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Lessons from Capillarity for Gravity ?

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based on work with Pierre Le Doussal and Kay Wiese,

hep-th/0606247 and in progress

String theory solves the ultraviolet problem of quantum gravity.

However, it has a host of infrared problems which hinder its confrontation with the observed low-energy physics. Most serious is the problem of vacuum stability after supersymmetry breaking, the related issue of moduli fixing with $\Lambda \geq 0$, and of course the infamous cosmological constant problem.

Many suspect that we are missing an important ingredient, beyond conventional QFT non-perturbative phenomena. One proposal is 'anthropic reasonning'. Others suspect that something is wrong with the usual ideas of locality and decoupling of scales.

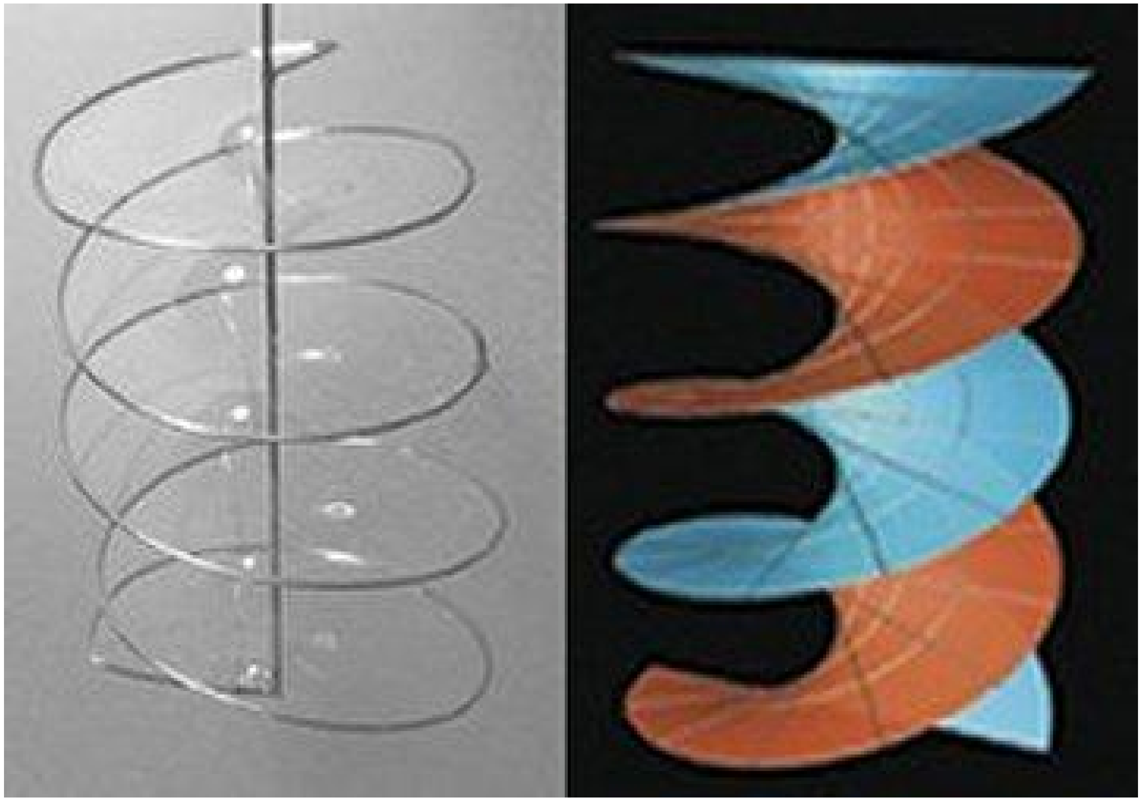
see e.g. talk by Elias Kiritsis

This talk is about a search for inspiration from condensed-matter physics. Contrary to its huge impact in QFT, this has been of little use up to now in helping to elucidate quantum-gravity phenomena.

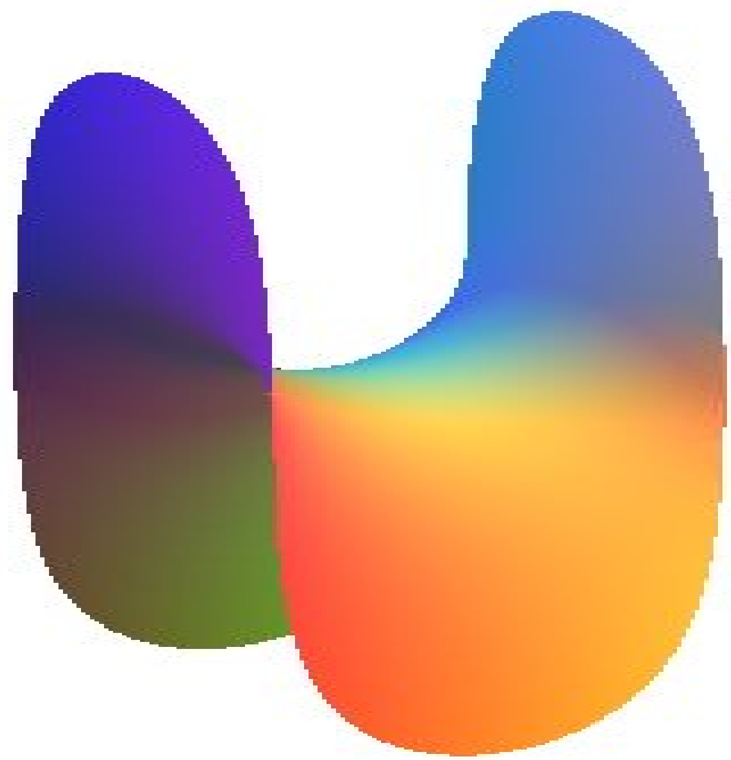
The theory of capillarity originates in the brilliant, almost simultaneous work of *Thomas Young* and of *Pierre Simon Laplace*, published in 1805.

