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IHP, 2006

Extended hybrid inflationary models

Partly based on works in collaboration with
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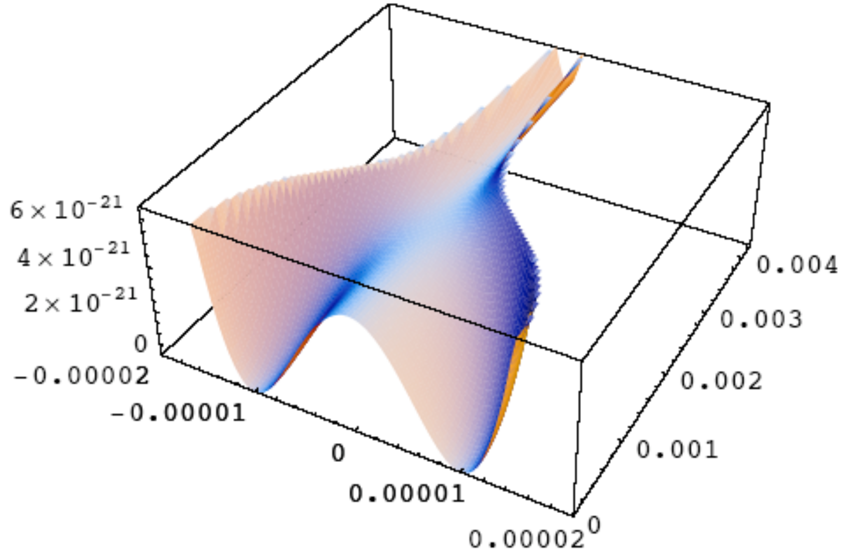
PRD**67** 121301, PRD**69** 063520, PRD**71** 063529, astro-ph/0604200

Hybrid inflation (Linde '93)

$$V(\varphi,\sigma) = V(\varphi) + \frac{\mu}{2}\left(\sigma^2 - \sigma_0^2\right)^2 + \frac{g}{2}\sigma^2\varphi^2$$



- For vev of fieds: vev of φ can be much smaller than Planck mass.
- From high energy physics BSMs, global and local susy, and superstrings (brane/antibrane collisions)

Global susy, in 1 page

- Ingredients:
  - Supermultiplet, minimum field content is one complex scalar field and one Majorana fermion (Wess Zumino model)
  - Superpotential, $W(\phi_i)$, Fayet-Iliopoulos term, ξ, charges, q's and coupling constant, g.
  - That leads to $\mathcal{L}_\psi = \mathcal{L}_{\text{cin.}} + \mathcal{L}_\psi - V_D - V_F - V_{1-boucle}$

$$\mathcal{L}_\psi = \overline{\psi}_i \frac{\partial^2 W}{\partial\phi_i \partial\phi_j} \psi_j \qquad V_F = \sum_i \frac{\partial W}{\partial \phi_i} \left(\frac{\partial W}{\partial \phi_i} \right)^* \qquad V_D = \frac{g^2}{2} \left(\sum_i q_i \phi_i \phi_i^* + \xi \right)^2$$

  - If susy is broken (Coleman-Weinberg formula),

$$V_{1-\text{boucle}} = \frac{1}{32\pi^2} \left(\sum_b m_b^2 - \sum_f m_f^2 \right) + \frac{1}{64\pi^2} \left[\sum_b m_b^4 \left(\log \frac{m_b^2}{\Lambda^2} - \frac{3}{2} \right) - \sum_f m_f^4 \left(\log \frac{m_f^2}{\Lambda^2} - \frac{3}{2} \right) \right]$$

F- and D-term hybrid models

- SUSY model from F-term (Dvali, Shafi Schaefer '94)

Superpotential: $$W = -\lambda \mu^2 S + \lambda S \Phi_+ \Phi_-$$

mass parameter

Dimensionless parameter

Chiral superfields

- SUSY model from D-term with nonzero g and ξ (Binétruy et Dvali '96)

Superpotential: $$W = \lambda S \Phi_+ \Phi_-$$

Dimensionless parameter

Chiral superfield with no charge

Superfields with charges +1 and -1

- F term inflation potentials

$$V = \lambda^2 \mathcal{S}^2 \left(|\eta|^2 + |\overline{\eta}|^2\right) + |\frac{1}{2}\lambda(\eta^2 - \overline{\eta}^2) - \mu^2|^2$$

$$V_{\text{infla.}} = \lambda^2 \mathcal{S}_c^4 \left(1 + \frac{\lambda^2}{16\pi^2} \log \frac{\mathcal{S}}{\mathcal{S}_c}\right)$$

$$\mu^2 \leq 10^{-6} \left(\frac{50}{N_e}\right)^{1/2}$$

- D-term inflation potentials

$$V = V_F + V_D = \lambda^2 |\mathcal{S}|^2 \left(|\overline{\phi}|^2 + |\phi|^2\right) + \lambda^2 |\overline{\phi}\phi|^2 + \frac{g^2}{2}\left(|\phi|^2 - |\overline{\phi}|^2 + \xi\right)^2$$

