

- **Preparation** of the data analysis of the gravitational wave space antenna.
 - 1) LISA (**L**aser **I**nterferometer **S**pace **A**ntenna) Why?
 - 2)How?



Frequency Limitation

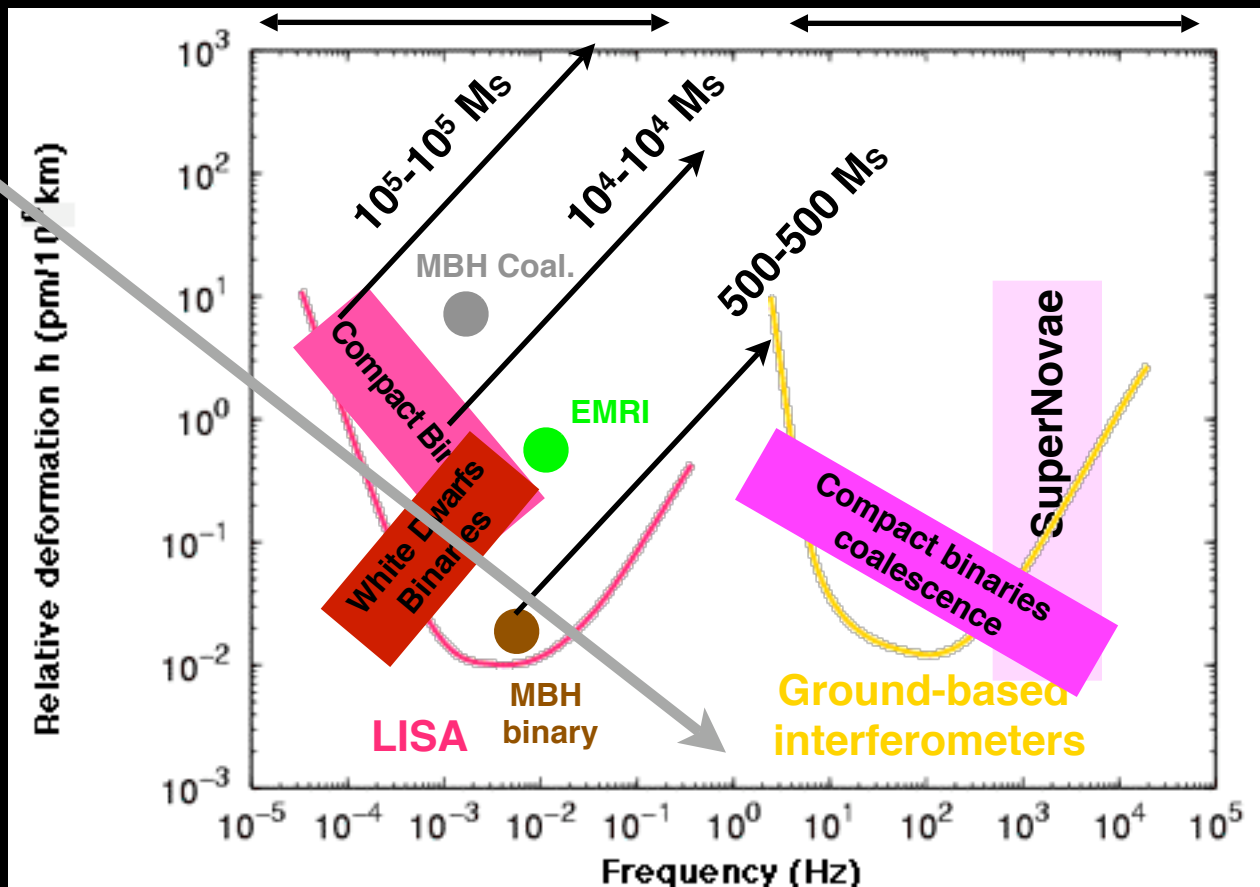
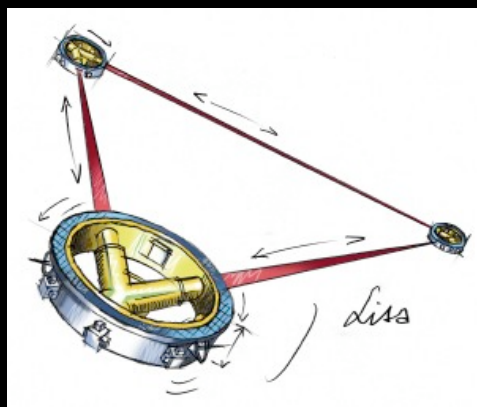


$$f = \frac{c}{\lambda} \approx \frac{c}{L}$$

Periodic sources

$L \approx 5 \cdot 10^6$ kms
burst sources

Seismic noise
cannot be
cancelled at
low-frequency



From the earth to the Stars





Science Goal

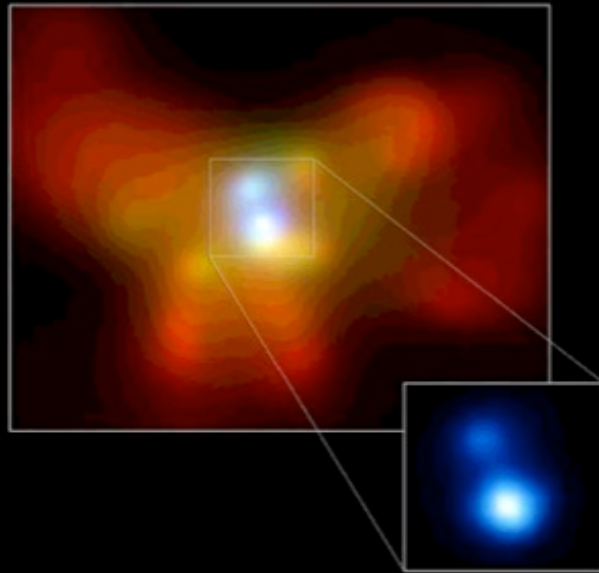
LISA frequency band : 10^{-4} - 10^{-1} Hertz

LISA science goal complementary to ground based interferometer

- Short-period known galactic binaries
- Mergers of massive (10^5 - $10^8 M_{\text{sun}}$) and intermediate mass (10^2 - $10^4 M_{\text{sun}}$)
- Compact objects (NS, BHs) spiralling into massive and intermediate mass BH
 - Astrophysical stochastic background : WD-WD galactic and extragalactic
 - Gravitational wave signals from the early universe