It was the first force to be clearly understood, besides gravity. It was also the first force to be given a geometric description, as a theory of minimal surfaces.

Minimal surfaces arise nowdays in many areas of physics and biology, and their study remains a very active branch of mathematics. They are usually visualized as soap films bounded by a wire-frame, a problem associated with the name of the 19th-century, blind Belgian physicist *Joseph Antoine Ferdinand Plateau*.



The helicoid



Scherk's surface: mirror symmetric to the saddle point of the Veneziano high-E, right-angle scattering amplitude *(CB, B.Pioline, hep-th/9909171)*.

The problem of capillarity and wetting differs from the problem of Plateau in that the boundary of the fluid surface is free to move along the (usually solid) substrate. Understanding the dynamics of wetting is of great current interest because of the wealth of potential applications. Here we will focus on the equilibrium problem, which is intriguing in its own right.

The problem of partial wetting

We consider a liquid with free surface of area $\mathcal{A}$ (liquid-air interface) wetting a solid over an area $\mathcal{A}'$ (liquid-solid interface). The setup may for instance consist of :

- a liquid inside a capillary tube $\Omega \times \mathbb{R}$, or
- a droplet resting on a solid plate.

The energy reads

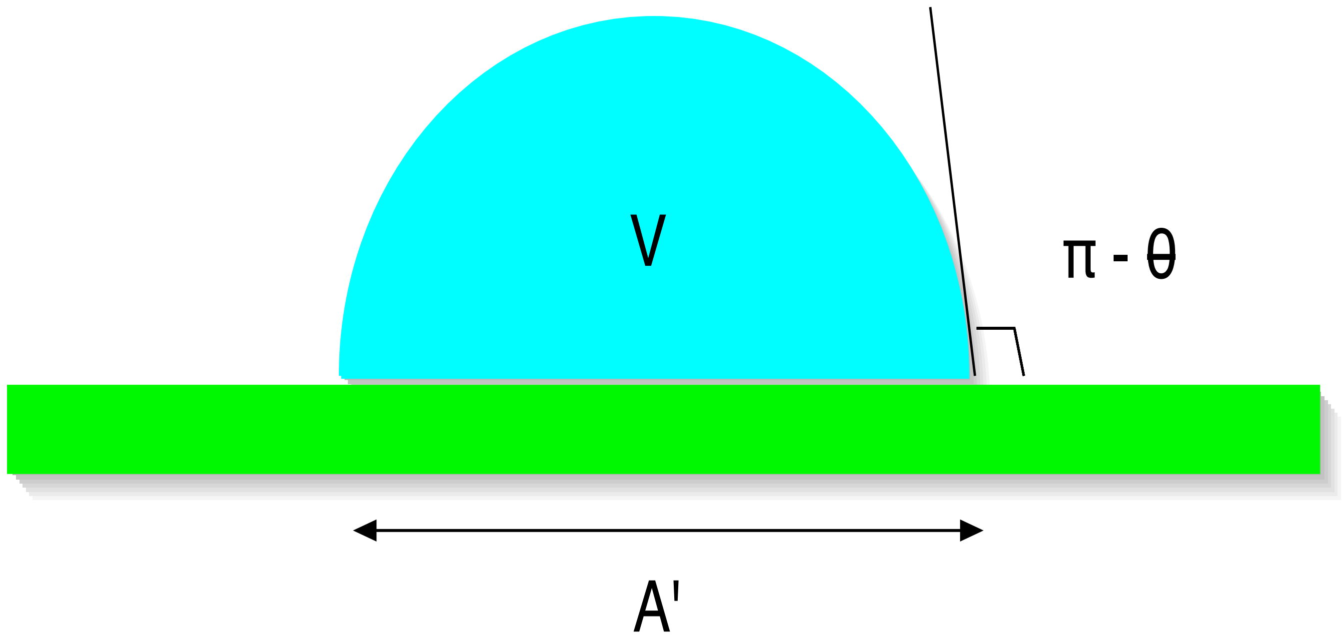
$$\mathcal{E} \;=\; \gamma\mathcal{A} - \gamma'\mathcal{A}' - pV \;\; + \; gravity \;,$$

where here :

$$\gamma \;\longrightarrow\; \text{liquid-air tension}$$

$$\gamma' = \gamma_{SA} - \gamma_{SL} \;\longrightarrow\; \text{solid-air minus solid-liquid tension}$$

$$p \;\longrightarrow\; \text{liquid-air pressure difference}$$



A droplet of fluid of volume V resting on a solid plate. The solid-fluid interface has area A'. The droplet makes a contact angle θ with the plate.

Gravity defines a length scale called the capillary length, $\kappa^{-1} = (\gamma/\rho g)^{1/2}$. Here ρ is the fluid mass density, and g is the gravitational acceleration. The capillary length introduces an infrared cutoff in the wetting problem. In normal everyday conditions $\kappa^{-1} \sim mm$, but it can be made much bigger

- in free-fall experiments *(NASA's Microgravity Lab)*
- if the air is replaced by a second (non-mixing) liquid.

This is, in practice, also achieved by focusing at distances much smaller than the millimeter, but much larger than the atomic-scale cutoff $\sim$ Angström .

e.g. Prevost, Rolley and Guthmann, 2002

Moulinet, Guthmann and Rolley, 2002

Henceforth we neglect gravity, and consider the limit $\kappa^{-1} \to \infty$.

