THEORY OF GRAVITATIONAL WAVE EMISSION

Luc BLANCHET

C.N.R.S., Institut d'Astrophysique de Paris

PART 1

EINSTEIN FIELD EQUATIONS

AND

QUADRUPOLE MOMENT FORMALISM

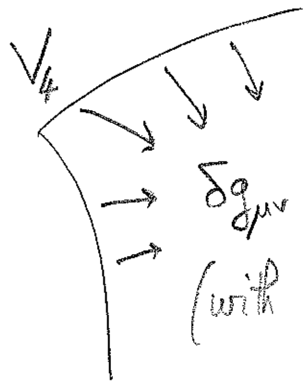
EINSTEIN FIELD EQUATIONS

They derive from the action

$$\boxed{S = \frac{c^3}{16\pi G}\int d^4x \sqrt{-g}\, R + S_m[g, \psi_m]}$$

Einstein-Hilbert action

matter action (all matter fields <u>universally coupled</u> to the metric $g_{\mu\nu}$)



(with $\delta g_{\mu\nu} = 0$ when $|x^\lambda| \to \infty$)

10 differential equations of second order

$$\underbrace{G^{\mu\nu}[g, \partial g, \partial^2 g]}_{\text{Einstein tensor}} = \frac{8\pi G}{c^4}\underbrace{T^{\mu\nu}}_{\text{stress-energy tensor}}[g]$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \qquad T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g_{\mu\nu}}$$

4 eqs. give the evolution of matter fields

$$\boxed{\nabla_\nu G^{\mu\nu} \equiv 0 \quad \Rightarrow \quad \nabla_\nu T^{\mu\nu} = 0}$$

contracted Bianchi identity (or Einstein identity)

Geometry is governed by 6 eqs., 4 eqs. can be imposed by a choice of coordinates

$$h^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

auxiliary Minkowski metric (signature $-+++$)

<u>Choice of coordinates</u> $\boxed{\partial_\nu h^{\mu\nu} = 0}$

Harmonic or de Donder

$$\boxed{\Box h^{\mu\nu} = \frac{16\pi G}{c^4} \tau^{\mu\nu}}$$

ordinary flat d'Alembertian $\Box = \eta^{\rho\sigma}\partial_\rho\partial_\sigma$

stress-energy <u>pseudo tensor</u> (actually a Lorentz tensor) of matter <u>and</u> gravitational fields (in harm. coordinates)

$$\tau^{\mu\nu} = |g|\, T^{\mu\nu} + \underbrace{\frac{c^4}{16\pi G} \Lambda^{\mu\nu}(h, \partial h, \partial^2 h)}_{\text{includes all non-linearities of Einstein's eqs.}}$$

$\Lambda^{\mu\nu} = \mathcal{O}(h^2)$

Harmonic coordinate condition is equivalent to matter equation

$$\boxed{\partial_\nu h^{\mu\nu} = 0 \iff \partial_\nu \tau^{\mu\nu} = 0 \iff \nabla_\nu T^{\mu\nu} = 0}$$

<u>NO-INCOMING RADIATION CONDITION</u>

Boundary conditions are imposed at <u>past null infinity</u> (case where $T^{\mu\nu}$ has a spatially compact support)

<u>Spatio-temporal infinities</u>

I^+ = future temporal infinity $\begin{pmatrix} t \to +\infty \\ r = \text{const} \end{pmatrix}$

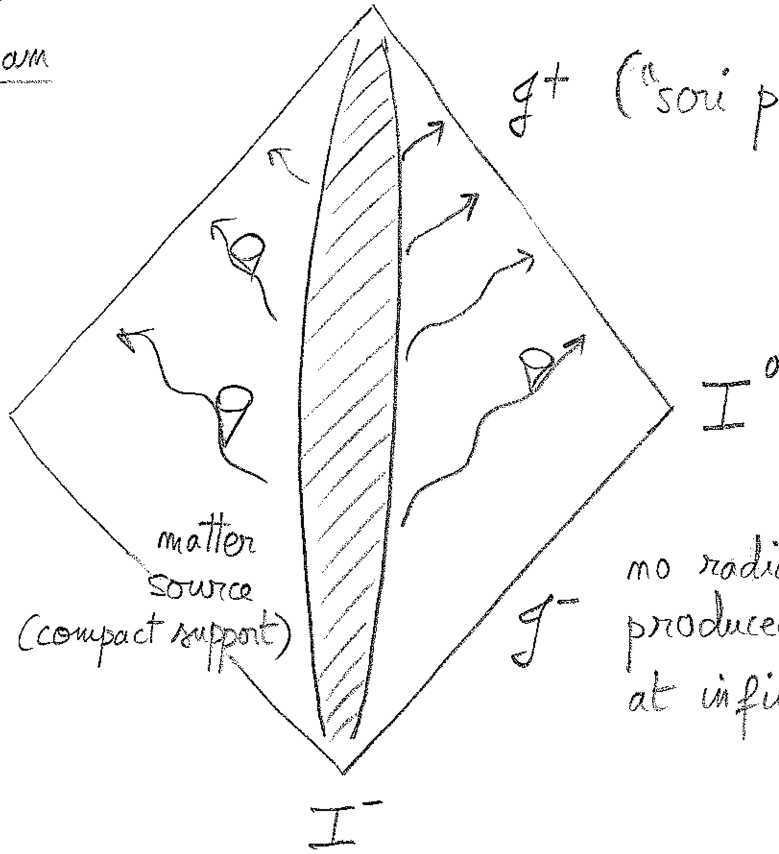
$\mathcal{J}^+$ = future null infinity $\begin{pmatrix} r \to +\infty \\ t - r/c = \text{const} \end{pmatrix}$