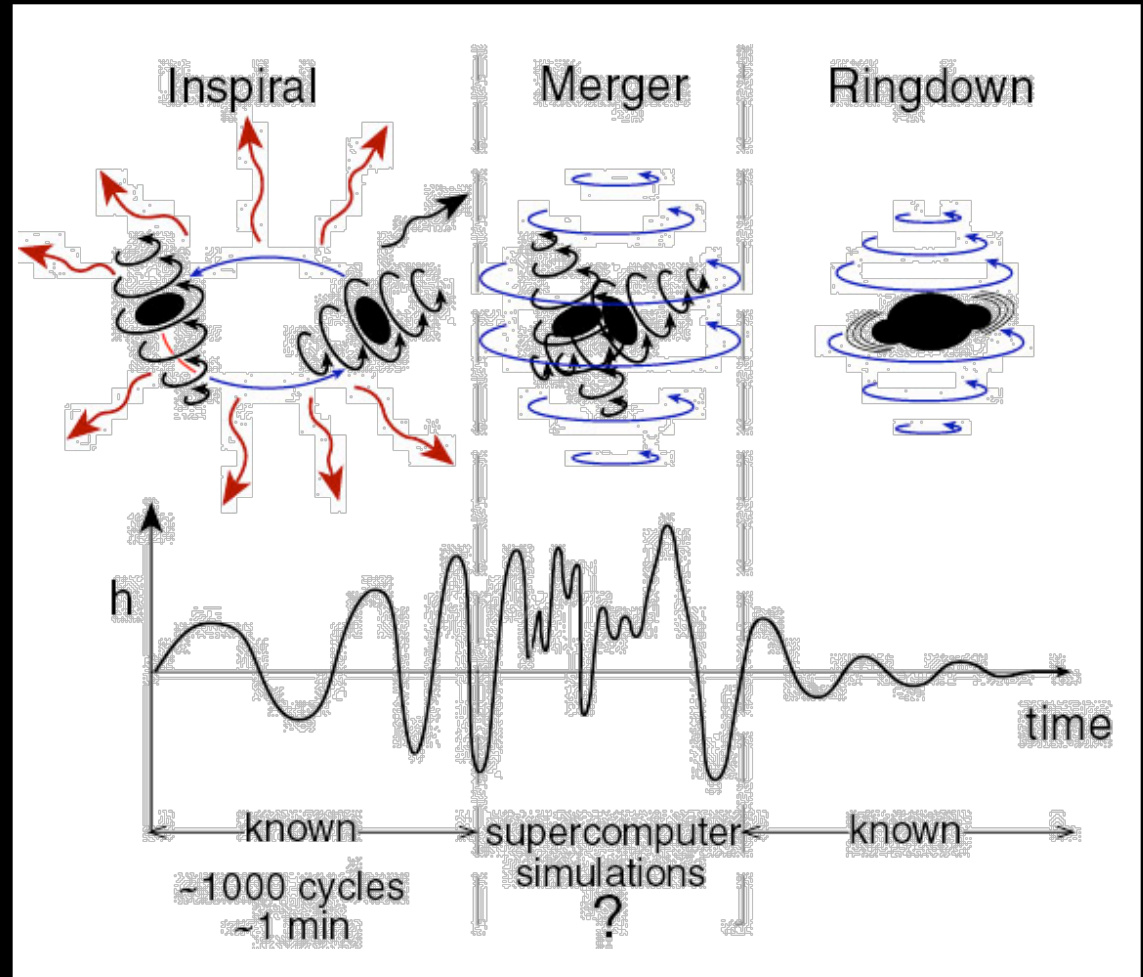
Massive Black Hole



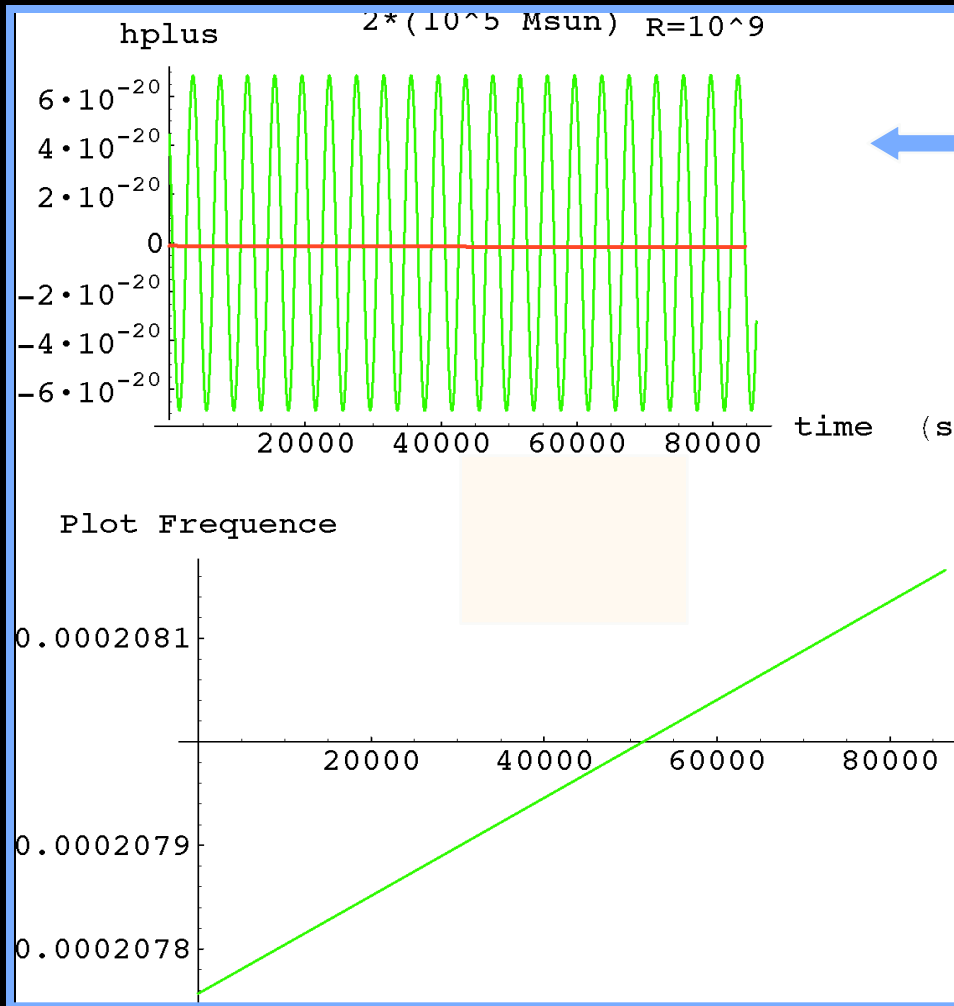
*Image of NC6240 taken by Chandra
Showing a butterfly shaped galaxy
Product of two smaller galaxies
(two active giant BH)*

Post Newtonian

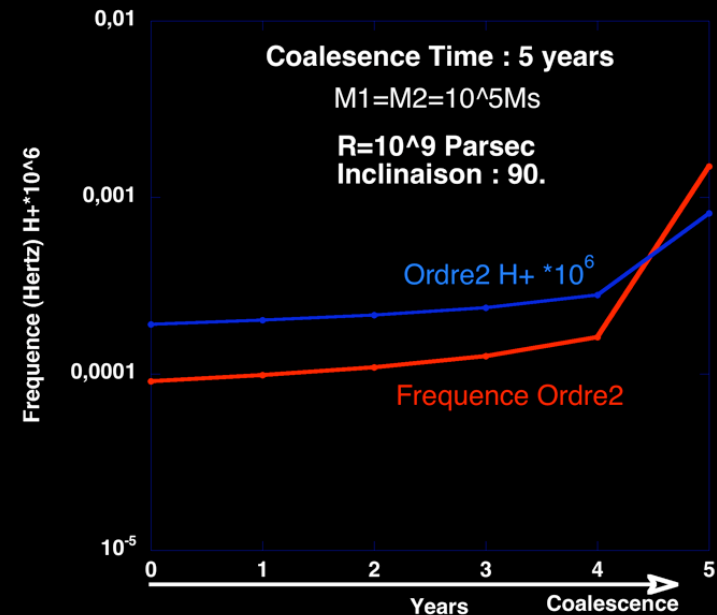
Numerical
relativity



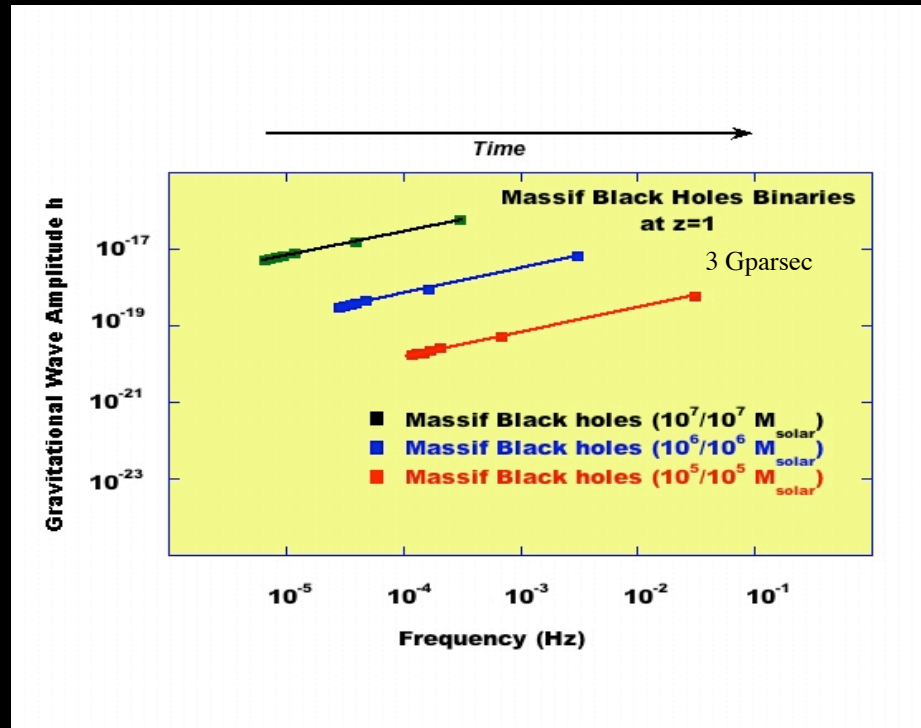
Massive Black Hole cont.



Post Newtonian calculation
10 days before coalescence



Massive Black Hole cont.



Relativity :

- From inspiraling post-Newtonian waveforms \rightarrow precision test of general relativity
- From merger waveforms (numerical relativity) \rightarrow test of non linear gravity.

Astrophysics :

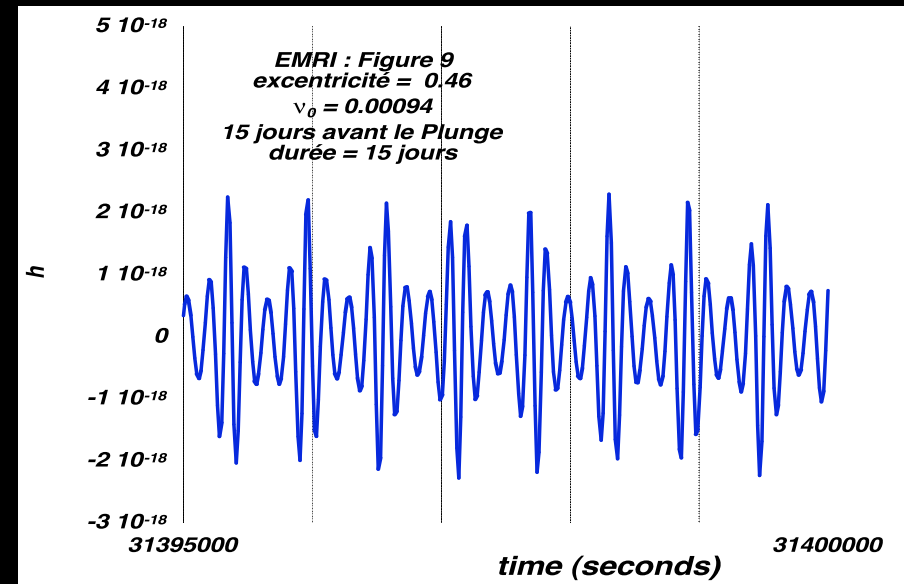
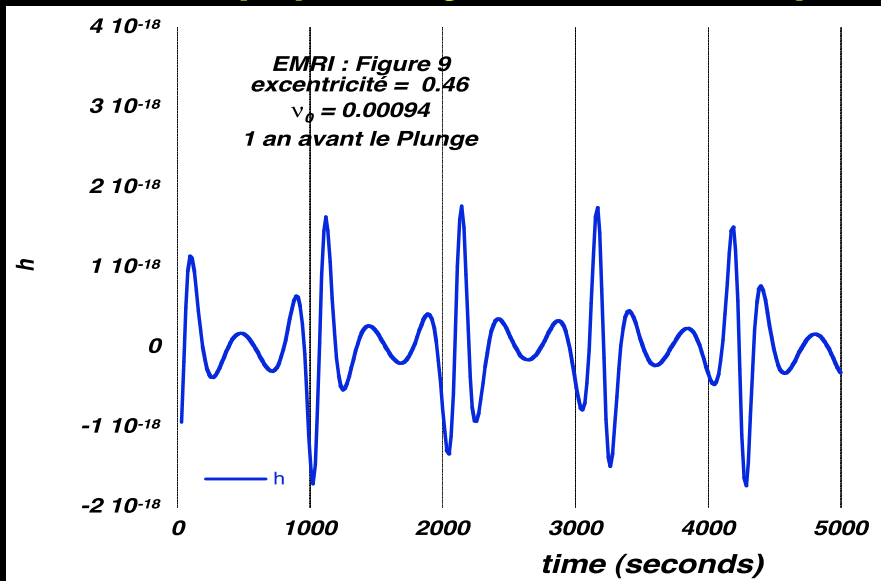
- Cosmic history of MBH's-MBH's

Events rates : 0.1 to 100 /years !

Extreme Mass Ratio in Spiral (EMRI)



Small body spiralling into central body of 10^5 to $10^7 M_{\text{sun}}$



Relativity :

- Relativistic orbits

Astrophysics :

- Probe astrophysics of dense cluster around MBH's
- Existence and population of IMBH

Events rates/years :

- 500-1000 for $10M_{\text{sun}} + 10^6M_{\text{sun}}$
- 10-90 for $0.6M_{\text{sun}} + 10^6M_{\text{sun}}$
- 1 for $100M_{\text{sun}} + 10^6M_{\text{sun}}$

Gérard Auger





Some properties of GW

For the experimentalist

- The wave is transverse, it is perpendicular to its direction of propagation.
- The deformation produced by a wave conserves surfaces.
- If the distance between two aligned masses increases, the distance between the two others masses along the perpendicular direction decreases.
- The wave is polarized.

