DARK ENERGY IN THE UNIVERSE

Modern paradigm of the Universe evolution

$$\cdots\to DS \to FRW\ RD \to FRW\ MD \xrightarrow{t_0} \widetilde{DS} \cdots\to$$

$a(t)$ $\quad e^{\int H dt}$ ($|\dot H| \ll H^2$) $\quad t^{1/2} \quad t^{2/3} \quad e^{\int H dt}$

$H \equiv \frac{\dot a}{a}$

Existense of dark energy – kinematical statement assuming the "Einsteinian interpretation"

$$R_i^k - \frac{1}{2}\delta_i^k R = -8\pi G\left(T_{i\,(m)}^k + \tilde T_{i\,(DE)}^k\right) \quad 4D$$

$T^k_{i\,(m)}$ – matter seen through its active gravitationa**l** mass (effect on motion of stars, galaxies and light)

$T^k_{i(DE);k} = 0$

$$\frac{\Delta\Phi}{a^2} = 4\pi G\,\delta\rho_m \Leftarrow$$

Remarkably $\tilde T^k_{i\,(DE)} \approx \varepsilon_{DE}\,\delta_i^k$

FRW symmetry: $\varepsilon_{DE}(z)$

$p_{DE}(z)$

$$\frac{G^2\hbar\,\varepsilon_{DE}}{c^7} = 1.25\cdot 10^{-123}\cdot\frac{\Omega_{DE}}{0.7}\left(\frac{H_0}{70}\right)^2$$

$$\rho_{DE} = \varepsilon_{DE}\,c^{-2} = 6.44\cdot 10^{-30}\ \mathrm{g\,cm^{-3}}\ (\ldots\ldots)$$

SCM : $\Lambda CDM + (\mathcal{K}=0) + \left(\begin{smallmatrix} n_s = 1 \\ \text{adiabatic} \end{smallmatrix}\right)$

$\sim 10\%$ accuracy

4 main fundamental constants and related theories:

1. $$\frac{\hbar^3 \varepsilon_\Lambda}{c^5 M_{Pl}^4} = \frac{G^2 \hbar\, \varepsilon_\Lambda}{c^7} = 1.25 \cdot 10^{-123} \cdot \frac{\Omega_\Lambda}{0.7} \cdot \left(\frac{H_0}{70}\right)^2$$

Theory of a cosmological constant (dark energy)

2. $$\frac{\varepsilon_b}{\varepsilon_m} = 0.150 \cdot \frac{\Omega_b h^2}{0.022} \cdot \left(\frac{70}{H_0}\right)^2 \cdot \frac{0.3}{\Omega_m} \qquad h = \frac{H_0}{100}$$

$$\varepsilon_m = \varepsilon_b + \varepsilon_{CDM}$$

Theory of dark non-baryonic matter

3. $$\frac{n_b}{n_\gamma} = 5.98 \cdot 10^{-10} \cdot \frac{\Omega_b h^2}{0.022} \cdot \left(\frac{2.73}{T_\gamma}\right)^3$$

Theory of baryogenesis

4. $$\Phi_0 = 2.8 \cdot 10^{-5} \cdot \left(\frac{A_s}{2.2 \cdot 10^{-9}}\right)^{1/2}$$

$\langle \Phi_{in}^2 \rangle = \Phi_0^2 \int \frac{dk}{k}$ at the MD stage

Theory of initial conditions – inflation

Comments about "cosmic coincidences" for Λ

1. Why small? Not known why so small, but all known dimensionless densities are very small

$$\rho_{DE} \sim \rho_{water}^{4/3} \sim m_\nu^4$$

2. Why now? Not an independent problem. Reduces to the first problem (plus relations between other fundamental constants) once "now" is defined in an objective way.

It is natural to use (weak) anthropic principle to define "now". However, in practice, very remote arguments are used.

Example. Let us, following Dicke, define $t_0 \sim t_{\substack{\text{active life} \\ \text{main sequence} \\ \text{star}}} \sim t_{Pl} \cdot \left(\frac{M_{Pl}}{m_p}\right)^3$

Then the "second coincidence problem" is reduced to the first one because of the empirical relation

$$\rho_{DE} \sim \left(\frac{m_p}{M_{Pl}}\right)^6$$

One constant — one problem ("coincidence")

H_0 - gives the present moment of time

$$3H_0^2 = \frac{8\pi G}{c^2}\left(\varepsilon_m + \varepsilon_\Lambda\right)$$

τ - optical width since recombination (calculable, in principle)

Interrelations ("coincidences")

1. $\Phi_0 \sim \Omega_{0\gamma}$ or $\Phi_0^{3/4} \sim \frac{n_\gamma}{n_b} \cdot \frac{\varepsilon_b}{\varepsilon_m} \cdot \left(\frac{\varepsilon_\Lambda \hbar^3}{c^5 M_{pe}^4}\right)^{1/4} \cdot \frac{M_{pe}}{m_p}$

The moment when $\left(\frac{\delta\rho}{\rho}\right)_m \sim 1$ at the cluster scale coincides with the moment when $\varepsilon_{CDM} \sim \varepsilon_\Lambda$

Not a consequence of the anthropic principle.

2. Weak anthropic principle removes the "second coincidence" ($\varepsilon_m \sim \varepsilon_\Lambda$ "now")

Let "now" be defined according to WAP: $t_0 \sim t_{pe} \cdot \left(\frac{M_{pe}}{m_p}\right)^3$ (Dicke)

$$\varepsilon_m \sim \frac{1}{G t_0^2} \sim \varepsilon_\Lambda \implies \frac{\hbar^3 \varepsilon_\Lambda}{c^5 M_{pe}^4} \sim \left(\frac{m_p}{M_{pe}}\right)^6$$

Investigation of dark energy

I. From observations to theory

Reconstruction of
(1998)

1) $H(z)$, $\varepsilon_{DE}(z)$
2) $q(z)$, $p_{DE}(z)$, $w_{DE}(z)$
3) $r(z)$, $\frac{dw_{DE}}{dz}$

1. Inversion of classical cosmological tests
2. CMB (acoustic peaks spacing, ISW)
3. $\left(\frac{\delta\rho}{\rho}\right)_m(z)$, $\Phi(z)$ from gravitational lensing, correlation of $\frac{\delta\rho}{\rho}$ with IS.

II. From theory to observations

Models (many of them!)
(qualitatively - the same as for inflation)

1. Fundamental constant
2. Scalar field (with $m \sim 10^{-33}$ eV)
3. Geometrical dark energy
(e.g., dark energy in scalar-tensor gravity

CLASSICAL COSMOLOGICAL TESTS AND THEIR INVERSION (RECONSTRUCTION OF H(Z))

1. High-z supernovae test

$$\mathcal{D}_L(z) = a_0(\eta_0 - \eta)(1+z), \qquad \eta = \int_0^t \frac{dt}{a(t)}$$

$$H(z) = \frac{da}{a^2 d\eta} = -(a_0 \eta')^{-1} = \left[\left(\frac{\mathcal{D}_L(z)}{1+z}\right)'\right]^{-1}$$

2. Angular size test

$$\theta(z) = \frac{d}{a(\eta)(\eta_0 - \eta)} = \frac{d(1+z)}{a_0(\eta_0 - \eta)}$$

$$H(z) = -(a_0 \eta')^{-1} = \left[d\left(\frac{1+z}{\theta(z)}\right)'\right]^{-1}$$

3. Volume element test

$$\frac{dN}{dz\,d\Omega} \propto \frac{dV}{dz\,d\Omega} = a^3 r^2 \left|\frac{dr}{dz}\right| =$$

$$= a^3 (r_0 - r)^2 \left|\frac{dr}{dz}\right| = f_V(z)$$

$$f_V(z) = \frac{1}{(1+z)^3 H(z)} \left(\int_0^z \frac{dz}{H(z)} \right)^2$$

$$H^{-1}(z) = \frac{d}{dz} \left\{ \left(3 \int_0^z f_V(z') (1+z')^3 dz' \right)^{1/3} \right\}$$

4. Ages of old objects at high z

$$T(z) > t_i(z)$$

$$T(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

$$H(z) = -\left((1+z) \frac{dT(z)}{dz} \right)^{-1}$$

5. High-z clustering tests

For $\lambda \ll \lambda_{\eta,\psi} \sim R_h$:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\frac{C}{a^3}\delta = 0$$

May be more complicated for geometric dark energy

$$\frac{d}{dt} = aH\frac{d}{da} \qquad C = \Omega_m H_0^2 a_0^3$$

$$H^2(a) = \frac{3C}{\delta'^2 a^6}\int_0^a \delta\delta' a\, da$$

$$a = \frac{a_0}{1+z}$$

$$\frac{H^2(z)}{H_0^2} = 3\Omega_m \frac{(1+z)^2}{\left(\frac{d\delta}{dz}\right)^2}\int_z^\infty \delta \left|\frac{d\delta}{dz}\right| \frac{dz}{1+z} =$$

$$= \frac{(1+z)^2\delta'^2(0)}{\delta'^2(z)} - \frac{3\Omega_m(1+z)^2}{\delta'^2(z)}\int_0^z \delta\delta' \frac{dz}{1+z}$$