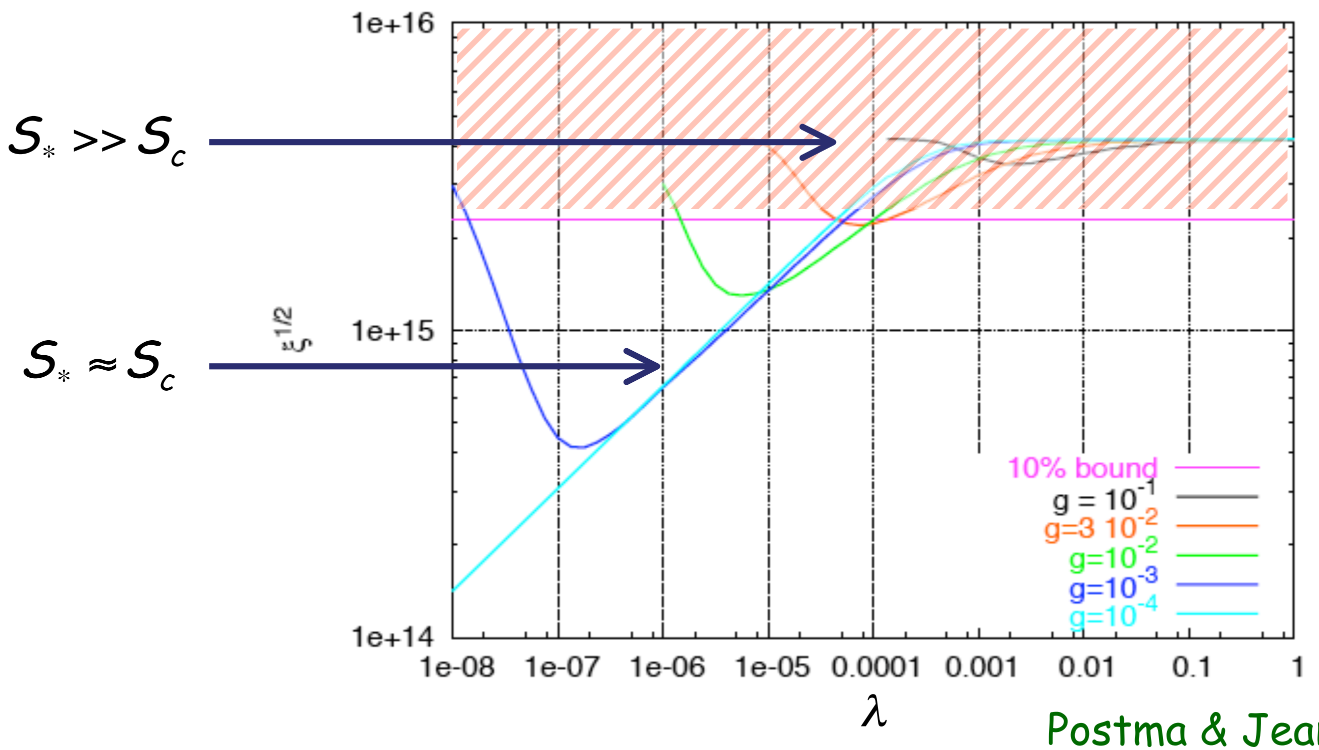
$$V_{\text{infla.}} = \frac{\lambda^4 \mathcal{S}_c^4}{2g^2} \left(1 + \frac{g^2}{8\pi^2} \log \frac{\mathcal{S}}{\mathcal{S}_c}\right)$$

$$\xi \leq 10^{-6} \left(\frac{50}{N_e}\right)^{1/2}$$

- End of inflation leads to cosmological defects (strings) of linear energy density μ or ξ

Constraints from cosmic-strings (D-term inflation) including Sugra corrections

FI-term $\sqrt{\xi}$ as function of λ



Postma & Jeannerot '06

Extensions ?

- Within global Susy (more fields):
  - Curvaton type models (Lyth & Wands '01 +…)
  - Multiple field inflations (FB in preparation)

- Within local Susy (Sugra)
  - Modulated inflation (Dvali, Gruzinov and Zaldarriaga '03; Kofman '03; FB, Kofman, Uzan '04)

Susy extensions

- What is happening if field content is extended ?

$$W = -\mu_i^2 S_i + \nu_i S_i^3 + \lambda_j \left(\alpha_i S_i\right)\left(\overline{\Phi}_j \Phi_j\right)$$

with only cubic terms

- Interesting cases are (in context of D-term inflation)
  - $W = \lambda S \overline{\Phi} \Phi + \mu^2 C$ curvaton type model
  - $W = \nu_i S_i^3 + \lambda \left(\alpha_i S_i\right) \overline{\Phi} \Phi$ multiple-field inflation