Consider for definiteness the capillary tube, with Ω a region of the (x, y) plane and $\mathbb{R}$ the z direction. In 'static' coordinates (assuming no overhangs) the fluid surface is defined by the height function $z(x, y)$ and the energy reads

$$\mathcal{E} \;=\; \mathcal{E}_{\text{bulk}} + \mathcal{E}_{\text{bnry}} \;=$$

$$= \int_{\Omega} \mathrm{d}x\, \mathrm{d}y \left(\gamma\sqrt{1 + (\partial_x z)^2 + (\partial_y z)^2} \;-\; pz\right) - \int_{\partial\Omega} \mathrm{d}l\, \gamma'(l)\, z \quad .$$

Note that gravity would have given a mass to z. Furthermore, we assumed that γ' depends on l, either because of impurities or by design. In the first case it will in general also depend on the height z, so that

$$\mathcal{E}_{\text{bnry}} \;= -\oint_{\partial\Omega} \mathrm{d}l \int_0^z \mathrm{d}\zeta\, \gamma'(l, \zeta) \;.$$

The minimum of the energy is a surface of constant mean curvature

$$\vec{\nabla} \cdot \left(\frac{\vec{\nabla} z}{\sqrt{1 + |\vec{\nabla} z|^2}} \right) = -\frac{p}{\gamma} \quad ,$$

with mixed Dirichlet/Neumann boundary conditions:

$$(x, y) \in \Omega \quad \text{and} \quad \left. \frac{\widehat{n} \cdot \vec{\nabla} z}{\sqrt{1 + |\vec{\nabla} z|^2}} \right|_{\partial\Omega} = \cos\theta(l) = \frac{\gamma'(l, z)}{\gamma} \ .$$

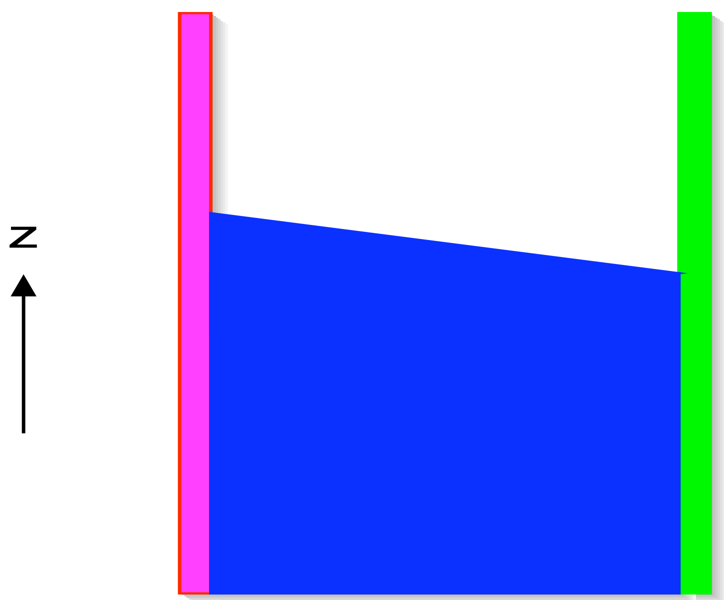
Here $\vec{\nabla} = (\partial_x, \partial_y)$ and $\widehat{n}$ is a unit vector normal to the boundary $\partial\Omega$.

The boundary equation is Young's equilibrium condition. It fixes the inclination angle $\theta(l)$ of the fluid surface, at each point of the contact line, as a function of the local adherence of the liquid to the container wall.

In the 'pure' case, where the chemical composition of the wall doesn't depend on height, the problem has generally no solution. A necessary (but not sufficient) condition for a solution to exist is the (global) tadpole cancellation:

$$p \times \mathrm{Area}(\Omega) + \oint_{\partial\Omega} \mathrm{d}l\, \gamma'(l) \;=\; 0 \ .$$

This requires a fine tuning of the composition and geometry of the capillary tube.



An example of tadpole cancellation:

$$p = 0 \quad \text{and} \quad \gamma' = -\gamma' \neq 0.$$

In a pure system, the average height $<z>$ is an undetermined modulus. It would run away if there was a non-zero tadpole. This degeneracy is lifted by wall-roughening, but it can lead to large effects at weak disorder.

The original motivation for our work was the apparent discrepancy between measurements of the contact-line fractal dimension (see later) and theoretical or numerical calculations .

LeDoussal, Wiese and Chauve, 2002

Rosso and Krauth, 2002

LeDoussal, Wiese, Raphael and Golestanian, 2004

A second motivation comes from the similarities between string perturbation theory and perturbation theory of the contact line. Are there any useful lessons for string theory to be learned?

Formal solution

Let us first set $p = 0$. The problem can be solved formally in two steps:

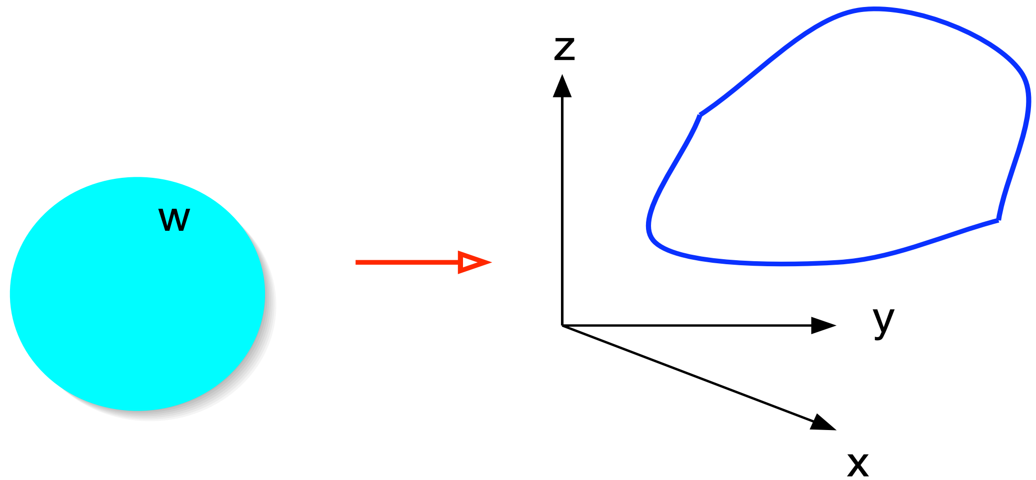
- The minimal energy is a generalized Legendre transform of the energy $\tilde{\mathcal{E}}(z(l))$ of the Dirichlet problem, with γ' a (field-dependent) source for $z(l)$.