I^0 = spatial infinity $\begin{pmatrix} r \to +\infty \\ t = \text{const} \end{pmatrix}$

$\mathcal{J}^-$ = past null infinity $\begin{pmatrix} r \to +\infty \\ t + r/c = \text{const} \end{pmatrix}$

I^- = past temporal infinity $\begin{pmatrix} t \to -\infty \\ r = \text{const} \end{pmatrix}$

Carter- Penrose diagram

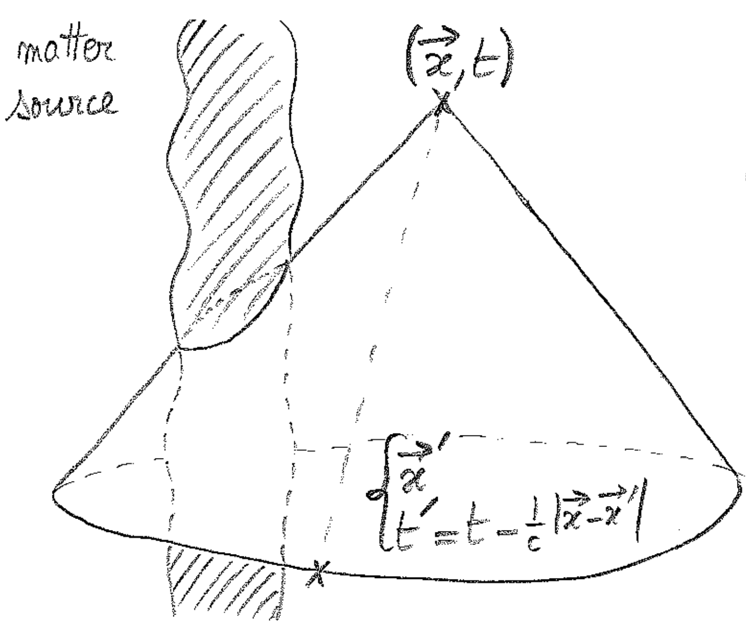


no radiation is produced by sources located at infinity

Kirchhoff's formula for the homogeneous sol. of

$$\Box\, h_{Hom} = 0$$

$$\boxed{h_{Hom}(\vec{x},t) = \lim_{|\vec{x}'|\to\infty} \iint \frac{d\Omega'}{4\pi}\left(\frac{\partial}{\partial r} + \frac{1}{c}\frac{\partial}{\partial t}\right)(r\, h_{Hom})\left(\vec{x}', t - \frac{|\vec{x}-\vec{x}'|}{c}\right)}$$



$(\vec{x},t)$ = field point

$(\vec{x}',t')$ = source point

No-incoming rad. cond. is

$$\boxed{\lim_{\mathscr{I}^-} \left(\frac{\partial}{\partial r} + \frac{1}{c}\frac{\partial}{\partial t}\right)(r\, h^{\mu\nu}) = 0}$$

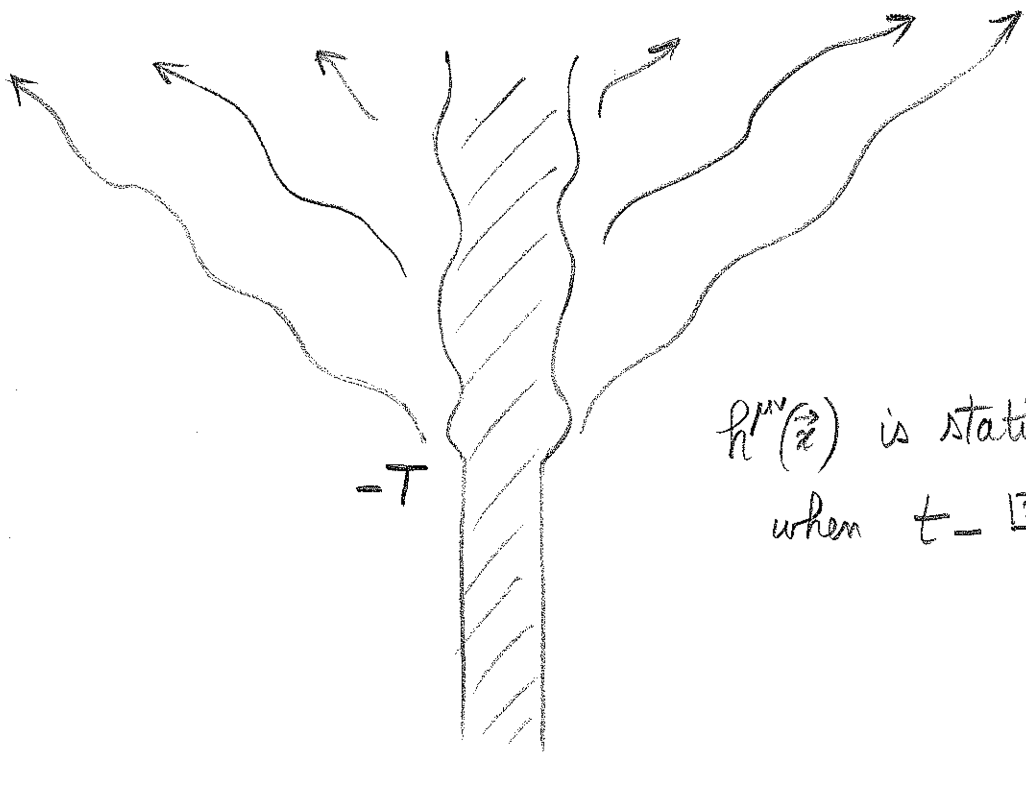
This excludes advanced waves $r\, h_{adv} \sim f(t + r/c)$ at $\mathscr{I}^-$