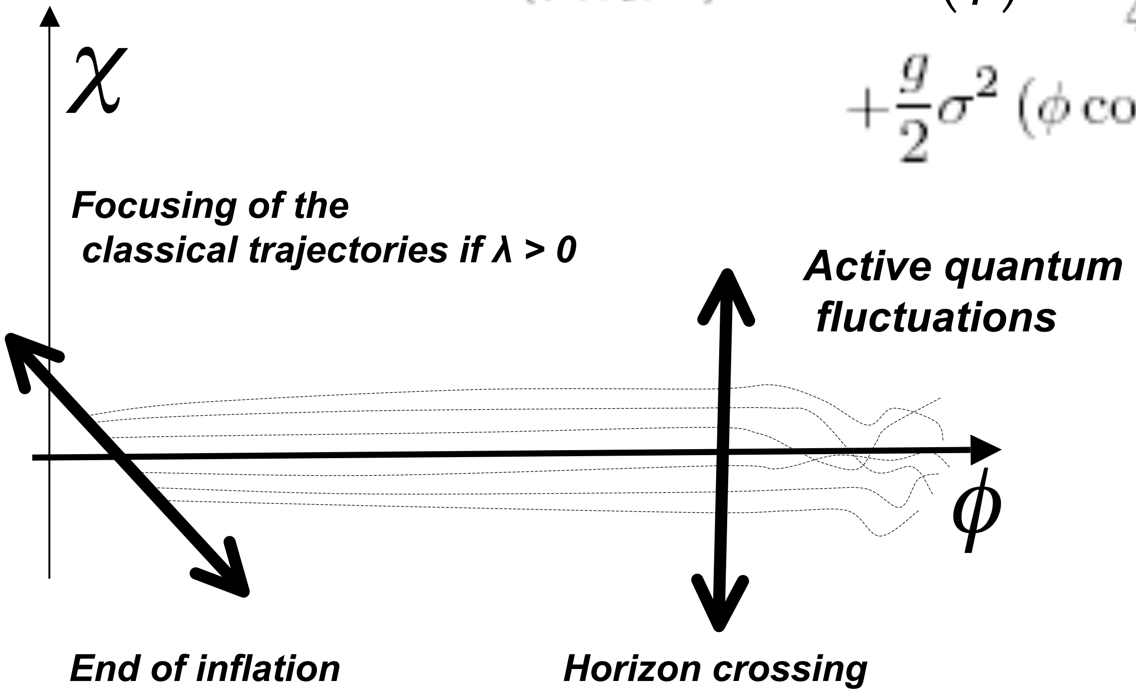
- Last example leads to a model of the form (FB & Uzan '03)

The inflaton

A light transverse field

A massive transverse field that eventually undergoes a phase transition

$$V(\phi,\chi,\sigma) = V(\phi) + \frac{\lambda}{4!}\chi^4 + \frac{\mu}{2}\left(\sigma^2 - \sigma_0^2\right)^2 + \frac{g}{2}\sigma^2\left(\phi\cos\alpha + \chi\sin\alpha\right)^2$$



Inflationary period stops at a time that depends on both the φ and χ values

Mode transfers

$$R \approx \int H\,\mathrm{d}t \approx \left. -\frac{3H^2}{V_{,\phi}}\right|_{\text{Horizon crossing}} \delta\phi \left. -\frac{3H^2}{V_{,\phi}}\right|_{\text{End of inflation}} \frac{\sin\alpha}{\cos\alpha}\,\chi$$

Standard adiabatic fluctuations

Transfer of isocurvature modes

Multiple field inflation

- You can have significant self coupling in transverse directions because slow-roll conditions do not apply: naïvely λ can be as large as unity
  - Is the mass protected against radiative corrections ?
  - What are the effects of quartic self-coupling on the statistical properties of the metric perturbations ?

  - ***Quantum field theory of a test field in (quasi) de sitter space time beyond linear theory…***

Quantum fields in de Sitter space

- de Sitter space $\mathrm{d}s^2 = \frac{1}{(H\eta)^2}\left(-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) \quad \eta = -\frac{1}{H}\mathrm{e}^{-Ht}$

$-\infty < \eta < 0^+$

- Quantification of scalar field

$$\widehat{v}_0(\mathbf{x},\eta) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\left[v_0(k,\eta)\widehat{b}_{\mathbf{k}}\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} + v_0^*(k,\eta)\widehat{b}_{\mathbf{k}}^{\dagger}\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$v_0'' + \left(k^2 - \frac{2}{\eta^2} - \cancel{\frac{m^2/H^2}{\eta^2}}\right)v_0 = 0 \quad \Longrightarrow \quad v_0(k,\eta) = \left(1 - \frac{\mathrm{i}}{k\eta}\right)\frac{\mathrm{e}^{-\mathrm{i}k\eta}}{\sqrt{2k}}$$

- Perturbation theory: the In-In formalism (Weinberg '05)

$$\mathcal{Q}(t) = (\mathcal{U}^{(I)})^{-1}(t,t_i)\,\mathcal{Q}^{(0)}(t)\,\mathcal{U}^{(I)}(t,t_i)$$

$$\mathcal{U}^{(I)}(t,t_i) = \left[\mathcal{T}\exp\left(-\mathrm{i}\int_{t_i}^{t} H^{(I)}\mathrm{d}t\right)\right]$$

