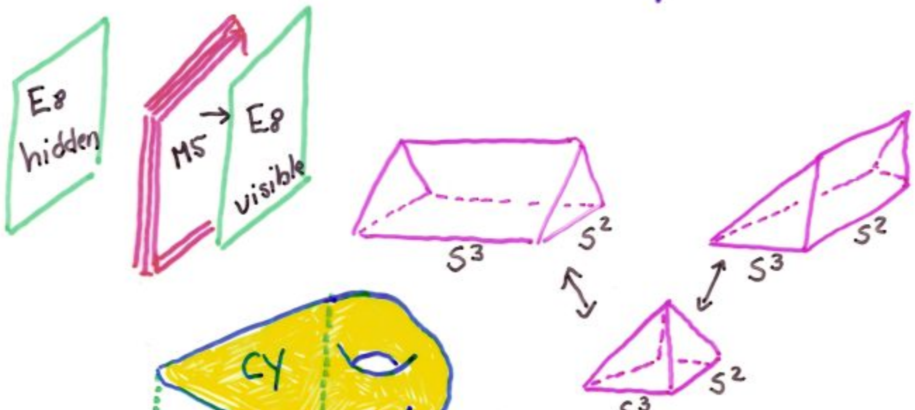
INFLATION, MODULI STABILIZATION AND STRING THEORY



MELANIE BECKER
PARIS, DECEMBER 06

INFLATION, MODULI STABILIZATION AND STRING THEORY

For many years string theory was considered to be inaccesible to experimental observations:

* Energy scales are far too high
* Moduli space problem of string theory

Many string theorists have high expectations that this situation is about to change:

* LHC (particle phenomenology)
* New cosmological data

From the theoretical side, the progress made in string theory for both particle physics and cosmology has one common origin FLUX COMPACTIFICATIONS!

* Moduli stabilization
* Lowering of the string scale

OVERVIEW

① Particle Phenomenology : hunting for the standard model

* Leading candidate approaches
* Some simple flux compactifications

② String theory and cosmology

* An inflationary model in heterotic M-theory
* Cosmic strings

REFERENCES : * K. Becker, M.B, J. Walcher, C. Vafa hep-th/0611001

* K. Becker, MB, X. Fu, L. Tseng + S.T. Yau, hep-th 0604137
* MB, Tseng, S.T Yau, to appear
* K. Becker, M.B, A. Krause hep-th/0510066
hep-th/0501130

HUNTING FOR THE STANDARD MODEL

Ever since the discovery of Calabi-Yau compactifications in the mid 1980's string theorists have tried to make contact with susy GUT theories and the MSSM

This turned out to be a very difficult question mainly because of 2 reasons:

(1) * Particle content + gauge group :

Just to reproduce the particle content and gauge group is challenging by itself.
In all models "exotic particles" emerge.

Encouraging news have come recently from B. Ovrut and collaborators as we shall see.

(2) * Moduli space problem : loss of predictive power.

WHAT ARE THE CURRENT LEADING MODELS AIMING TO REPRODUCE THE MSSM?

Traditionally this question has been addressed in the context of the $E_8 \times E_8$ heterotic string. However, our current understanding of string dualities allows us to search for an answer also in the Type II context

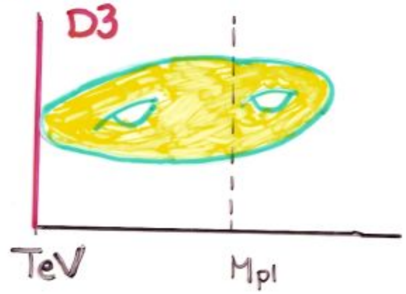
① INTERSECTING D-BRANE MODELS

Orbifolds like $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ and branes wrapping cycles of these orbifolds and intersecting at angles are used to construct "semirealistic" standard like models. (exotic particles!)

Cvetic, Shiu, Uranga, Lüst, Ibañez, ...

② D3-BRANE AT A DEL-PEZZO SINGULARITY

A standard like model can be constructed from a quiver gauge theory that lives on a D3-brane localized on a del Pezzo-8 singularity of a Calabi-Yau manifold.
The resulting theory has an extended Higgs sector



H. Verlinde, Wijnholt

These are rather interesting Type II models.
I shall focus on the more traditional heterotic approach...