- The reduced-energy functional is quadratic in conformal gauge:

$$\tilde{\mathcal{E}} \;=\; \frac{i\gamma}{2}\int_0^{2\pi} \mathrm{d}\phi \left(\vec{r}_+ \cdot \frac{\mathrm{d}\vec{r}_-}{\mathrm{d}\phi} - \vec{r}_- \cdot \frac{\mathrm{d}\vec{r}_+}{\mathrm{d}\phi}\right) \;=\; 2\pi\gamma \sum_{n=1}^{\infty} n\,|\vec{r}_n|^2 \;.$$

where $\vec{r} = (x, y, z)$, $w = re^{i\phi}$ is a conformal coordinate on the unit disk, and one decomposes as usual into positive- (negative-) frequency parts:

$$\vec{r} \;=\; \vec{r}_+ + \vec{r}_- \;=\; \sum_{n=1}^{\infty} (\vec{r}_n w^n + \vec{r}_{-n} \bar{w}^n)$$



The problem is not quite yet solved, because the boundary curve $\vec{\mathrm{v}}(s)$ is not in general parametrized by the (special) coordinate ϕ. The reparametrization $\phi = f^{-1}(s)$, is fixed implicitly by the conformal-gauge condition:

$$\frac{\mathrm{d}\vec{r}_+}{\mathrm{d}\phi}\cdot\frac{\mathrm{d}\vec{r}_+}{\mathrm{d}\phi} = 0 \qquad \text{with} \qquad \frac{\mathrm{d}\vec{r}_+}{\mathrm{d}\phi} \;=\; -\frac{i}{8\pi}\oint \mathrm{d}\phi'\, \frac{\vec{\mathrm{v}}(f(\phi'))}{\sin^2(\frac{\phi-\phi'}{2})}\ .$$

Note that from these relations f (and hence also the energy $\tilde{\mathcal{E}}$) is a non-linear and non-local functional of the boundary data $\vec{\mathrm{v}}(s)$.

The conformal conditions can be solved constructively by the *Weierstrass parametrization* of the minimal surface

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathrm{Re} \int^{w} f \mathrm{d}w' \times \begin{pmatrix} 2g \\ -i(1+g^2) \\ 1-g^2 \end{pmatrix} \quad ,$$

where g is an analytic function and $f\mathrm{d}w$ a 1-form (on the disk the latter is always exact). Another parametrization is in terms of two analytic sections of spin structures:

D. Sullivan, 1989

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathrm{Re} \int^{w} \mathrm{d}w' \times \begin{pmatrix} 2\psi_1\psi_2 \\ -i(\psi_1^2+\psi_2^2) \\ \psi_1^2-\psi_2^2 \end{pmatrix} \quad ,$$

The problem is then to determine (f, g) or (ψ_1, ψ_2) from the boundary data $\vec{\mathrm{v}}(s)$.

For non-zero pressure, i.e. surfaces of constant mean curvature, the analyticity requirement is replaced by the following integrability condition:

$$\bar{\partial}\,(\psi_1, \psi_2) = \frac{p}{\gamma}(\bar{\psi}_2, -\bar{\psi}_1)(|\psi_1|^2 + |\psi_2|^2)\ .$$

This can be related to the sinh-Gordon equation. Alternatively, the $p \neq 0$ problem is the same as the non-critical $SU(2)$ *Wess-Zumino-Witten* model, in the limit of infinite radius for the sphere and fixed density of flux.

Either way, the bulk equations are classically integrable *(F. Helein,* ETH Lecture Notes 2000) , but the boundary-value problem becomes even more formidable. In any case, p will not play a special role in what follows.