- Thus we have:

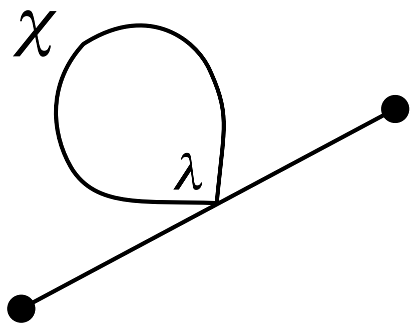
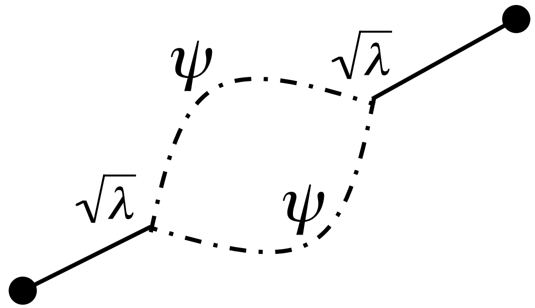
$$\langle \chi_{\mathbf{k}_1}\ldots\chi_{\mathbf{k}_n}\rangle \equiv \langle 0|U^{-1}(\eta_0,\eta)\;\chi_{\mathbf{k}_1}\ldots\chi_{\mathbf{k}_n}\;U(\eta_0,\eta)|0\rangle$$

Free vacuum

Radiative corrections to scalar mass

(Brunier, FB, Uzan PRD, hep-ph/0412186)

- Case of a scalar field imbedded in a chiral super-multiplet (Wess-Zumino model)

$\delta m^2 =$  $+$ 

$$\delta m^2 = \delta m_{\rm B}^2 + \delta m_{\rm F}^2 = \frac{3\lambda\ H^2}{2\pi^2}\ln\left(\frac{\Lambda}{\mu}\right) + \frac{\lambda\ H^2}{\pi^2}\ln\left(\frac{\mu}{\Lambda_{\rm IR}}\right)$$

$$\Rightarrow m_{eff.}^2 = \lambda H^2$$

"classical IR divergence"

Self-coupled scalar field in de Sitter space time

- The motion equation of a test scalar field is the following,

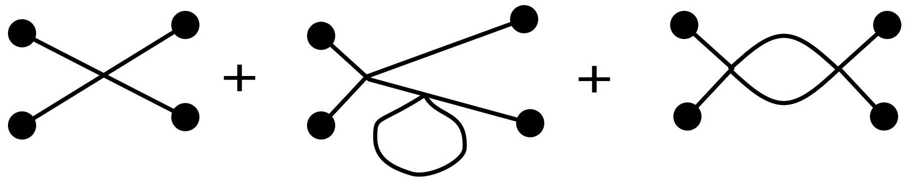
$$\ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\Delta\chi = -V'$$

Negligible at super-Hubble scales

Non-linear source term.

- For a quartic potential, the first non trivial high-order correlator is the fourth

$$\langle \chi(\mathbf{k}_1)\ldots\chi(\mathbf{k}_4)\rangle_c =$$



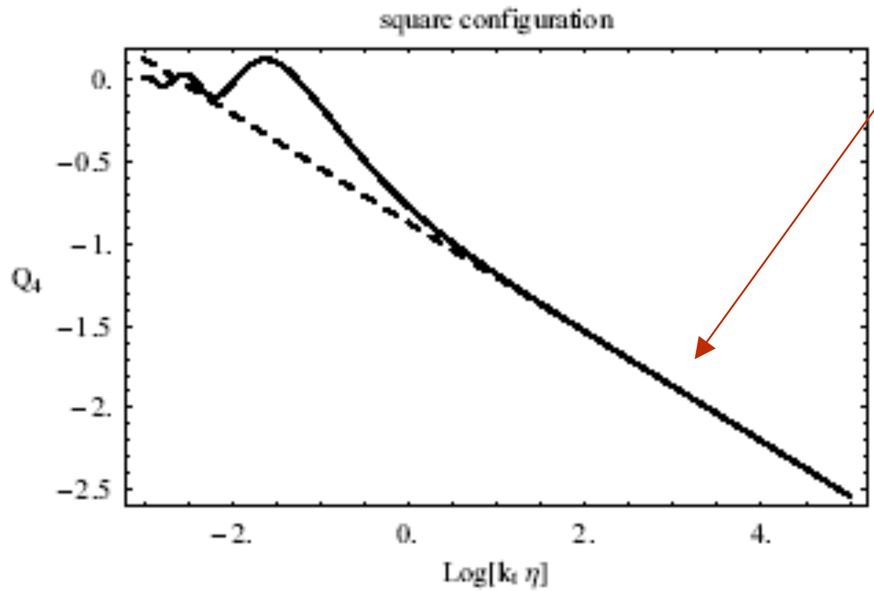
Exact results from quantum theory

For a quartic coupling and in the super-horizon limit

(FB, Brunier & Uzan '03)

$$\langle \chi(\mathbf{k}_1)\dots\chi(\mathbf{k}_4)\rangle_c = -\frac{\lambda\, H^4}{24}\frac{\delta_{\rm Dirac}(\sum \mathbf{k}_i)}{\prod k_i^3}\left[-\sum k_i^3\left(\gamma+\zeta(\{k_i\})\underbrace{+\log\left[-\eta\sum k_i\right]}_{=N_{efolds}}\right)\right]$$

$$Q_4(k_1,k_2,k_3,k_4) = \frac{P_4(k_1,k_2,k_3,k_4)}{P_2(k_1)P_2(k_2)P_2(k_3)+\mathrm{sym.}}$$