Einstein field eqs. can be solved (in an iterative way) by means of standard retarded integral in 3+1 dimensions

$$\boxed{h^{\mu\nu}(\vec{x}, t) = -\frac{4G}{c^4} \iiint \frac{d^3\vec{x}'}{|\vec{x} - \vec{x}'|} \, \tau^{\mu\nu}\left(\vec{x}', t - \frac{1}{c}|\vec{x} - \vec{x}'|\right)}$$

note this is infact an integro-differential equation because $\tau^{\mu\nu}$ depends on $h, \partial h, \partial^2 h$

<u>Stationarity in the past</u> (simple way to implement the no-incoming rad. condition)



$h^{\mu\nu}(\vec{x})$ is stationary (ind. of t) when $t - \frac{|\vec{x}|}{c} \lesssim -T$

LINEARIZED GRAVITATIONAL WAVES IN VACUUM

$$\begin{cases} \Box h^{\mu\nu} = 0 \\ \partial_\nu h^{\mu\nu} = 0 \end{cases} \qquad \left(\text{we neglect } \mathcal{O}(h^2)\right)$$

Gauge transformation preserving the harmonic cond. $\partial h = 0$

$$h'^{\mu\nu} = h^{\mu\nu} + \partial^\mu \xi^\nu + \partial^\nu \xi^\mu - \eta^{\mu\nu} \partial_\rho \xi^\rho$$

where $\Box \xi^\mu = 0$

Fourier decomposition $\boxed{h^{\mu\nu}(x) = \int d^4x\, H^{\mu\nu}(k)\, e^{i k_\lambda x^\lambda}}$

Fourier amplitude of monochromatic wave $k_\lambda = \begin{pmatrix}\text{wave}\\ \text{vector}\end{pmatrix}$

$$\boxed{\begin{aligned} &k^2 \equiv \eta_{\mu\nu} k^\mu k^\nu = 0 \\ &k_\nu H^{\mu\nu} = 0 \end{aligned}}$$

Can perform a gauge transf. with any $\xi^\mu(x) = \int d^4x\, \mathcal{E}^\mu(k) e^{i k\cdot x}$

TT coordinates u^μ four-vector constant (independent of x) and not orthogonal to k_μ (i.e. $u_\mu k^\mu \neq 0$) for instance

u^μ = four velocity of an observer (time-like)

There exists a gauge such that (at once)

$$\boxed{\begin{aligned} u_\nu H^{\mu\nu} &= 0 \\ H \equiv \eta_{\mu\nu} H^{\mu\nu} &= 0 \end{aligned}}$$

$\longleftarrow$ transverse (T) condition

$\longleftarrow$ traceless (T) condition

Proof: perform a gauge transf. in Fourier domain

$$H^{\mu\nu} = H_0^{\mu\nu} + i k^\mu \varepsilon^\nu + i k^\nu \varepsilon^\mu - i \eta^{\mu\nu} k_\rho \varepsilon^\rho$$

Then TT conditions are satisfied with gauge vector

$$\varepsilon^\mu = \frac{i}{(uk)} \left[u_\nu \bar{H}_0^{\mu\nu} - \frac{k^\mu}{2(uk)} u_\rho u_\sigma \bar{H}_0^{\rho\sigma} \right]$$

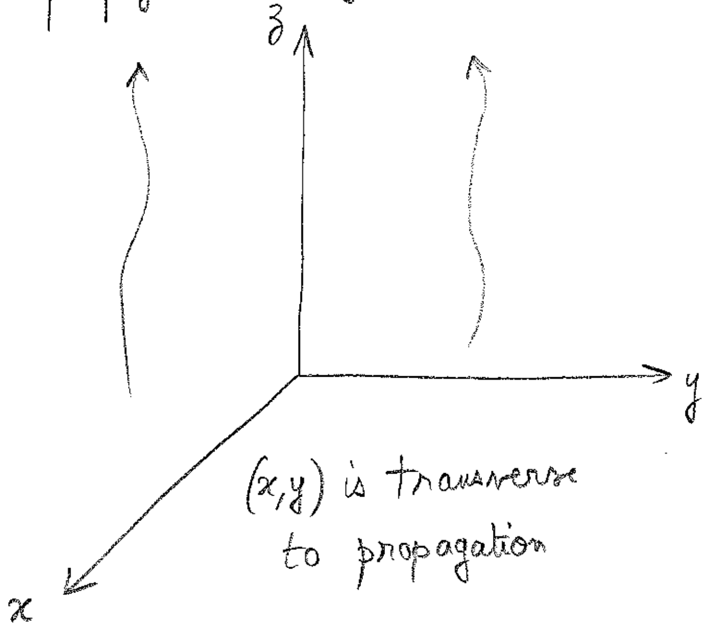
where $\bar{H}_0^{\mu\nu} = H_0^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} H_0$

$10 - 4 - (4-1) - 1 = 2$ independent components of $H^{\mu\nu}$

2 polarization states

$u^\mu = (1, \vec{0})$ in rest frame of observer

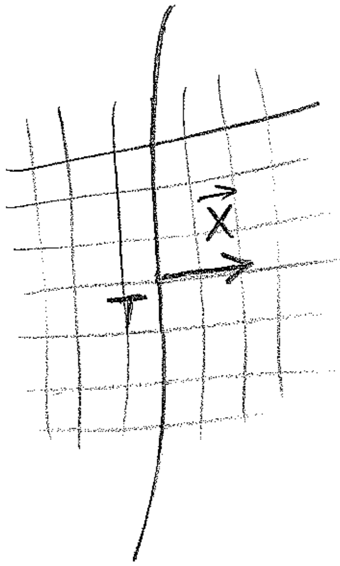
propagation in z-direction



$$h_{\mu\nu}^{TT} = \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t-z/c) & h_\times(t-z/c) & 0 \\ 0 & h_\times(t-z/c) & -h_+(t-z/c) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

