

**The Multiverse as a Reduced Phase Space :
The Meta-Universe as a Kolmogorovian Probability Space**

G W Gibbons

DAMTP

There is a lively debate at present among cosmologists involving the “probability “ of various observations/models. These discussions often entail use of the Anthropic Principle in some form or other/

Roughly speaking there are two different approaches which I would like to associate with two different words. *

*Beware: not everybody uses these words in the sense described below

The Multiverse

This may be defined as the abstract and timeless set of all possible universes, i.e. of all connected spacetimes satisfying the Einstein Equations. This space, $M_{\text{Multiverse}}$ at least in mini-superspace examples, is even dimensional $\dim M_{\text{Multiverse}} = 2n - 2$ and carries, by virtue of it being a reduced phase space, a natural symplectic structure, i.e. a closed 2-form ω and hence measure

$$\frac{1}{(n-1)!} (-1)^{\frac{1}{2}(n-1)(n-2)} \omega^{n-1} \quad (1)$$

This is measure originally advocated by Gibbons, Hawking, Stewart and revised recently by Gibbons and Turok.

The main difficulty is that the total measure of $M_{\text{Multiverse}}$ diverges

$$\int_{M_{\text{Multiverse}}} \frac{1}{(n-1)!} (-1)^{\frac{1}{2}(n-1)(n-2)} \omega^{n-1} = \infty \quad (2)$$

and Probabilities cannot be normalised. Gibbons and Turok have made a suggestion for solving this problem.

The Meta-Universe

As introduced by Vilenkin, this is a single connected 4-dimensional spacetime $M_{\text{Meta-Universe}}$ possibly containing many causally disjoint regions. Points of $M_{\text{Meta-Universe}}$ are called spacetime events and the probability of a set $U \subset M_{\text{Meta-Universe}}$ of such events is taken to be proportional to their spacetime volume

$$\int_U \sqrt{|g|} d^4 x \quad (3)$$

Again the main problem is one of normalizability,

$$\int_{M_{\text{Meta-Universe}}} \sqrt{|g|} d^4 x = \infty. \quad (4)$$

It is amusing nevertheless that the Meta-Universe mirrors in many ways the standard Kolmogorovian notion of a Probability Space.

This is first of all a Measure Space, that is a triple $\{\Omega, \mathcal{F}, \mu\}$, where Ω is a set called the *sample space*, \mathcal{F} a collection of subsets of Ω called *measurable subsets* carrying the structure of a σ -algebra, that it is closed under denumerable unions and intersections, and μ is a non-negative countably additive set function on \mathcal{F} .

To get a Probability Space $\{\Omega, \mathcal{F}, P\}$, one adds the requirement that μ is normalised to unity.

Note that the measurable subsets are not all possible subsets of Ω

To make contact with current discussions it is tempting to identify Ω with $M_{\text{Meta-Universe}}$ and μ with $\sqrt{|g|}d^4x$.

A natural suggestion for \mathcal{F} the measurable subsets or at least a basis for \mathcal{F} would be the *Alexandrov Open Sets* or *Causal Diamonds* of the form

$$I^+(p) \cap I^-(q), \quad (5)$$

. where p and q are two spacetime events and I^\pm indicates chronological future and past.

Note that in probability theory elements of \mathcal{F} , i.e. measurable subsets are called *events* and care must be taken to avoid confusion.

This raises the purely mathematical question of whether Alexandrov open sets really do form a basis for measurable sets. This almost certainly imposes restrictions on the causal structure of the Meta-Universe.

A more significant problem is the issue of normalisation. One possibility, is to Weyl rescale the spacetime metric

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, \quad (6)$$

so as to render the total volume finite

$$\int_{M_{\text{Meta-Universe}}} \sqrt{|\tilde{g}|} d^4 x < \infty. \quad (7)$$

This leaves invariant the Causal Diamonds, but there is no obviously canonical choice for the conformal factor $\Omega(x)$ *.

*not to be confused with the sample space

Myrheim's Formula for the volume of a causal diamond is

$$V = \frac{2\pi}{3}T^4 \left(1 - \frac{1}{90}(3R_{\mu\nu} - 2Rg_{\mu\nu})T^\mu T^\nu\right) + \dots \quad (8)$$

where $T^\mu T_\mu = -\frac{1}{4}T^2$ and $T = T(p, q)$ is the proptime between p and q .

Matter causes the diamonds to shrink in volume.

Why should one want a probability theory for cosmology in the first place?

One motivation is to apply **Bayesian Reasoning** to Cosmological Observations.

That is the viewpoint I will take from now on.

A priori probability distributions , “Priors” are integral part of the scientific method.

They are needed to to assess the probability that a hypothesis is true given a set of observations or measurements.

They are also required to assess the reliability and information content of predictions since if a large measure of hypotheses give the same observations, then those observations don't tell us much.