H_x and H_x

- Important properties :

$$\frac{\delta L}{L} = \frac{1}{2} h$$

Geometrical interpretation of general relativity. L and ∂L should be interpreted as a propre distance. h is indeed a distance





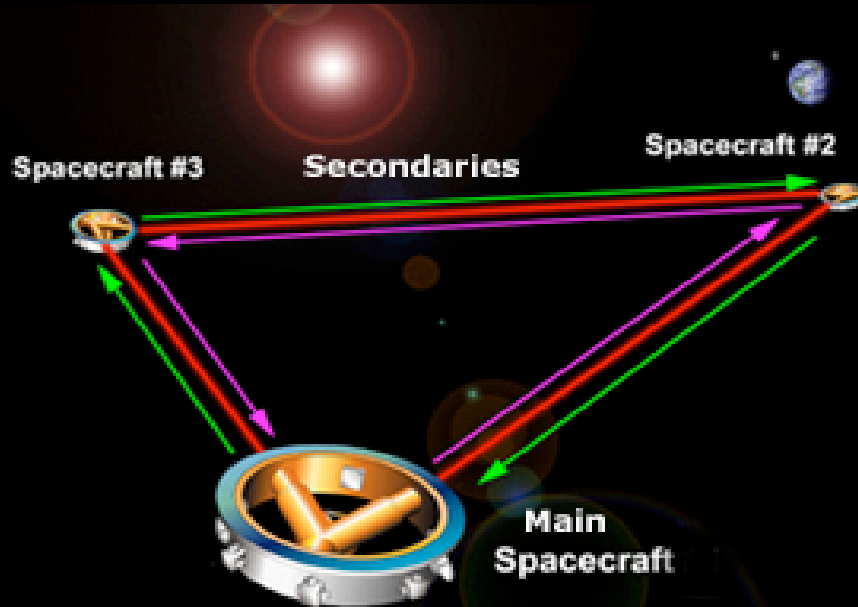
How to measure

- Six free falling "mirrors"
- Use interferometry for measuring
- LISA frequency band : 10^{-4} - 10^{-1} Hertz

$$\frac{\delta L}{L} = \frac{1}{2} h$$

$$f = \frac{c}{\lambda} \approx \frac{c}{L} \rightarrow L \approx 5 \cdot 10^6 \text{ kms}$$

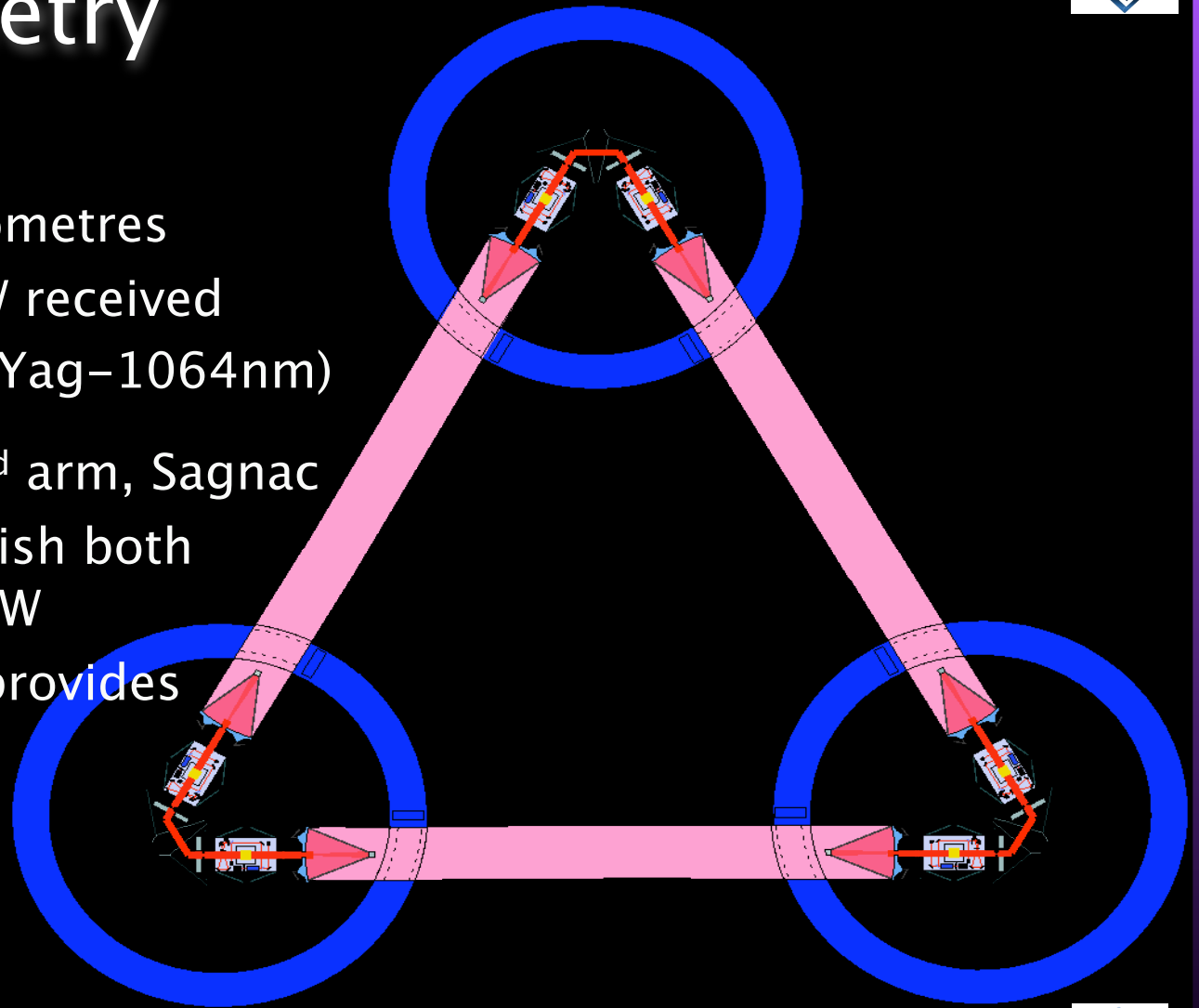
$$\partial L \approx 10^{-12}$$



Interferometry

Diffraction widens the laser beam to many kilometres

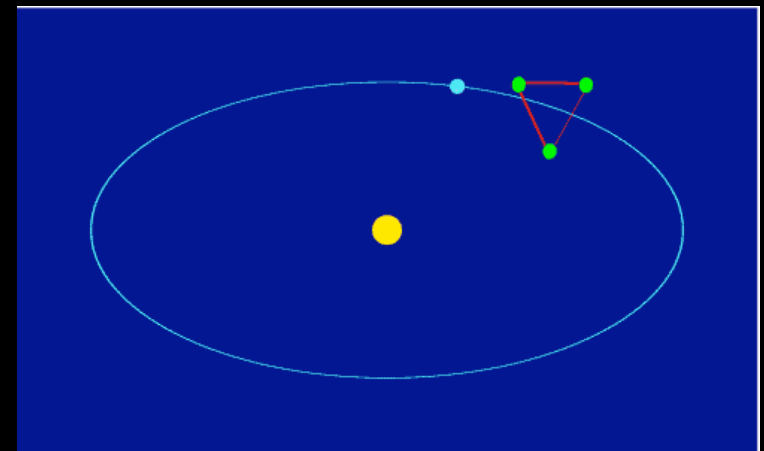
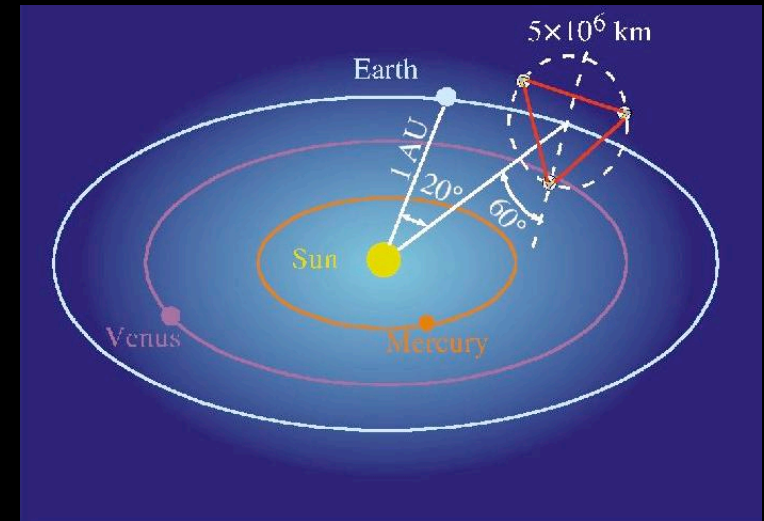
- 0.7 W sent, 70 pW received
- Need 6 lasers (NdYag-1064nm)
- Michelson with a 3rd arm, Sagnac
- Capable to distinguish both polarizations of a GW
- Orbital movement provides directionality



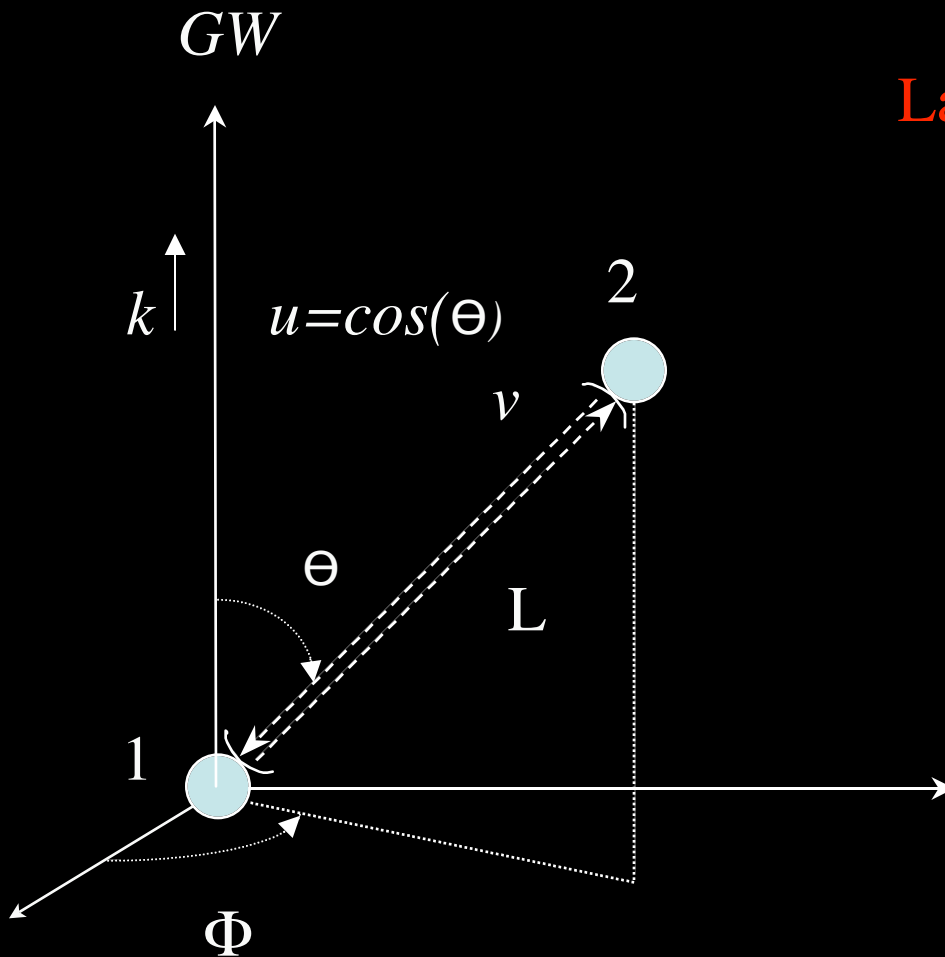
Orbiting



- 3 heliocentric orbits
- LISA centre follow Earth to 20° .
- Angle between LISA frame and ecliptic frame is 60° .
- Variation of LISA during the year
 - ⇒ **Directional** information of GWs.
 - Multi Michelson ⇒ **Polarization** of GW