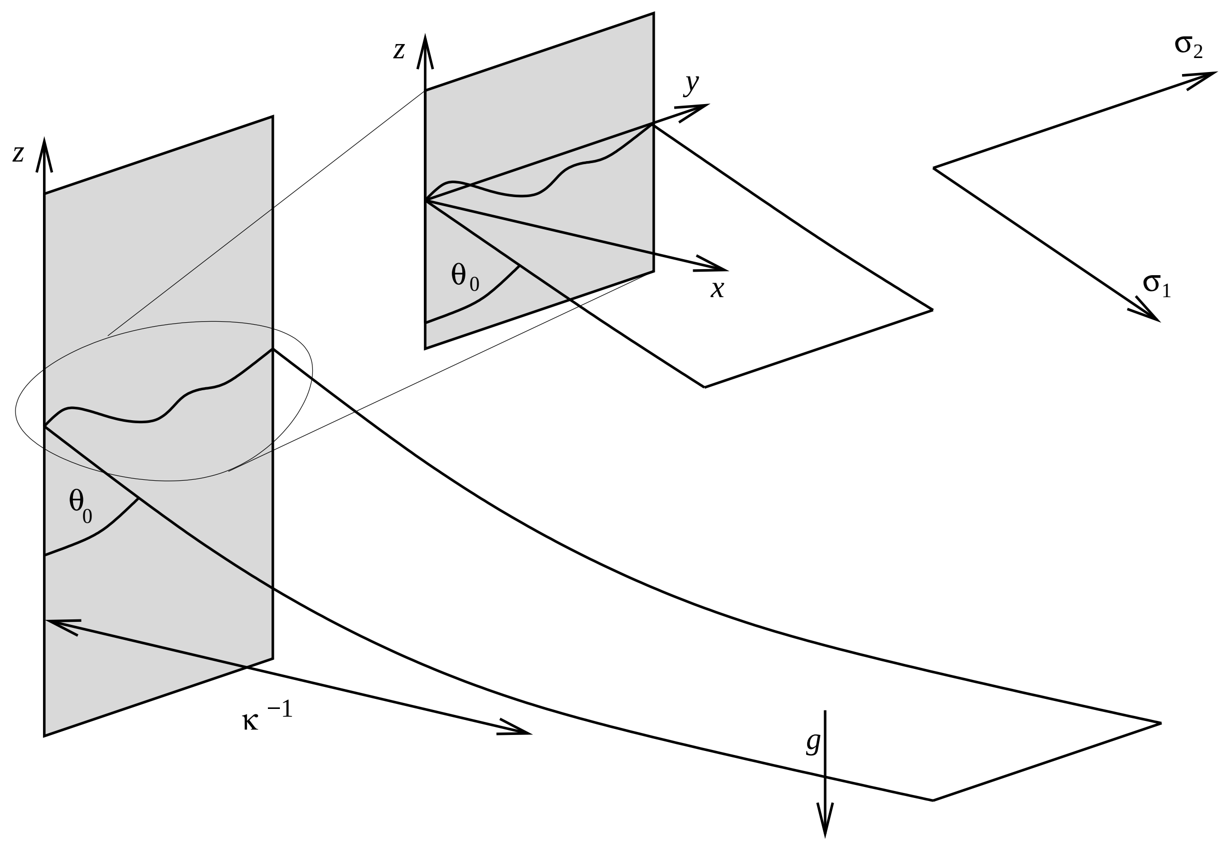
From now on we will therefore set $p = 0$.

Quasilocal perturbation theory

The reduced energy functional defines a reparametrization-invariant, parameter-free and background-independent theory for the contact line. The background is fixed by the shape and composition of the container walls. These introduce new (infrared) length-scales in the problem, in addition to κ^{-1} and to γ/p.

One question we set out to address is whether these infrared scales decouple from the (universal?) physics of the contact line at shorter distances, i.e. from the response of the contact line to localized forces and/or localized disorder. This is not a priori obvious, because of the fine tuning required for tadpole cancellation. As I will now try to explain, the answer to this question within perturbation theory is affirmative, but the story is less clear beyond.

The scale-invariant background consists of a homogeneous planar wall at $x = 0$, meeting a planar liquid surface at fixed contact angle $\theta_0 = \arccos(\gamma'_0/\gamma)$. We will be interested in the energy $\tilde{\mathcal{E}}$ of the deformed contact line $z = h(y)$ or, for fixed chemical disorder $\Delta\gamma'(y, z)$, in its generalized Legendre transform $\mathcal{E}$.



It is convenient to choose a conformal coordinate $2w = \sigma + i\tau$ defined in the upper-half plane $\tau \geq 0$, rather than in the interior of the unit disk. Using the fact that $x(\sigma, \tau)$ is harmonic, we may further set $x = \sin\theta_0\, \tau$. This is analogous to the proper-time gauge of string theory. The perturbed surface is then

$$\vec{r} \;=\; \vec{r}_0 + \Delta\vec{r} \;=\; (\sin\theta_0\; \tau,\, \sigma,\, -\cos\theta_0\, \tau) + (0, \tilde{y}, \tilde{z}) \ ,$$

where $\tilde{y}$ and $\tilde{z}$ are also harmonic functions. They are thus determined by their restrictions to the real axis, which obey the coupled equations:

$$\tilde{z}(\sigma) = h\Big(\sigma + \tilde{y}(\sigma)\Big) \ \ ,$$

$$\frac{\mathrm{d}\tilde{y}_+}{\mathrm{d}\sigma} + i\cos\theta_0\, \frac{\mathrm{d}\tilde{z}_+}{\mathrm{d}\sigma} \;=\; -\left(\frac{\mathrm{d}\tilde{y}_+}{\mathrm{d}\sigma}\right)^2 - \left(\frac{\mathrm{d}\tilde{z}_+}{\mathrm{d}\sigma}\right)^2 \ .$$

Here $\tilde{y}_+$ and $\tilde{z}_+$ are the positive-frequency parts of $\tilde{y}$ and $\tilde{z}$. This system of equations can now be solved by a formal expansion in powers of $h(y)$.

To calculate the energy one must use a gauge-invariant infrared cutoff, e.g. a capillary tube with Ω a rectangle of size $L_x \times L_y$. For tadpole cancellation we must choose opposite tensions on the $x = 0$ and $x = L_x$ walls, and zero tension for the walls at $y = \pm L_y/2$. One must furthermore define $\tilde{\mathcal{E}}$ so as to include the boundary contribution of the pure system. The final answer is:

$$
\begin{aligned}
\tilde{\mathcal{E}}[h] - \tilde{\mathcal{E}}[0] \;&=\; \frac{\gamma}{2}\int_{-\infty}^{\infty} \mathrm{d}\sigma \left[i\tilde{y}_+ \frac{\mathrm{d}\tilde{y}_-}{\mathrm{d}\sigma} + i\tilde{z}_+ \frac{\mathrm{d}\tilde{z}_-}{\mathrm{d}\sigma} - \cos\theta_0\, \tilde{z}\frac{\mathrm{d}\tilde{y}}{\mathrm{d}\sigma} + c.c. \right] \\
&= \gamma \int_0^{\infty} \frac{\mathrm{d}k}{2\pi}\, k \left(|\tilde{y}_k + i\cos\theta_0 \tilde{z}_k|^2 + \sin^2\theta_0\, |\tilde{z}_k|^2 \right) \,.
\end{aligned}
$$

The energy is quadratic in $\tilde{y}$ and $\tilde{z}$, where $y(\sigma) = \sigma + \tilde{y}(\sigma)$ relates the natural to the conformal parameterization of the contact line. As explained before, the problem is non-linear because this change of coordinate depends explicitly on the pinning profile.

