Center for Gravitational Wave Astronomy
Department Physics & Astronomy
The University of Texas
at Brownsville

Black holes, Gravitational Waves and Numerical Relativity

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NR @UTB

with
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Institut Henri Poincare
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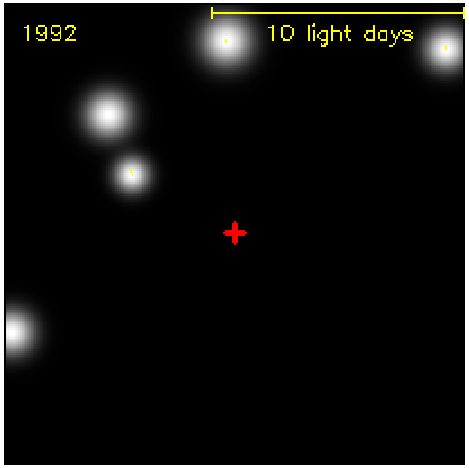
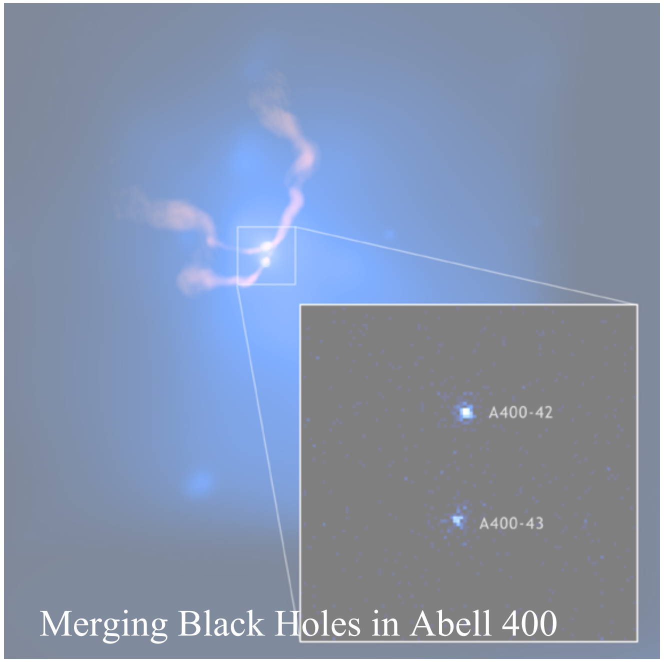
Black Holes in Astrophysics

- Some evidence of stellar mass BHs (e.g. Cyg X-1):
  - Stellar collapse: $M \sim 25\, M_{\odot}$
  - no evidence about stellar size BBH mergers yet …
- Observational evidence of supermassive BHs (MBHs) residing at the centers of most galaxies (e.g. SgrA*):
  - Huge mass in small volume: $M \sim 10^6 - 10^9\, M_{\odot}$
  - Formation is not quite clear … (Hint: related to the birth and history of the host galaxy)
- Most galaxies are expected to merge one or more times → MBH binaries merge …
  - Expect ~ several events/year
  - MBH mergers trace galaxy mergers
- Observing GWs from MBH mergers can probe early stages of structure formation …

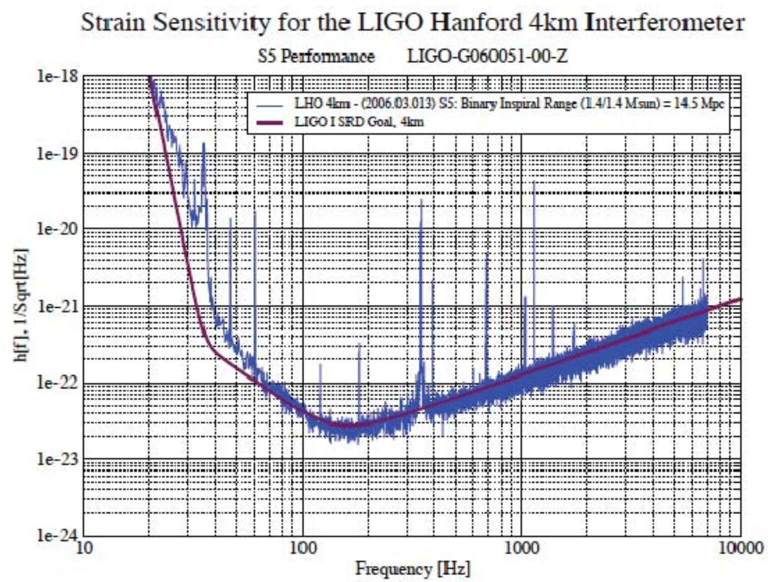




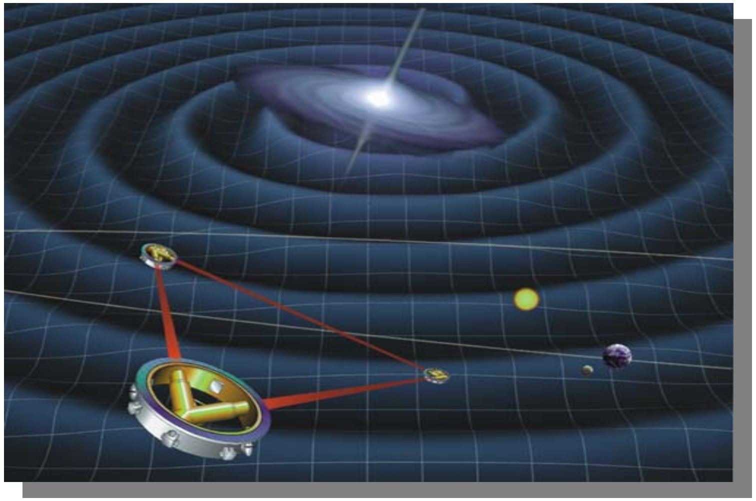
(X-ray: NASA/CXC/AIfA/D.Hudson & T.Reiprich et all;Radio:NRAO/VLA/NRL)

Ground-based interferometers . . .

- Current projects:
  - LIGO: Hanford (WA) and Livingston (LA); $L = 4$ km
  - VIRGO: France/Italy, near Pisa; $L = 3$ km
  - GEO600: Germany/Britain, Hanover; $L = 600$ m
- LIGO (NSF) is taking data now at design sensitivity …
- Its goal is to detect high frequency GW (broad band) $$10\,\mathrm{Hz} \leq f_{\mathrm{GW}} \leq 10^4\,\mathrm{Hz}$$
- Typical sources: NS/NS, NS/BH , BH/BH
  - BH → M~5-20 $M_\odot$



Space-based interferometer

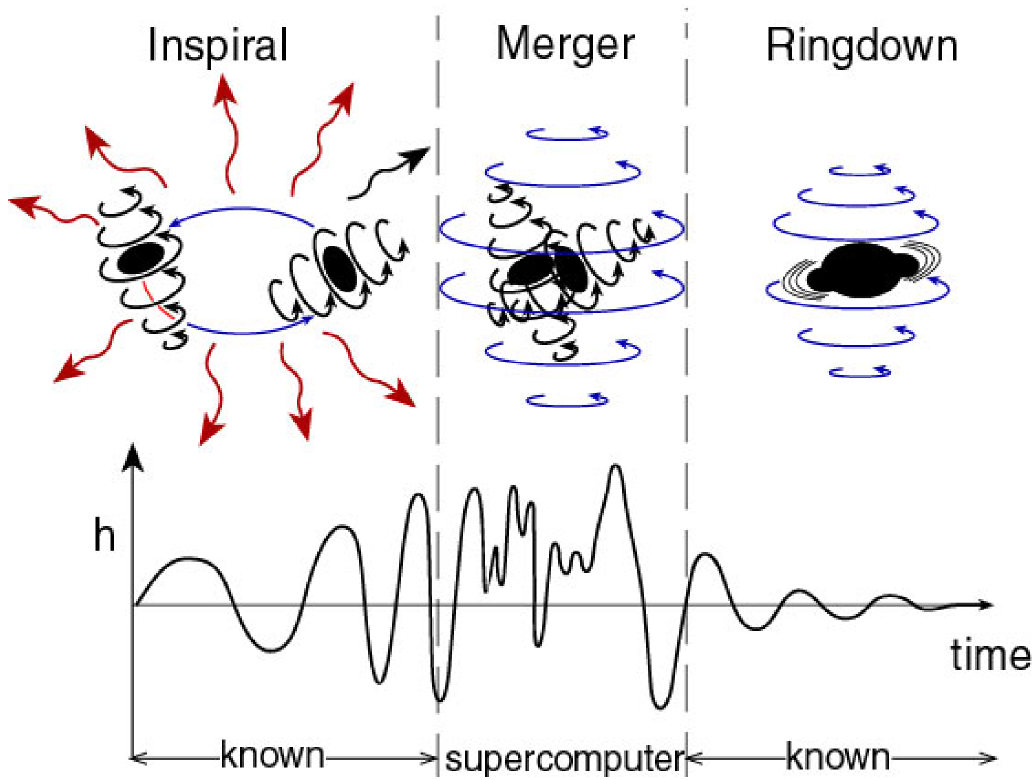


- LISA (Laser Interferometric Space Antenna) is a NASA/ESA space mission, consisting of 3 spacecraft:
  - equilateral triangle
  - orbits Sun at 1 AU
  - 20^o behind Earth in its orbit
  - arm length $L = 5 \times 10^6$ km
  - optical transponders receive and re-transmit phase locked light
- launch date ~ 2015 …

- Its goal is to detect low frequency GW $10^{-4}\ \mathrm{Hz} \leq f_{\mathrm{GW}} \leq 1\mathrm{Hz}$
- Typical sources: MBH/MBH, Galactic binaries, NS/MBH, BH/MBH
  - MBH→ $M \sim 10^6 - 10^9\ M_\odot$
  - $L_{GW} \sim 10^{23}\ L_\odot$
- Detectable by LISA to z ~ 10 …

The collision of binary black hole binary

- The final coalescence proceeds in 3 stages: inspiral, merger and ringdown
- Strong-field (nonlinear) merger stage requires Numerical Relativity to calculate dynamics & gravitational radiation emission (e.g. waveforms)