Determination of Ω_m and q_0 from $\delta(z)$:

$$\Omega_m = \frac{\delta'^2(0)}{3\left|\int_0^\infty \delta\delta' \frac{dz}{1+z}\right|}$$

The textbook expression

(Weinberg, Peebles, etc.)

$$\delta(z) \propto H(z) \int_z^{\infty} \frac{(1+z')\,dz'}{H^3(z')}$$

Theorem

It is valid if and only if

$$H^2(z) = C_1 + C_2(1+z)^2 + C_3(1+z)^3$$

$$\longleftrightarrow$$

The super-Hubble solution $(k \ll aH)$

$$\delta(z) \propto a\left(1 - \frac{H}{a}\int a\,dt\right)$$

Valid for subhorizon scales if

$$H^2(z) = C_1 + C_2(1+z)^3$$

$$\longleftrightarrow$$

From 2dF survey: $\left.\dfrac{d\ln\delta}{d\ln(1+z)}\right|_{z=0.15} = -0.51 \pm 0.11$

How to determine $\delta(z)$

a) Evolution of clustering with z

$$r_0(z)$$

b) Evolution of rich cluster abundance with z

$$n(\geq M)(z)$$

c) Weak gravitational leasing of galaxies and CMB

$$\Phi(z)$$

6. CMB tests

a) Spacing between acoustic peaks

$$R \equiv \sqrt{\Omega_{m0}}\, H_0 \int_0^{z_{dec}} \frac{dz}{H(z)} = 1.70 \pm 0.03$$

(Wang & Mukherjee, 2006)

Precise but degenerate test

b) Correlation between $\frac{\Delta T}{T}$ and LSS (due to the ISW effect)

7. Sakharov oscillations in $P_0(k)$

$$\sqrt{\Omega_{m0}} \left(\frac{H_0}{H(z_1)}\right)^{1/3} \left(\frac{1}{z_1}\int_0^{z_1} dz \cdot \frac{H_0}{H(z)}\right)^{2/3} = 0.469 \pm 0.017$$

$z_1 = 0.35$

(Eisenstein et al., 2005)

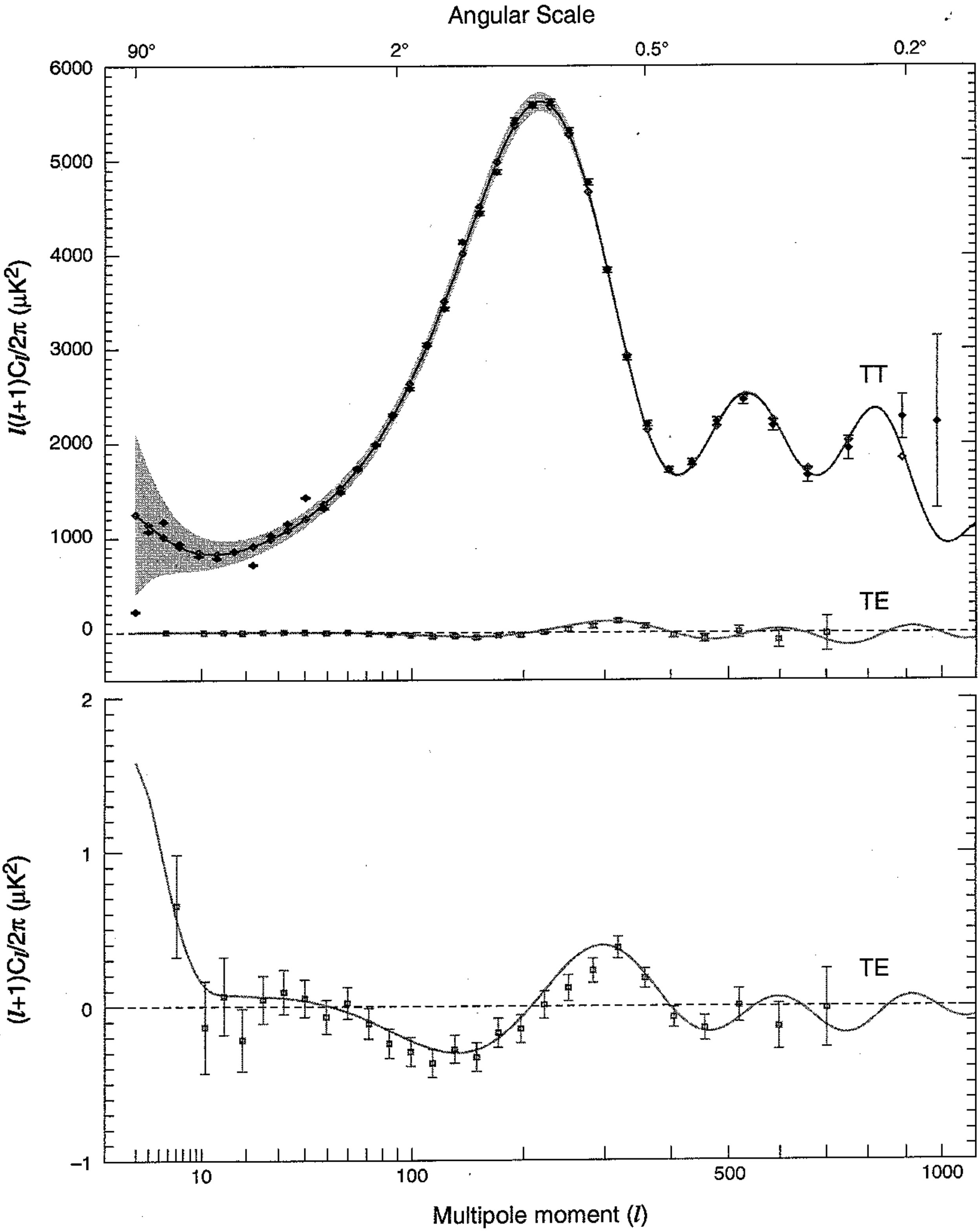


Fig. 22.— Angular power spectra C_l^{TT} & C_l^{TE} from the three-year *WMAP* data. *top*: The TT data are as shown in Figure 16. The TE data are shown in units of $l(l+1)C_l/2\pi$, on the same scale as the TT signal for comparison. *bottom*: The TE data, in units of $(l+1)C_l/2\pi$. This updates Figure 12 of Bennett et al. (2003b).