Physics at super-horizon scales

- General : the computation of the four point correlation function (at tree order for a scalar quantum field in de Sitter space) is possible;
- **After horizon crossing** one has to deal with a classical stochastic field that follows a well defined evolution equation,

$$\ddot{\underline{\delta s}} + 3H\,\dot{\underline{\delta s}} = \mathcal{S}(\underline{\delta s})$$

- For $V(\delta s) = \frac{\lambda}{4!}\,\delta s^4$ isocurvature fluctuations are bounded

- The non-linear evolution of a classical stochastic field can be described by a perturbation theory approach (FB & Uzan '03); KG with a non-linear source term.

Gaussian

$$\underline{\delta s}(t) = \underline{\delta s}^{(0)} + \underline{\delta s}^{(1)} + \ldots$$

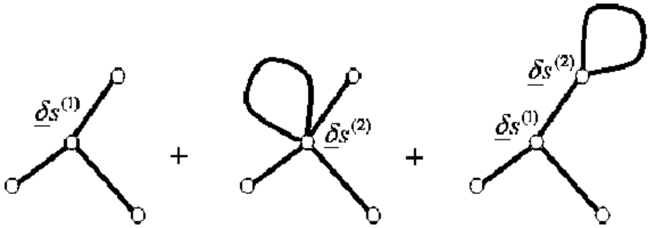
Leading order in λ

$$\underline{\delta s}^{(1)}(t) = \underline{\delta s}^{(\mathrm{ng})} + (t - t_{\mathrm{H}})\frac{\mathcal{S}(\underline{\delta s}^{(0)})}{3\,H} = -\frac{\lambda}{18}\,N_R\frac{\left[\underline{\delta s}^{(0)}\right]^3}{H^2}$$

A classical perturbation theory approach

- Cumulant computation (at tree order) following PT techniques

(Peebles, Fry, Bernardeau, Scoccimarro, etc…)

$\langle \underline{\delta s}^4 \rangle_c =$ 

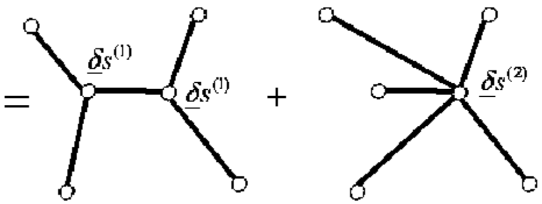
+ sym. + …

Loop terms are ill defined (sub-horizon effects)

- Tree order calculation of the four point function

$$\langle \underline{\delta s}^4 \rangle_c = 4 \left\langle \underline{\delta s}^{(1)} \left[\underline{\delta s}^{(0)} \right]^3 \right\rangle_c = -\frac{4\lambda}{3} \frac{N_R}{H^2} \left\langle \left[\underline{\delta s}^{(0)} \right]^2 \right\rangle^3$$

- Tree order calculation of the six point function…

$\langle \underline{\delta s}^6 \rangle_c =$ 

+ sym. + …

$$\langle \underline{\delta s}^6 \rangle_c = 30 \left\langle \left[\underline{\delta s}^{(1)} \right]^2 \left[\underline{\delta s}^{(0)} \right]^4 \right\rangle_c + 6 \left\langle \underline{\delta s}^{(2)} \left[\underline{\delta s}^{(0)} \right]^5 \right\rangle_c$$

… good up to a PDF reconstruction

The complete PDF of the curvature fluctuations can be obtained from the resolution of the motion equation for $\underline{\delta s}(t)$