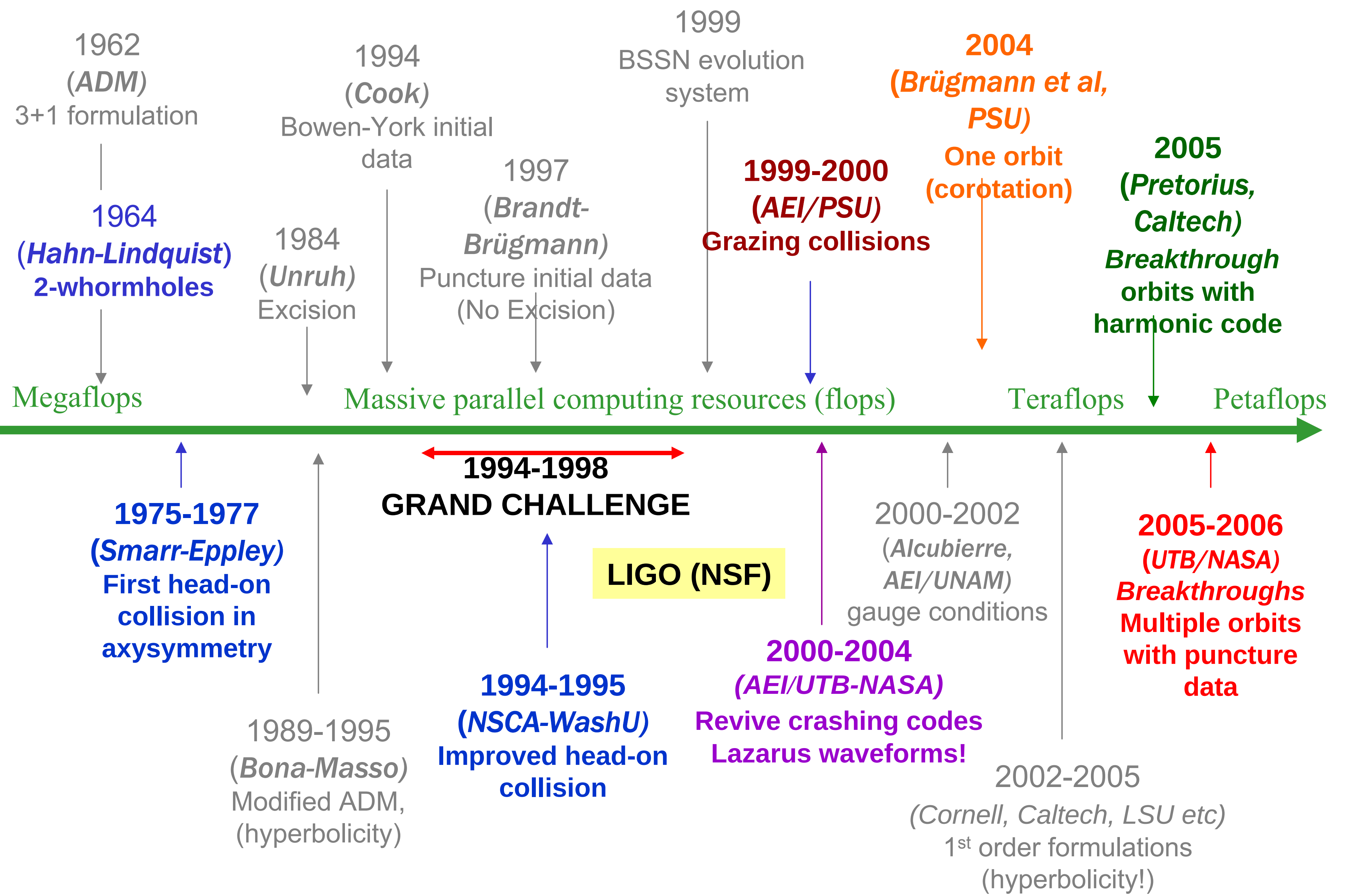


> 1000 cycles (PN theory)

BH perturbation theory

(graphic courtesy of Kip Thorne)

Numerical Relativity: 30-40 years of challenges



The Lazarus results

Baker, Bruegmann, Campanelli, Lousto, Takahashi, PRL (2001). [gr-qc/0102037]

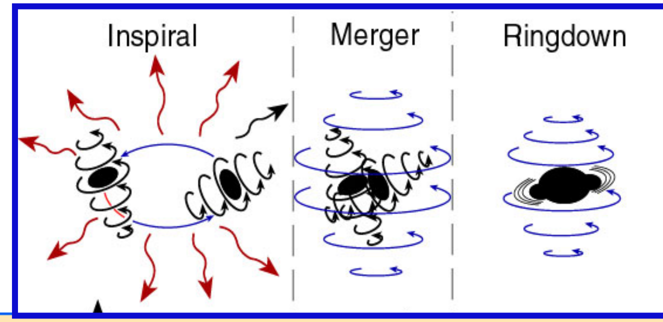
• New hybrid method which uses NR combined with black hole perturbation theory in the ringdown phase

• The first waveforms (for equal-mass, non-spinning BBH) are relatively simple …

• The energy and angular momentum losses during the plunge phase of equal mass non spinning holes are respectively ~ 3% and 12%

• The rotation parameter of the final Kerr hole is $a/M\sim 0.7\text{-}0.75$ (non-spinning, moderately spinning holes)

• Lazarus: a success, but concerns remain about accuracy (complexity of the interfaces) and the choice of initial data …



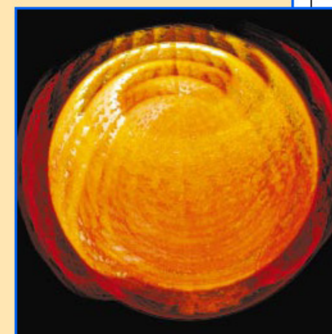
Gravitational physics

Black hole blockbuster

What happens when one black hole hits another black hole? Does it get more black? Seriously, though, such questions are being tackled by scientists seeking to detect gravity waves. These invisible waves are ripples in the fabric of space-time, predicted by Einstein's theory of relativity, but that remain undetected. They are thought to be emitted in copious amounts by colliding black holes, and a new simulation by researchers in Germany (J. Baker *et al. Phys. Rev. Lett.* **87**, 121103; 2001) gives an advance preview.

Collisions between other astronomical giants, such as galaxies, produce light and other radiation, but black-hole collisions generate only gravity waves. Black hole binaries are thought to emit gravity waves all the time, but only when they collide are the waves strong enough to be detected on Earth. Three detectors are expected to start collecting data soon: the US LIGO and German–British GEO600 projects in 2002, and the Italian–French VIRGO detector in 2003.

To fully simulate the merger of two black holes, the German team merged — appropriately enough — two different approaches for calculating what might happen before and after the collision. In the computer-generated image shown here, spherical shells of intense gravity waves move outwards from the centre of the collision. The authors estimate that 3% of the total mass of the black holes is released as energy by the collision — higher than expected.



W. BENGER, AEI/ZIB

These calculations should provide experimenters with a rough estimate of what to look out for, and guide more advanced simulations that take into account, for example, the likelihood that the black holes are also spinning. Prepare for a glimpse of the darkest corners of the Universe. **Sarah Tomlin**

NATURE|Vol.413|4October 2001|www.nature.com

The last orbit of binary black holes

M.Campanelli, C.Lousto & Y.Zlochower (UTB), PRD RP (2006) [gr-qc/0601091]

ψ_4

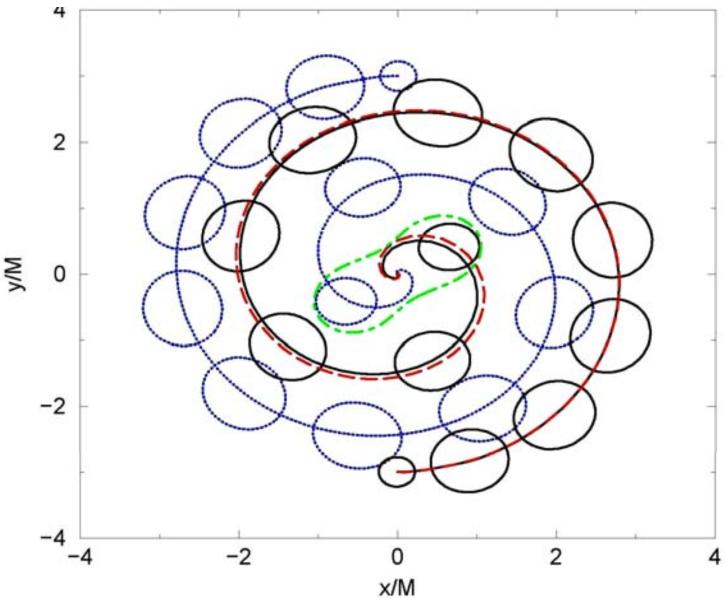
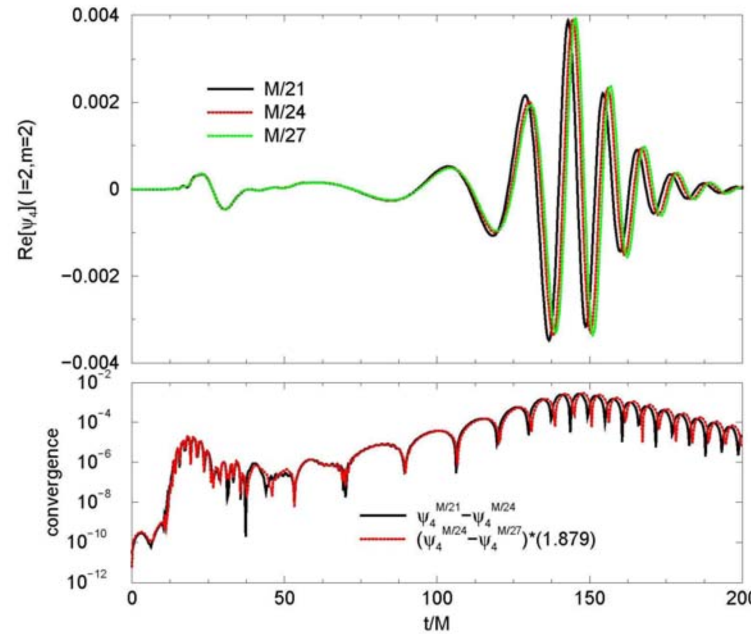


Figure 1: AH's every $10M$, puncture tracks at $h = M/21$ and $h = M/27$.