The expansion of the energy reads $\tilde{\mathcal{E}}[h] - \tilde{\mathcal{E}}[0] = \sum_{n=2}^{\infty} \tilde{\mathcal{E}}_n$. The leading term is the non-local, linear-elasticity energy of Joanny-de Gennes :

$$\tilde{\mathcal{E}}_2 = \gamma \sin^2\theta_0 \int_0^\infty \frac{\mathrm{d}k}{2\pi}\, k\,|h_k|^2 \quad = \quad \frac{\gamma}{4\pi} \sin^2\theta_0 \iint \mathrm{d}\sigma\,\mathrm{d}\sigma' \,\frac{[h(\sigma)-h(\sigma')]^2}{(\sigma-\sigma')^2}\ .$$

Joanny and de Gennes, 1984

Pommeau and Vannimenus, 1985

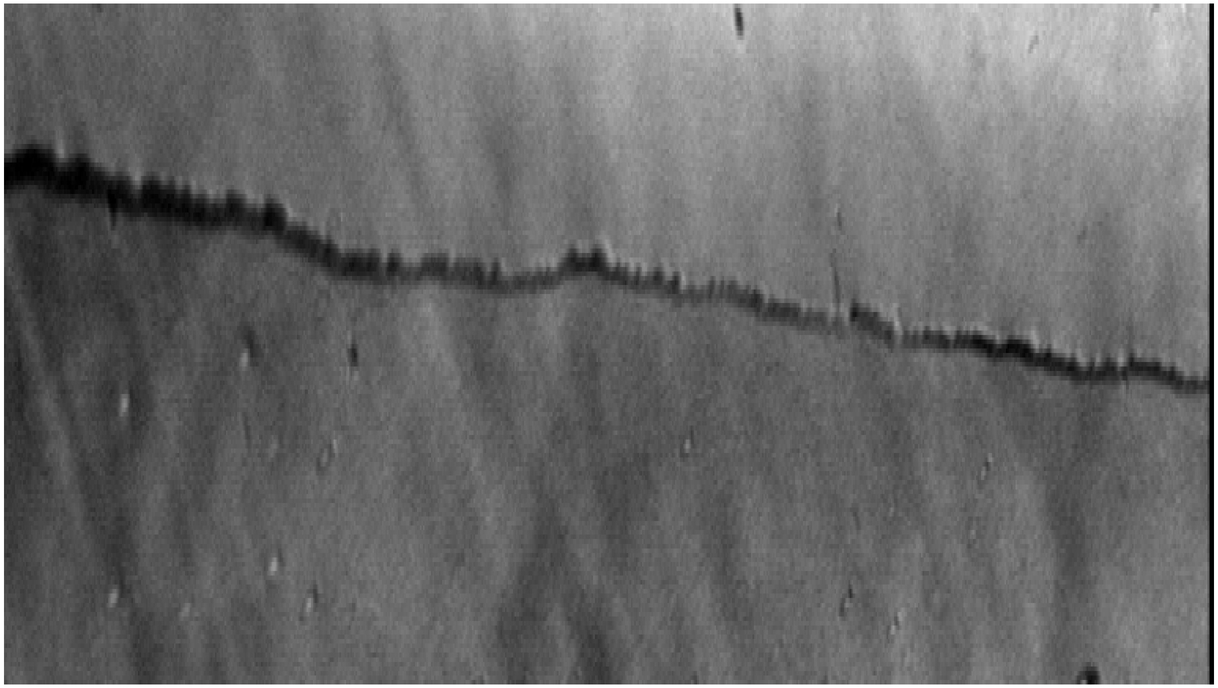
It is invariant under $SL(2,\mathbb{R})$ and arises in many other contexts, e.g. in models of dissipative quantum mechanics.

Caldeira and Legget, 1981

Callan and Thorlacius, 1990

The linear dispersion relation implies that the deformations of the contact line decay and propagate at constant velocity. This behaviour has been experimentally verified by Ondarcuhu and Veyssie.

de Gennes, 1986; Ondarcuhu and Veyssie, 1991



A contact line (above an example of Helium on Cesium at $1.93K$, from *Guthmann, Rolley et al*) is characterized by a roughness exponent : width $\sim$ (length)$^{\zeta}$. The JdG energy $\tilde{\mathcal{E}}_2$ predicts correctly $\zeta = 1/3$ for weak disorder. It seems however to fail at the depinning threshold, where $\zeta_{\text{exp}} = 0.5$ while $\zeta_{\text{th}} = 0.4$. A possible explanation is that one cannot neglect the higher-order terms *(Golestanian and Raphaël, 2002).*

The corrections to the JdG elastic energy break the accidental $SL(2,\mathbb{R})$ symmetry, but preserve the scale covariance of the problem:

$$\tilde{\mathcal{E}}[h^{(\lambda)}] \;=\; \lambda^2\,\tilde{\mathcal{E}}[h] \qquad \text{if} \qquad h^{(\lambda)}(y) \equiv \lambda\, h(\lambda^{-1}y) \quad .$$