③ THE HETEROTIC STRING

Encouraging news from $E_8 \times E_8$ heterotic string (Ovrut, Braun, Donagi, Pantev ....)

- * Standard model gauge group
- * 3 families of quarks + leptons each containing a right handed neutrino
- * No "exotic" matter
- * 6 geometric moduli and a small number of vector bundle moduli

$$E_8 \to \underset{\substack{\uparrow \\ \text{gauge instanton}}}{SU(4)} \oplus \underset{\substack{\uparrow \\ \text{massive neutrinos}}}{Spin(10)}$$

The model: $E_8 \times E_8$ heterotic string

compactified on a Calabi-Yau 3-fold (elliptic fibration over del Pezzo surface)

DRAWBACK: No predictions for coupling constants, as the internal manifold is CY.

$\Rightarrow$ FLUX COMPACTIFICATIONS ?

A SIMPLE HETEROTIC FLUX COMPACTIFICATION

Flux compactifications can lift many (if not all) the moduli fields.
Such compactifications are crucial not only for particle phenomenology but also for cosmology....

GOAL:
- What is a flux compactification?
- Consider a simple example

TORSIONAL CONSTRAINTS

A. Strominger 1980's

More "exotic" type of manifolds can be considered that are not Calabi-Yau but non-Kähler manifolds with torsion.

$$H_{MNP} = \text{Torsion}$$

Demanding $N=1$, $D=4$ allows us to derive the constraints that these manifolds satisfy.

In the 10D string frame the action for the heterotic string is

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{2} H^2 - \frac{\alpha'}{4} \operatorname{tr} F^2 \right)$$

$$H = dB + \frac{\alpha'}{4} \Omega(A) \; ; \; \Omega(A) = \operatorname{tr}\left(A \wedge dA - i \frac{2}{3} A \wedge A \wedge A \right)$$

$$F = dA - i\, A \wedge A$$

Susy transformations :

$$[\gamma^N, \gamma^P] = 2\gamma^{NP}$$

$$\delta\psi_M = \nabla_M \eta + \frac{1}{8} H_{MNP} \gamma^{NP} \eta$$

$$\delta\lambda = \gamma^M \partial_M \phi\, \eta + \frac{1}{12} H_{MNP} \gamma^{MNP} \eta$$

$$\delta\chi = \gamma^{MN} F_{MN} \eta$$

$$\{\gamma_{MN}, \gamma^P\} = 2\gamma_{MN}{}^{P}$$

supersymmetric configuration : $\delta(\text{fermions}) = 0$

$\Rightarrow$ constraints on internal geometry

To evaluate these constraints decompose the spinor and all fields into internal and external components.

METRIC :

Calabi-Yau

$$g_{MN}(x,y) = e^{2\phi}\begin{pmatrix} g_{mn}(y) & 0 \\ 0 & g_{\mu\nu}(x) \end{pmatrix}$$

Einstein frame

$e^{2\phi(y)}$: for torsional manifold. (warp factor)
Depends only on internal coordinates

Other fields are decomposed too.
Also the spinor :

$$\eta(x,y) = \underset{\substack{\uparrow \\ \text{4D spinor}}}{\varepsilon(x)} \otimes \underset{\substack{\uparrow \\ \text{6D spinor}}}{\eta(y)}$$

Forms that entirely characterize the background geometry :

$$J^{(1,1)}_{mn} = -i\,\eta^{+}\gamma_{mn}\eta \quad \text{hermitian form}$$

$$\Omega^{(3,0)}_{mnp} = e^{-2\phi}\,\eta^{T}\gamma_{mnp}\eta \quad \text{holomorphic 3-form}$$

CALABI-YAU	TORSIONAL GEOMETRY
$dJ = 0$	$d(\lVert\Omega\rVert^2 J\wedge J) = 0$
Kähler condition	conformally balanced
$d\Omega^{(3,0)} = 0$	$d\Omega^{(3,0)} = 0$
$H = 0$	$H = i(\bar{\partial}-\partial)J$
$\phi = \text{const}$	$e^{-2(\phi-\phi_0)} = \lVert\Omega\rVert$

HERMITIAN YANG-MILLS :

$F_{2,0} = F_{0,2} = 0$ — holomorphic

$F_{mn}J^{mn} = 0$ — primitive

ANOMALY CANCELLATION :

Gives crucial information for torsional manifolds

$$dH = 2i\,\partial\bar{\partial}J = \frac{\alpha'}{4}\left(\mathrm{tr}\,R\wedge R - \mathrm{tr}\,F\wedge F\right)$$

* Relates gauge bundle with geometry

* For Calabi-Yau: $H=0$ "standard embedding"

  Torsional manifolds: $dH \neq 0$ no standard embedding.