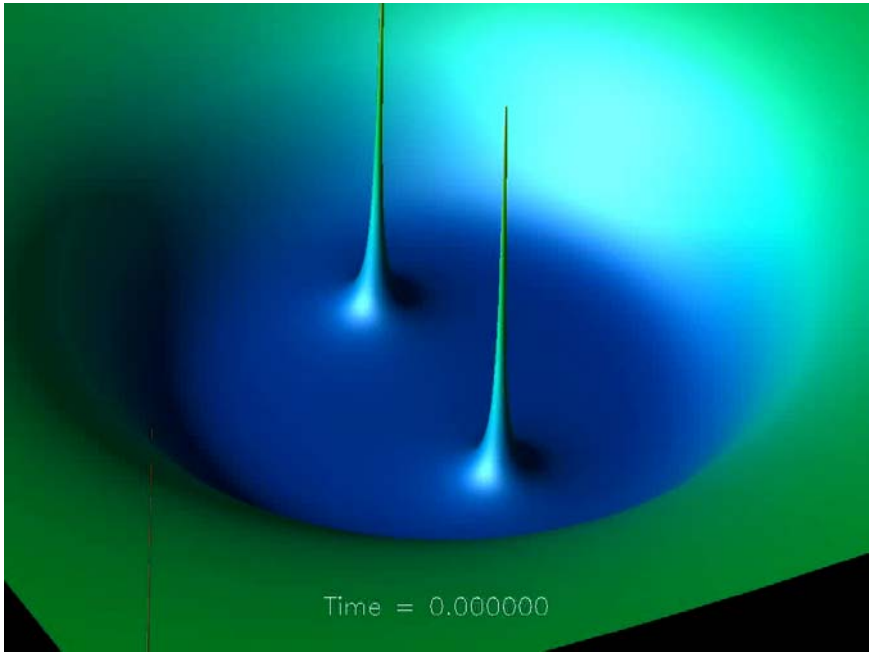


Waveforms simple (universal) show a relatively short burst of spurious radiation due to the non-ideal choice of the initial data

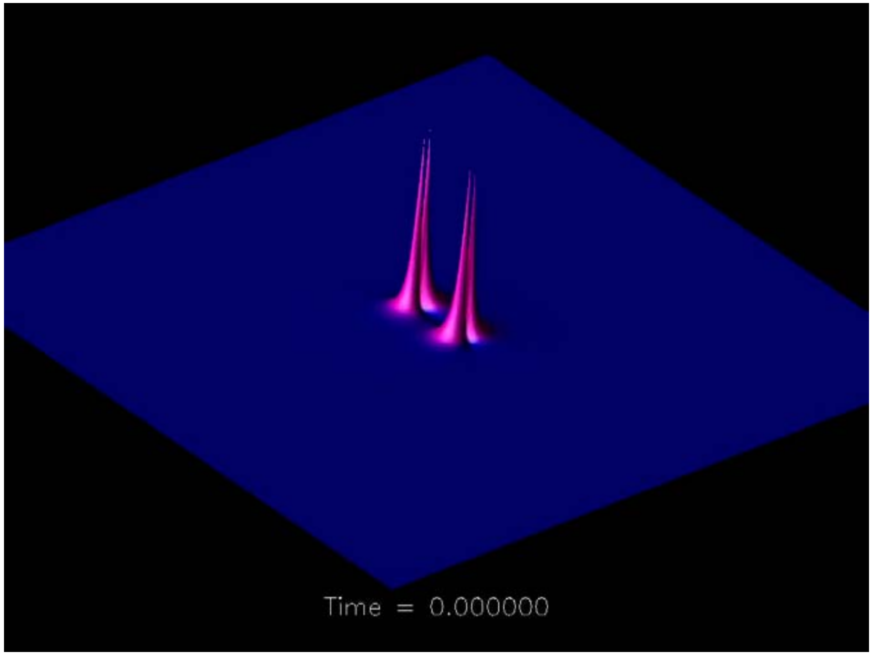
Table 1: Results of the evolution as determined from the waveform and the remnant horizon.

Method	E_{rad}/M_{ADM}	J_{rad}/J_{ADM}	T_{cah}/M	$a/M_{\mathcal{H}}$
Radiation	$(3.18 \pm 0.2)\%$	$(24.3 \pm 2)\%$	≈ 121	0.673 ± 0.002
Horizon	$(3.3 \pm 0.2)\%$	$(24.7 \pm 0.4)\%$	≈ 125	0.688 ± 0.001

UTB: the last orbit of equal-mass non-spinning binary black holes



Conformal factor ψ



$\Psi_4 \sim d^2/dt^2[h_+ + ih_x]$

Note that each simulation (equal-mass, non-spinning BBH) takes about 1-2 weeks on a 64-nodes Linux cluster …

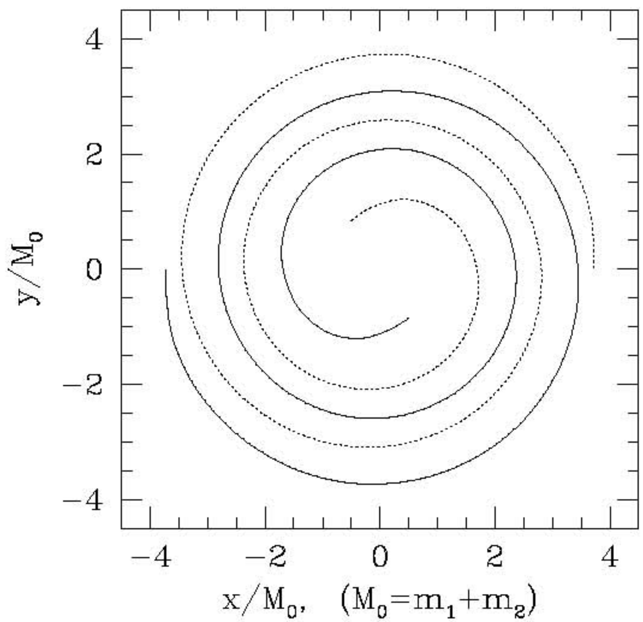
M.Campanelli, C.Lousto & Y.Zlochower (UTB), *The last orbit of binary black holes*, Rapid Communication PRD (2006)

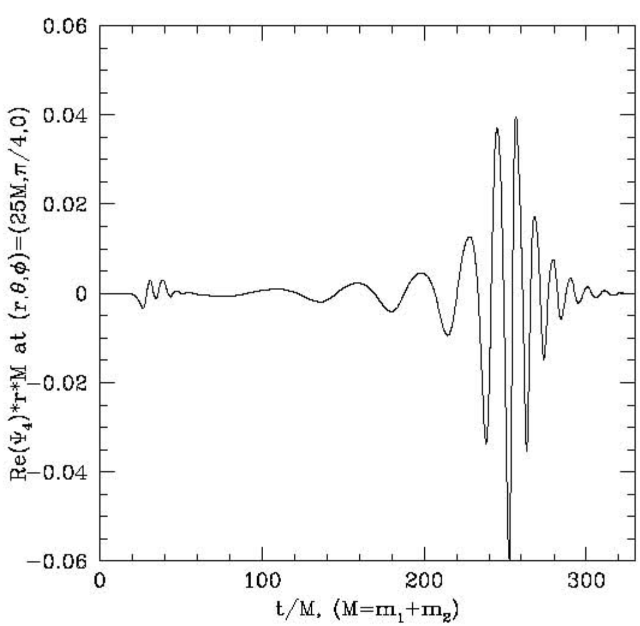
Pretorius breakthrough: different …

In early 2005 Pretorius (Caltech/Alberta, PRL 2005) demonstrates that the binary black hole problem 'can be done' in Numerical Relativity (orbits, waveforms etc) but uses a *completely different* system to the standard

- Not 3+1 formalism but evolve directly the 4 metric
- Generalized harmonic coordinates
- Excision, `Constraint damping', AMR, compactified spatial infinity

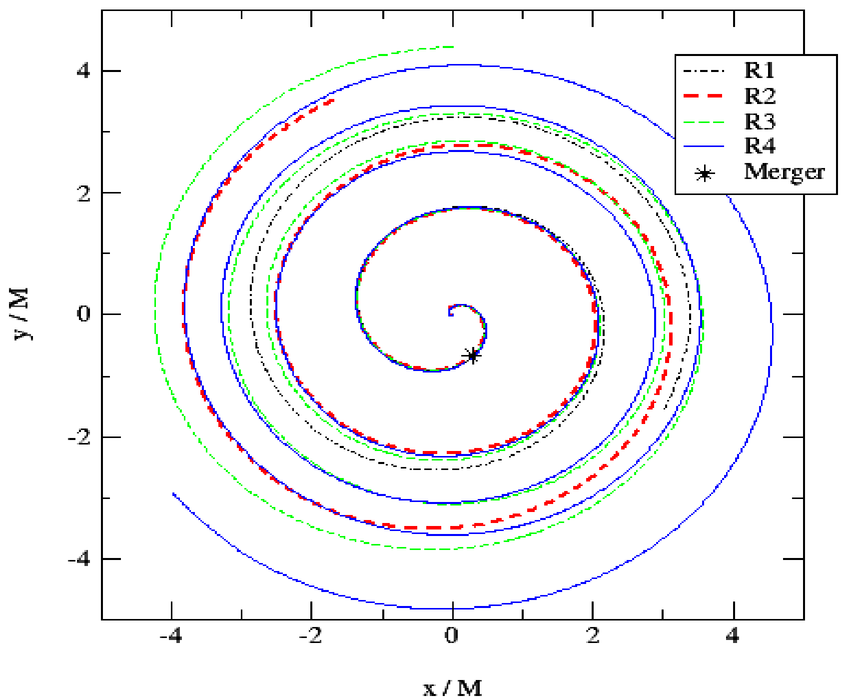
but *what are the key ingredients*?

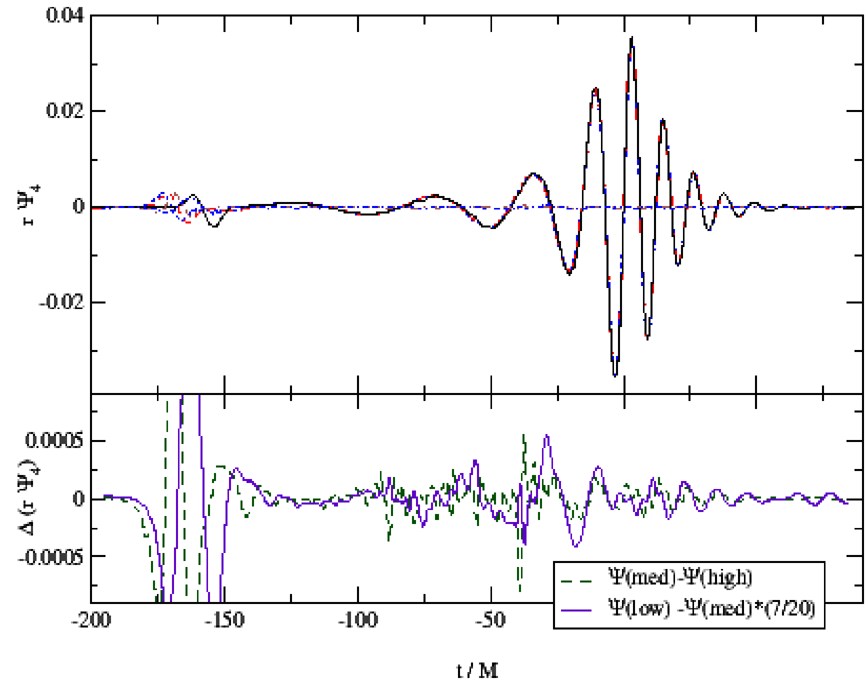




Multiple orbits from equal-mass non-spinning black hole binaries

Baker et al, PRD (2006) [gr-qc/0602026]