These terms can be generated systematically with the help of diagrammatic rules as follows: first integrate out the bulk fields keeping their values on the boundary fixed, then eliminate the field z with the help of a Lagrange-multiplier field that imposes $z = h(y)$. This leads to the variational area functional

$$\tilde{\mathcal{A}}(\alpha, \tilde{y}; h) \;=\; \frac{1}{2}\int_k |k|\,\tilde{y}_k\,\tilde{y}_{-k} \,-\, \frac{1}{2}\int_k \frac{1}{|k|}\,\tilde{\alpha}_k\,\tilde{\alpha}_{-k} \,+\, \int_k \alpha_{-k} H_k \quad ,$$

where $\alpha_k = \cos\theta_0\, 2\pi\delta(k) + \tilde{\alpha}_k$ and H_k is the Fourier transform of $h(\sigma + \tilde{y}(\sigma))$:

$$H_k = h_k + \int ik_1\, h_{k_1}\,\tilde{y}_{k_2} + \frac{1}{2}\int (ik_1)^2 h_{k_1}\,\tilde{y}_{k_2}\,\tilde{y}_{k_3} + \cdots \quad .$$

The integrals in the last expression run over $\sum k_j = k$, consistently with momentum conservation. The reduced energy functional can then be computed by summing tree-level diagrams of this non-local one-dimensional theory.

The result for the cubic and quartic terms, for instance, reads:

$$\tilde{\mathcal{E}}_3 = \gamma \cos\theta_0 \, \sin^2\theta_0 \int h_{k_1} h_{k_2} h_{k_3} \frac{|k_1| k_2 k_3}{|k_3|} \equiv -\gamma \cos\theta_0 \, \sin^2\theta_0 \int h_{k_1} h_{k_2} h_{k_3} \, k_1 k_2 \, \Theta(k_1 k_2)$$

$$\tilde{\mathcal{E}}_4 = \frac{\gamma}{2} \int h_{k_1} h_{k_2} h_{k_3} h_{k_4} \, k_1 k_2 k_3 k_4 \left[\sin^4\theta_0 \left\{ \frac{\Theta(k_1 k_2)\Theta(k_3 k_4)}{|k_1 + k_2|} + \frac{2k_1}{|k_1|} \frac{\Theta(k_3 k_4)}{(k_3 + k_4)} \right\} \right.$$

$$\left. + \sin^2\theta_0 \cos^2\theta_0 \left\{ \frac{k_1 k_4}{|k_2 k_3 k_4|} + \frac{{k_2}^2 - {k_1}^2}{|k_1||k_4||k_1 + k_2|} \right\} \right] .$$

This expansion is potentially both UV and IR divergent. Ultraviolet finiteness can be established if and only if $k^2 h_k \to 0$ as $k \to \infty$. Thus the contact line must be continuously differentiable (this excludes in particular kinks and cusps). The result follows by a scaling argument, using the scale covariance of p. 24.

Infrared finiteness is subtler, because individual diagrams diverge. The necessary and sufficient requirement is that $kh_k \to 0$ as $k \to 0$, i.e. h must vanish at $y \to \pm\infty$. The proof can be established iteratively, from the perturbative expansion of the coupled equations in p. 20.

Finally, the Legendre transformation of the energy $\tilde{\mathcal{E}}[h]$ is manifestly finite for any sufficiently-smooth and localized source $\delta\gamma'$.

This shows that a decoupled, quasilocal perturbation theory can be indeed defined at scales between the container size and the atomic scale, as claimed.

Interaction of two contact lines

To illustrate the IR decoupling explicitly, consider the interaction of two contact lines on parallel walls at $x = 0$ and $x = L$, and with opposite solid-fluid tensions (barring disorder). The equilibrium surface is thus an inclined plane, with contact angles $\pm\theta_0$. Letting $h_1(y)$ and $h_2(y)$ be the deformed contact lines, one finds the following leading-order energy :

$$\tilde{\mathcal{E}}_2^{\,\text{strip}} = \gamma \sin^2\theta_0 \int_0^\infty \frac{\mathrm{d}k}{2\pi}\, k \Bigg(\left(|h_{1,k}|^2 + |h_{2,k}|^2\right) \frac{\cosh(kL/\sin\theta_0) - 1}{\sinh(kL/\sin\theta_0)} + |h_{1,k} - h_{2,k}|^2 / \sinh(kL/\sin\theta_0) \Bigg) \; .$$

For wavevectors $k \gg \sin\theta_0/L$ the interaction decays exponentially :

$$\tilde{\mathcal{E}}_2^{\,\text{strip}} \simeq \tilde{\mathcal{E}}_2(h_1) + \tilde{\mathcal{E}}_2(h_2) + \mathcal{O}\left(\exp(-2kL/\sin\theta_0)\right) \; .$$

In the opposite limit of a thin strip, or short-wavelength deformations:

$$\tilde{\mathcal{E}}_2^{\text{strip}} \simeq \gamma \sin\theta_0 L \int_0^\infty \frac{\mathrm{d}k}{2\pi} \left[|h_{1,k} - h_{2,k}|^2 \left(\frac{\sin^2\theta_0}{L^2} - \frac{k^2}{6} \right) \;+\; \frac{k^2}{2} \left(|h_{1,k}|^2 + |h_{2,k}|^2 \right) \;+\; O(k^4) \right] \;.$$

The leading term is proportional to the increase in area of a planar strip, whose boundaries undergo a relative displacement $h_1 - h_2$ along the walls, with which it made initially an angle θ_0. If $h_1 = h_2$, the next term in the above energy is that of an elastic rod with effective tension $\gamma_{\text{eff}} = \gamma L \sin\theta_0$. This has also a simple geometric interpretation: The rod is a thin surface strip of width $L/\sin\theta_0$, which is deformed by an amount $h_1(y)\sin\theta_0$ in the transverse direction.

The exact expression for $\tilde{\mathcal{E}}_2^{\text{strip}}$ thus extrapolates continuously between local elasticity and the JdG behavior of the contact line.