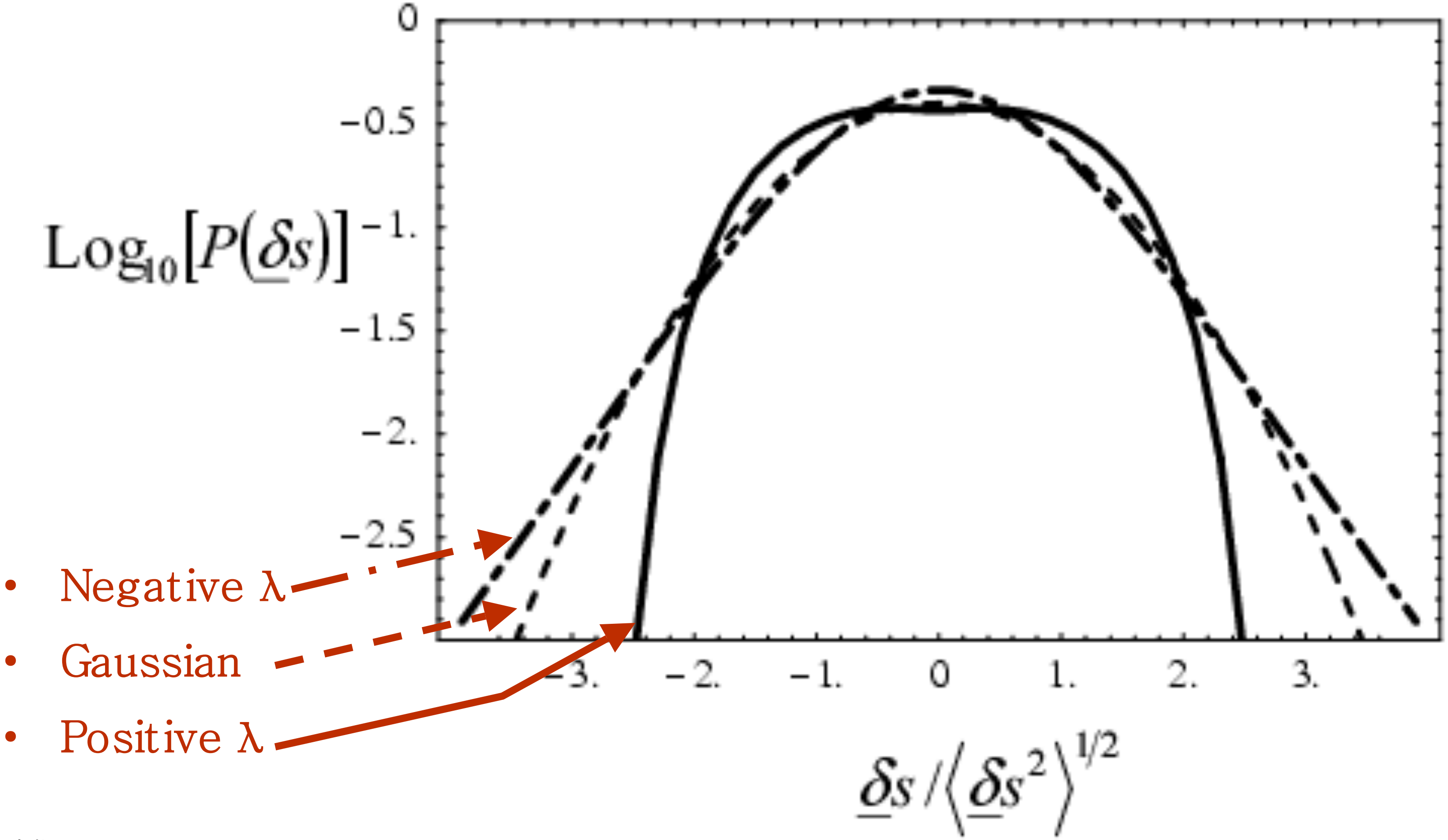
Motion equation in the Slow Roll limit : $3H\,\dot{\underline{\delta s}}(t) = -\frac{\lambda}{3!}\underline{\delta s}(t)^3$

$$\Rightarrow \quad \underline{\delta s}(t) = \frac{\underline{\delta s}^{(0)}}{\sqrt{1-\nu_3(t)\left[\underline{\delta s}^{(0)}\right]^2/3}} \quad \text{with} \quad \nu_3(t) = -\frac{\lambda\,t}{3\,H} = -\frac{\lambda N_R}{3\,H^2}$$

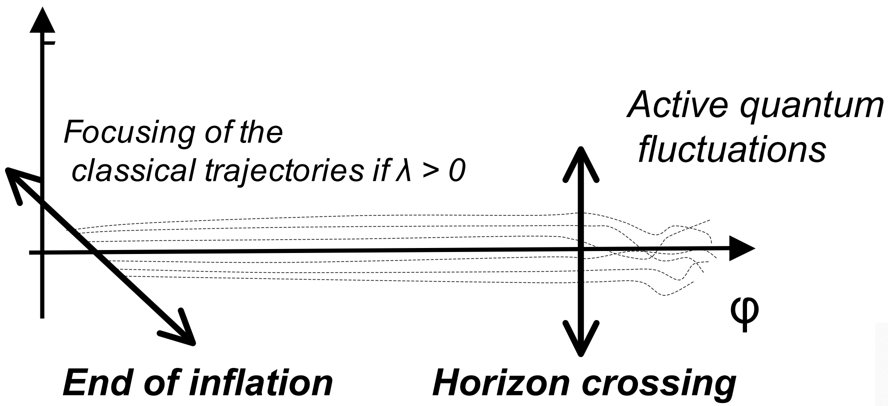
The PDF for $\underline{\delta s}(t)$ can then be obtained from a simple nonlinear transform, or from an inverse Laplace transform of the cumulant generating function if one wants to keep only the tree order contribution. The cumulant generating function is obtained from the vertex generating function through a Legendre transform.