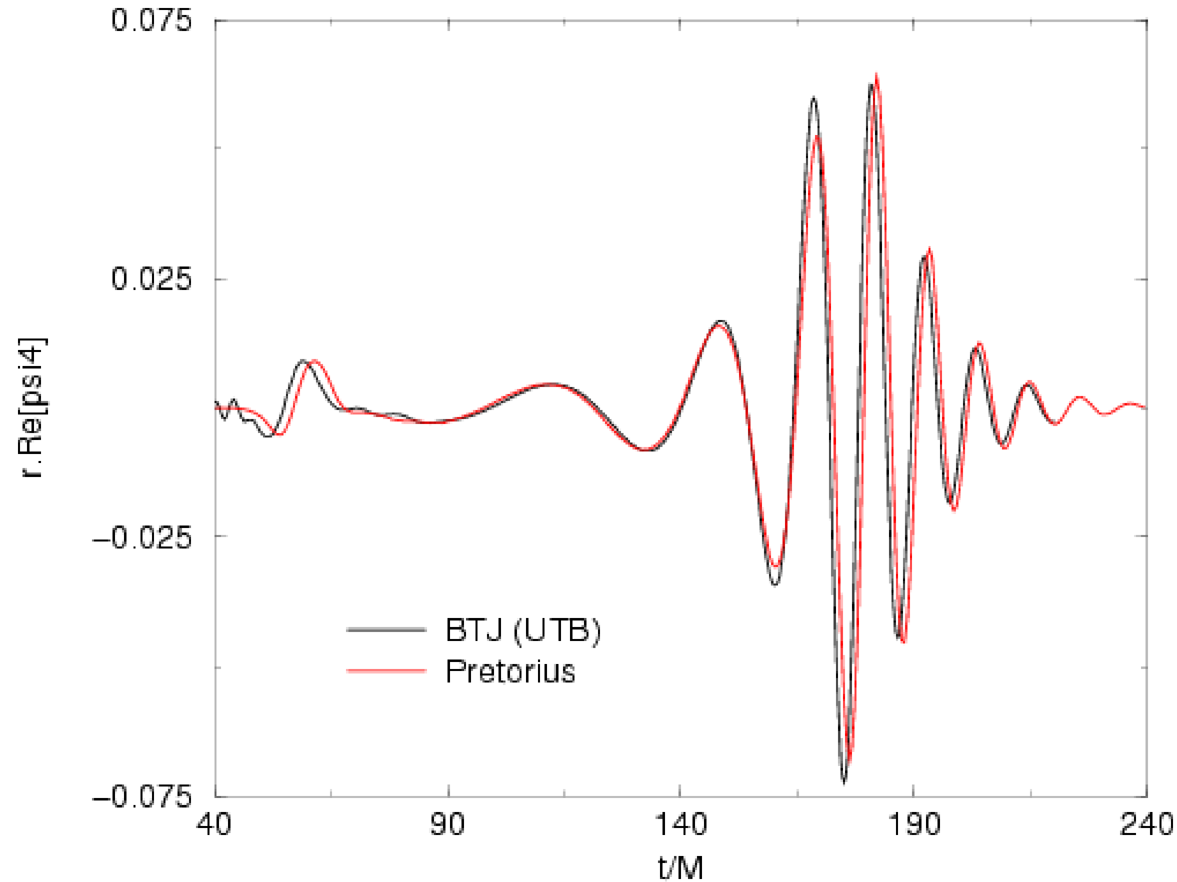




NASA simulations showing multiple orbits of black holes. When shifted in time, the late time behavior is the same for different initial separations.

These results confirm the picture of the `universality of the merger waveform'

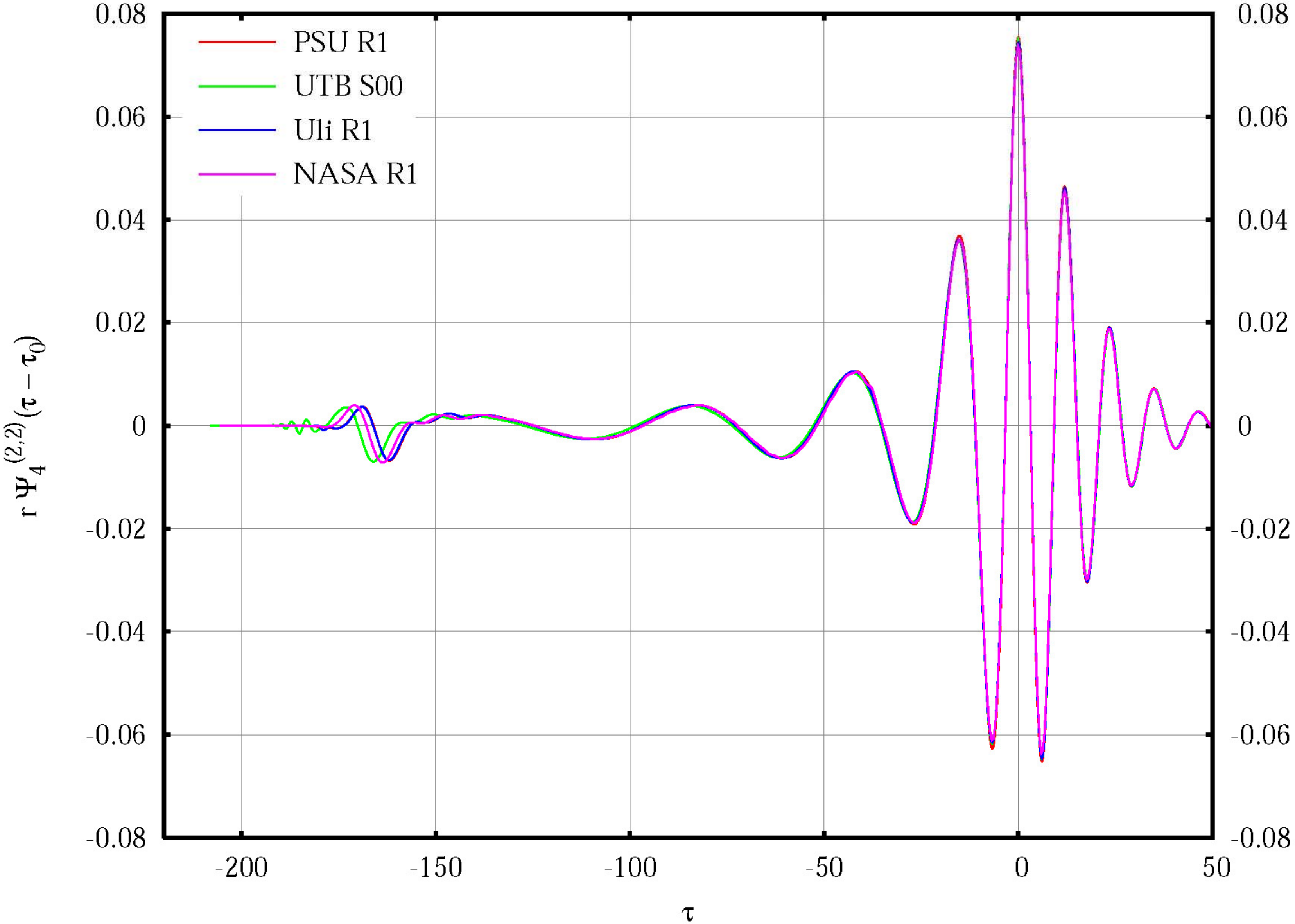
Universality of merger waveforms: a first (crude) comparison …



- First comparison done at the AANR meeting in Mexico (Guanajuato, May 2006):
  - UTB using puncture initial data for equal mass non-spinning BBH
  - Pretorius using excision thin-sandwich initial data with equal mass (small) corotation spins (small phase differences due to spins)
- Different resolutions and evolution systems, but same merger waveforms!?

- Confirm this with UTB, NASA, PSU and Jena results (see NRwaves.org)

NRwaves.org



PSU R1
UTB S00
Uli R1
NASA R1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-200
-150
-100
-50
0
50
τ
$r\,\Psi_4^{(2,2)}(\tau-\tau_0)$

DATA ANALYSIS AND NUMERICAL RELATIVITY CHALLENGES

- DA ‘require’ the knowledge of GW strain: h+, h×
- Source Modeling groups use models (PN, NR, CL, etc) to compute the GW strain
  - NR calculations provide: $\psi_4 = d^2/dt^2\,(h_+ - i\,h_\times)$, easy conversion in h+, h×
  - ψ_4 is generally integrated over spheres at large radii (source location and direction with respect the detector)
  - Currently extract mostly l=±m=2 modes, but higher modes important (different masses) and require additional accuracy …
- But ‘target’ waveforms containing *all three stages* of binary coalescence, but may too expensive to compute starting from very large separations and a larger parameter space:
  - Simulation costs (equal mass, non spinning, AMR) ~1000 CPU hours, 18GByte of RAM
  - Simulation costs (non-equal mass, non-spinning, AMR) > 5,000 CPU hours
  - Simulation costs (non-equal mass, spinning, AMR) > 40,000 CPU hours

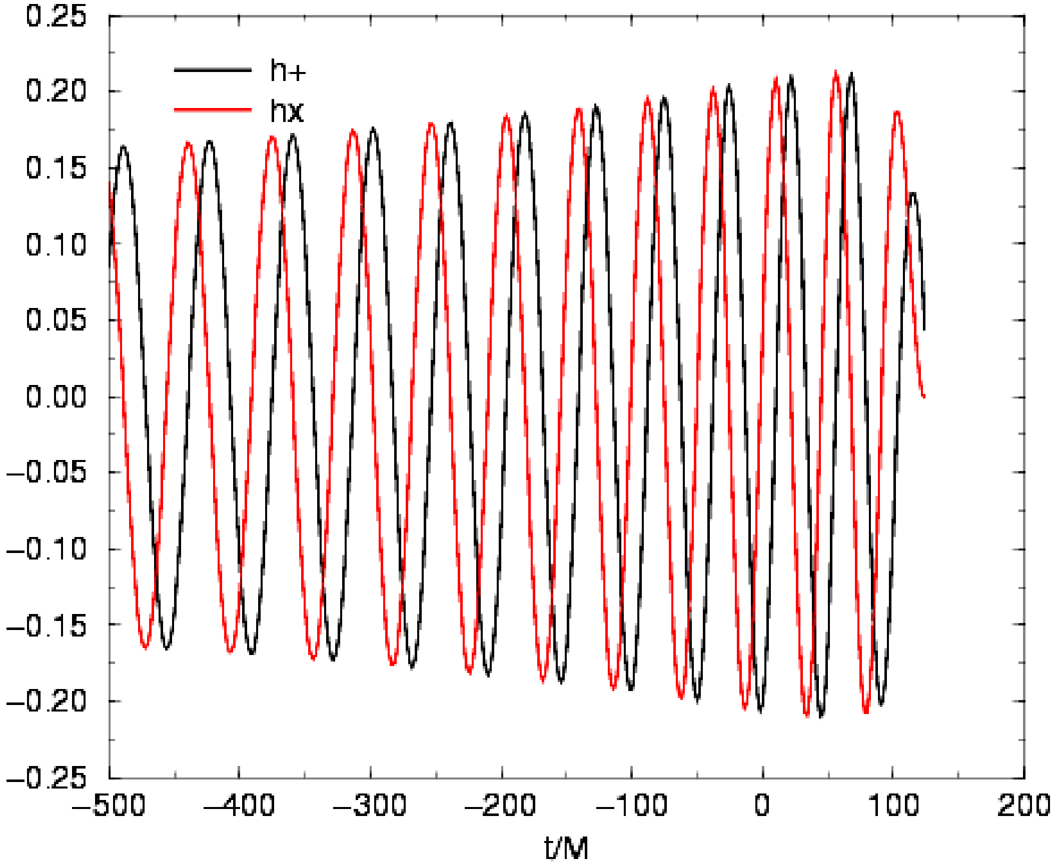
- Construct hybrid analytical/numerical models e.g. waveform:
  - Match PN inspiral waveforms with the NR waveforms over a region (more than a period long) where both the calculations are presumably valid …
  - How consistent is the matching procedure?
  - NR accuracy is important to validate PN theory ⟶