A capillary wormhole

Perturbation theory breaks down for $h'(y) \sim o(1)$. One potential cause are coordinate singularities, which can be seen to arise in numerical solutions of the coupled equations for $\tilde{y}$ and $\tilde{z}$. Indeed, the choice of proper-time gauge amounts to setting $f \propto g^{-1}$ in the Weierstrass formula. This is globally allowed as long as g does not develop zeros in the upper half plane.

Besides coordinate singularities, capillary minimal surfaces can exhibit many gauge-invariant non-perturbative phenomena. These include topology change, continuous and discontinuous phase transitions, and a geometrical obstruction known as the wedge phenomenon.

see R. Finn, Notices of the AMS 46

This can be understood as follows:

Integrate the bulk equation over some subregion $\Omega' \subset \Omega$:

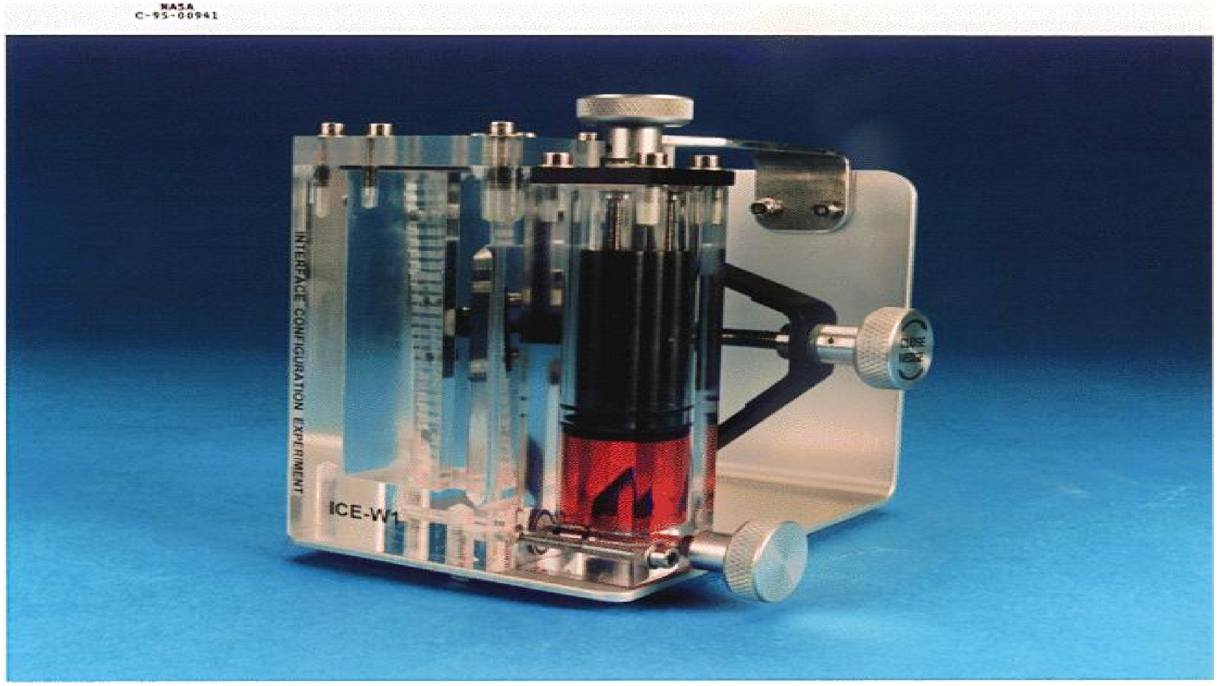
$$\gamma\,|\partial\Omega'_{\text{interior}}| \;\geq\; \left|\gamma \int_{\partial\Omega'_{\text{interior}}} \frac{\widehat{n}\cdot\vec{\nabla}z}{\sqrt{1+|\vec{\nabla}z|^2}}\right| \;=\; p\,|\Omega'| \;+\; \gamma'\,|\partial\Omega'_{\text{exterior}}|\;.$$

Take Ω' to be an infinitesimal triangle of opening angle 2α. Then there is no solution to the problem if

$$\gamma \sin\alpha < \gamma' = \gamma \cos\theta_0$$

In this case the contact line cannot close locally, and the minimal surface is forced to develop a second sheet !

This is a local obstruction similar to a wormhole, in that it forces the fluid surface to develop (mathematically) a second infinite sheet. In NASA's ICE experiment, the fluid could be seen to creep up the wedge and out of the tube.



Capillary tubes from the Interface Configuration Experiment *(Concus, Finn and Weislogel, 1995)*. One of NASA's motivations was the need to understand fuel behavior in low-gravity conditions inside reservoirs.

The wedge and other non-perturbative phenomena pose a threat to the UV/IR decoupling of the contact-line theory, since the microscopic structure of $\partial\Omega$ can affect the global properties of the fluid surface. From the mathematical viewpoint the problem lies with the Legendre transformation: indeed, in the pure Dirichlet problem small-scale fluctuations can be neglected thanks to the scaling property of $\tilde{\mathcal{E}}[h]$, even if the perturbative series does not converge. In the transformed variables, on the other hand, perturbation theory breaks down.

From the physical point of view, these divergenences are presumably cured by locality in field (target) space: normal roughening will never produce a wedge extending all along the z direction. 'Small perturbations of the (x, y) loop $\partial\Omega$ must be localized in the z direction.

Should we be thinking about string-perturbation theory that is 'local' in dilaton, moduli etc space?

Conclusions

- Capillarity and wetting phenomena are complex phenomena of great interest for engineering and applied physics. Many of their aspects are still ill-understood.

- Capillarity is a geometric theory, which may hold lessons for theories of gravity.

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