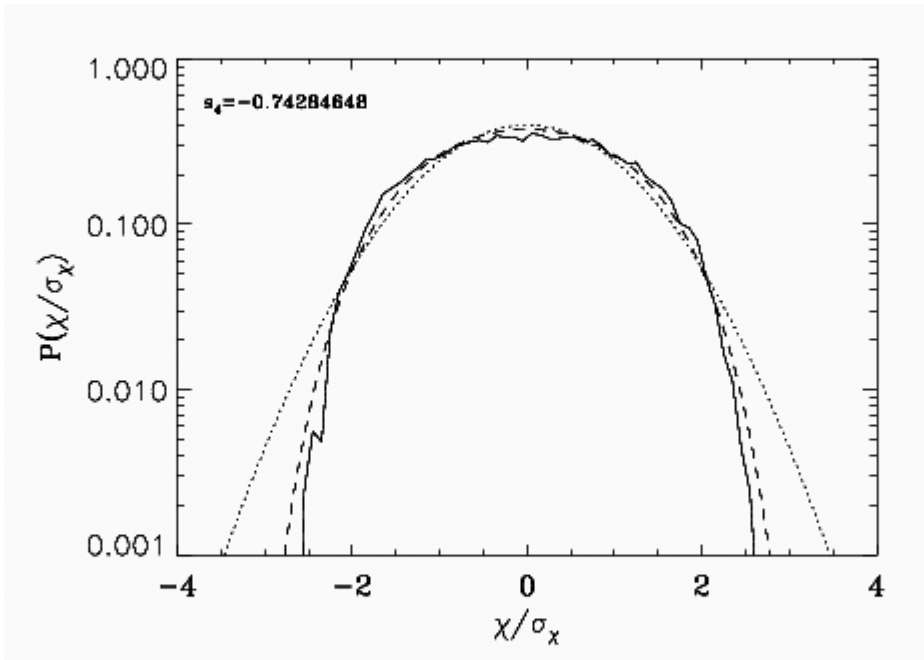
Reconstructed PDF shape:

$$P(\underline{\delta s})\mathrm{d}\underline{\delta s} = \sqrt{\frac{3}{2\,\pi}\left|\frac{1-\underline{\delta s}^2\nu_3}{(3+\underline{\delta s}^2\,\nu_3)^3}\right|}\exp\left[-\frac{3\,\underline{\delta s}^2}{(6+2\,\underline{\delta s}^2\,\nu_3)\sigma_s{}^2}\right]\frac{\mathrm{d}\underline{\delta s}}{\sigma_s}.$$



The curvature PDF evolution





Evolution of the curvature PDF, a numerical experiment

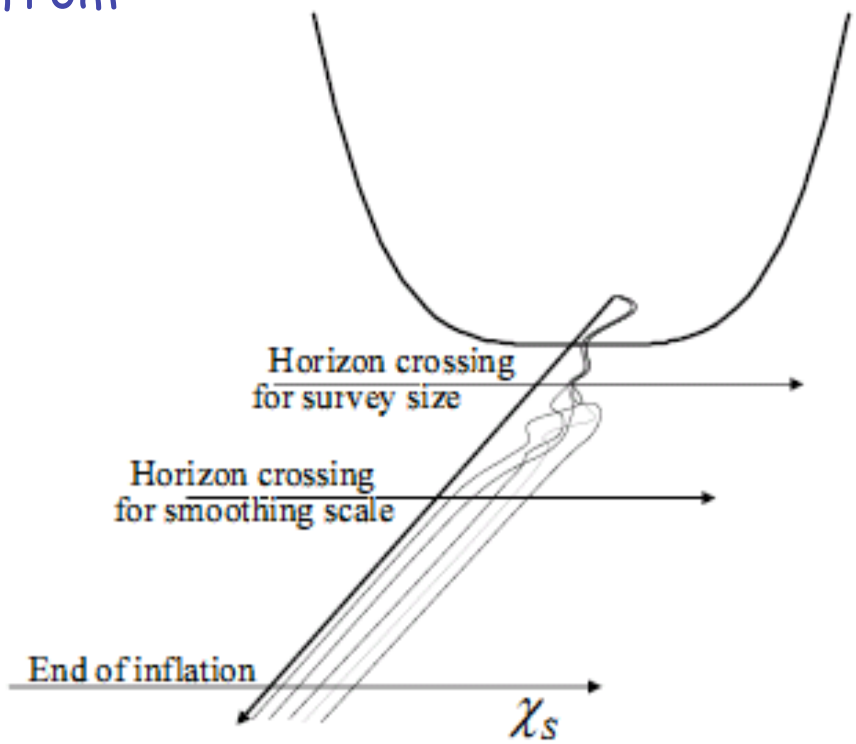
Phenomenological consequences

- Extended models of hybrid inflation can lead to a richer phenomenology …
  - Breaking of the relation between tensor and scalar metric fluctuations
  - Possibility of having Non-Gaussian adiabatic fluctuations
  - Part of effects is due to finite volume effects