NR meets PN, San Louis Feb 2007

3.5 PN Waveforms

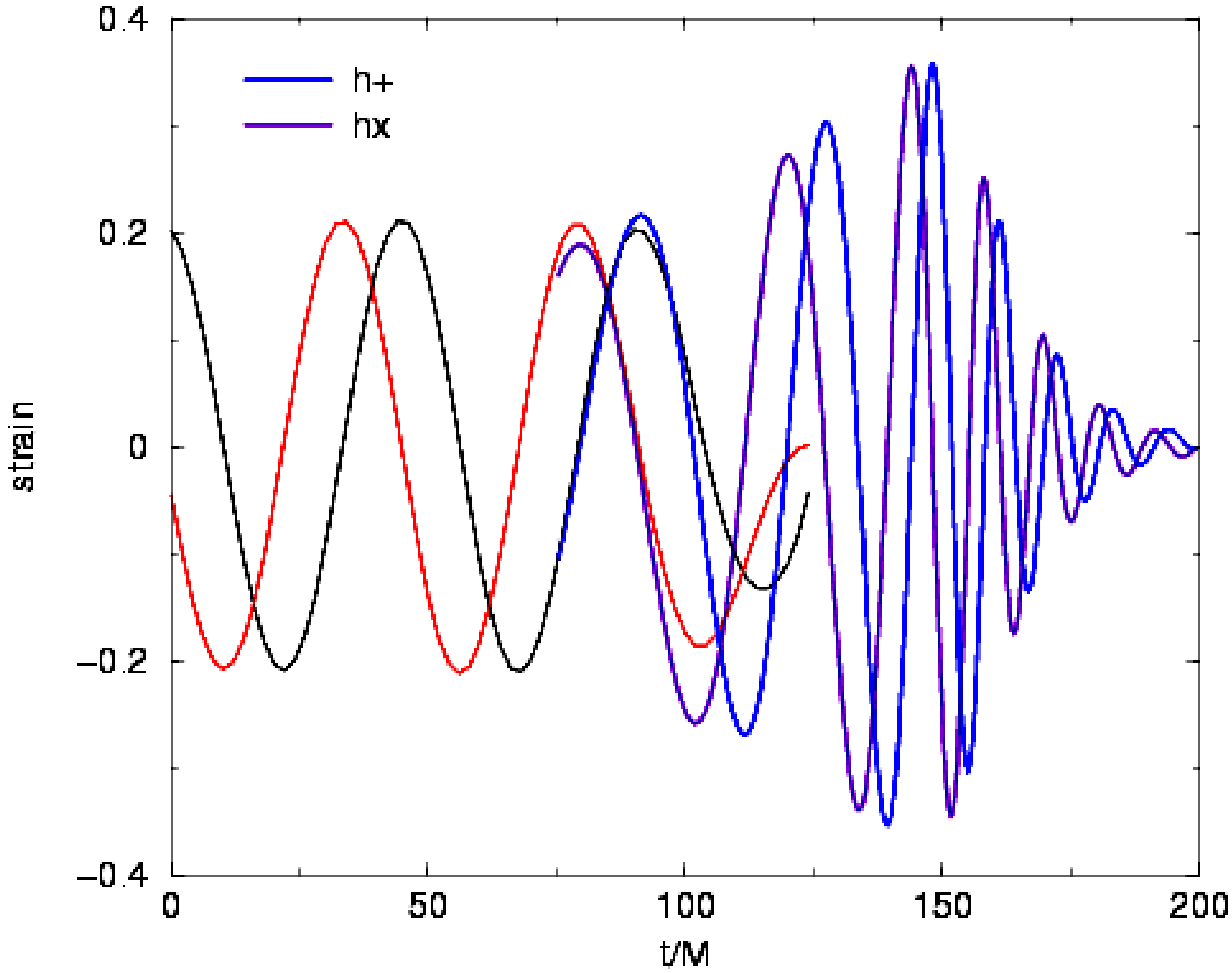
$$
\begin{aligned}
r.h_+(t) &= -2(M\dot{\varphi}(t))^{2/3}\cos(2\varphi(t)) \\
r.h_\times(t) &= -2(M\dot{\varphi}(t))^{2/3}\sin(2\varphi(t))
\end{aligned}
$$

$$t_c = 114M, \quad \lambda = -\frac{1987}{3080}, \quad \theta = -\frac{11831}{9240}$$

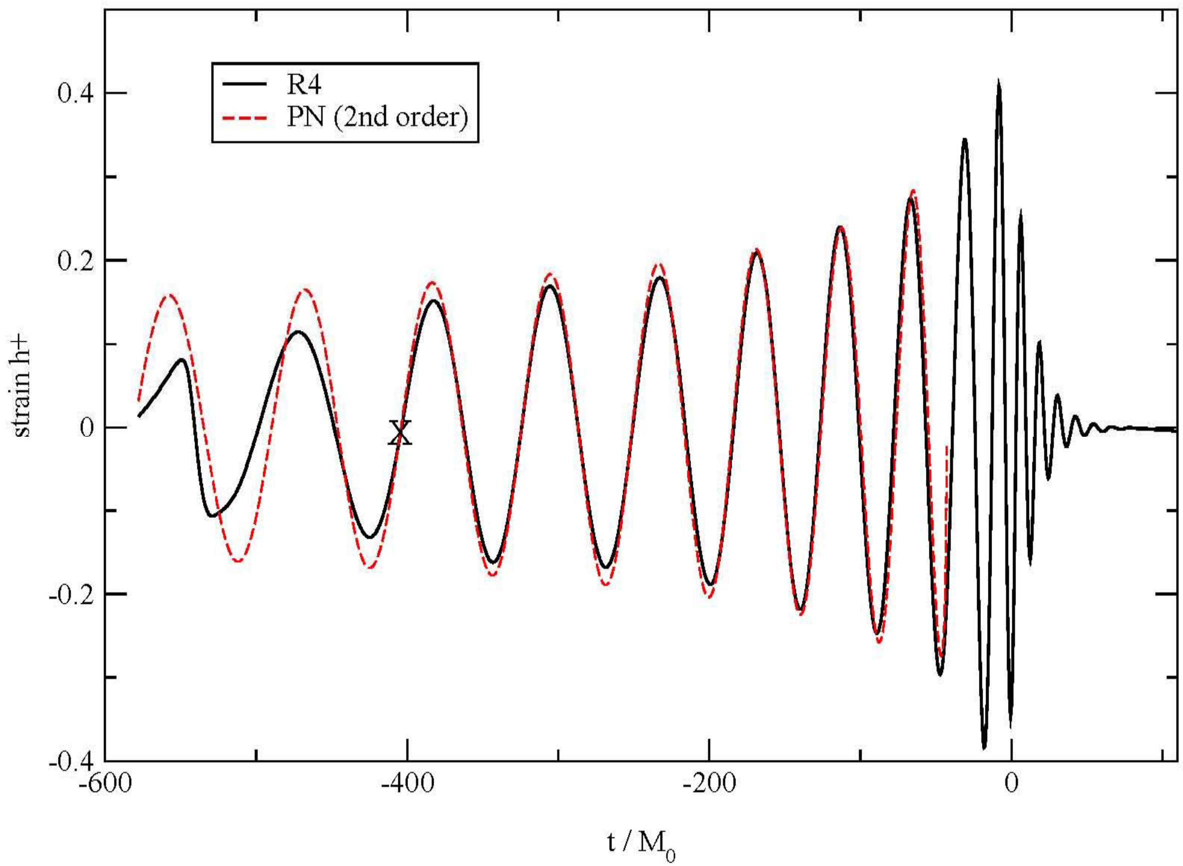


L. Blanchet, G. Faye, B.R. Iyer, and B. Joguet, Phys. Rev. D 65, 061501(R) (2002).

3.5 PN + Full Numerical: Matching Waveforms



NASA Goddard : A second attempt to match with (2) PN waveforms



Some ellipticity in the initial data …

Plot from Baker’s talk, AANR workshop, Guanajuato, May 6-11, (2006)