M. Tegmark et al., ApJ 606, 702 (2004)

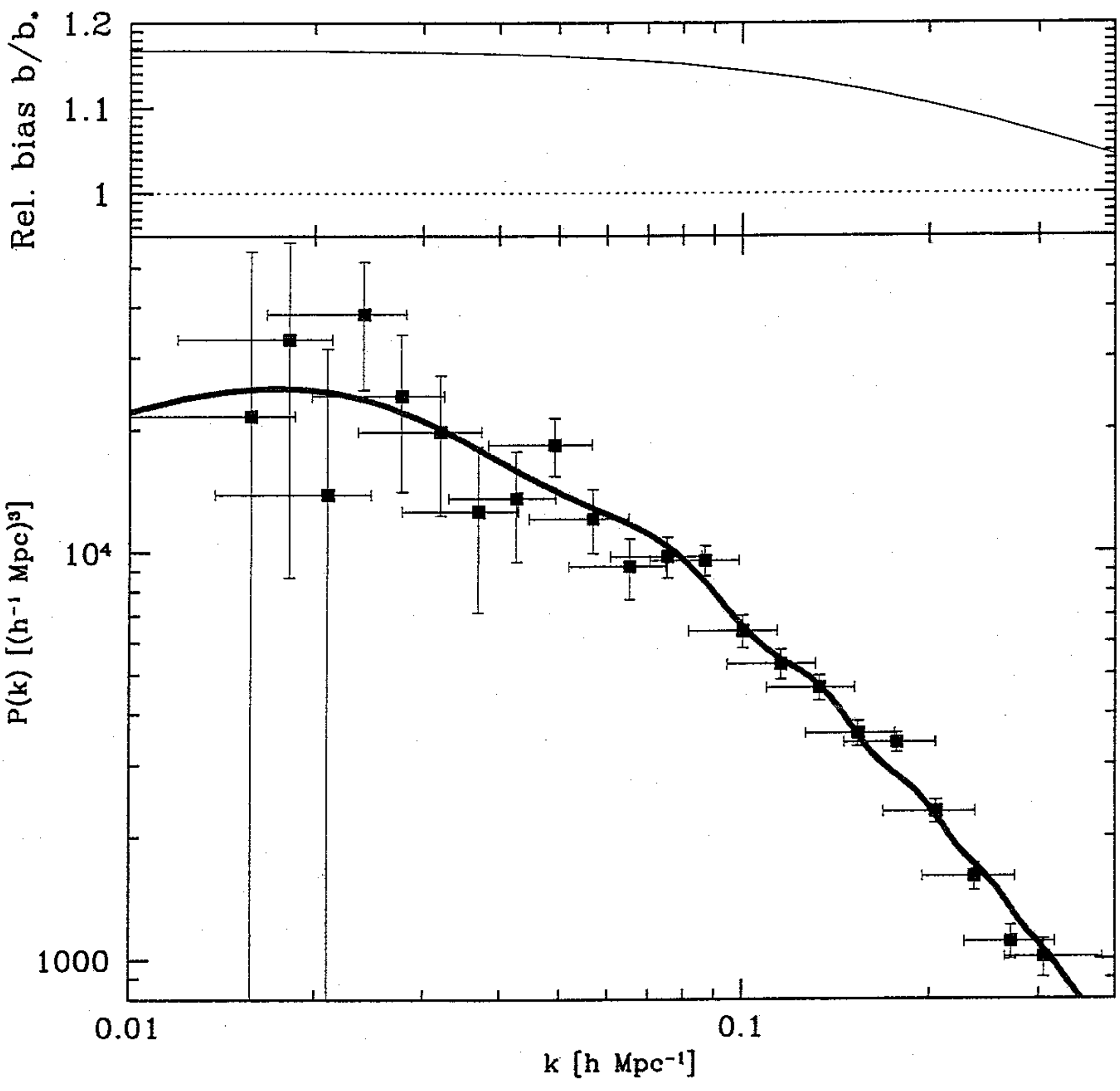


FIG. 22.— The decorrelated real-space galaxy-galaxy power spectrum using the modeling method is shown (bottom panel) for the baseline galaxy sample assuming $\beta = 0.5$ and $r = 1$. As discussed in the text, uncertainty in β and r contribute to an overall calibration uncertainty of order 4% which is not included in these error bars. To remove scale-dependent bias caused by luminosity-dependent clustering, the measurements have been divided by the square of the curve in the top panel, which shows the bias relative to L_* galaxies. This means that the points in the lower panel can be interpreted as the power spectrum of L_* galaxies. The solid curve (bottom) is the best fit linear ΛCDM model of Section 5.

D. E. Eisenstein et al., astro-ph/0501171

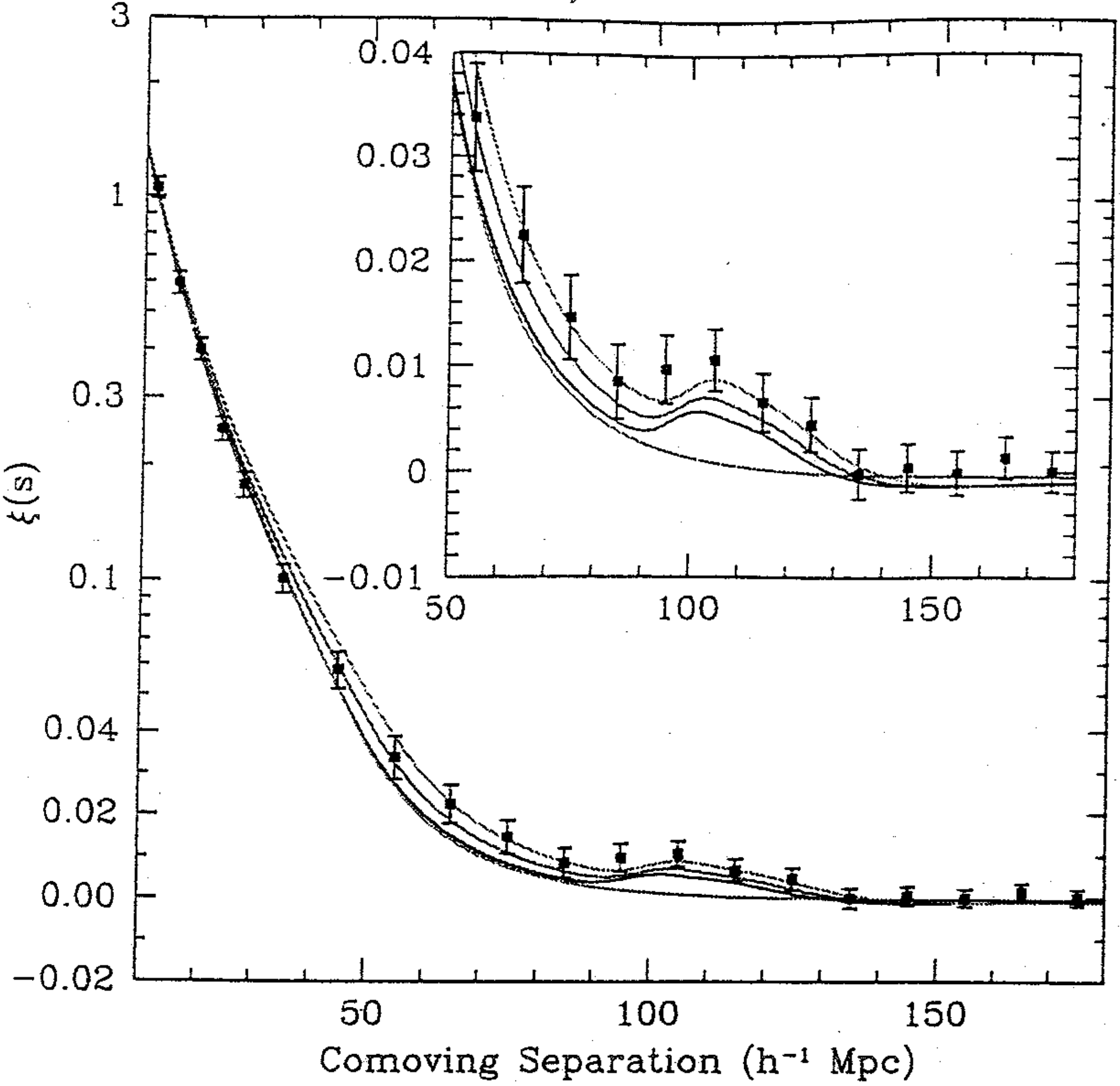


FIG. 2.— The large-scale redshift-space correlation function of the SDSS LRG sample. The error bars are from the diagonal elements of the mock-catalog covariance matrix; however, the points are correlated. Note that the vertical axis mixes logarithmic and linear scalings. The inset shows an expanded view with a linear vertical axis. The models are $\Omega_m h^2 = 0.12$ (top, green), 0.13 (red), and 0.14 (bottom with peak, blue), all with $\Omega_b h^2 = 0.024$ and $n = 0.98$ and with a mild non-linear prescription folded in. The magenta line shows a pure CDM model ($\Omega_m h^2 = 0.105$), which lacks the acoustic peak. It is interesting to note that although the data appears higher than the models, the covariance between the points is soft as regards overall shifts in $\xi(s)$. Subtracting 0.002 from $\xi(s)$ at all scales makes the plot look cosmetically perfect, but changes the best-fit χ^2 by only 1.3. The bump at $100h^{-1}$ Mpc scale, on the other hand, is statistically significant.

$$q_0 = -1 + \left.\frac{d\ln H}{d\ln(1+z)}\right|_{z=0}$$

$$\Lambda = \text{const} \longrightarrow H^2(z) = H_0^2\left(1-\Omega_m + \Omega_m(1+z)^3\right)$$

$$\longrightarrow \quad q_0 = \frac{3}{2}\Omega_m - 1$$

II. Reconstruction of $V(\varphi)$ from $H(a$

$$\begin{cases} 8\pi G V = aH\dfrac{dH}{da} + 3H^2 - \dfrac{3}{2}\Omega_m H_0^2 \cdot \left(\dfrac{a_0}{a}\right)^3 \\ 4\pi G\, a^2 H^2\left(\dfrac{d\varphi}{da}\right)^2 = -aH\dfrac{dH}{da} - \dfrac{3}{2}\Omega_m H_0^2\left(\dfrac{a_0}{a}\right)^3 \end{cases}$$

Necessary condition ($\varepsilon_{DE} + p_{DE} \geqslant 0$)

$$w_{DE} \equiv \frac{p_{DE}}{\varepsilon_{DE}} \geqslant -1$$

$$\frac{dH^2}{dz} \geqslant 3\,\Omega_m H_0^2 (1+z)^2$$

$$H^2 \geqslant H_0^2\left(1 + \Omega_m(1+z)^3 - \Omega_m\right)$$

In particular: $\quad q_0 \geqslant \frac{3}{2}\Omega_m - 1$

No such a condition in case of scalar-tensor gravity

Example of the reconstruction

Let $\Lambda \propto a^{-q}$ or $P_\Lambda = \left(-1+\frac{q}{3}\right)\varepsilon_\Lambda$

$q = \text{const} < 3$

$$H^2(z) = \Omega_m H_0^2 (1+z)^3 + (1-\Omega_m) H_0^2 (1+z)^q$$

$$w = \frac{q-3}{3} < 0$$

$$\frac{a}{a_0} = \left(\frac{\Omega_m}{1-\Omega_m}\right)^{\frac{1}{3-q}} \sinh^{\frac{2}{3-q}}\left((3-q)\sqrt{\frac{2\pi G}{q}}\,(\varphi-\varphi_0+\varphi_1)\right)$$

$$V(\varphi) = \frac{3-\frac{q}{2}}{8\pi G}\cdot\frac{(1-\Omega_m)^{\frac{3}{3-q}}}{\Omega_m^{\frac{q}{3-q}}}\,H_0^2\,\times$$

$$\times\,\frac{1}{\sinh^{\frac{2q}{3-q}}\left((3-q)\sqrt{\frac{2\pi G}{q}}\,(\varphi-\varphi_0+\varphi_1)\right)}$$

$$\varphi_1 = \varphi_1(\Omega_m, q) = \frac{1}{3-q}\sqrt{\frac{q}{2\pi G}}\,\ln\frac{1+\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}$$