Response function of one arm



Laser frequency

$c=1$

$$\frac{\Delta \nu(t)}{\nu_0} \Big|_{21}^{GW} = \frac{(1-\mu)}{2} [h(t-(1+\mu)L) - h(t)]$$

$$\frac{\Delta \nu(t)}{\nu_0} \Big|_{12}^{GW} = \frac{(1+\mu)}{2} [h(t-L) - h(t-\mu L)]$$

$$h(t) \equiv h_+(t) \cos(2\phi) + h_\times(t) \sin(2\phi)$$

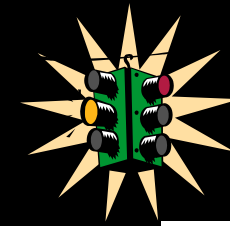
$$\nu(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt}$$



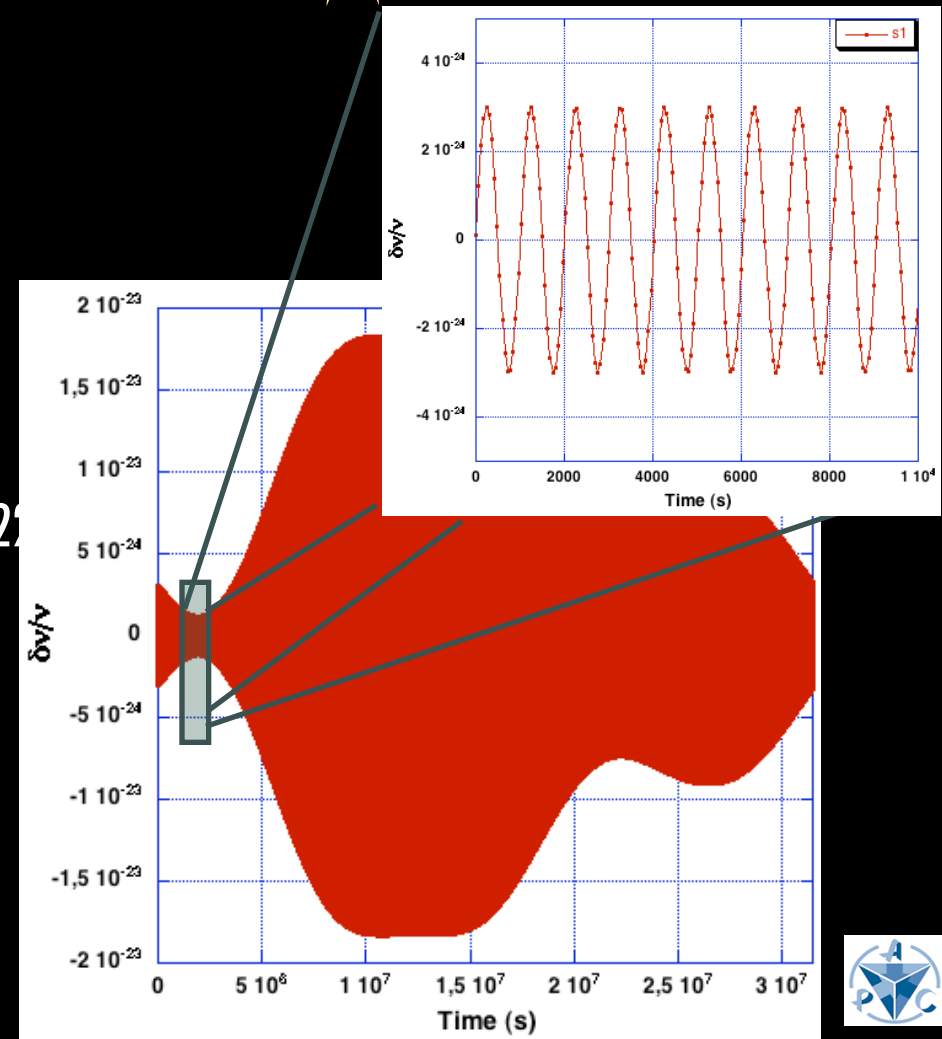
Michelson Response of binaries



- $\lambda = 298^\circ$, $\beta = 27^\circ$,
- $\psi = 228^\circ$,
- $f = 10^{-3}$ Hz,
- $h_+ = 3.5 \times 10^{-22}$, $h_x = 3.5 \times 10^{-22}$
- $\phi_{0h_+} = 4.21$, $\phi_{0h_x} = 5.78$.



No noise

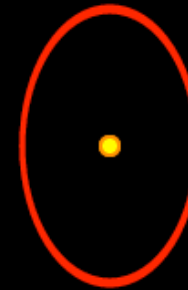


Michelson Response of binaries

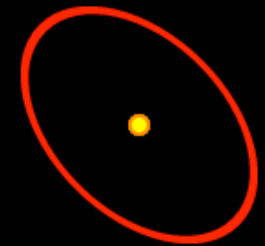
Monochromatic GW polarization Hx



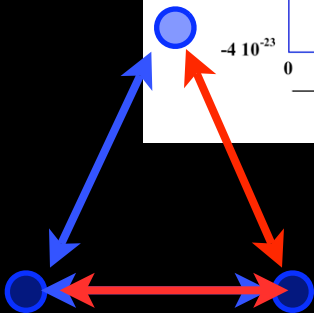
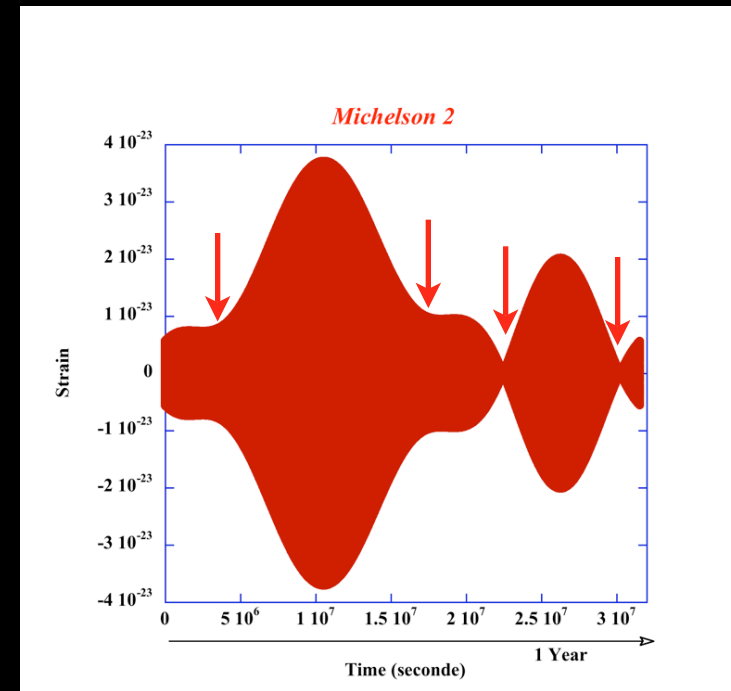
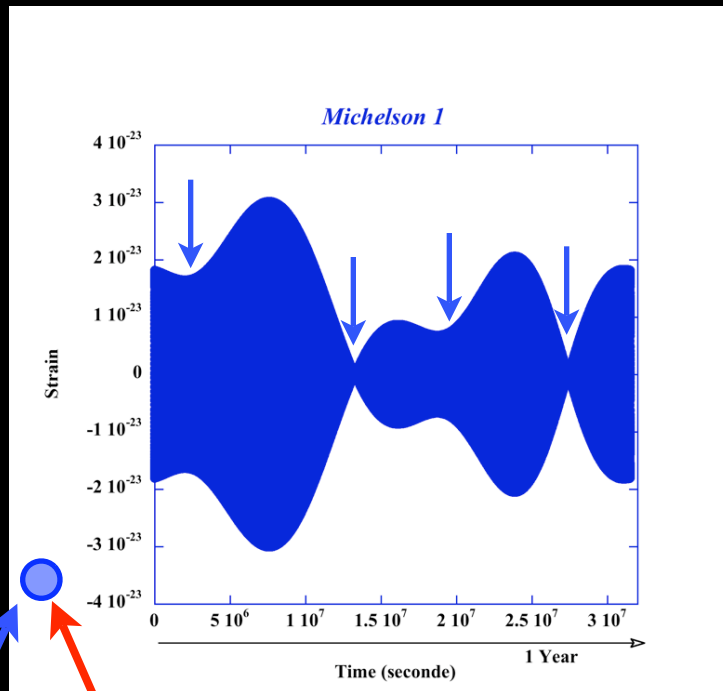
No noise



+ polarization



× polarization



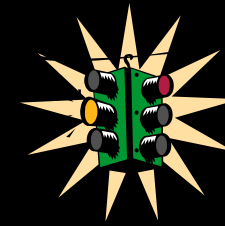
Frequency : 0.0009930348535 Hertz

λ : 297.9 β : 27.16

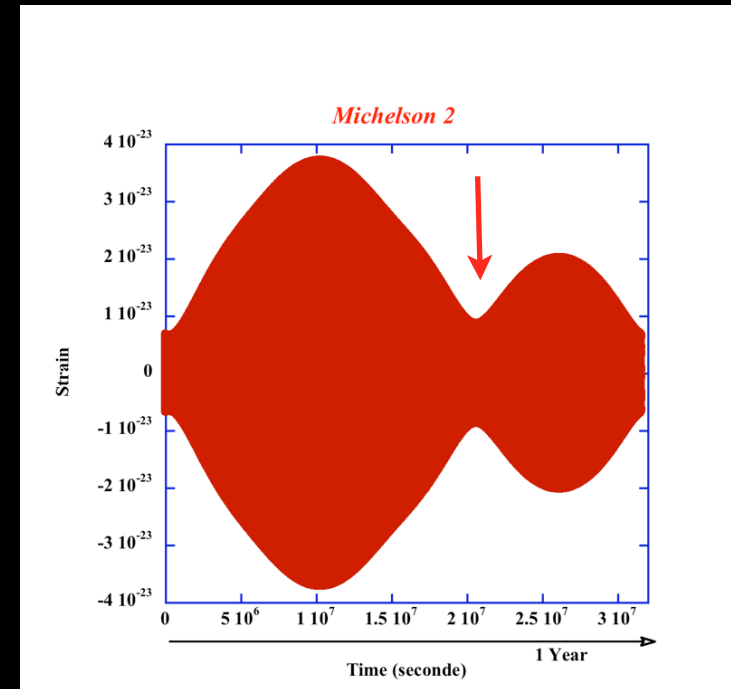
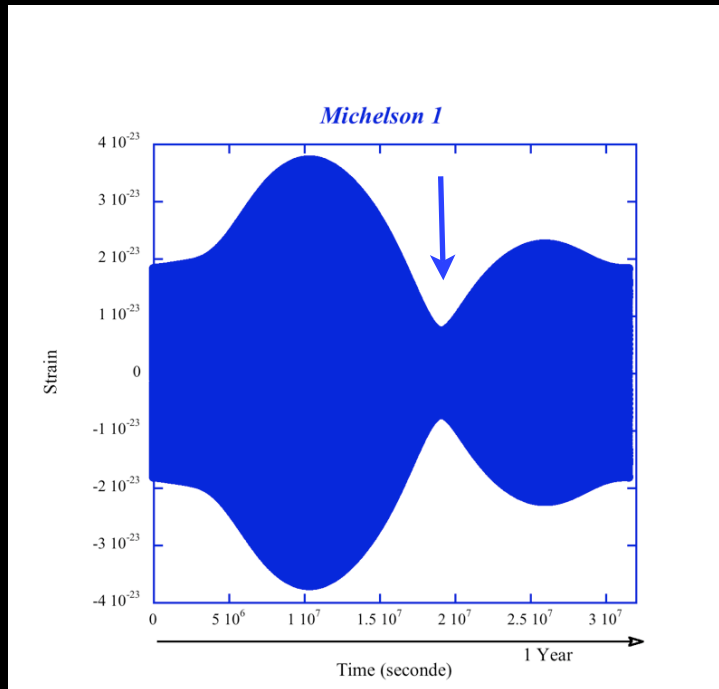