  This will give very interesting gauge symmetry breaking patterns

* Anomaly cancellation is a highly non-linear differential eqn for the dilaton. The existence of a solution was recently proven by Fu and Yau.

* Torsional manifolds are almost unexplored territory for mathematicians.

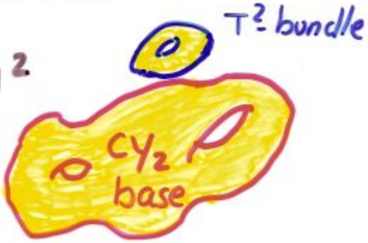
A SMOOTH COMPACT EXAMPLE : FSY GEOMETRY

Fu, Strominger + Yau

Smooth compact examples that solve the torsional constraints have been called "FSY geometries"

The model we constructed is a T^2 bundle over a Calabi-Yau 2-fold base

$$ds^2 = e^{2\phi(y)} \underbrace{ds_S^2}_{\text{base}} + |\underbrace{dz}_{\text{torus coordinate}} + \underbrace{\alpha(y)}_{\text{twist}}|^2$$



Introduce :

$$\theta = dz + \alpha(y)$$

$$d\theta = \omega = \omega_S^{(2,0)} + \omega_A^{(1,1)}$$

This characterizes the geometry

Hermitian form

$$J = e^{2\phi} J_S + \frac{i}{2}\, \theta \wedge \bar{\theta}$$

The manifold is conformally balanced if θ is primitive wrt the base

$$J_S \wedge \theta = 0$$

Holomorphic 3-form

$$\Omega = \Omega_S \wedge \theta \qquad \text{is closed}$$

ANOMALY CANCELLATION

We have imposed susy constraints but not anomaly cancellation:

* Base is K3 or T4
* Infinite number of solutions $\frac{\omega}{2\pi\sqrt{\alpha'}} \in H^2(S, \mathbb{Z})$
* Dilaton is not determined

Anomaly cancellation has something to say about all 3 statements

$$2i\,\partial\bar{\partial}J = \frac{\alpha'}{4}\left(\mathrm{tr}\,R\wedge R - \mathrm{tr}\,F\wedge F\right) = (*)$$

① Allows us to exclude a T^4 base

$$\int_X J\wedge(*) \quad \Rightarrow$$

$$\int_X \mathrm{tr}\,R_S\wedge R_S\wedge J = \frac{2}{\alpha'}\int_X \overbrace{e^{-4\phi}\left(\|\omega_S\|^2+\|\omega_A\|^2\right)J^3}^{>0}$$

$$- \underbrace{\int_X \mathrm{tr}\,F\wedge *F}_{>0}$$

$\Rightarrow$ T^4 not allowed

② Now integrate $\int_{K3}(*)$ to get a topological constraint

first Pontryagin

$$-\frac{P_1(F)}{2} + \int_{K3}\frac{1}{\alpha'}\left(\|\omega_S\|^2+\|\omega_A\|^2\right)\frac{J_S^2}{2} = 24$$

Only finite number of ω's allowed !

③ Fu + Yau existence theorem :

Anomaly cancellation gives a highly non-linear second order differential equation for the dilaton :

$$D_2(\phi) = \psi \leftarrow \text{source term}$$

$$\psi = \frac{1}{\alpha'} \left(\|w_S\|^2 + \|w_A\|^2 \right) J_S^2 - \frac{1}{2} \left(\mathrm{tr}\, R_S \wedge R_S - \mathrm{tr}\, F \wedge F \right)$$

Integrability condition :

$$\int_{K3} \psi = 0 \qquad \text{(topological constrain)}$$

$\Rightarrow$ Differential eqn is elliptic

Number of solutions is finite.