Another interesting example:

$$\varepsilon_\Lambda = \varepsilon_\varphi + \varepsilon_1 \left(\frac{a_0}{a}\right)^3$$

$$H^2 = H_0^2 \left(1 - \Omega_m - \Omega_1 + (\Omega_m + \Omega_1)\left(\frac{a_0}{a}\right)^3\right)$$

$$\varepsilon_V = \frac{3H_0^2}{8\pi G}\left(1 - \Omega_m - \Omega_1\right)$$

$$\varepsilon_1 = \frac{3H_0^2}{8\pi G}\,\Omega_1$$

$$V(\varphi) = \frac{3H_0^2}{8\pi G}\left(1 - \Omega_m - \Omega_1 + A\sinh^2\left(B(\varphi - \varphi_0 + \varphi_1)\right)\right)$$

$$A = \frac{1}{2}\,\frac{\Omega_1(1 - \Omega_m - \Omega_1)}{\Omega_m + \Omega_1}\ ; \quad B = \sqrt{\frac{6\pi G(\Omega_m + \Omega_1)}{\Omega_1}}\ ;$$

$$\varphi_1 = \sqrt{\frac{\Omega_1}{(\Omega_m + \Omega_1)\,24\pi G}}\ \ln \frac{1 + \sqrt{\Omega_m + \Omega_1}}{1 - \sqrt{\Omega_m + \Omega_1}}$$

$t \to \infty$: $a \to \infty$, $\varphi \to \varphi_0 - \varphi_1$

De Sitter with $m_\Lambda = \frac{3}{2} H_\infty$

$$\boxed{\Omega_1 \lesssim 0.05\,\Omega_m}$$

$$
\begin{tikzcd}
\mathcal{D}_L(z) \arrow[dr] & & \\
& H(z) \arrow[r, "\Omega_m"] & V(\varphi) \\
\left(\frac{\delta\rho}{\rho}\right)_{CDM}(z) \arrow[ur, "H_0"'] & &
\end{tikzcd}
$$

Reconstruction of $\delta(z)$ from $\mathcal{D}(z)$

$G_{eff}(z) = \text{const}$ assumed

Test if DE is physical or geometrical

$$E(z) = H_0 \mathcal{D}_L(z)/(1+z) \Rightarrow z(E)$$

$$\delta(E(z)) = \delta(0) + \delta'(0) \int_0^E (1+z(E))\, dE$$

$$+ \frac{3}{2} \Omega_m \int_0^E (1+z(E_1))\, dE_1 \int_0^{E_1} \delta(E_2)\, dE_2$$

Given $\delta'(0)/\delta(0)$, $\delta(z)/\delta(0)$ can be found iteratively

No differentiation of data!

Basic quantities

Order	Geometrical	Physical
1	$H(z) \equiv \frac{\dot a}{a}$ $H(0) = H_0$	$\varepsilon_m = \frac{3H_0^2}{8\pi G} \cdot \Omega_{m0}(1+z)^3$ $\varepsilon_{DE} = \frac{3H^2}{8\pi G} - \varepsilon_m$
2	$q(z) \equiv -\frac{\ddot a a}{\dot a^2}$ $= -1 + \frac{d \ln H}{d \ln(1+z)}$ $q(0) = q_0$ For $\varepsilon_\Lambda = $ const: $q(z) = -1 + \frac{3}{2}\Omega_m(z)$	$V(z)$; $T(z) \equiv \frac{\dot\varphi^2}{2}$ $\Omega_V = \frac{8\pi G V}{3H^2}$; $\Omega_T = \frac{8\pi G T}{3H^2}$ $\Omega_V = \frac{2-q}{3} - \frac{H_0^2}{2H^2}\Omega_{m0}(1+z)^3$ $\Omega_T = \frac{1+q}{3} - \frac{H_0^2}{2H^2}\Omega_{m0}(1+z)^3$
3	$r(z) \equiv \frac{\dddot a a^2}{\dot a^3}$ $r(0) = r_0$ For $\varepsilon_\Lambda \equiv$ const: $r \equiv 1$	$\Pi(z) \equiv \dot\varphi V'$ $\Omega_\Pi = \frac{8\pi G \dot\varphi V'}{3H^3}$ $\Omega_\Pi = \frac{1}{3}\left(2 - 3q - r + \frac{9H_0^2}{2H^2}\Omega_{m0}(1+z)^3\right)$

Derivative quantity:

$$w = \frac{T-V}{T+V} = \frac{2q-1}{3\left(1 - \frac{H_0^2}{H^2}\Omega_{m0}(1+z)^3\right)}$$

Practical reconstruction of $H(z)$, $w(z)$, etc. from $D_L(z)$

Explicit or implicit smoothing over some interval Δz is required!

1. Top-hat smoothing
2. Gaussian smoothing MNRAS 366, 1081 (2006)
   (A. Shafieloo et al., astro-ph/0505329)
3. The principal components method
4. Parametric fits (implicit smoothing!)

a) $$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m(1+z)^3$$

$$A_0 + A_1 + A_2 + \Omega_m = 1$$

This fit does not exclude a possibility $\varepsilon_{DE} < 0$!

U. Alam et al. MNRAS 354, 275 (2004) [astroph/0311364]

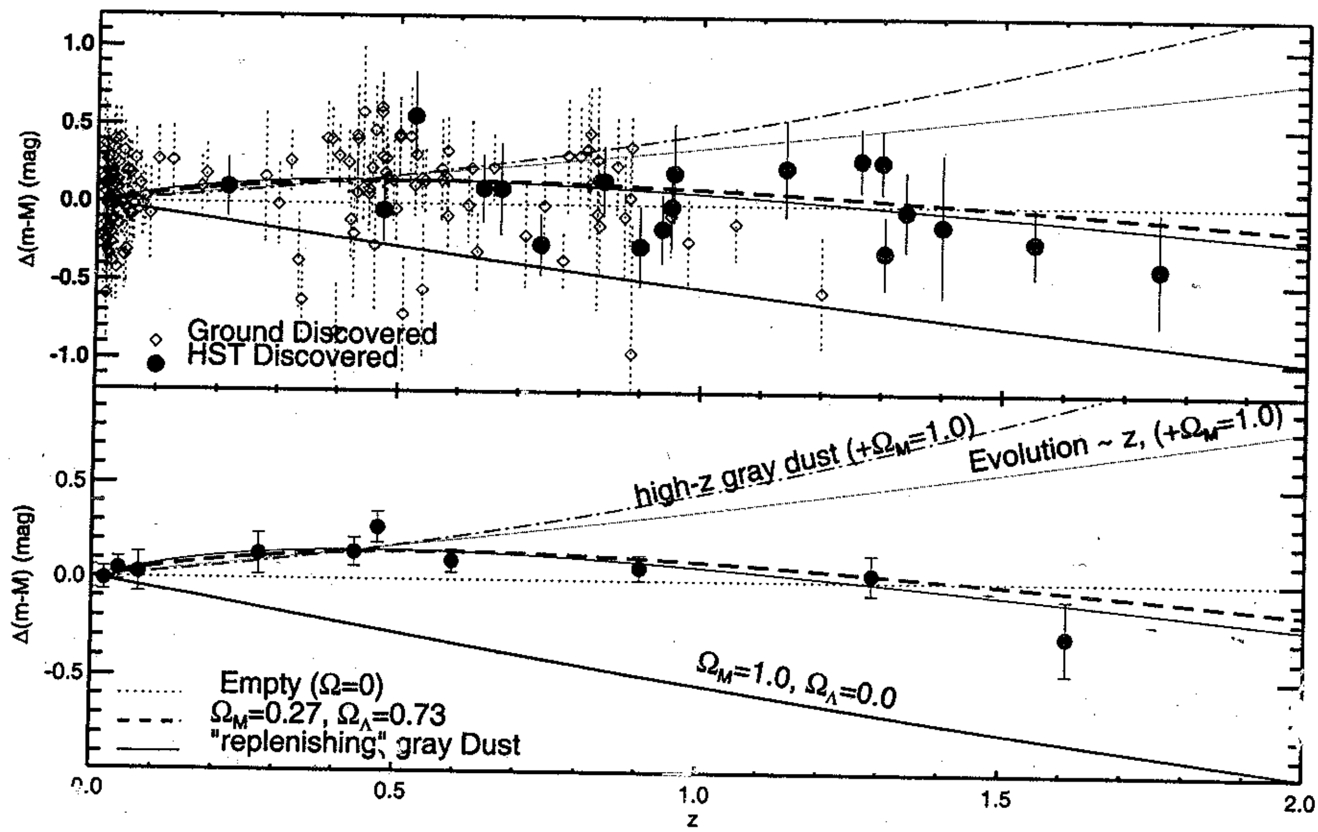
U. Alam et al. JCAP 0604, 008 (2004) [astro-ph/0403687]

b) The CPL fit

(Chevallier - Polarski - Linder)

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

A. Riess et al., astro-ph/0402512



Recent enlargement of the "Gold" sample:

A. Riess et al., astro-ph/0611572

U. Alam et al. JCAP 0604 (2004) 008 (astro-ph/0403687)

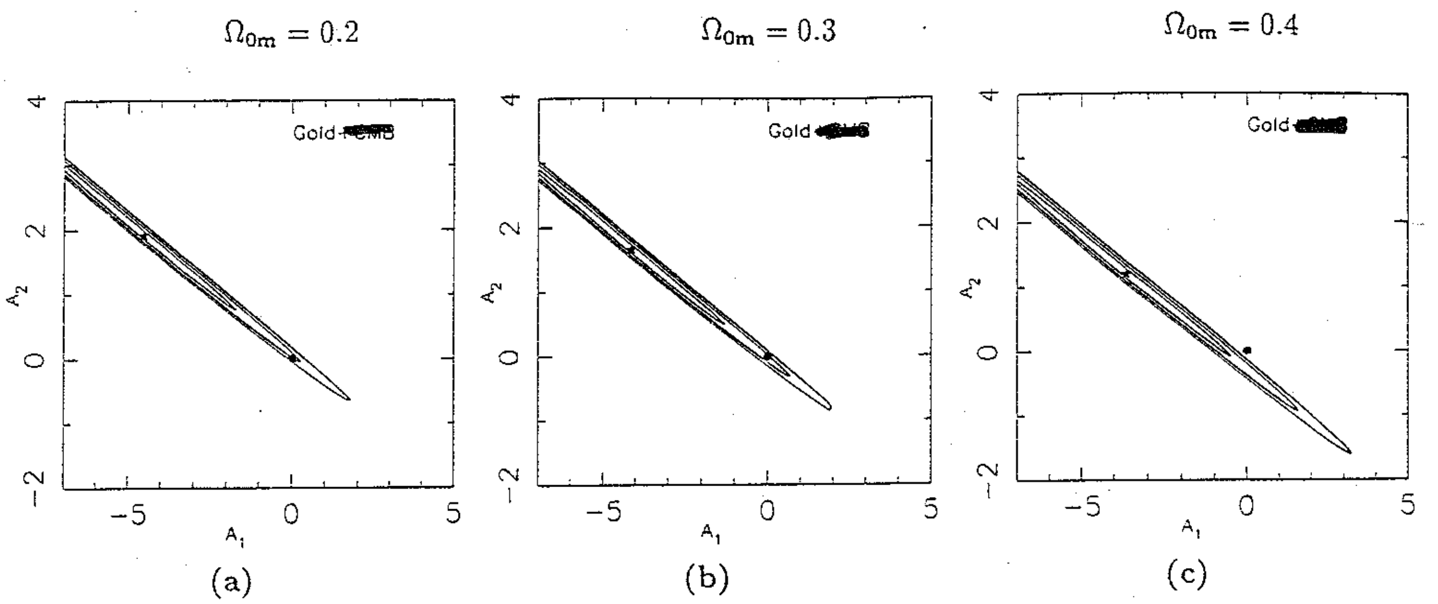


Figure 4. The (A_1, A_2) parameter space for the ansatz (5) for different values of Ω_{0m}, using the 'Gold' sample of SNe from [18]. The star in each panel marks the best-fit point, and the solid contours around it mark the $1\sigma, 2\sigma, 3\sigma$ confidence levels around it. The filled circle represents the ΛCDM point. The corresponding χ^2 for the best-fit points are given in table 1.

Table 1. χ^2 per degree of freedom for best-fit and ΛCDM models for analysis using the 'Gold' sample of SNe from [18]. w_0 is the present value of the equation of state of dark energy in best-fit models.

	Best-fit		ΛCDM
Ω_{0m}	w_0	χ^2_{min}	χ^2
0.20	−1.20	1.036	1.109
0.30	−1.35	1.034	1.053
0.40	−1.59	1.030	1.086

$$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m(1+z)^3$$

U. Alam et al. JCAP 0604(2004)008 (astro-ph/0403687)

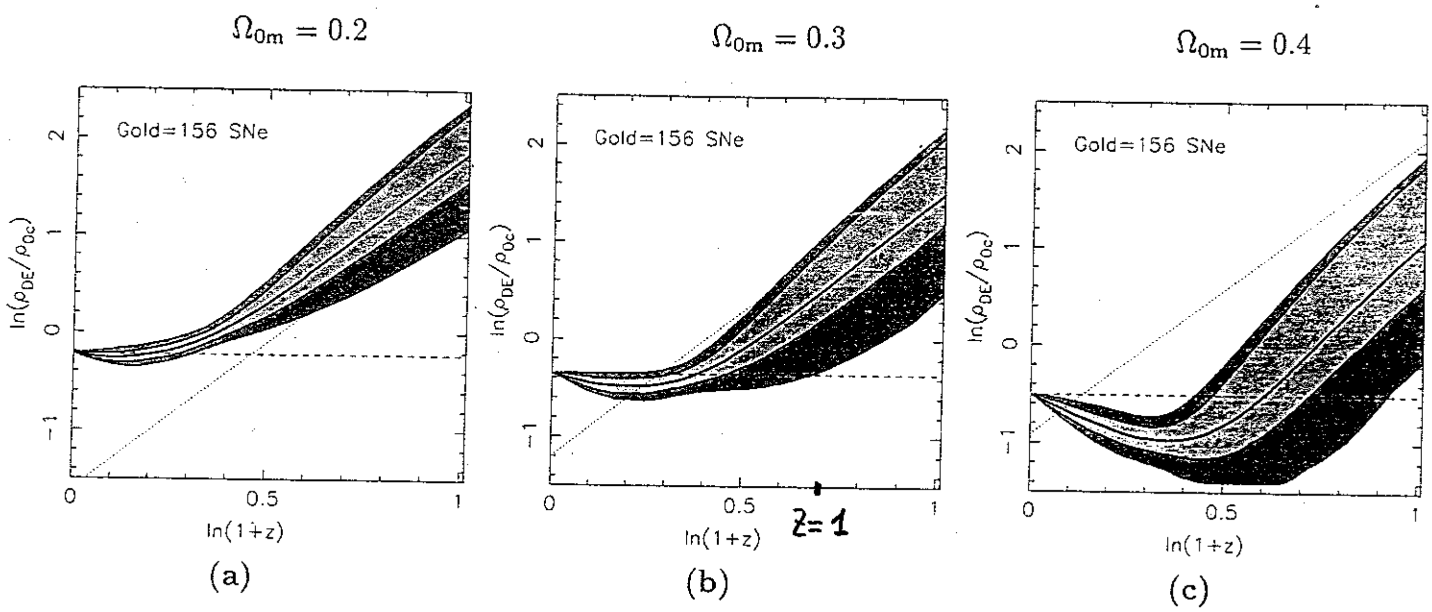


Figure 5. The logarithmic variation of dark energy density $\rho_{\mathrm{DE}}(z)/\rho_{0c}$ (where $\rho_{0c} = 3H_0^2/8\pi G$ is the present critical energy density) with redshift for different values of Ω_{0m}, using the 'Gold' sample of SNe from [18]. The reconstruction is done using the polynomial fit to dark energy, ansatz (5). In each panel, the thick solid line shows the best-fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best-fit. The dotted line denotes matter density $\Omega_{0m}(1+z)^3$, and the dashed horizontal line denotes ΛCDM.

Table 2. The weighted average $\bar{w}$ (eq 10) over specified redshift ranges for analysis using the 'Gold' sample of SNe from [18]. The best-fit value and 1σ deviations from the best-fit are shown.