The effect of BBH mergers on LIGO

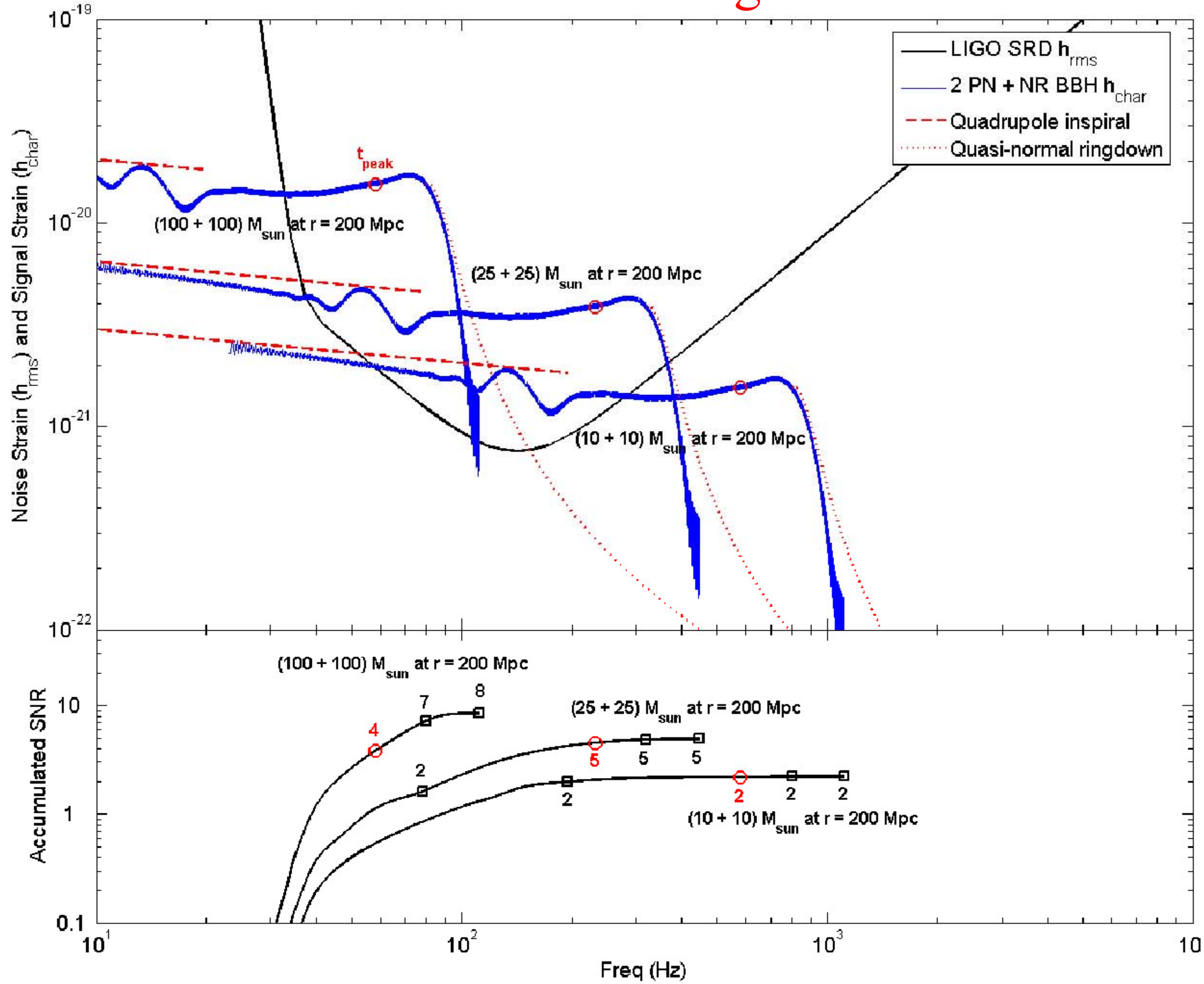
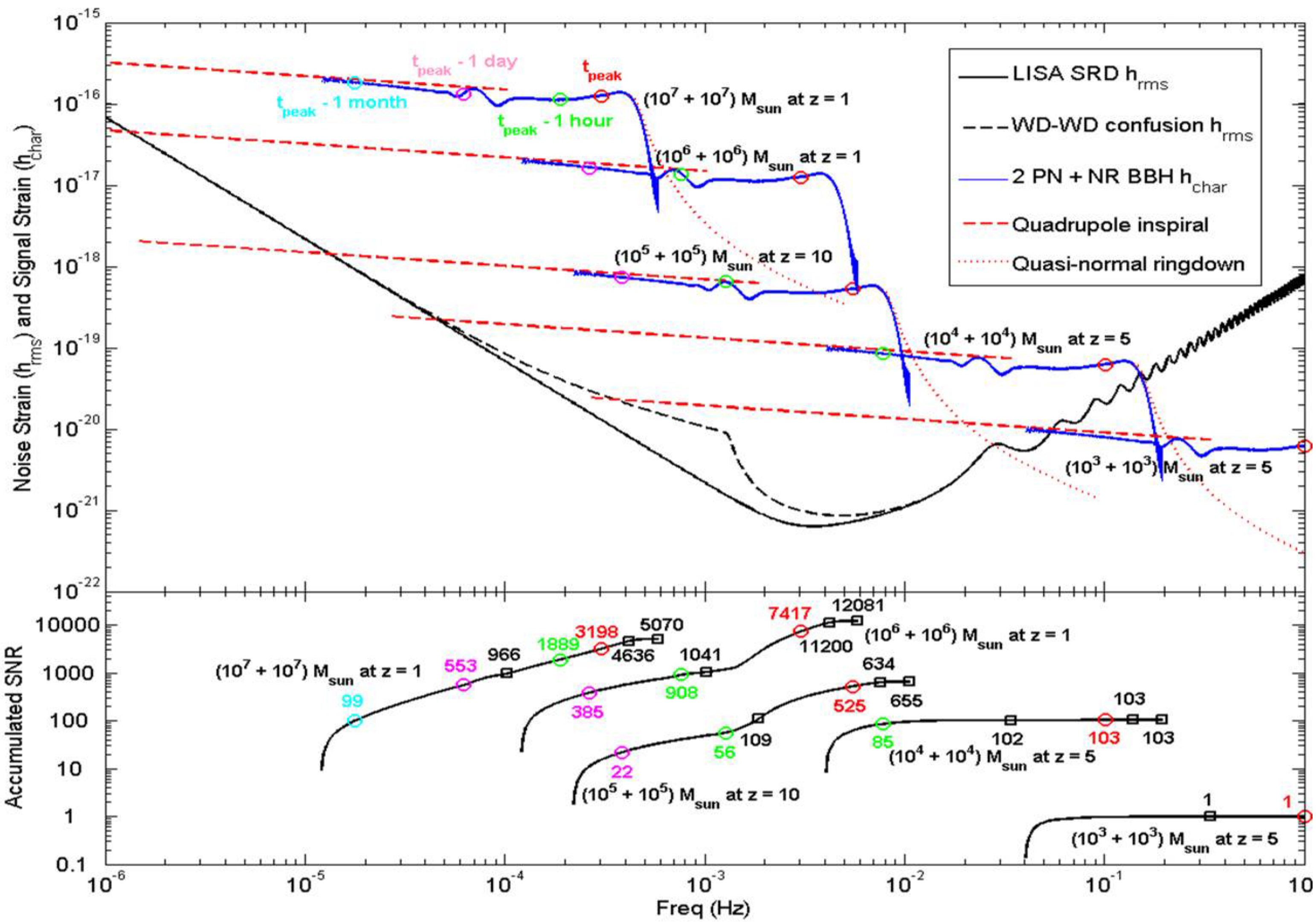


Figure 1. LIGO Noise h_{rms} and the h_{char} of a 2 PN inspiral tied to an NR merger and ringdown, with the corresponding accumulated SNR.

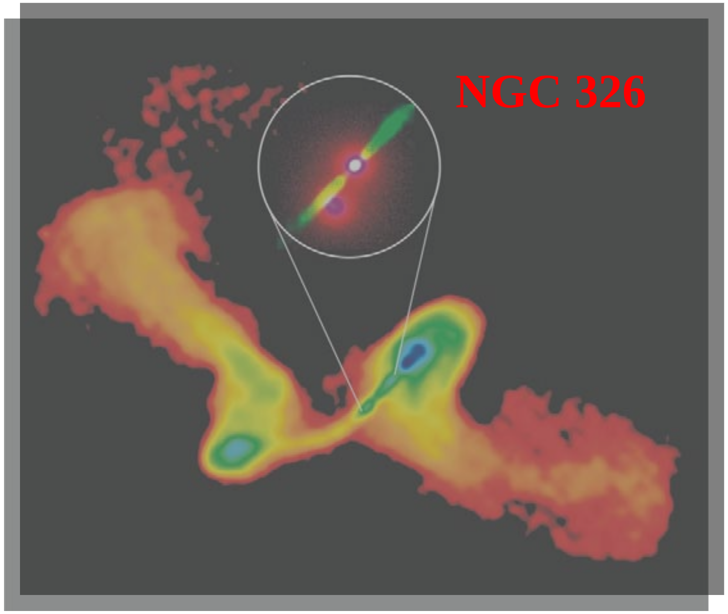
S. Williams & J. Baker, NASA-GSFC (poster Lisa6 simposium)

The effect of BBH mergers on LISA



The science of MBH mergers

- Final merger of MBHs occurs in the arena of very strong gravity …
- Observing gravitational waves allows direct tests of general relativity in the very dynamical, strong field regime…
- Gravitational waves encode dynamics of massive objects …
- When $m_1 \neq m_2$, GW emission is asymmetric producing recoil velocity (kick) …
- If this kick is large enough, it could eject the merged remnant from the host structure… and affect the rates of merger events
- MBHs are expected to be spinning (with a relatively high spin) …
- MBH mergers could produce interesting spin dynamics and couplings (e.g. spin-flip)



Spin-flip in X-shaped radio morphologies induced by merger?

Merritt & Eckers, Science 297 (2002)

(Image NRAO/AUI & Inset: STScI)

Spinning black-hole binaries: the orbital hang-up

Campanelli, Lousto, Zlochower, PRD. [gr-qc/0604012]

Equal masses, a/m= -0.75 (S- -), 0.0 (S00), +0.75 (S++) with total $J/M^2>1$

Initially $M\Omega = 0.05$ → $T_{\text{orbital}} \sim 125M$ (other orbital parameters from 3PN)

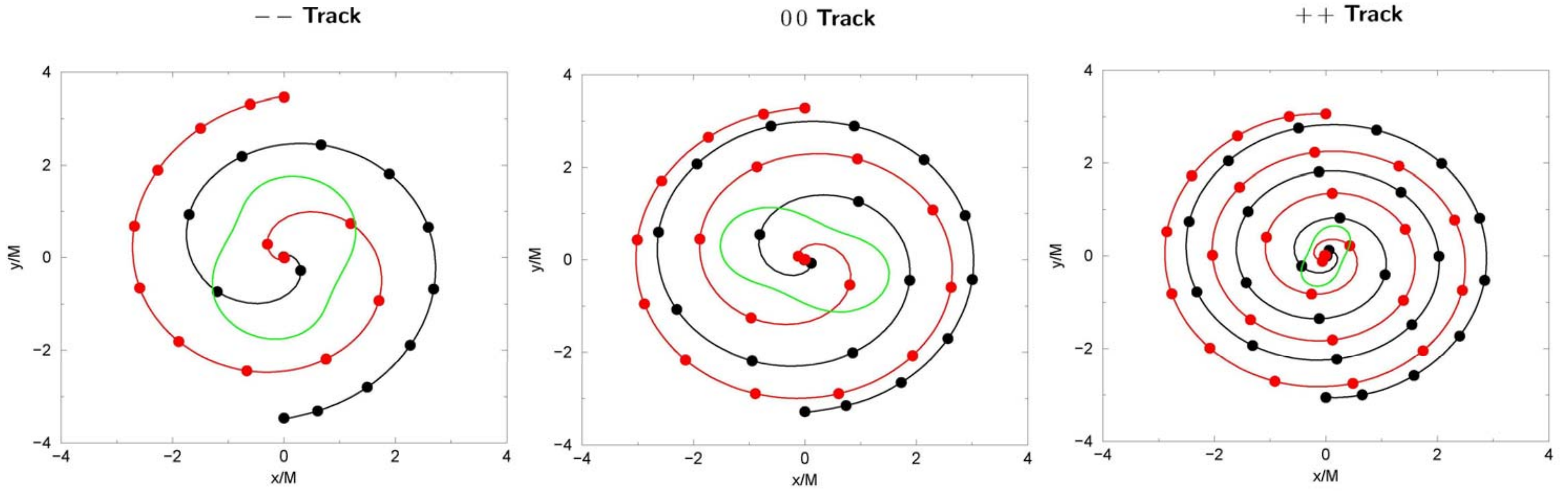


Figure 4: Puncture tracks for the − − configuration.

Figure 5: Puncture tracks for the 0 0 configuration.

Figure 6: Puncture tracks for the + + configuration.