Michelson Response of binaries

Monochromatic GW polarization H_+ , H_x



No noise



Frequency : 0.0009930348535 Hertz

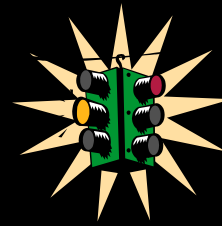
λ : 297.9 β : 27.16

Gérard Auger 16

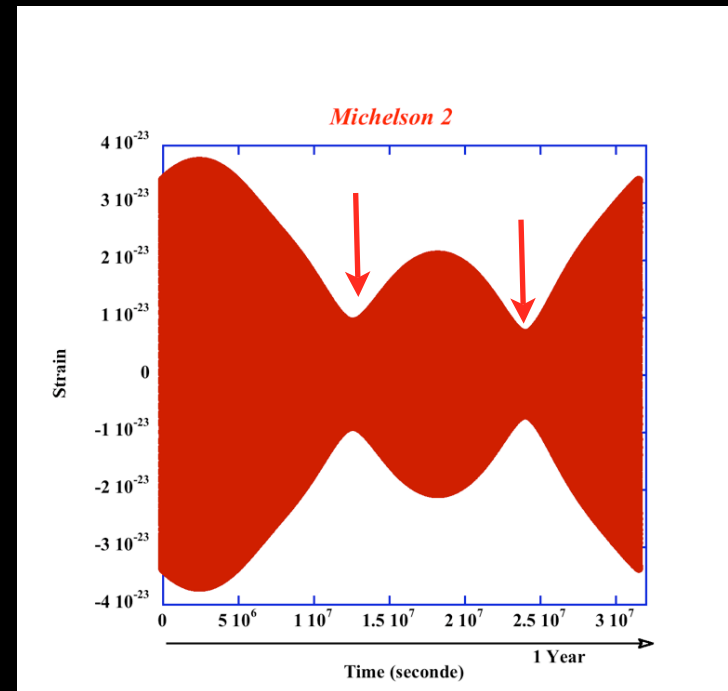
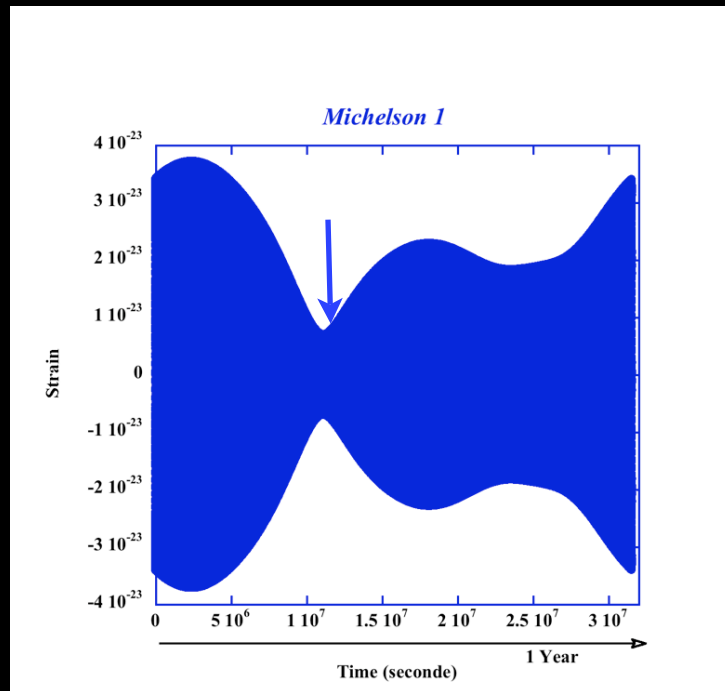


Michelson Response of binaries

Monochromatic GW same polarization



No noise

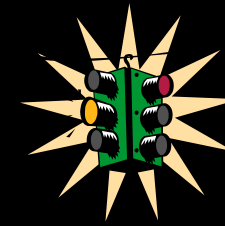


Frequency : 0.0009930348535 Hertz

$\lambda : +90$ $\beta : +90$



Frequency response



No noise

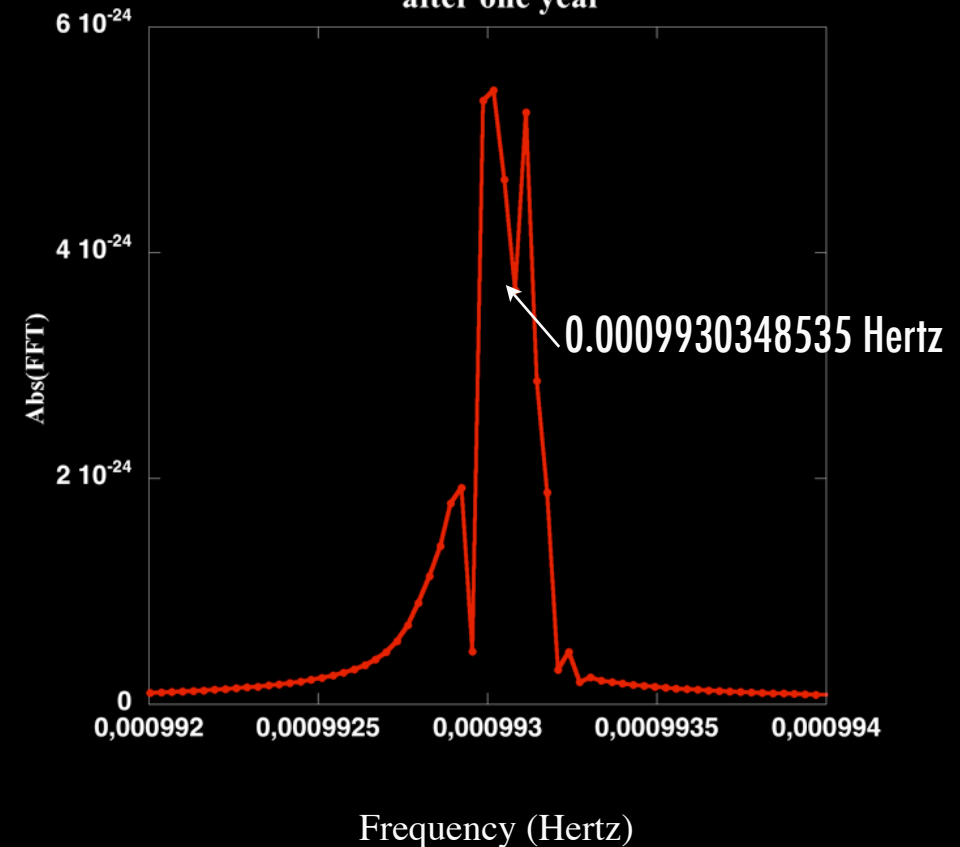


- Modulation
- Doppler shift

Frequency : 0.0009930348535 Hertz


$\lambda : 297.9$ $\beta : 27.16$

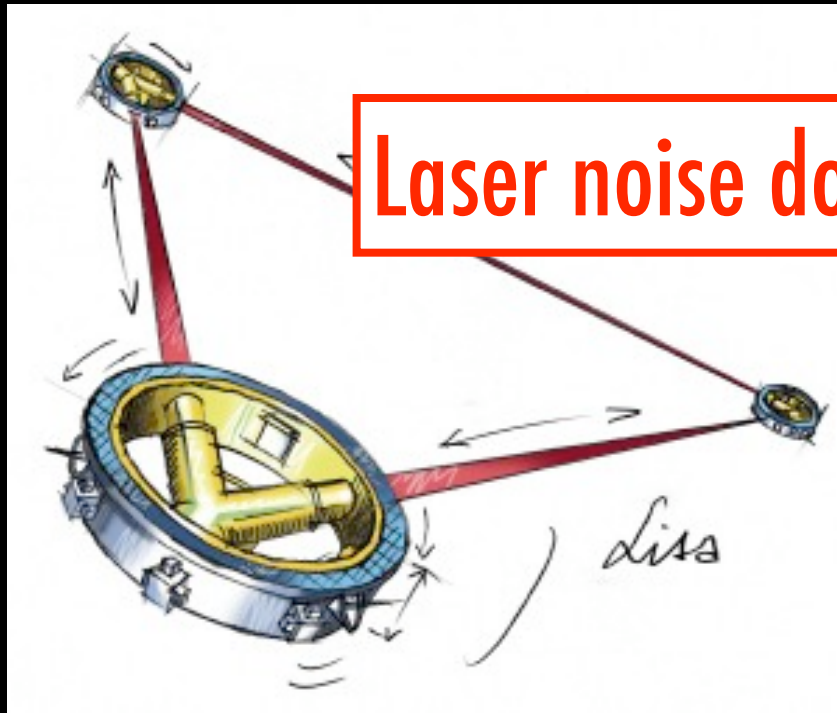
Fourier Transform of monochromatic GW
after one year