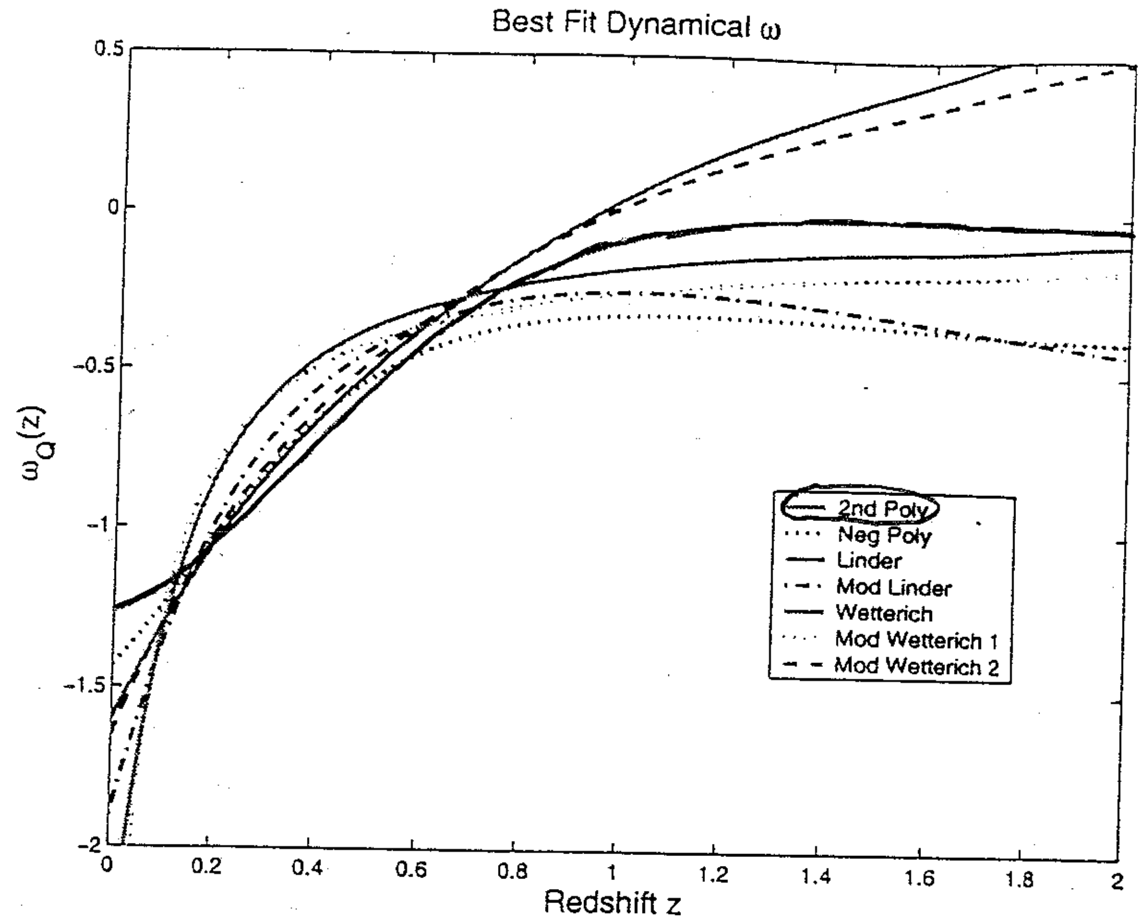
	$\bar{w}$		
Ω_{0m}	$\Delta z = 0 - 0.414$	$\Delta z = 0.414 - 1$	$\Delta z = 1 - 1.755$
0.2	$-0.847^{+0.019}_{-0.043}$	$-0.118^{+0.280}_{-0.211}$	$0.089^{+0.067}_{-0.039}$
0.3	$-1.053^{+0.089}_{-0.070}$	$-0.159^{+0.319}_{-0.259}$	$0.118^{+0.073}_{-0.041}$
0.4	$-1.310^{+0.220}_{-0.179}$	$-0.210^{+0.452}_{-0.340}$	$0.215^{+0.081}_{-0.050}$

$$1+\bar{w} \equiv \frac{1}{\Delta \ln(1+z)} \int (1+w(z)) \frac{dz}{1+z} = \frac{1}{3} \frac{\Delta \ln \tilde{\rho}_{DE}}{\Delta \ln(1+z)}$$

"w-probe" A. Shafieloo et al. MNRAS, 366, 1081 (2006) (astro-ph/0505329)

Comparison of different parametrizations for dark energy evolution

Y. Gong, astro-ph/0405446



→ used by Alam et al. (2003, 2004)

FIG. 10: The evolution of ω_{DE} for different parameterizations. The parameters are the best fit parameters with the prior $\Omega_{m0} = 0.3 \pm 0.04$ to the 157 gold sample SNe.

S. Nesseris, L. Perivolaropoulos, astro-ph/0610092

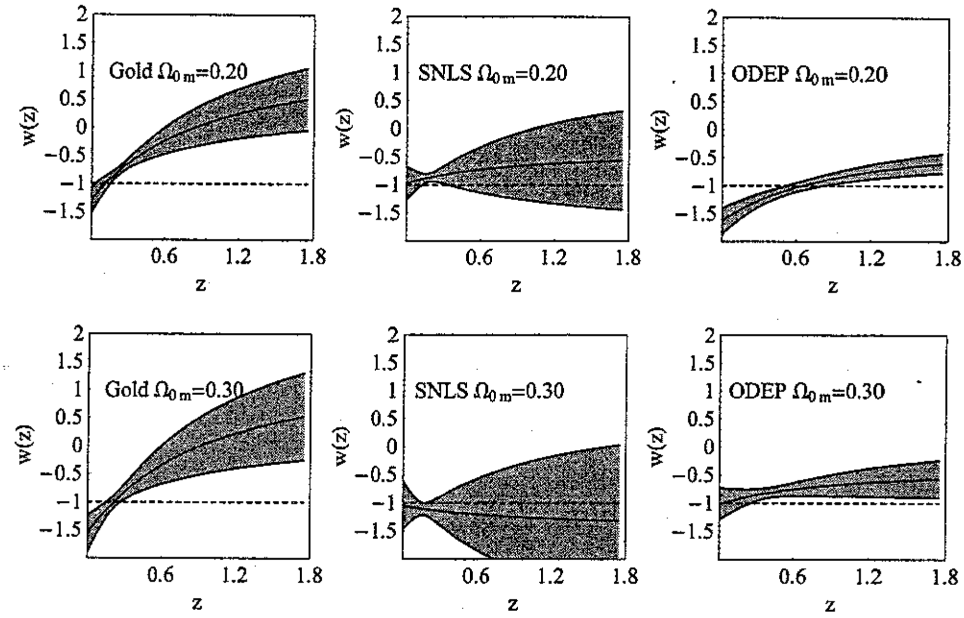


FIG. 7: The best fit form of $w(z)$ for each dataset category for both $\Omega_{0m} = 0.2$ and $\Omega_{0m} = 0.3$ along with the 1σ errors (shaded region). The categories are: Gold dataset (column 1), SNLS (column 2) and Other Dark Energy Probes (ODEP column 3).

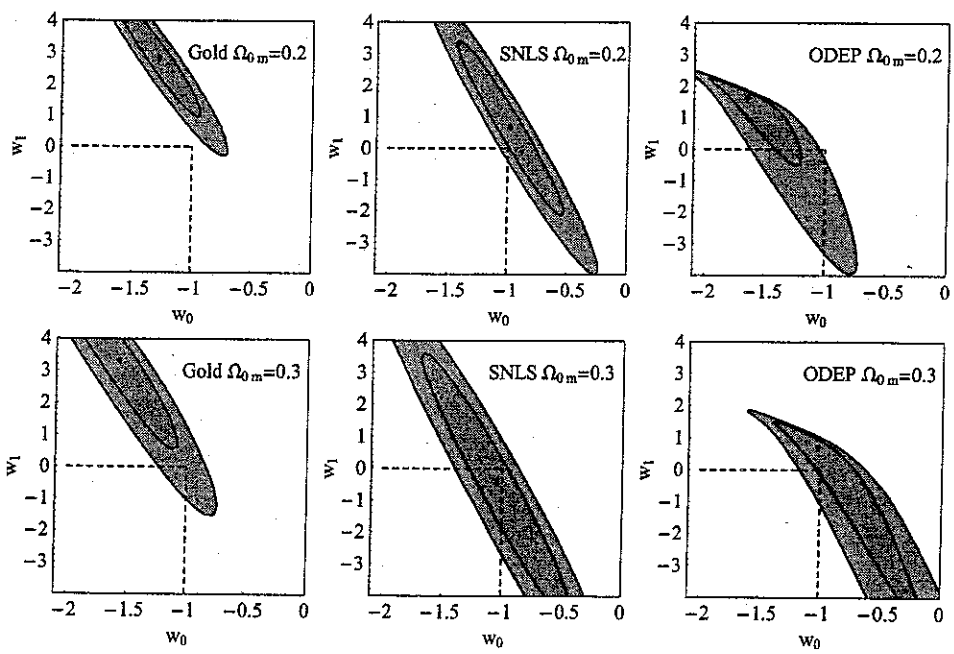


FIG. 8: The 68% and 95% χ^2 confidence contours in the $w_0 - w_1$ parameter space for each dataset category for both $\Omega_{0m} = 0.2$ and $\Omega_{0m} = 0.3$. Notice that for the SNLS dataset ΛCDM is within the 1σ region.

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

RESULTS AND CONCLUSIONS

1. In the first approximation, dark energy is well described by a cosmological constant Λ $(w \equiv -1)$.

2. Account of CMB, LSS and Ly-α data, in addition to SNe data, shrinks error bars around $\Lambda = \mathrm{const}$.
   However, for a geometrical dark energy, results from LSS and Ly-α ~~may~~ be reconsidered.

3. If $w = \mathrm{const}$ is assumed, then

$$|w+1| \lesssim 0.1$$

No evidence for a "permanent" phantom.
No evidence for the "Big Rip" in future ($\Delta T > 50$ byl. y.)
U. Seljak et al., astro-ph/0604335: $w = -1.04 \pm 0.06$

4. Without this assumption, some place for a "temporary" phantom exists for low redshifts $z \lesssim 0.2$, but $\overline{w}(0 < z \lesssim 0.4) \simeq -1$.

Conclusion for models with "phantoms": necessity of consideration of the "phantom boundary" ($w=-1$) crossing

5. $w(0) \gtrsim -1.4 = 1/\Omega_\Lambda$

No sign of WEC violation for DM+DE!

6. SNe data alone admit considerable softening of w ($-1 < w < 0$) for $0.4 \lesssim z \lesssim 1$. This is restricted by CMB, LSS and Ly-α data, but even with them, increase of ε_{DE} in 2 times by $z \sim 1$ is not excluded at 95% level.