ACTION OF GRAVITATIONAL WAVES ON MATTER

central geodesics ($X^i = 0$)



Fermi coordinates ($X^i; T$) in the neighborhood of central geodesics

$T =$ proper time along central geodesics

$$\boxed{g_{\mu\nu}(\vec{X}, T) = \eta_{\mu\nu} + \underbrace{F_{\mu\nu ij}(T)}_{\text{function of time } T} X^i X^j + \mathcal{O}(|\vec{X}|^3)}$$

Geodesic equ. in vicinity of central geodesic ($|\vec{X}| \ll \lambda^{GW}$)

$$\frac{d^2 X^i}{dT^2} = -c^2 \frac{\partial \Gamma^i_{00}}{\partial X^j}(T, \vec{0}) X^j = -c^2 R^i_{\cdot 0j0}(T, \vec{0}) X^j$$

(to first order in X^i)

Riemann in Fermi coord.

($-c^2 R_{i0j0}$ is a relativistic version of the tidal tensor $\partial_i \partial_j U$)

$$R^i_{\cdot 0j0} = \frac{\partial X^i}{\partial x^\lambda} \frac{\partial x^\mu}{\partial X^0} \cdots \overset{TT}{R}{}^\lambda_{\cdot \mu\nu\rho} \approx \overset{TT}{R}{}^i_{\cdot 0j0} \approx -\frac{1}{2c^2} \frac{\partial^2 h^{TT}_{ij}}{\partial t^2}$$

Riemann in TT coordinates

$$\boxed{\frac{d^2 X^i}{dT^2} = \frac{1}{2} \frac{\partial^2 h^{TT}_{ij}}{\partial t^2}(T, \vec{0}) X^j}$$

acceleration in Fermi coord.

wave form in TT coord. evaluated on central geodesic

$$\boxed{X^i(T) = X^i(0) + \frac{1}{2} h^{TT}_{ij}(T, \vec{0}) X^j(0)}$$

(to first order in h)

position before passage of GW

QUADRUPOLE MOMENT FORMALISM

Matter source is

- <u>isolated</u> ($T^{\mu\nu}$ has a compact support)
- <u>post-Newtonian</u> $\boxed{\varepsilon \approx \frac{v}{c} \ll 1}$
- <u>self-gravitating</u>: internal motion is due to gravitational forces

$$\gamma \sim \frac{v^2}{a} \sim \frac{GM}{a^2}$$

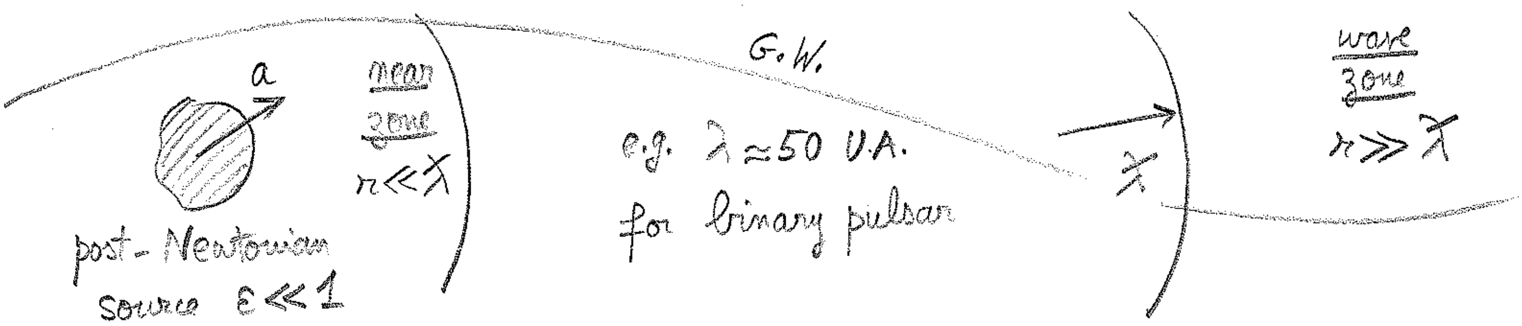
a = size of source
M = its mass

Period of motion $P \sim \frac{2\pi a}{v}$

Gravitational wave length $\lambda = cP$

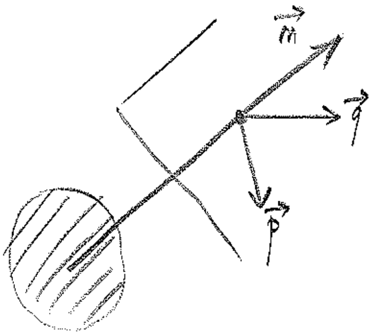
$$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$$

$$\boxed{\frac{R}{\lambda\!\!\!^{-}} \sim \frac{v}{c} \approx \varepsilon}$$



The near zone ($r \ll \lambda\!\!\!^{-}$) covers entirely the post-Newtonian source

$$Q_{ij}(t) = \int_{\text{source}} d^3x \, \rho(\vec{x},t)\left(x_i x_j - \frac{1}{3}\delta_{ij}\vec{x}^2\right)$$



$$\boxed{h_{ij}^{TT} = \frac{2G}{c^4 r} P_{ijkl}(\vec{n}) \left\{ \ddot{Q}_{kl}\left(t - \frac{r}{c}\right) + \mathcal{O}(\varepsilon) \right\} + \mathcal{O}\left(\frac{1}{r^2}\right)}$$