LISA : Noises

 **Laser noise** : Interference between two lasers which are not perfectly stable.



Laser noise dominates the signal

30 Hz.Hz^{1/2}

 **Inertial masses**
Imperfection of drag free system.

 **Shot Noise** Measurement noise on the photodiode

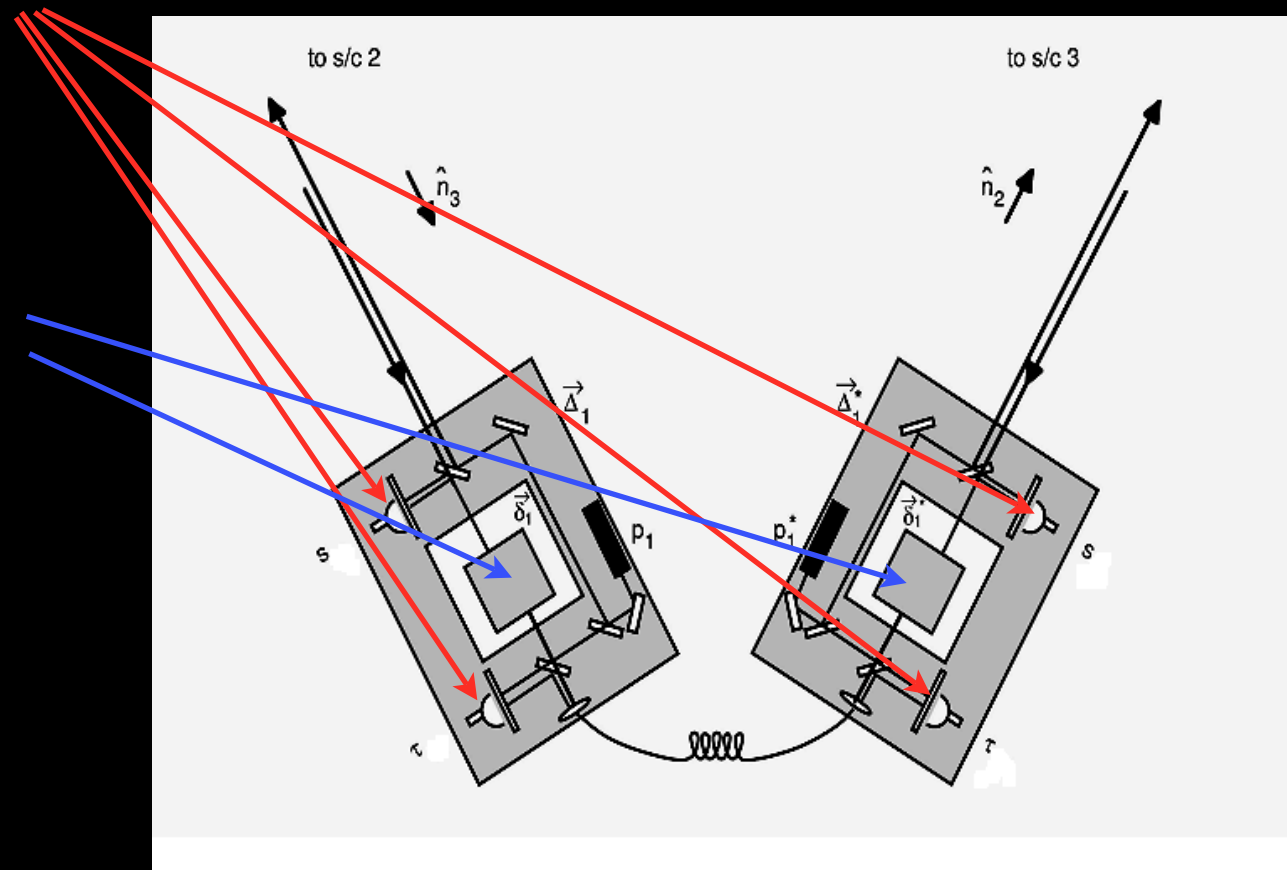


Laser noise



- By spacecraft 4 measurements

2 "mirrors"
inertial masses



Laser noise T.D.I (Time Delay Interferometry)



numerical interferometry

Laser frequency

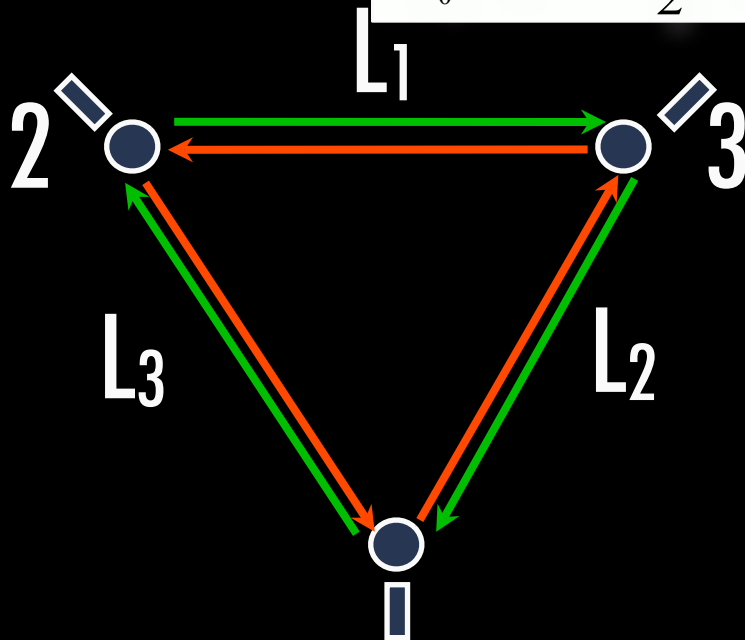
$$\frac{\Delta \nu(t)}{\nu_0} \Big|_{21}^{GW} = \frac{(1-\mu)}{2} [h(t-(1+\mu)L) - h(t)]$$

$$\frac{\Delta \nu(t)}{\nu_0} \Big|_{12}^{GW} = \frac{(1+\mu)}{2} [h(t-L) - h(t-\mu L)]$$

of LISA as a closed array
delay lines between the

test masses.

This approach allows us to reconstruct the unequal-arm Michelson interferometer, as well as new interferometric combinations, which offer advantages in hardware design, in robustness to failures of single links, and in redundancy of data.



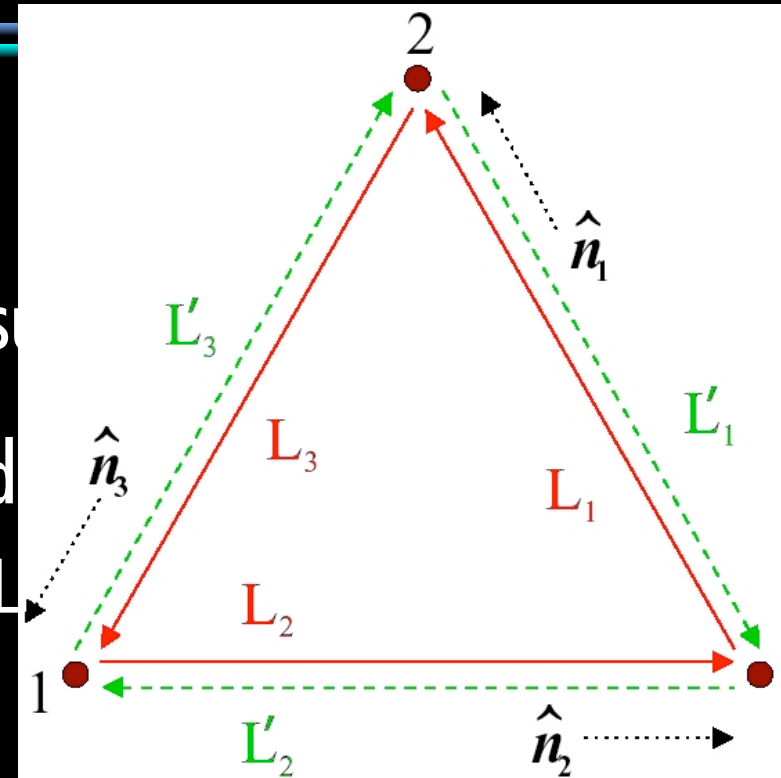
$$D = \left(t - \frac{L}{c}\right)$$

$$\alpha = s_1 + D_3 s_2 + D_3 D_1 s_3 - (s'_1 + D_2 s'_3 + D_2 D_1 s'_2) = 0$$



Laser noise T.D.I

- Phase shift between the two beams measured
- Beams from an external spacecraft, are delayed
 - delay operator D_i : $D_i x(t) = x(t-L_i)$
- The measurements :



$$s_1 = S_1^{GW} + s_1^{ShotNoise} + D_3 p'_2 - p_1 + \nu_0 \left(-2 \hat{n}_3 \cdot \vec{\delta}'_1 + \hat{n}_3 \cdot \vec{\Delta}'_1 + \hat{n}_3 \cdot D'_3 \vec{\Delta}'_2 \right)$$

$$\tau_1 = p'_1 - p_1 - 2\nu_0 \hat{n}_2 \cdot \left(\vec{\delta}'_1 - \vec{\Delta}'_1 \right) + \mu_1$$

$$s'_1 = s'_1{}^{GW} + s'_1{}^{ShotNoise} + D'_2 p_3 - p'_1 + \nu_0 \left(2 \hat{n}_2 \cdot \vec{\delta}'_1 - \hat{n}_2 \cdot \vec{\Delta}'_1 - \hat{n}_2 \cdot D_2 \vec{\Delta}'_3 \right)$$

$$\tau'_1 = p_1 - p'_1 + 2\nu_0 \hat{n}_3 \cdot \left(\vec{\delta}'_1 - \vec{\Delta}'_1 \right) + \mu_1$$

$$s_1 = s_1^{GW} + D_3 p'_2 - p_1$$

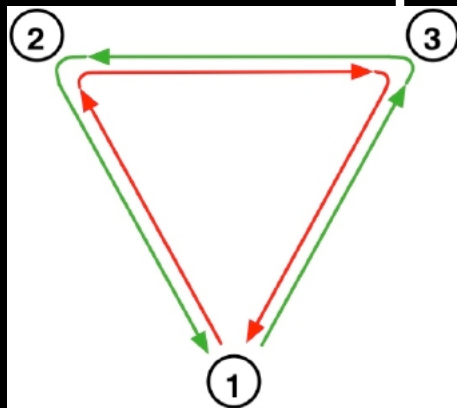
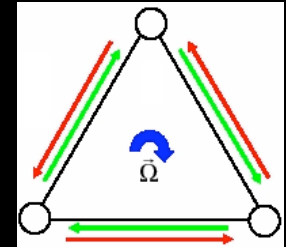
$$s'_1 = s'_1{}^{GW} + D'_2 p_3 - p'_1$$

With only the laser noise :

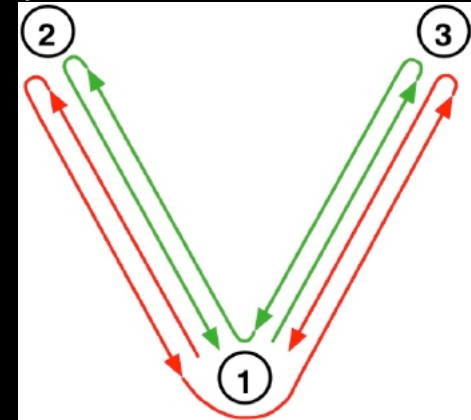
Laser noise T.D.I



- Many groups of TDI generators
 - 1st generation : fixed LISA configuration.
 - 2nd generation : consideration of flexing and Sagnac effect.
- Geometric representation by beam loops :



$$\alpha = -s_1 - D_3 s_2 - D_1 D_3 s_3 + s'_1 + D_{2'} s'_3 + D_{1'} D_{2'} s'_2 \approx 0$$



$$X = -s_1 - D_3 s'_2 - D_3 D_{3'} s'_1 - D_3 D_{3'} D_{2'} s_3 + s'_1 + D_{2'} s_3 - D_{2'} D_2 s_1 - D_{2'} D_2 D_3 s_3 \approx 0$$