Future data will do much better. A place for dynamical dark energy (especially, a geometrical one) still exists!

Recent review on the reconstruction approach:

V. Sahni, A.A. Starobinsky, astro-ph/0610026

Interpretation of results

1. Conservative.

$\varepsilon_{DE} = \text{const} = \varepsilon_0$ agrees with all SN + CMB data (inside 2σ or better)

Especially good with Ly-α data from SDSS added and for SNLS data.

Models.

a) Casimir energy or vacuum polarization from additional compact or ~~curved~~ non-compact spatial dimensions

$$\varepsilon_{DE} = \frac{C}{R_d^4}$$

$D = 4 + d$
$d = 2$ flat compact
$d = 1$ AdS

$R_d < 5 \cdot 10^{-3}$ cm $\rightarrow$ $0 < C < 0.1$ (D.J. Kapner et al. hep-ph/0611184)

Deviation from the Newton law at $R \lesssim R_d$ – the most crucial test for these class of models

b) String theory de Sitter vacua – have appeared in fantastically large amount recently (the "third" string revolution?)

Models of dynamical dark energy

Practical use of the remarkable similarity between primordial DE (supporting inflation) and present DE: the same models may be used for description of both inflation and present dark energy

Single inflation	Quintessence
$R + R^2$ model	$F(R)$ model
Extended inflation	Scalar-tensor DE
k-inflation	k-essence
Brane inflation	Brane DE
String inflation	String DE

Model requirements

1. Stability of the Minkowski space-time with respect to perturbations with $\omega^2 \gg H_0^2$
   a) absence of tachyons,
   b) absence of ghosts
2. Solar System tests
3. Stability of the MD-stage

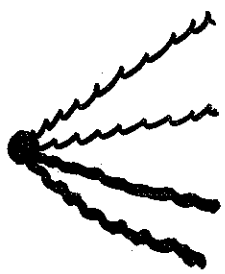
Example: $\mathcal{L} = F(R) \Rightarrow \begin{cases} \frac{dF}{dR} > 0 \\ \frac{d^2F}{dR^2} \geq 0 \end{cases}$

4. MD- and RD-stages should be generic

What if recent phantom behaviour of dark energy will be confirmed by observations?

Ghost phantom models of dark energy are bad.

1. Quantum instability



2. At the classical level:

does not explain homogeneity and isotropy of the Universe

E.g.: for a given $\bar{H} = \frac{1}{3}\frac{d}{dt}\ln abc$, it is much more probable to have very different $\frac{\dot a}{a}, \frac{\dot b}{b}, \frac{\dot c}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem.

Physical DE models

1. Quintessence = minimally coupled scalar field with some potential ("inflaton today")

$$\varepsilon = \frac{\dot{\varphi}^2}{2} + V\,, \quad p = \frac{\dot{\varphi}^2}{2} - V$$

No crossing of $w=-1$ line

If $V \propto \varphi^{-n} \Longrightarrow n < 1$

Slow-roll regime: $w \simeq -1 + \frac{\dot{\varphi}^2}{V} \simeq -1$

2. The Chaplygin gas model

$$p = -\frac{\varepsilon_0^2}{\varepsilon}$$

$$c_s^2 \equiv \frac{dp}{d\varepsilon} = \frac{\varepsilon_0^2}{\varepsilon^2} > 0$$

$\varepsilon > \varepsilon_0$ for the "usual" model $\left(\begin{matrix} \varepsilon + p > 0 \\ c_s^2 < 1 \end{matrix}\right)$

$$\varepsilon = \sqrt{\varepsilon_0^2 + C(1+z)^6}$$

Unifies DM and DE : can describe both the MD stage in the past and the transition to the Λ-dominated stage today

Equivalent field-theoretical models:

a) Quintessence with

$$V(\varphi) = \frac{\varepsilon_0}{2}\left(\cosh(2\sqrt{6\pi G}\,\varphi) + \frac{1}{\cosh(2\sqrt{6\pi G}\,\varphi)}\right)$$

Equivalence for some solutions (see V. Gorini et al., PRD 72, 103518 (2005); astro-ph/0504576)

b) k-essence $\mathcal{L} = -\varepsilon_0\sqrt{1 - T_{,\mu}T^{,\mu}}$

Equivalence for all solutions with $T_{,\mu}T^{,\mu} > 0$

However, λ_J is too large!

$\lambda_J \propto c_s t \propto t^3$ at the MD stage

Perturbations stop growing for

$z \sim 3$ for the present comoving scale 100 Mpc

$z \sim 14$ —— " —— " —— " —— 1 Mpc

Wrong $P(k)$ today!

Geometrical $F(R)$ model of dark energy

$$S = \frac{1}{16\pi G}\int F(R)\sqrt{-g}\, d^4x + S_m$$

$$F(R) = R + f(R) \qquad\qquad R \equiv R_i^i$$

$$\frac{F}{2}\delta_i^k - F' R_i^k - (\Box\delta_i^k - \nabla_i\nabla^k)F' = -8\pi G\, T_i^k$$

$$-R_i^k + \frac{1}{2}\delta_i^k R = 8\pi G\, T_i^k + \underbrace{-f' R_i^k + \frac{f}{2}\delta_i^k - (\Box\delta_i^k - \nabla_i\nabla^k) f'}_{8\pi G\, T_{i,DE}^k}$$

Particle content: graviton + massive scalar particle (called "scalaron" in A.S., 1980)

No ghosts if

① $F' > 0 \qquad f' > -1$

② $F'' \geq 0 \qquad f'' \geq 0$

Classically for FRW model $\begin{cases} F'(R_0) = 0 & \text{- loss of homogeneity} \\ F''(R_0) = 0 & \text{- non-analytical behaviour of } R(t)\ (R(t) = R_0 + R_1 t \ln^{1/2} t + \end{cases}$

Equivalence between $\mathcal{L} = \frac{1}{16\pi G}[R + f(R)]$ and $\mathcal{L} = \frac{R}{16\pi G} + \frac{1}{2}\Phi_{,i}\Phi^{,i} - V(\Phi)$ theories

1^{st} class includes the (symplifed) Starobinsky's inflationary model (1980); 2^{nd} class includes the original Guth's "old" inflation (1981), "new" inflation (1982), Linde's chaotic inflation (1983), power-law inflation (1984).

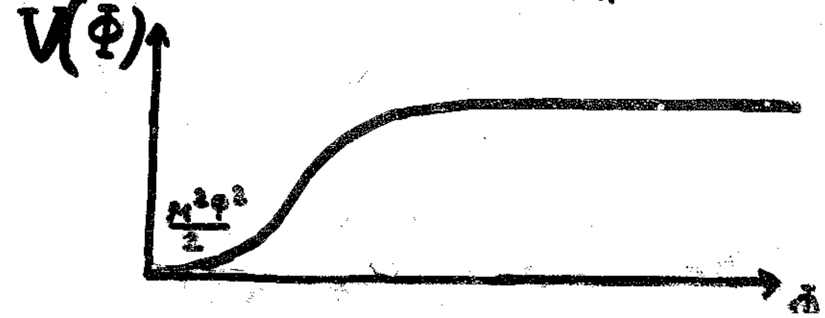
Theorem. There is one to one correspondence between solutions of the both theories for which $\left(1 + \frac{df}{dR}\right) \neq 0$:

$$(g_{ik}, R(g_{ik})) \longleftrightarrow (\tilde{g}_{ik}, R(\tilde{g}_{ik}), \Phi)$$

where
$$\tilde{g}_{ik} = \left|1 + \frac{df}{dR}\right| g_{ik}$$
$$\Phi = \sqrt{\frac{3}{16\pi G}} \ln\left|1 + \frac{df}{dR}\right| \quad \Longrightarrow R = R(\Phi)$$
$$V(\Phi) = \frac{1}{16\pi G} \frac{\left(R\frac{df}{dR} - f\right)}{\left(1 + \frac{df}{dR}\right)^2} \operatorname{sign}\left(1 + \frac{df}{dR}\right)$$

Examples.

1. $f(R) = \frac{R^2}{6M^2}$ $\qquad V(\Phi) = \frac{3M^2}{32\pi G}\left[1 - \exp\left(-\sqrt{\frac{16\pi G}{3}}\Phi\right)\right]^2$



$\boxed{f(R) = R^2/6M^2}$

Internally self-consistent inflationary model with a graceful exit to the subsequent FRW stage (first, MD; second, after scalaron decay, RD)

$\tau \sim \frac{M_{pl}^2}{M^3}$ $(\to N \sim 53)$ A. S., PLB (1980)

$M = 2.7 \times 10^{-6} M_{pl} \cdot \frac{53}{N}$ $(A_s = 2.2 \times 10^{-9})$

$n_s = 1 - \frac{2}{N} \simeq 0.96$, $r = \frac{12}{N^2} \simeq 4 \cdot 10^{-3}$

$\longleftrightarrow$

Recent idea:

F(R) model with $F(R) \to \infty$ for $R \to 0$ as a model of dark energy at present time

Capozziello 2002
Capozziello et al. 2003
Carroll et al. 2003 ....

