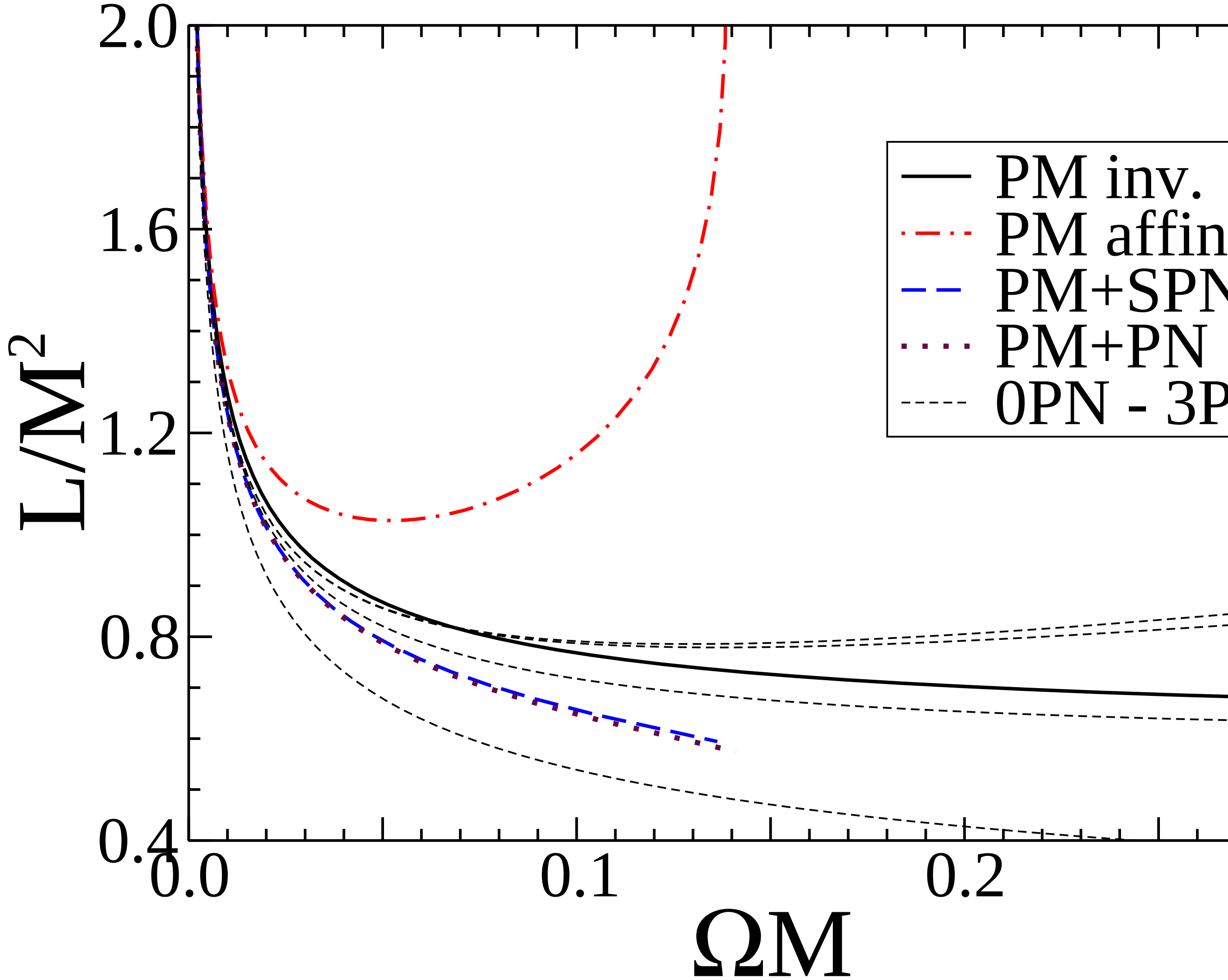
⋆ The first law $\delta E = \Omega \delta J$ is proved to be satisfied for
circular solutions.

◦ PM and PM+PN solutions for circular orbits.





1.00
0.99
0.98
0.97
0.96
0.95
E/M
0
0.1
0.2
ΩM
PM inv.
PM affine
PM+SPN
PM+PN
0PN - 3PN



PM inv.
PM affin
PM+SPN
PM+PN
0PN - 3P
L/M²
ΩM
2.0
1.6
1.2
0.8
0.4
0.0
0.1
0.2