Laser noise T.D.I

- In summary: there are 6 optical benches, 6 lasers, and a total of 12 Doppler time series observed.
- The 6 beams exchanged between distant spacecraft contain the information about the GW signal (s_{ij}); the other 6 signals (t_{ij}) are for comparison of the lasers and relative optical bench motions within the spacecraft.
- The functional space of interferometric combinations can be generated with the 4 generators $\alpha, \beta, \gamma, \xi$

$$\xi - \xi_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$

$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \xi$$

$$P = \xi - \alpha_{,1}$$

$$E = \alpha_{,1} - \xi_{,1}$$

$$U = \gamma_{,1} - \beta$$

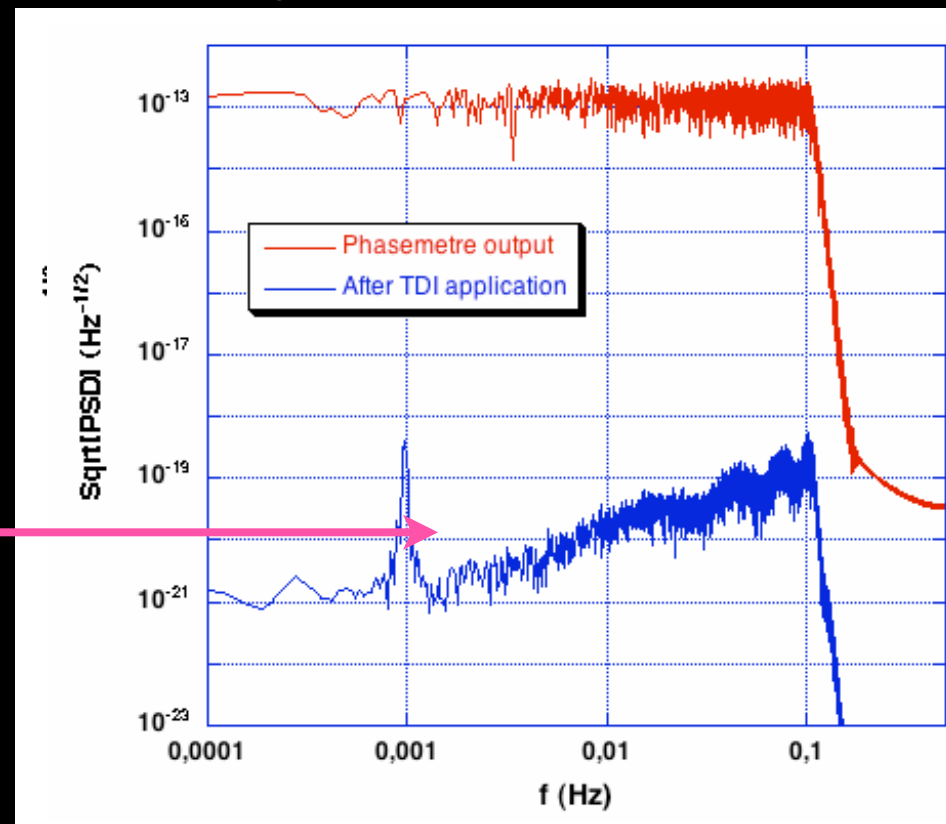


Laser noise T.D.I

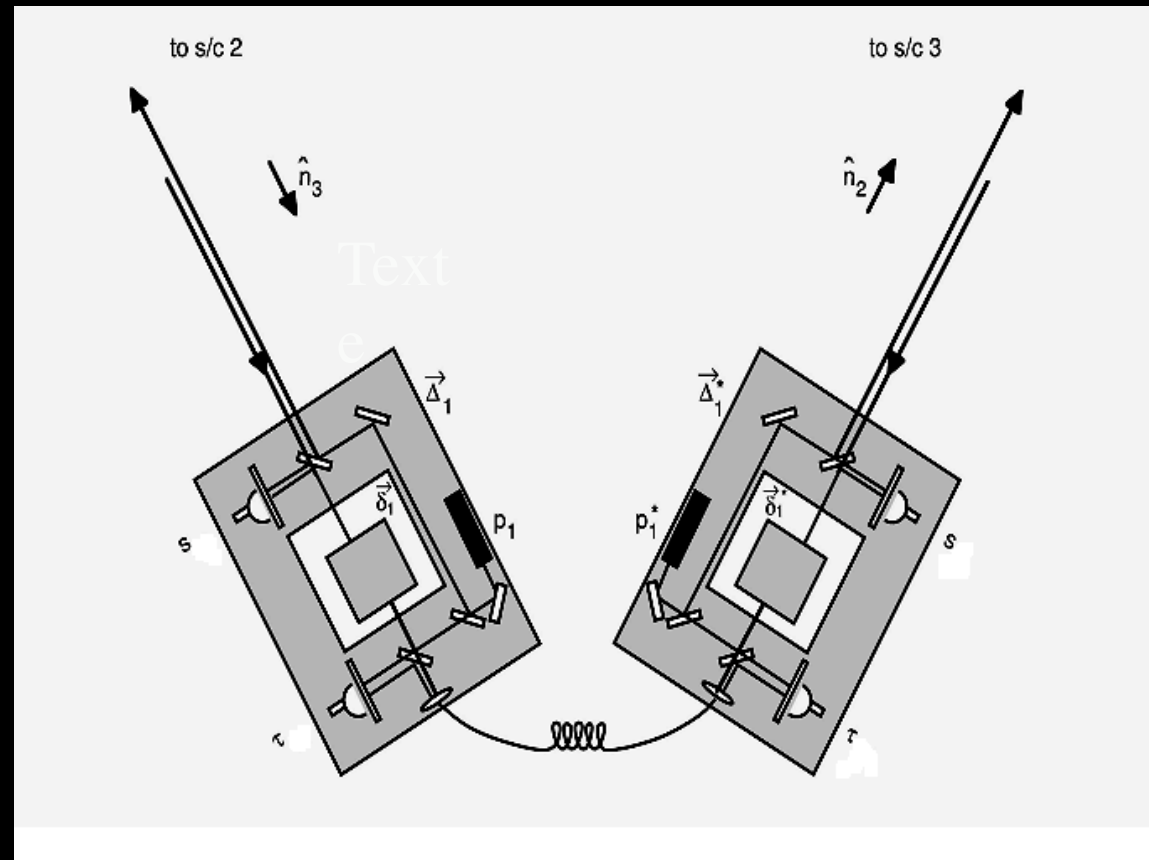
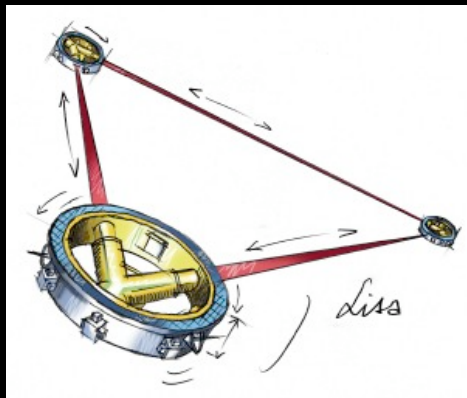
- The laser noise is modeled by a bandwidth limited white noise at $30 \text{ Hz} \cdot \text{Hz}^{-1/2}$.
- The application of TDI recovers the GW signal.

A Gw is
hidden in
there !

Here it is !



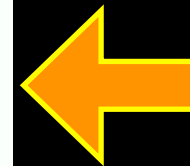
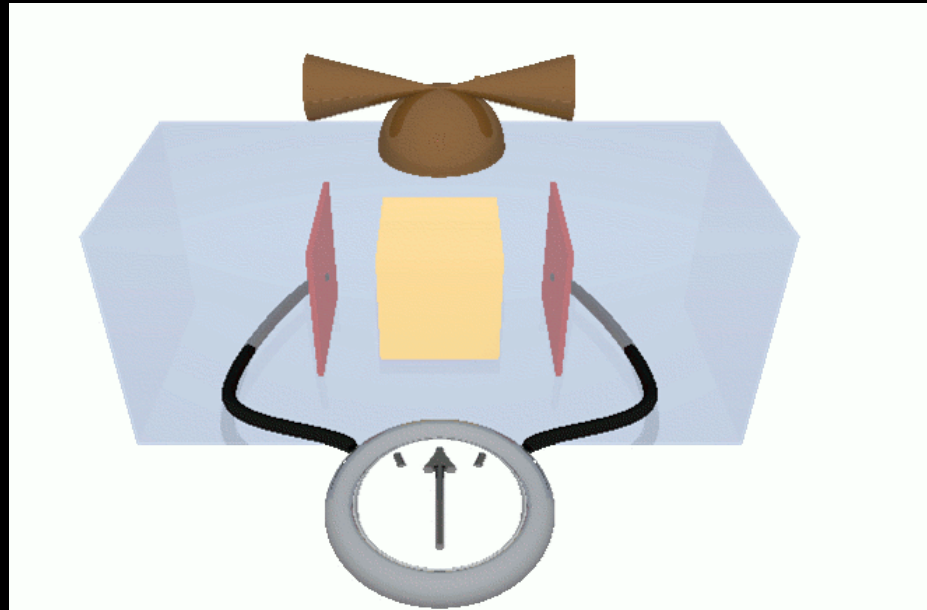
Inertial masses



Inertial masses

Free Fall in Space

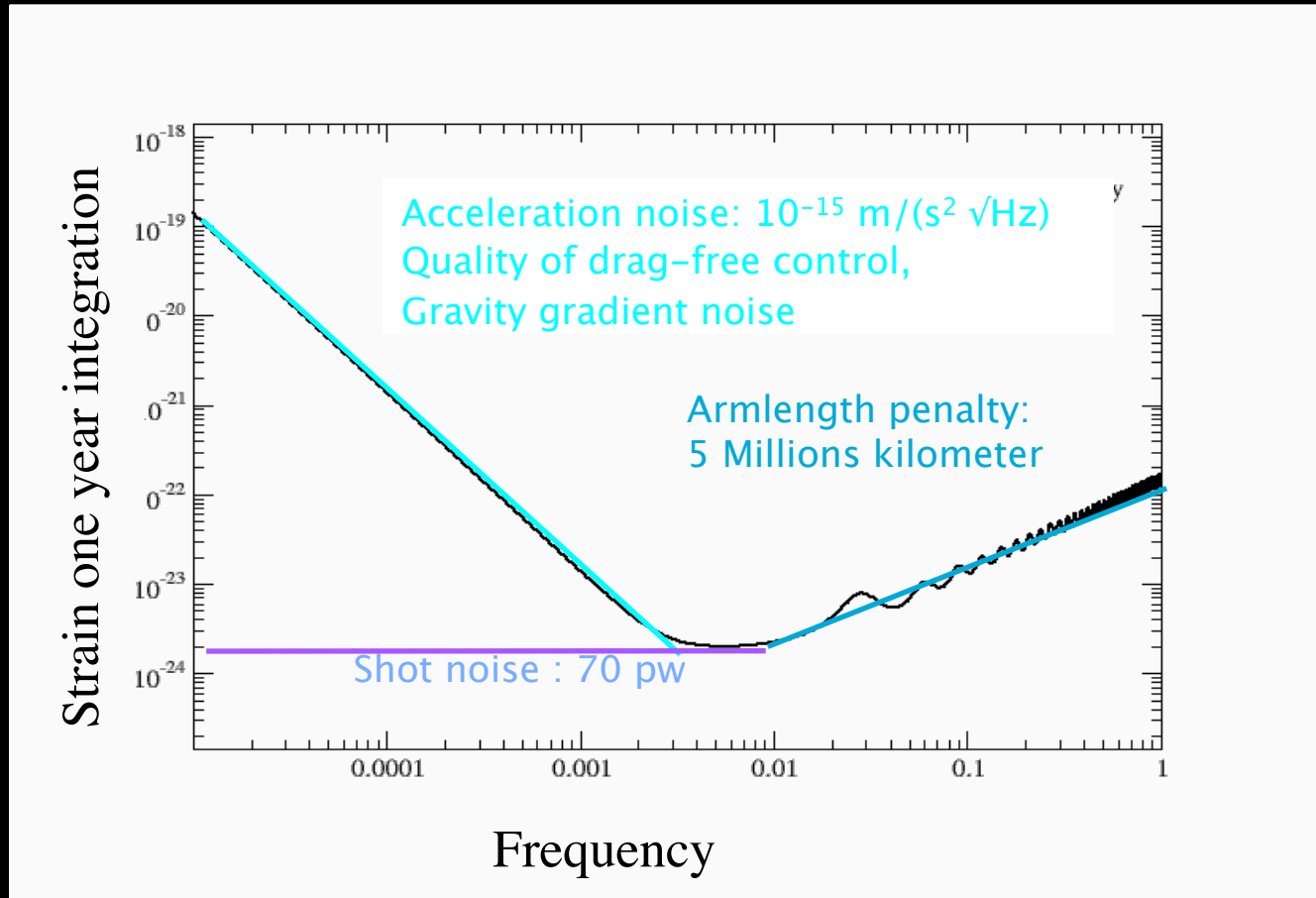
Drag free control



The resulting motion of the solar wind would be 10^4 times larger than the tiny motion due to GW