TT projection operator $P_{ijkl} = P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl}$ where $P_{ij} = \delta_{ij} - n_i n_j$

Polarization states w.r.t. $\vec{p}, \vec{q}$

$$h_+ = \frac{p_i p_j - q_i q_j}{2} h_{ij}^{TT}$$

$$h_\times = \frac{p_i q_j + p_j q_i}{2} h_{ij}^{TT}$$

$\vec{p}, \vec{q}$ polarization vectors

$$\boxed{\mathcal{F}^{GW} \equiv \left(\frac{dE}{dt}\right)^{GW} = \frac{G}{5c^5}\left\{\dddot{Q}_{ij}\,\dddot{Q}_{ij} + \mathcal{O}(\varepsilon^2)\right\}}$$

Einstein quadrupole formula

order of magnitude of radiation reaction
$\mathcal{O}(\varepsilon^5)$ called also 2.5PN

Typically $Q \sim Ma^2$ $\quad \dddot{Q} \sim M a^2 \omega^3$ $\quad \omega = \frac{2\pi}{P}$

Self-gravitating source $\omega^2 \sim \frac{GM}{a^3}$

$$\boxed{\mathcal{F}^{GW} \sim \left(\frac{c^5}{G}\right)\left(\frac{GM\omega}{c^3}\right)^{10/3}}$$

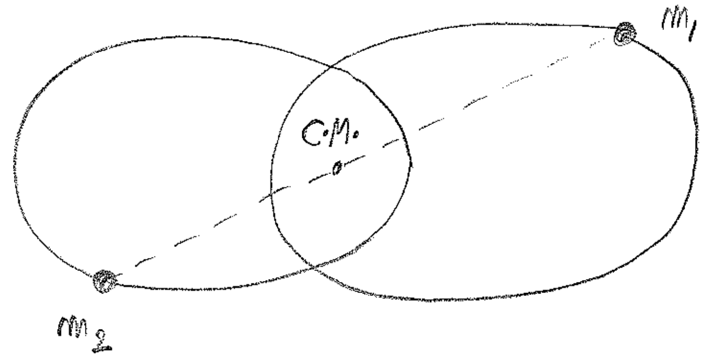
Ultra-relativistic source $v \sim c$ or $\frac{GM\omega}{c^3} \sim 1$

$$\underbrace{\mathcal{F}^{GW}\Big|_{\substack{\text{ultra}\\\text{relativistic}}} \sim \frac{c^5}{G} = 3.63\ 10^{52}\,W}_{\text{value independent of source}}$$

GW has typically the frequency $\omega \sim \frac{c^3}{GM}$

$M \sim 1\,M_\odot$	$\omega \sim 10^3\,Hz$	bandwidth of LIGO/VIRGO
$M \sim 10^6\,M_\odot$	$\omega \sim 10^{-3}\,Hz$	bandwidth of LISA

PETERS & MATHEWS FORMULA



Two compact objects (without spin) on a Keplerian ellipse

a = semi-major axis

e = eccentricity

$$M = m_1 + m_2$$

$$\mu = \frac{m_1 m_2}{M}$$

$$\boxed{\nu = \frac{\mu}{M} \text{ such that } 0 < \nu \leq \frac{1}{4}}$$

test-mass limit

equal masses

$$\boxed{\langle \mathcal{F}^{GW} \rangle \equiv \frac{1}{P} \int_0^P dt\, \mathcal{F}^{GW}(t) = \frac{32}{5} \frac{c^5}{G} \nu^2 \left(\frac{GM}{ac^2}\right)^5 \frac{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4}{(1-e^2)^{7/2}}}$$

eccentricity dependent "enhancement" factor $f(e)$

Energy balance argument

$$\frac{dE}{dt} = - \langle \mathcal{F}^{GW} \rangle \quad \text{with} \quad E = - \frac{GM^2 \nu}{2a}$$

$$GM = \omega^2 a^3$$

$$\boxed{\dot{P} = - \frac{192\pi}{5c^5} \left(\frac{2\pi GM}{P}\right)^{5/3} \nu f(e) = -2.4\ 10^{-12}\ s/s}$$

Binary pulsar PSR 1913+16

in agreement with observations (Taylor et al).

INSPIRALLING COMPACT BINARIES

Evolution of eccentricity $e(t)$

Orbit's energy and angular momentum

$$\boxed{\begin{aligned} \frac{E}{\nu} &= -\frac{GM^2}{2a} \\ \frac{J}{\nu} &= \sqrt{GM^3 a(1-e^2)} \end{aligned}}$$

$$\nu \equiv \frac{\mu}{M}$$

Apply quadrupole formulas for both E and J

$$\dot{E} = -\left\langle \frac{G}{5c^5} \dddot{Q}_{ij} \dddot{Q}_{ij} \right\rangle$$

$$\dot{J}^i = -\left\langle \frac{2G}{5c^5} \varepsilon_{ijk} \ddot{Q}_{jl} \dddot{Q}_{kl} \right\rangle$$

$$\boxed{\frac{e^2}{(1-e^2)^{19/6}} \left(1 + \frac{121}{304} e^2\right)^{\frac{145}{121}} = \left(\frac{\omega}{\omega_0}\right)^{-\frac{19}{9}}}$$

gives $e(t)$ as a function of $\omega(t)$ during the inspiral (ω_0 is determined from initial conditions) ($e^2 \sim P^{19/9}$ for small e)

For the binary pulsar $e_{now} = 0.617$

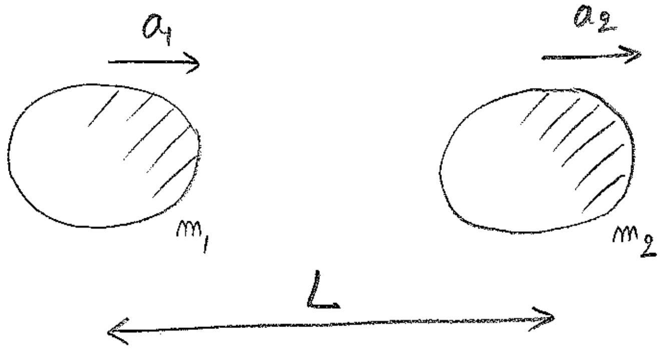
$\omega_{now} = 2.24 \; 10^{-4}$ Hz

hence GWs are visible by VIRGO/LIGO when

$$\boxed{\omega \sim 30 \text{ Hz} \quad \Rightarrow \quad e \sim 5 \; 10^{-6}}$$

eccentricity is negligible in general.