Spin-orbit coupling effects:

- S - - (unaligned) case: early merger ~ 1 orbit → a/M=0.44
- S00 (non-spinning) case: complete ~1.75 orbits → a/M=0.68
- S++ (aligned) case: hang-up ~ 3.2 orbits → a/M=0.89
- Extrapolating to maximal individual spins → a/M=0.97

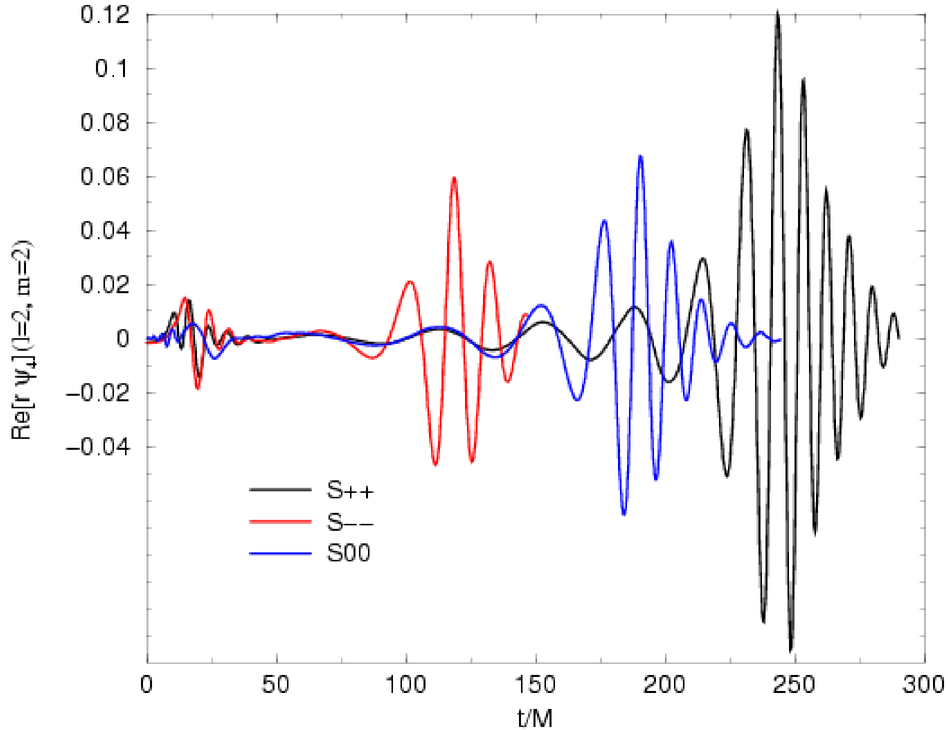


Table 1: Results of the evolution as determined from the waveform and the remnant horizon.

Config	E_{rad}/M_{ADM}	J_{rad}/J_{ADM}	T_{cah}/M	$a/M_{\mathcal{H}}$
$++$	$(6.5 \pm 0.1)\%$	$(33.8 \pm 1.5)\%$	≈ 232	0.892 ± 0.002
$0\,0$	$(3.51 \pm 0.01)\%$	$(26.9 \pm 0.1)\%$	≈ 161	0.688 ± 0.001
$--$	$(2.1 \pm 0.1)\%$	$(26 \pm 2)\%$	≈ 105	0.44 ± 0.01

Extrapolating to maximal individual spins we get $a/M^2 = .976$ (linear) and $a/M^2 = .952$ (quadratic).

The cosmic censorship is respected … unfortunately!

Spin-orbit interactions in black hole binaries

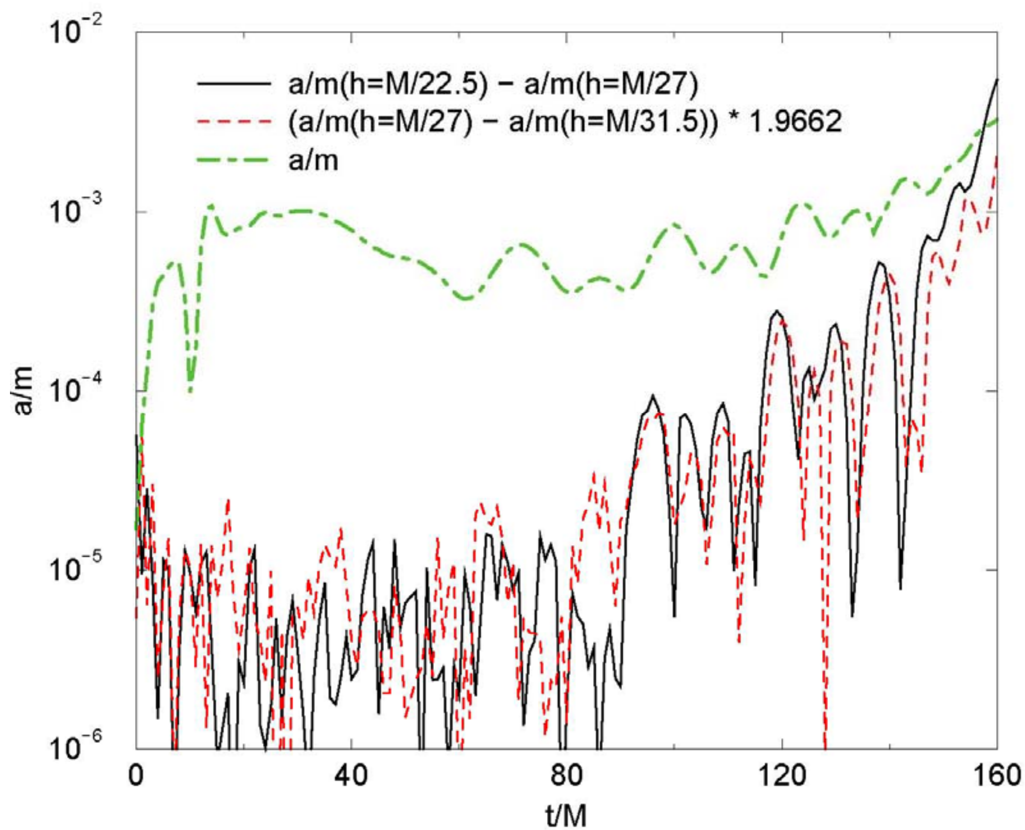
Campanelli, Lousto, Zlochower, PRD [astro-ph/0608275]

Can tidal effects spin-up the holes to the orbital frequency, or equivalently lock the spins of the holes to a corotation state? Tidal effects stronger in the merger stage …

We calculate the spin-up of the holes with the isolated horizon algorithm developed by Dreyer et al, PRD (2003) [qr-qc/0206008]:

$$S = \frac{1}{8\pi}\oint_{AH}(\varphi^a R^b K_{ab})\,\mathrm{d}^2V.$$

We can measure spins of the order of a/M~10^{-3} with an accuracy of 1% or better for L≥4.5M and of 20% for L~3M

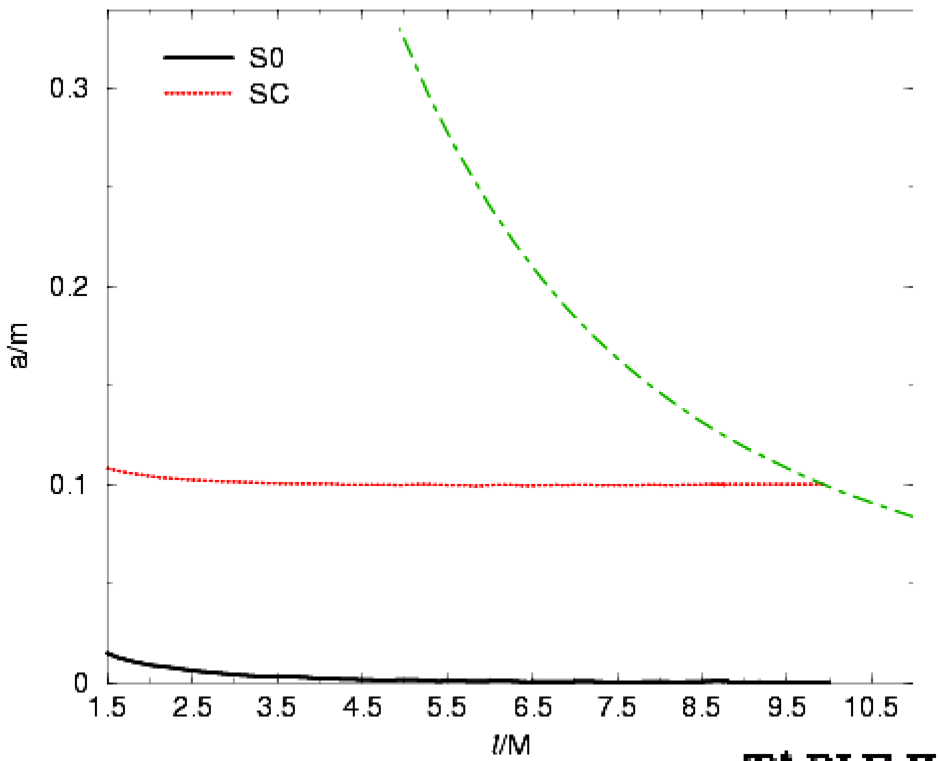


Spin-up of the individual black-hole horizons in the S00 (0 initial spin)

Name	S/M^2	Y/M	P/M	J/M^2	$M\Omega$	m_p/M	l/M
S00	0.000	3.280	0.1336	0.876	0.0500	0.4848	10.01
S0.1	0.025757	3.2534	0.132957	0.917	0.0500	0.483115	9.93

Spin-orbit interactions in black hole binaries

Campanelli, Lousto, Zlochower, PRD [astro-ph/0608275]



The values that we obtain for the spin-up of the binary holes (in the S00 and S0.1 cases) are two order of magnitude smaller that those expected for a corotation state!

TABLE III: Merger time, T_{CAH}/M, for the S0 and SC configurations versus resolution. Horizon searches were performed every $0.3M$ and $0.2M$ for the S0 and SC configuration respectively.

resolution	S0	SC
M/22.5	160.7 ± 0.3	168.6 ± 0.2
M/27	166.0 ± 0.3	174.2 ± 0.2
M/31.5	168.3 ± 0.3	176.6 ± 0.2
extrapolation	172 ± 2	179 ± 2

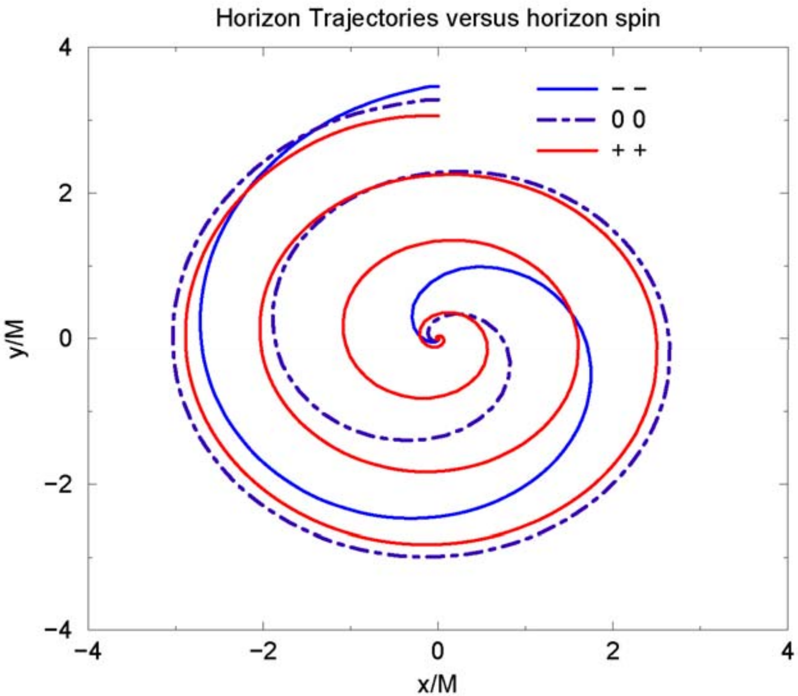
• The values that we obtain for the spin-up of the binary holes (in the S00 and S0.1 cases) are two order of magnitude smaller that those expected for a corotation state.