Main problem: effective coupling to usual matter in the Einstein frame

F(R) theory is equivalent to the $\omega=0$ scalar tensor theory:

$$S = \int \left(A(\varphi) R + B(\varphi) \right) \sqrt{-g}\, d^4x$$

If scalaron is very light, $\gamma = 1/2$ (though $\beta = 1$)

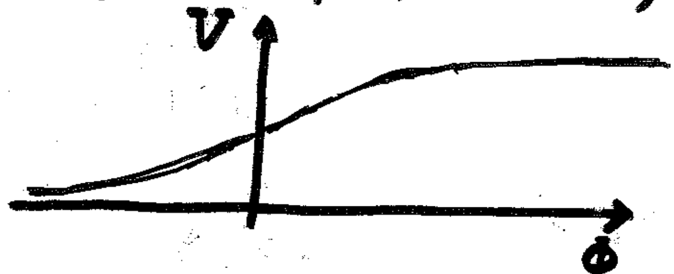
Possible way out: make scalaron massive for large R by taking $F(R) \sim \frac{R^2}{6M^2}$ for $|R| \to \infty$.

However, for $F(R) \sim \begin{cases} \pm |R|^{-m} & |R| \to 0 \\ R^2 & |R| \to \infty \end{cases}$

either ① or ② are violated

New way out: $F(R) \sim |R|^{\alpha}$, $1 < \alpha < 2$, $|R| \to 0$

(c.f. Capozziello et al., astro-ph/0307018)
$\alpha \simeq 1.4$



New problem: existense and stability of the long MD-stage

STABILITY OF THE MD STAGE IN THE $f(R)$ THEORY

Requires $F''(R) \geq 0$ for $|R| > H_0^2$

The model $F(R) = R - R_0\left(\frac{R}{R_0}\right)^{\alpha}$,

$$R_0 \sim H_0^2, \quad 0 < \alpha < 1$$

satisfies this condition. For oscillations:

$$\overline{w}_{osc} = 1 - \frac{\alpha}{2} \quad \text{during MD stage}$$

It also has the stable late-time DS attractor with

$$R = R_0 \cdot (2-\alpha)^{1/(1-\alpha)}$$

and $F'(R) \geq 0$.

$\alpha < 0$: the stable model is

$$F(R) = R + R_0\left(\frac{R}{R_0}\right)^{\alpha}$$

sign opposite to that used in Carroll et al.

No dS attractor

General theory

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + f(R)\right)$$

$$|f| \ll |R| \,, \quad \left|\frac{df}{dR}\right| \ll 1 \,, \quad \left|R\,\frac{d^2 f}{dR^2}\right| \ll 1$$

$$R = \underbrace{8\pi G T}_{R^{(0)}} + R^{(1)} + \delta$$

non-oscillating term

$R^{(1)} \sim f(R^{(0)})$

$$\delta \propto a^{-3/2} (f'')^{-3/4} \exp\left(i \int \frac{dt}{\sqrt{3f''}} \right)$$

Stability : $f'' \equiv \frac{d^2 f}{dR^2}(R^{(0)}) \geq 0$

Effective EMT of these oscillations ('scalarons') : dust-like but with variable mass (so $p \neq 0$)

$$\varepsilon \sim \frac{f''}{G}\delta^2 \sim \frac{m_{eff}}{a^3} \qquad m_{eff}^2 \propto \frac{1}{f''}$$

If $R^{(0)}$ is produced by massive particles

$R^{(0)} \propto a^{-3} \longrightarrow m_{eff} \propto a^{\frac{3}{2}\alpha - 3}$ for $f \propto |R|^{\alpha}$

$$\overline{w}_{osc} = 1 - \frac{\alpha}{2}$$

Valid during the RD stage, too

DE models with "gravity leaking to higher dimensions"

The simplest model (Dvali et al., 2000)
Gravity in the $D=5$ bulk + induced gravity on the brane

$$H^2 = \left(\sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2$$

$$H_0 r_c = \frac{1}{1-\Omega_m} \simeq 1.4 \quad (1.25 \text{ for } \Omega_m = 0.2)$$

$\Omega_m = 0.3$

$$a = a_0 \sinh^{2/3}\varphi$$

$$\frac{3t}{2r_c} = \varphi - \frac{e^{-2\varphi}}{2} + \frac{1}{2}$$

$$\Omega_m(t) = e^{-2\varphi}$$

$$q(t) = \frac{2\Omega_m - 1}{1+\Omega_m}$$

$$r(t) = 1 - \frac{9\Omega_m^2(1-\Omega_m)}{(1+\Omega_m)^3}$$

$\Omega_m = 0.3$: $\quad q_0 = -0.31,\ r_0 = 0.74$

Sahni & Shtanov (2002, 2004) – generalization admitting $w_{DE} < -1$ and/or $\varepsilon_{DE} < 0$

This model is on verge to be falsified, (using $\frac{\Delta T}{T}$ and $\frac{\delta\rho}{\rho}$)
but still not!

Has a ghost (see, e.g., hep-th/0610282)

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese,
D. Polarski, A. S.
Phys. Rev. Lett. 85, 2236 (2000)
$\varepsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{L} = \frac{1}{2}\left(F(\varphi)R + Z(\varphi)\varphi_{,\mu}\varphi^{,\mu}\right) - V(\varphi) + \mathcal{L}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$Z(\varphi) = 1$ $\qquad \omega^{-1}(\varphi) = F^{-1}\left(\frac{dF}{d\varphi}\right)^2$

Two independent observable cosmological functions are required for reconstruction of $F(\varphi)$ and $V(\varphi)$

$$\begin{array}{ccc} D_L(z), & & \delta(z) \\ \downarrow & & \downarrow \\ H(z) & \longrightarrow & F(z) \end{array}$$

Properties of scalar-tensor models of dark energy

R. Gannouji, D. Polarski, A. Ranquet, A. S.
JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w=-1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.
"Curvature induced phantomness"

2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w<-1$ exist if

$$F = \alpha \varphi^2 , \quad \varphi \to \infty$$

(Barrow & Maeda 1990)

$$V = V_0 |\varphi|^n , \quad 2 < n < 4$$

Then $a(t) \propto (t_0 - t)^q$ $\qquad q = \dfrac{2\left(n+2+\frac{1}{\alpha}\right)}{(n-2)(n-4)} < 0$

$\varphi(t) \propto (t_0 - t)^r$ $\qquad r = \dfrac{2}{2-n} < 0$

However, for these solutions $|w+1| \leq \frac{\alpha}{3} \sim \frac{1}{\omega_{BD}}$ and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$$

Bertotti et al., 2003 → Cassini mission

$$\beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4}$$

Pitjeva, 2005 → ephemerides of planets

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5) \cdot 10^{-13} y^{-1}$$

$$\beta_{PN} - 1 = (1.2 \pm 1.1) \cdot 10^{-4}$$

Williams et al., 2005 → lunar laser ranging

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{d\varphi}\right)^2} \right)_0 > 4 \cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \dots$$

$$\frac{V(z)}{3 F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \dots$$

$$\frac{H^2(z)}{H_0^2} = 1 + h_1 z + h_2 z^2 + \dots$$

$$F_0^{-1/2} \varphi'(z) = \varphi_0' + \varphi_1' z + \varphi_2' z^2 + \dots$$

$$w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \dots$$

$$|F_1| < 10^{-2}$$

$$\varphi_0'^2 = 6(1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{BD,0}}$) ?

$$F_2 < 0\ , \quad |F_2| \sim 1 \gg |F_1| \qquad (|F_1| < 10^{-2})$$

$$|F_2| > 3(\underbrace{\Omega_{DE,0}}_{1-\Omega_{m,0}} - \Omega_{V,0}) > 0$$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{V,0})}{3\Omega_{DE,0}} < 0$$

h_1 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{PN} - 1 = -\frac{F_1^2}{6(\Omega_{DE,0} - \Omega_{V,0})} < 0$$

$$\beta_{PN} - 1 = -\frac{F_1^2 F_2}{72(\Omega_{DE,0} - \Omega_{V,0})} > 0$$

$$-4 < \frac{\gamma_{PN} - 1}{\beta_{PN} - 1} = \frac{12(\Omega_{DE,0} - \Omega_{V,0})}{F_2} < 0$$

However, $|\gamma_{PN} - 1|$ and $|\beta_{PN} - 1|$ may be much smaller than $|1+w|$ if F_1 is very small

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = H_0 F_1 \left(1 - \frac{F_2}{3(\Omega_{DE,0} - \Omega_{V,0})}\right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$ may be a strong argument for significant phantomness of present dark energy. Negative detection tell us nothing.

5. Correct asymptotic behaviour for large z ($\varphi'^2 \geq 0$, $w_{DE} \leq 0$) requires non-zero and non-constant $V(\varphi)$

Example: $F(\varphi) \to F_\infty < F_0$, $V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0\Omega_{u,\infty}}}\,\varphi\right)$ for $z, \varphi \to \infty$

CONCLUSIONS

1. Expected deviations ($\Delta G_{eff}/G_{eff}$, $\max|w+1|$) less than 10%; At least 1% accuracy level is needed for further progress.
2. Still it is not guaranteed that any deviation of dark energy from the cosmological constant will be discovered (that, of course, does not mean that the present dark energy is stable and eternal)
3. More possibilities to find deviation from Λ for $\delta(z)$