Finite size effects



Look for influence of quadrupole moments Q_1 and Q_2 induced by tidal interactions between non-spinning compact objects

$$\boxed{Q_1 = k_1 m_2 \frac{a_1^5}{L^3} \qquad Q_2 = k_2 m_1 \frac{a_2^5}{L^3}}$$

$k_{1,2}$ = Love numbers (depend on internal structure)

$Q_{1,2}$ scale like L^{-3} because of tidal field $\partial_{ij} U \sim \frac{1}{L^3}$

Introduce the compacity parameters

$$K_1 = \frac{2Gm_1}{a_1 c^2} \qquad K_2 = \frac{2Gm_2}{a_2 c^2}$$

The quadrupoles modify the energy and GW flux and the orbital frequency ω and phase $\phi = \int \omega dt$

$$\dot{E} = -\mathcal{F}^{GW} \quad \Rightarrow \quad \phi = -\int \frac{\omega dE}{\mathcal{F}^{GW}}$$

Effect of quadrupoles is

depends on internal structure

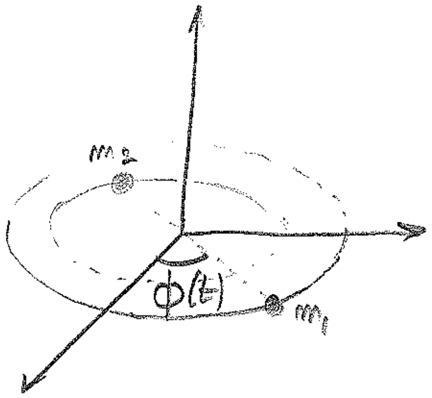
$$\boxed{\phi^{\text{finite-size}} = \underbrace{\phi_0 - \frac{1}{8x^{5/2}}}_{\text{point-mass result}} \left\{ 1 + (\text{const}) \left(\frac{x}{K}\right)^5 \right\}}$$

$x \equiv \left(\frac{GM\omega}{c^3}\right)^{2/3}$ Since $K \sim 1$ for compact objects the formal order of magnitude of the finite-size effect is 5PN (namely $x^5 \sim \frac{1}{c^{10}}$)

Orbital phase evolution $\phi(t)$

(same as for binary pulsar, i.e. based on

$$\frac{dE}{dt} = -\mathcal{F}^{GW})$$



where $\quad \frac{E}{M} = -\frac{c^2}{2}\nu x$

$$\mathcal{F}^{GW} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$\boxed{x = \left(\frac{GM\omega}{c^3}\right)^{2/3}} = \text{PN parameter } \mathcal{O}(\varepsilon^2)$$

$$\dot{E} = -\mathcal{F}^{GW} \Rightarrow \dot{x} = \frac{64}{5}\frac{c^3}{G}\frac{\nu}{M}x^5 \Rightarrow x(t) = \left[\frac{256}{5}\frac{c^3}{G}\frac{\nu}{M}(t_c - t)\right]^{-1/4}$$

t_c = instant of coalescence

$$\phi(t) = \int \omega\, dt = \frac{5}{64\nu}\int x^{-7/2}dx \Rightarrow \boxed{\phi(t) = \phi_c - \frac{x(t)^{-5/2}}{32\nu}}$$

Number of orbital cycles left till coalescence from time t

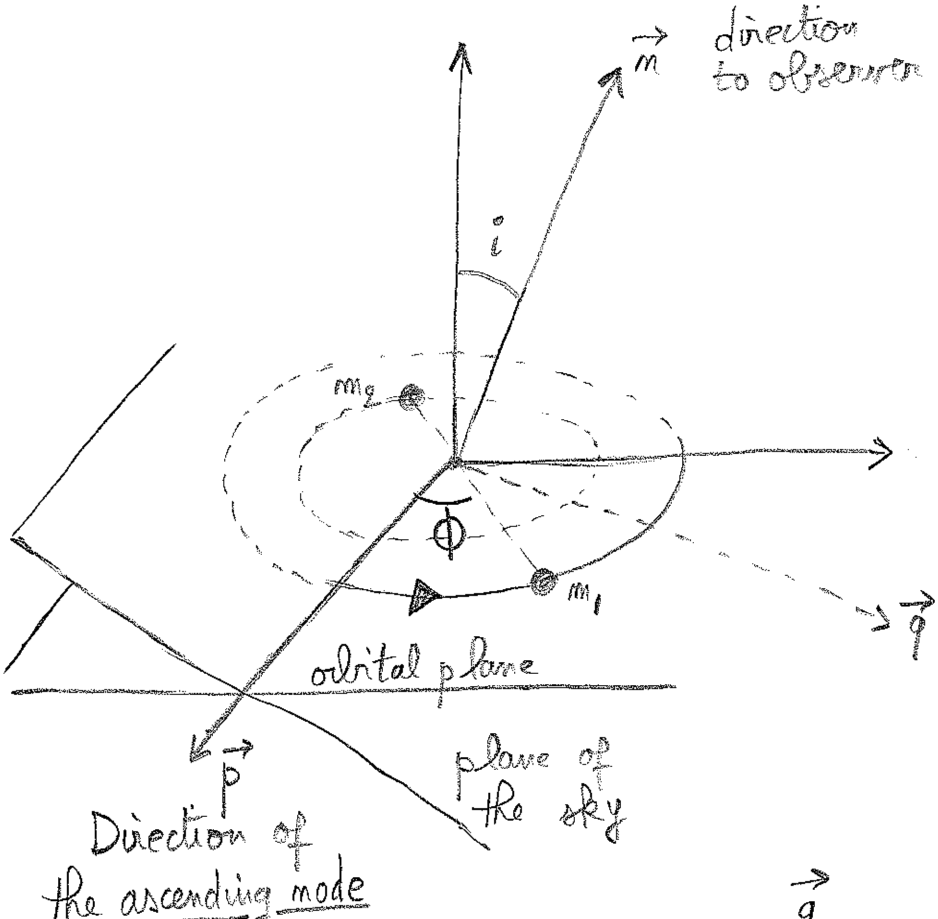
$$\mathcal{N} = \frac{\phi_0 - \phi(t)}{\pi} = \underbrace{\frac{1}{32\pi\nu}\left(\frac{GM\omega}{c^3}\right)^{-5/3} = \mathcal{O}(\varepsilon^{-5})}_{\text{inverse of order of radiation reaction } \varepsilon^{-5} \sim \left(\frac{c}{v}\right)^5}$$