Finite volume effects

- Super-horizon value of fields is non-zero
  - Different observable quantities share a common history, e.g. originate from the same value of $\bar{\chi}$
  - The typical excursion values of the field can be obtained from a Langevin equation

$$\dot{\chi}_{\rm H} = -\frac{1}{3H}\frac{{\rm d}V(\chi_{\rm H})}{{\rm d}\chi_{\rm H}} + \xi_{\rm Q}(\mathbf{x},t)$$



Finite Volume effects for multiple-field models

Metric fluctuations are, $\phi_{\mathbf{k}} = \alpha \phi_{\mathbf{k}}^{\mathrm{SF}} + \beta \chi_{\mathbf{k}}$, what is observed is $\delta\Phi = \Phi - \bar{\Phi}$

Average over whole sky

What is measured is measured for a fixed value of $\bar{\chi}$

Consequences : non-zero third-order correlations

$$\langle \chi(\mathbf{k}_1)\chi(\mathbf{k}_2)\chi(\mathbf{k}_3)\rangle_{\bar{\chi}} \simeq \langle \chi(\mathbf{k}_1)\chi(\mathbf{k}_2)\chi(\mathbf{k}_3)\bar{\chi}\rangle_c \; \frac{\bar{\chi}}{\sigma_{\bar{\chi}}^2}$$
$$= -\frac{\lambda\bar{\chi}}{3H^2}(2\pi)^{9/2} \log\left[\left(\sum_i k_i\right)\eta\right] [P(k_1)P(k_2) + \mathrm{perm.}]\; \delta\left(\sum_i \mathbf{k}_i\right)$$

and a non-zero skewness

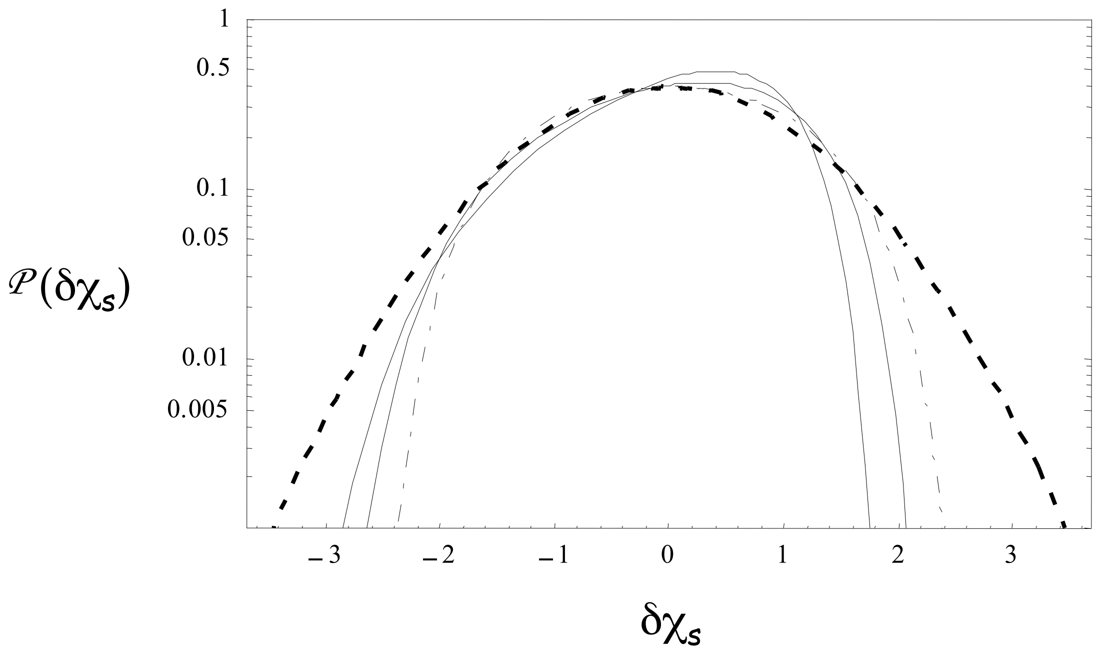
$$\langle X_3\rangle_{\bar{\chi}} = -\lambda\;\bar{\chi}\;\log\left(2\eta k_{\mathrm{S}}^{2/3} k_{\mathrm{H}}^{1/3}\right)\frac{\sigma_\delta^4}{H^2}$$

$\bar{\chi}$

Consequences for quartic couplings

Late time expression of the PDF of

$$\mathcal{P}_{\rm eq}(\chi_{\rm H}) = \frac{1}{2\Gamma(5/4)H}\left(\frac{\pi^2\lambda}{9}\right)^{1/4}\exp\left[-\frac{\pi^2\lambda}{9H^4}\chi_{\rm H}^4\right]$$



Conclusion

- Hybrid inflation can be extended in ways that lead to a rich phenomenology.

- Connection with potentials motivated by super-strings ?