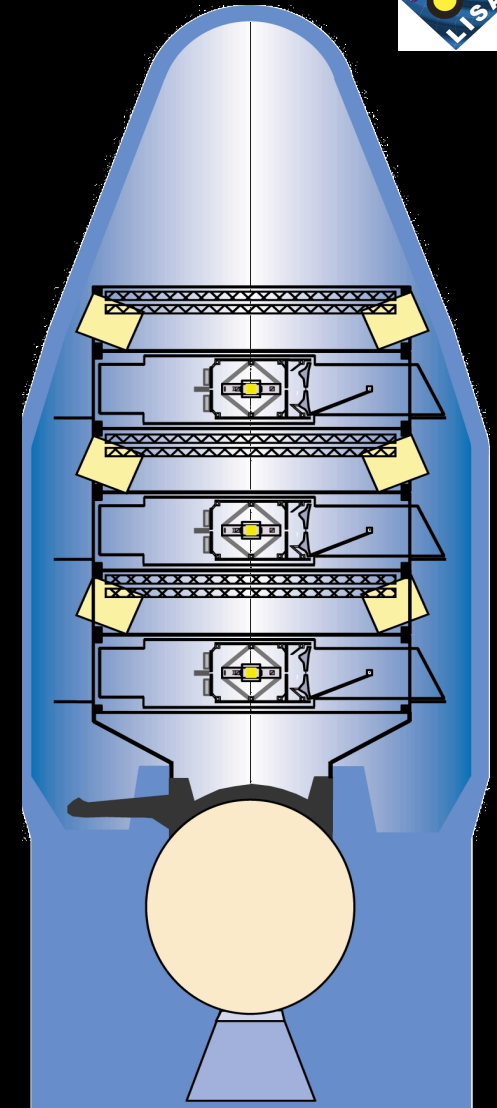


Noise Limitation



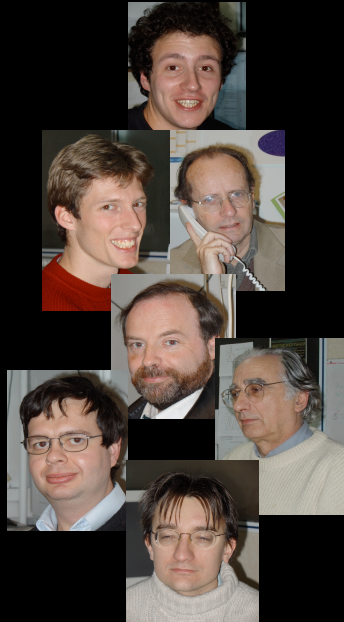
Summary

- Proposed to ESA 1993, approved as a Cornerstone Mission 1996
- Collaborative ESA/NASA mission with a 50/50 sharing ratio
 - ESA: Responsibility for the payload I&T, 50% of the payload (nationally funded)
 - NASA: 3 S/C, launcher, ground segment (DSN), mission ops
 - Science ops will be shared
 - Data analysis by two independent teams (Europe and US)
- Launch foreseen in the 2014/??? timeframe





Team LISA APC



- Contribution to the interferometry of LISA Pathfinder.
- Development of a simulator for the LISA mission (LISA Code).
- R&D in Laser frequency stabilization (Iodine molecular line)

<http://www.srl.caltech.edu/lisa/documents/PrePhaseA.pdf>

<http://www.apc.univ-paris7.fr/LISA-France/biblio.phtml>

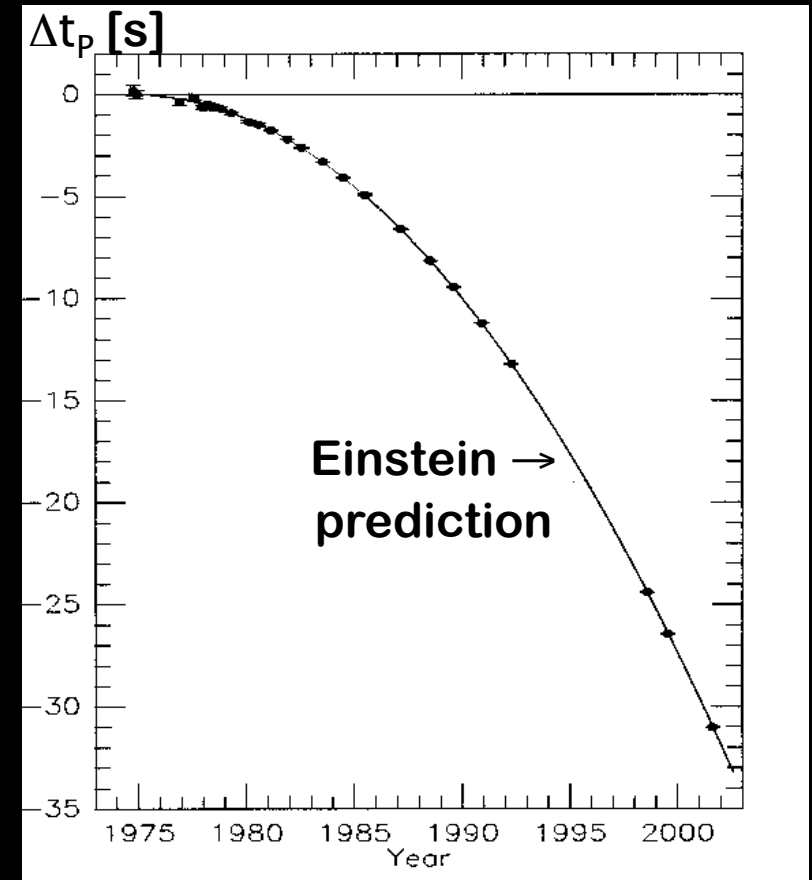
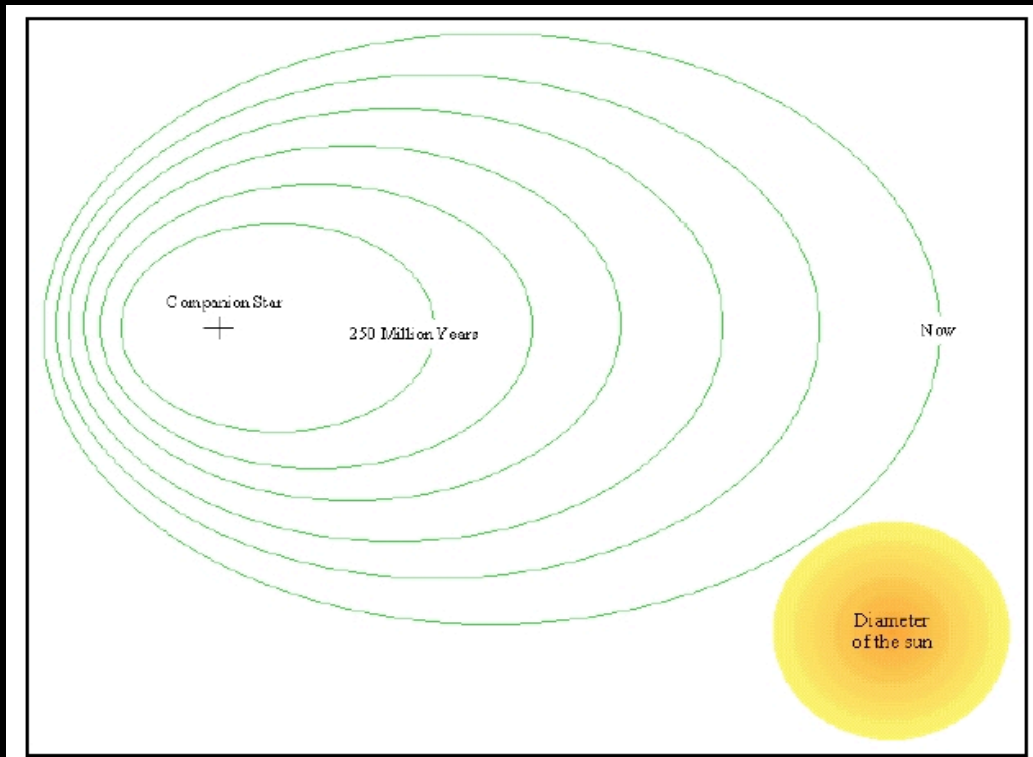




End



Indirect proof

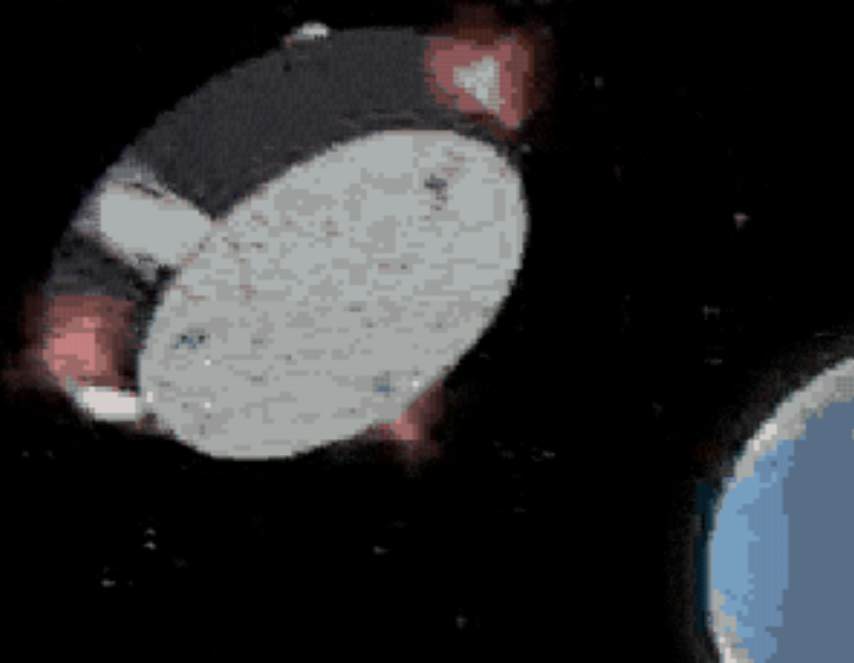
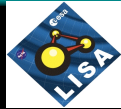


Hulse–Taylor Binary PSR1913+16 (1974)
Nobel Price (1993)

Remark : Outside of LISA Gérard Auger



Gérard Auger



Gérard Auger