But $\mathcal{N}$ should be monitored in LIGO/VIRGO with precision

$$\delta\mathcal{N} \sim 1$$

so it is evident that PN corrections in the phase will play a crucial role up to at least the 2.5PN order. Detailed analysis show that <u>good templates for inspiralling compact binaries should have 3PN accuracy.</u> Current theoretical prediction is 3.5PN.

Wave form of inspiralling compact binaries (ICBs)

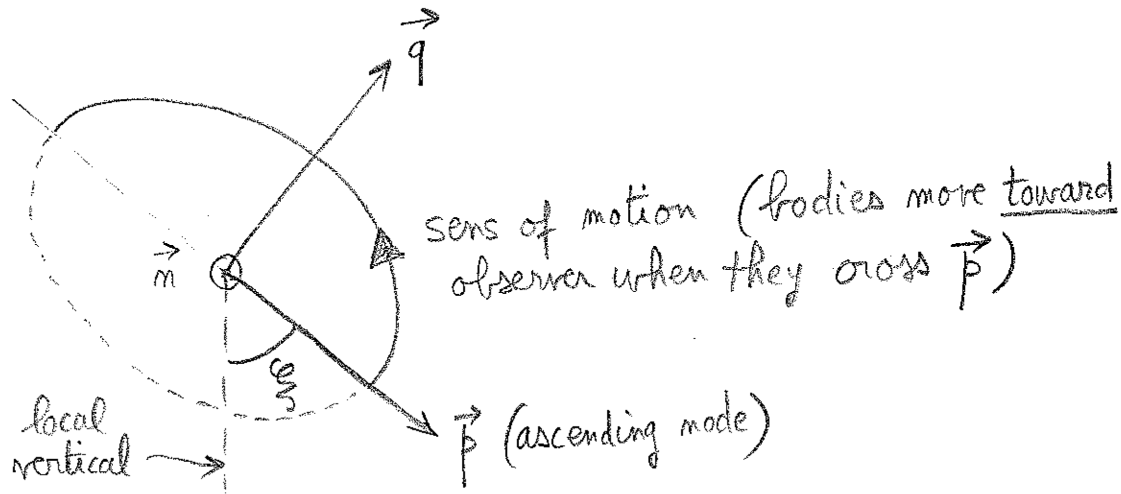


$\vec{p}, \vec{q}$ = polarization vectors (in the plane of sky)

i = inclination angle

$\phi(t)$ = orbital phase

As seen from observer:



ξ = polarization angle (between $\vec{p}$ and local vertical of observer)

Response of detector $\boxed{h \equiv \frac{2\delta L}{L} = F_+ h_+ + F_\times h_\times}$