ANOMALY CANCELLATION

~~HERMITIAN YANG MILLS EQUATION~~

Topological constrain:

$$-\frac{P_1(F)}{2} + \frac{1}{\alpha'} \int_{K3} \left(\|\omega_S\|^2 + \|\omega_A\|^2 \right) \frac{J_S^2}{2} = 24$$

In general the ω's will be combinations of self-dual and anti-self dual 2-forms of K3:

$$\omega_S = m\, \Omega_S \leftarrow \text{holomorphic 2-form}$$

$$\omega_A = \sum_{I=1}^{19} m^I K_I \leftarrow \text{19 anti-self-dual 2-forms}$$

GAUGE BUNDLE

The gauge bundle needs to satisfy the hermitian Yang-Mills eqn:

$$F_{2,0} = F_{0,2} = 0 \qquad F_{m\bar{m}} J^{m\bar{m}} = 0$$

Stable bundles on X can be obtained by lifting up stable bundles on K3.

Stable bundles on K3 are very well understood mostly due to the work of Mukai.

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr}\, R\wedge R - \mathrm{tr}\, F\wedge F\right)$$ Anomaly cancellation

No standard embedding! Consider stable $SU(N)$ bundles:

$N=2$ $\quad E_8 \to SU(2) \otimes E_7$

$N=3$ $\quad E_8 \to SU(3) \otimes E_6$

$N=4$ $\quad E_8 \to SU(4) \otimes \boxed{SO(10)}$

$N=5$ $\quad E_8 \to SU(5) \otimes \boxed{SU(5)}$

GUT groups!

The model is still a toy model (zero # of generations! ⇒ Del Pezzo base (in progress)

MODULI FIELDS OF HETEROTIC FLUX COMPACTIFICATIONS (in progress)

What are the moduli fields of heterotic flux compactifications?
Answer: all deformations of the hermitian form J and the holomorphic 3-form Ω that leave invariant

* Conformally balanced condition
* Hermitian Yang-Mills
* Anomaly cancellation

Ideally the moduli fields become massive as a superpotential in terms of the flux is generated:

$$W = \int (H + i\,dJ) \wedge \Omega$$ SUPERPOTENTIAL

This might be expected by analogy with Type IIB:

$$\boxed{W = \int G \wedge \Omega}$$

Becker², Vafa + Walcher

HETEROTIC M-THEORY COSMOLOGY

POWER LAW INFLATION

Lucchin, Matarrese (1985)

Inflation is defined as a period in the evolution of our universe during which the scale factor of the 4D FRW universe

$$\underset{\uparrow \text{ time}}{\ddot{a}(t)} > 0$$

Such a solution can be realized in the presence of a single scalar field φ (inflaton) with a potential

$$\boxed{U(\varphi) = U_0 \, e^{-\sqrt{\frac{2}{p}} \frac{\varphi}{M_{Pl}}}}$$

POWER LAW

This leads to a power law solution

$$a(t) = a_0 t^p \quad ; \quad p>1 \text{ inflation (shallow potentials)}$$

For power law inflation the inflaton behaves as

$$\varphi(t) = \sqrt{2p}\ M_{pl} \log\left(\sqrt{\frac{U_0}{p(3p-1)}}\ \frac{t}{M_{pl}}\right)$$

Also valid for $1/3 < p < 1$ but then not inflationary.

Power law inflation has simple (constant) slow roll parameters :

* $\varepsilon = \frac{M_{pl}^2}{2}\left(\frac{U'}{U}\right) = \frac{1}{p} < 1$

* $\eta = M_{pl}^2\ \frac{U''}{U} = \frac{2}{p} < 1$

Constant slow roll parameters mean that there is no exit from inflation
When embedded into M-theory this presents no problem ....
Additional exponential contributions will guarantee for an exit ...

We are interested in heterotic M-theory, so that after reheating we can eventually make contact with the MSSM (as discussed before).

HETEROTIC M-THEORY AND OPEN MEMBRANE INSTANTONS

In heterotic M-theory we compactify

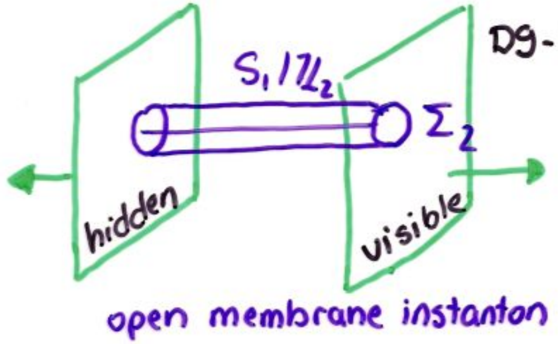
$$\frac{\text{M-theory}}{\underset{\text{warped}}{\underset{\uparrow}{M_6 \otimes}} \; \underbrace{S_1/\mathbb{Z}_2}_{}} \quad \leftarrow \text{interval}$$

Take M_6 to be a Calabi-Yau 3-fold $h_{11}=1$ (1 Kähler modulus) for illustrative purposes...