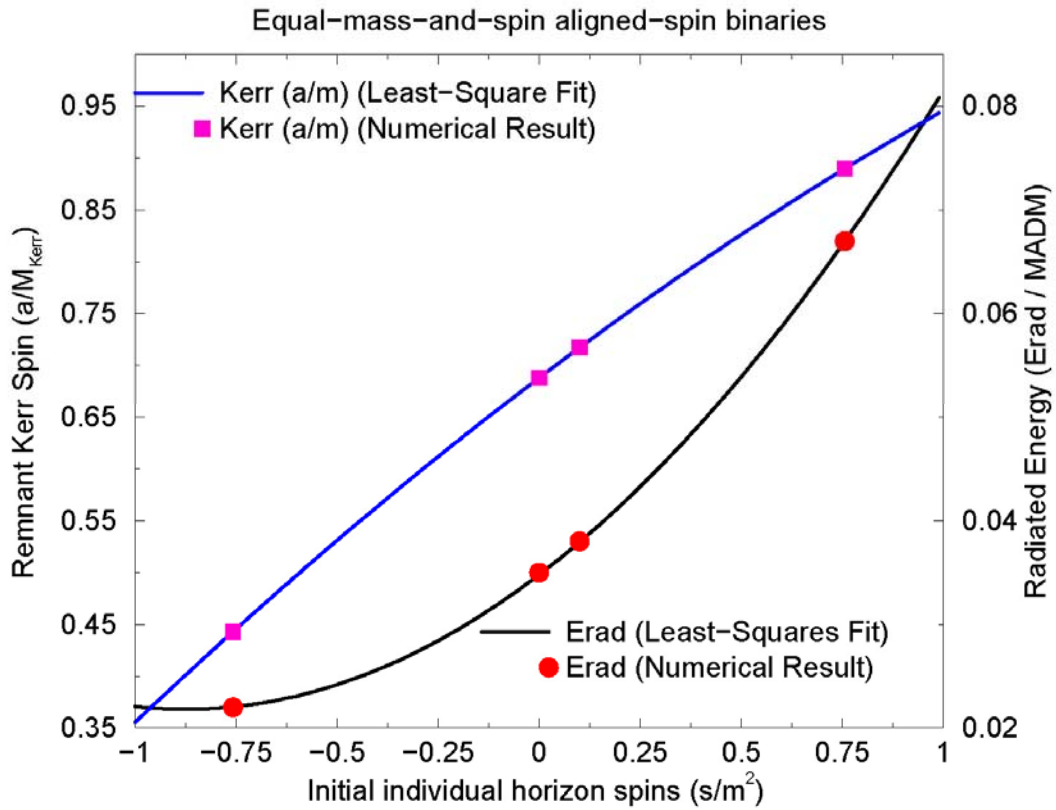
• Results for NS-NS and NS-BH inspirals were computed by Bilsten and Cutler, ApJ (1992)

TABLE VI: The measured remnant horizon specific spin $(a/M_H)|_R$ and energy radiated E_{rad}, as well as the predicted remnant horizon specific spin $(a/M_H)|_{pred}$ and energy radiated $E_{rad}/M|_{pred}$ (based on a least-squares fit) for quasi-circular, equal-mass, equal-spin binaries with initial specific spins $(a/m)|_I$.

$(a/m)\|_I$	$(a/M_H)\|_R$	$(a/M_H)\|_{pred}$	E_{rad}/M	$E_{rad}/M\|_{pred}$
-0.757	0.443 ± 0.001	0.4430	$(2.2 \pm .01)\%$	2.20%
0.000	0.688 ± 0.001	0.6878	$(3.5 \pm 0.1)\%$	3.48%
0.1001	0.717 ± 0.001	0.7169	$(3.8 \pm 0.1)\%$	3.79%
0.757	0.890 ± 0.001	0.8900	$(6.7 \pm 0.2)\%$	6.70%
-1.0	* * *	0.355	* * *	2.2%
$+1.0$	* * *	0.946	* * *	8.1%

The effect of spins …



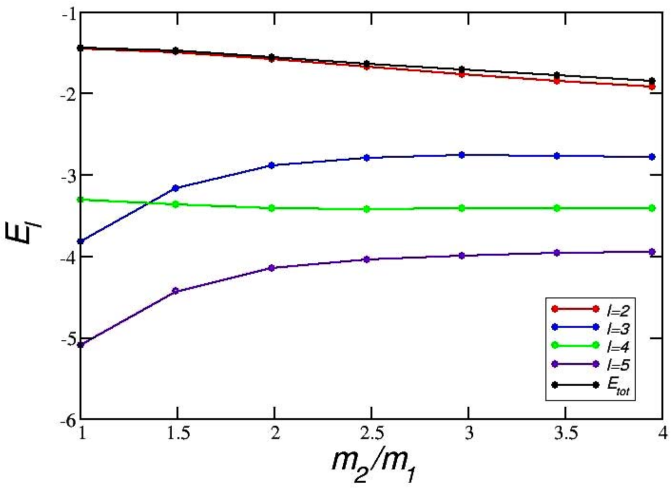


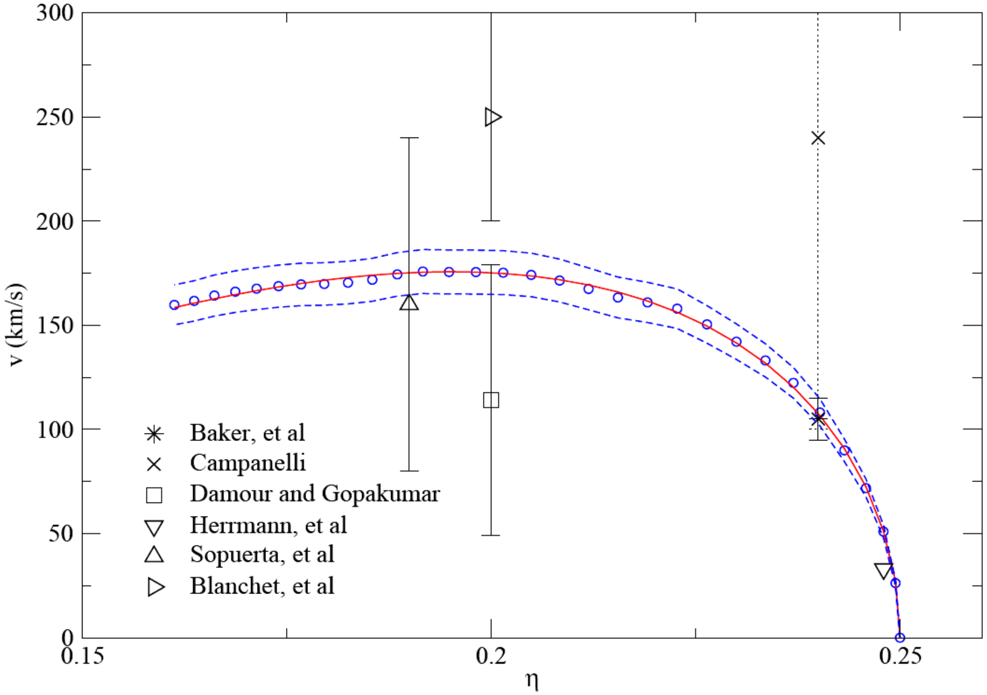
- Gravitational radiation and merger time are strongly affected by the value and direction of each individual BH spins (Campanelli, Lousto, Zlochower, gr-qc/060412, astro-ph/0608275)

- Note that the GW energy emitted for highly spinning binaries (with aligned spins) can increases by almost a factor 3, while inspiral last at least twice as long as in the non spinning case …

(FSU) Jena

- Gonzalez et al '06:
  - Parameter study: $\frac{M_1}{M_2} = \frac{1}{1} \dots \frac{1}{4}$
  - 150,000 CPU hours
  - Kick, Maximum: $175.7 \pm 11\,\mathrm{km/s}$ at $\eta = 0.195 \pm 0.005$
  - 2nd order convergence
  - radiated E, J; Spin of final hole





Conclusions

- Remarkable progress in last year:
  - Moving punctures approaches (UTB and NASA) quickly adopted with very minor changes by several groups, including PSU, FAU, Jena, LSU, AEI, and UNAM
  - A variation of the harmonic approach (Pretorius) now adopted by Caltech/Cornell groups (adapted to 1^{st} order formulation, spectral code etc)
- Waveforms for equal-mass non-spinning BBH merger appear to be 'universal'
  - The merger is relatively insensitive to small changes of the initial data parameters!
  - Not true for the orbital dynamics (small ellipticity in all initial data)
- Multiple orbits (five-ten) are necessary to explore overlapping with PN results
  - Accuracy in the phase important …
  - Work in progress at UTB/FAU to built PN initial data for puncture evolution
- We also started to explore the parameter space:
  - NASA, PSU, Jena etc → unequal-mass BBH mergers
  - UTB etc → spinning BBH mergers
- Most groups are now limited by:
  - Computational resources …
  - Sophisticated software algorithms to improve accuracy (AMR)
- Numerical relativity is finally entering a golden age of applications!

Supercomputers:

UTB used a 70-node Linux cluster, Funes, built at the beginning of 2004, thanks to the support of a NASA University Research grant and NSF grid computing projects. Each node is dual Pentium Xeon 3.2 Ghz processors with 8 Gb of RAM, 2 x 120 Gigabyte hard drives, and is interconnected through a gigabit network.

NASA used Columbia, the fourth fastest supercomputer in the world, which consists of a 10,240-processor SGI Altix system comprised of 20 nodes, each with 512 Intel Itanium 2 processors, interconnected with InfiniBand network. Columbia has 440 terabytes of Fibre Channel RAID storage and running a Linux operating system.

CONCLUSION AND DISCUSSION

- Recent advances in NR now make it now possible to utilize late-inspiral and merger waveforms:
  - Recent calculations for equal-mass, non-spinning mergers now accurate up to 2% …
  - Simulations of unequal mass, spinning already exists …
- There is a great deal of work yet required to yield accurate full coalescence waveforms which can densely populate the physical parameter space before AdvLIGO is on line, but the NR is now ready to take this challenge!
- Extraction of the most exciting science from GWs is now dependent on having the numerical infrastructure (Efficient AMR, Spectral Codes, computational resources, etc) needed to compute accurate merger waveforms
- But it may be still too expensive …
- One can build hybrid PN+merger waveforms now:
  - Accurate waveforms models for last part of the inspiral are needed to validate PN theory.
  - Currently NR spectral codes are in the best position to do this!
- How to efficiently one can create a dynamical catalog of waveforms templates?
  - Distribution of tasks in the NR community may be difficult to coordinate (physics).
  - Feedback from data analysis will be crucial for parameter ranges.
- Find a way to systematically and efficiently communicate with the LSC