$F_{+,\times}$ = detector's pattern functions

depend on $-\vec{n}$ (direction of source) and ξ

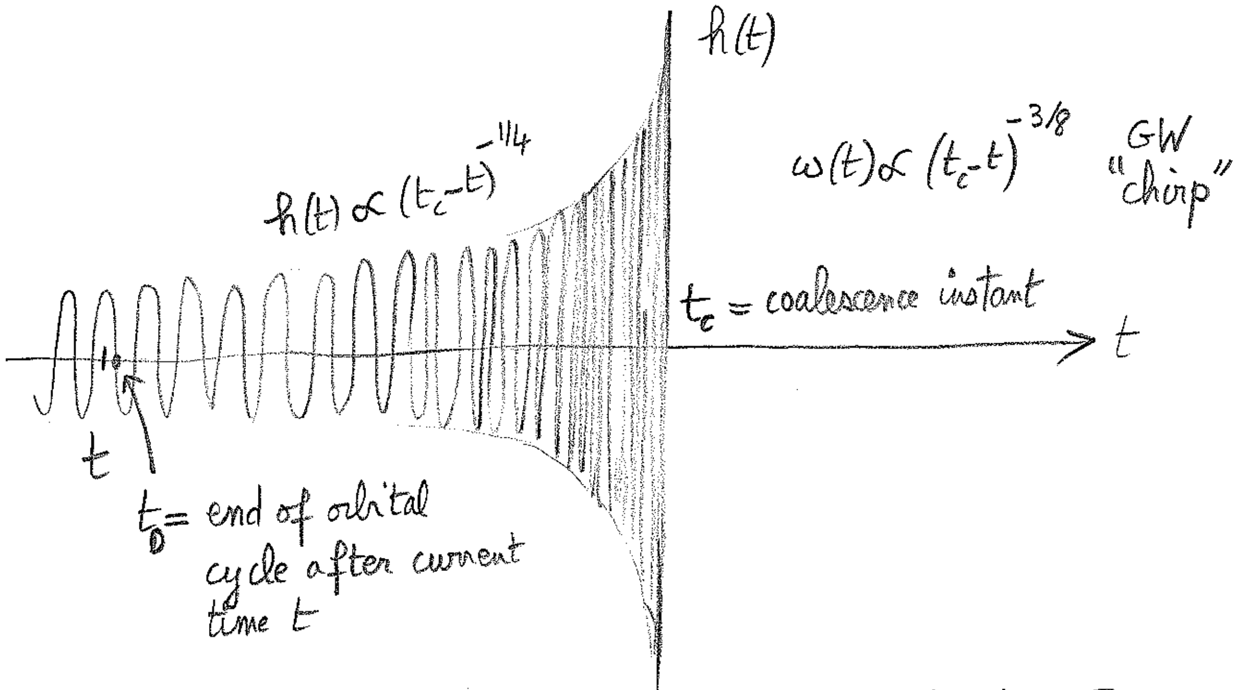
In quadrupole approximation

$$\boxed{\begin{aligned} h_+ &= \frac{2G\mu}{c^2 D}\left(\frac{GM\omega}{c^3}\right)^{2/3}(1+\cos^2 i)\cos(2\phi) \\ h_\times &= \text{———}\quad (2\cos i)\sin(2\phi) \end{aligned}}$$

D = distance of source = luminosity distance in cosmology

where

$$\boxed{\begin{aligned} \phi(t) &= \phi_c - \frac{1}{\nu}\left(\frac{\nu c^3}{5GM}(t_c - t)\right)^{5/8} \\ \omega(t) &= \frac{c^3}{8GM}\left(\frac{\nu c^3}{5GM}(t_c - t)\right)^{-3/8} \end{aligned}}$$



Suppose current time t is such that $t_c - t \gg P$ (non-relativistic limit, two bodies are well-separated)

$$t_c - t = (t_c - t_0)\left[1 + \frac{t_0 - t}{t_c - t_0}\right] \quad \text{with} \quad \frac{t_0 - t}{t_c - t_0} \ll 1$$

$$\phi(t) \approx \phi_c - \frac{1}{\nu}\left(\frac{\nu c^3}{5GM}(t_c - t_0)\right)^{5/8}\left[1 + \frac{5}{8}\frac{t_0 - t}{t_c - t_0} + \cdots\right]$$

$$\approx \phi_0 + \frac{5}{8\nu}\left(\frac{\nu c^3}{5GM}\right)^{5/8}(t_c - t_0)^{-3/8}\, t + \cdots$$

thus $\boxed{\phi(t) \approx \phi_0 + \omega_0 t + \cdots}$ constant orbital motion in the non relativistic limit

Orders of magnitude

$$h \sim \frac{GM\nu}{c^2 D}\left(\frac{GM\omega}{c^3}\right)^{2/3}$$

Number of cycles around frequency ω

$$n = \frac{\omega^2}{\dot{\omega}} \sim \frac{1}{\nu}\left(\frac{GM\omega}{c^3}\right)^{-5/3} = \mathcal{O}(\varepsilon^{-5})$$ inverse of rad. reaction order

Effective amplitude after matched filtering

$$\boxed{h_{eff} = h\sqrt{n} \sim \frac{GM\sqrt{\nu}}{c^2 D}\left(\frac{GM\omega}{c^3}\right)^{-1/6}}$$

Example: coalescence of two supermassive BHs in LISA

Characteristic frequency $\omega_c \sim \omega_{I.C.O.}$

innermost circular orbit (defined by the minimum of the energy function)

$$\frac{GM\omega_c}{c^3} \sim 0.1 \quad \Rightarrow \quad f_c \sim 10^4 \, Hz \left(\frac{M_\odot}{M}\right)$$

(from 3PN theory) For LISA $f_c \in [10^{-4}\,Hz, 10^{-1}\,Hz]$

Hence LISA should observe

$$\boxed{10^5 M_\odot \lesssim M \lesssim 10^8 M_\odot}$$

$$\boxed{h_{eff} \sim 10^{-14} \left(\frac{1\,Gpc}{D}\right)\left(\frac{\nu}{0.25}\right)^{1/2}\left(\frac{M}{10^7 M_\odot}\right)^{-5/6}\left(\frac{f}{10^{-4} Hz}\right)^{-1/6}}$$

Separation of BHs ($M \sim 10^7 M_\odot$) at entry frequency of LISA

$$r = \left(\frac{GM}{\omega^2}\right)^{1/3} \sim 1 \text{ A.U.}$$

Time left till coalescence

$$T = \frac{5GM}{\nu c^3}\left(\frac{8GM\omega}{c^3}\right)^{-8/3} \sim 10 \text{ days}$$

The signal-to-noise of the supermassive BH coalescence in LISA is enormous

$$\boxed{\frac{S}{N} = \left(\int_{-\infty}^{+\infty} d\omega \frac{|\tilde{h}(\omega)|^2}{S_n(\omega)}\right)^{1/2} \sim \frac{h_{eff}}{\sqrt{\omega S_n(\omega)}} \sim 10^4}$$

$S_n(\omega) \sim 10^{-34}\,Hz^{-1}$ for LISA