In principle M_6 would be a torsional manifold

BOUNDARIES OF SPACE-TIME

Compactifying the 11D space-time on $S_1/\mathbb{Z}_2$ means that it has 2 boundaries (fixed planes):



Becker[2] + Strominger

Open membrane instantons cause the boundaries to repell each other

$$W = h\, e^{-T}$$ ← volume of the instanton

Field is not canonically normalized!

$$\varphi_T = M_{Pl}\sqrt{\frac{3}{2}}\,\log\,(T+\bar{T})$$

Potential is a double exponential, too steep!

ASSISTED INFLATION (Liddle, Mazumdar, Schunk 1998)

This type of inflation is based on N scalar fields

$$U = U_0 \, e^{-\sqrt{\frac{2}{p}} \frac{\varphi_i}{M_{Pl}}} \qquad i = 1, \dots, N$$

The eom have a late time attractor

$$\boxed{\varphi_1 = \varphi_2 = \cdots = \varphi_N}$$

The multifield problem can then be mapped to a single field problem which resembles power law inflation:

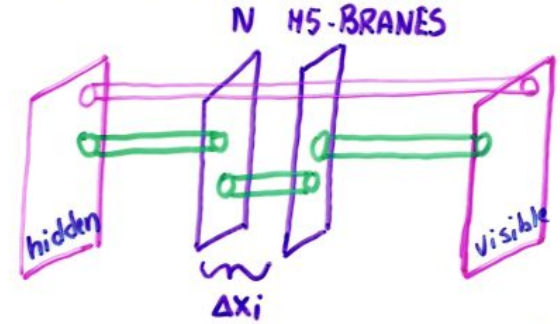
$$a(t) = a_0 \, t^{P(N)} \quad \text{with} \quad p(N) = N \cdot p$$

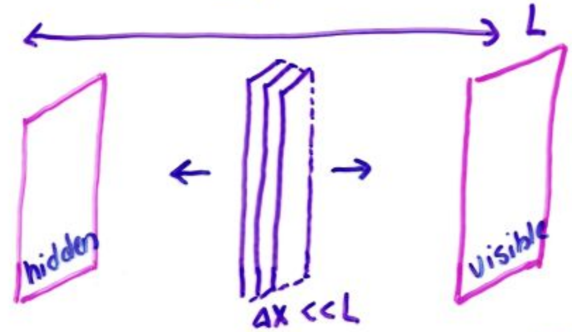
This leads to slow roll inflation for large enough N

$$P(N) > 1$$

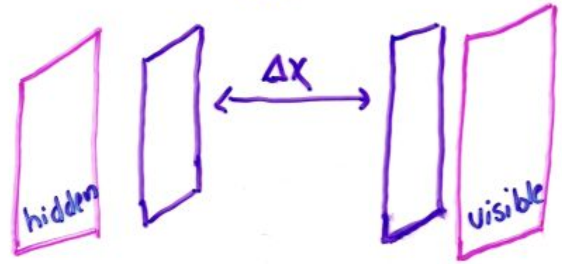
Even though a single exponential is too steep to support slow roll inflation, for large enough N (many exponentials), ε, η are small.

M5-BRANES IN HETEROTIC M-THEORY





inflationary era



inflation ends

$\Delta x \approx \Delta L$

COSMOLOGICAL DATA

The mapping to assisted inflation is possible

$$\boxed{19 < N < 195}$$

We can compare with recent ~~as~~ cosmological data within this range:

SPECTRAL INDEX :

Quantum fluctuations of the inflaton result in a spectrum of density perturbations, which explains the large structure of our universe.

According to the most recent data (astro-ph/0407372, Seljak et. al)

$$\boxed{n_{exp} = 0.98 \pm 0.02}$$

Can be realized with $\boxed{N \approx 90}$ M5-BRANES

NUMBER OF E-FOLDS : $N_e \approx 345$