Conversely if only small measure of hypotheses predict a set of well verified observations, one may have high confidence in those hypotheses.

However much confusion can result if different people use different priors, or more misleadingly, don't clearly articulate the that they priors they are using.

This problem is particularly prevalent in cosmology where there is at present no consensus on a suitable *a priori* measure on initial conditions.

In choosing *a priori* probabilities, it seems safest to assume as little as possible, consistent with making any progress. Indeed progress consists of using the results of observations and experiments to refine and update what are initially very flat *a priori* distributions so that they peak at some approximation for the real world.

In M/String theory **all of physics is subsumed into history**. Thus all coupling constants, and low energy laws of physics are determined by initial conditions. Issues of **naturalness** reduce to the **naturalness of initial conditions**.

Thus if M/String theory is to qualify as a science it requires convincing ***a priori*** measures.

More formally if

$P(O|U)$ is the probability of making an observation O in universe U ,
(**Likelihood**)

$P(U|O)$ is the probability we are in universe U having made an observation O (***A posteriori probability***)

$P(O)$ is the probability of making an observation O in *any* universe

$P(U)$ is the probability that the universe is U (***A priori probability***)

$P(O \cap U)$ is the probability of making an observation O and the universe is actually U

Then

$$P(U|O)P(O) = P(O \cap U) = P(O|U)P(U) \quad (9)$$

whence the **Cosmic Bayes's Theorem** tells us that

$$P(U|O) = \frac{P(O|U)P(U)}{\int_M P(O|U)P(U)dU} \quad (10)$$

where

the integral is over the **Multiverse** and dU is a measure on the multiverse.



Thomas Bayes

quodque solum, certa nitri signa præbere, sed plura concurrere debere, ut de vero nitro producto dubium non relinquatur.

LII. *An Essay towards solving a Problem in the Doctrine of Chances. By the late Rev. Mr. Bayes, F. R. S. communicated by Mr. Price, in a Letter to John Canton, A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763. I Now send you an essay which I have found among the papers of our deceased friend Mr. Bayes, and which, in my opinion, has great merit, and well deserves to be preserved. Experimental philosophy, you will find, is nearly interested in the subject of it; and on this account there seems to be particular reason for thinking that a communication of it to the Royal Society cannot be improper.

He had, you know, the honour of being a member of that illustrious Society, and was much esteemed by many in it as a very able mathematician. In an introduction which he has writ to this Essay, he says, that his design at first in thinking on the subject of it was, to find out a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circum-

Philosophical Transactions of the Royal

Society 53 (1763) 269-271

If we adopt **Laplace's Principle of Indifference** then $P(U)$ is independent of U .



Any other choice of $P(U)$ is a **proposal for the state of the the universe**

We can define Kolmogorov's measure of the **information content** of a proposal $P(U)$ via

$$\int P(U) \ln P(U) dU \quad (11)$$

which should be least for Laplace's Proposal

But what is dU ?

In the rest of this talk, based on work with Neil Turok, (hep-th/0609095) I shall revisit an old proposal of John Stewart and Stephen Hawking and myself (A Natural Measure On The Set Of All Universes *Nucl.Phys.* **B281** (1987) 736) on the probability of inflation in a scalar field cosmology , as judged using the Liouville measure.

We claimed that the probability of inflation was high. That work was criticised by Don Page and Stephen Hawking (HOW PROBABLE IS INFLATION? *Nucl.Phys.* **B298**(1988) 789) on the grounds that the total measure was divergent and hence probabilities are not well defined.

Neil and I have a suggestion for solving the divergence problem, and we find that if one accepts it, then the probability of inflation with N e-folds is low, typically being proportional to e^{-3N} .

In cosmology one is often interested in the likelihood or otherwise of initial conditions. If one restricts attention to consistent * finite dimensional truncations, so-called mini-superspace models, then we are faced with a standard problem in Hamiltonian mechanics, and it is natural to bring to bear on the problem the standard techniques of statistical mechanics which have been used so successfully in all other areas of physics, for example in evaluating one's chances of success in a dice game.

*in the technical sense

The Einstein and matter equations provide a Hamiltonian flow in a $2n$ -dimensional phase space P , equipped with a symplectic form ω which may be written in local Darboux coordinates as

$$\omega = dp_i \wedge dq^i. \quad (12)$$

This gives the Liouville volume element on P

$$\frac{(-1)^{\frac{1}{2}n(n-1)}}{n!} \omega^n = d^n p d^n q. \quad (13)$$



Liouville

However we need a measure on the space M of **dynamical trajectories**. The Hamiltonian \mathcal{H} is constrained to vanish and so the flow lines lie on a $2n - 1$ dimensional **Constraint submanifold**

$$C = \mathcal{H}^{-1}(0). \quad (14)$$

This is odd-dimensional but to take the so-called **Symplectic or Marsden-Weinstein quotient**, sometimes called the **reduced phase space**,

$$M = C/R = \mathcal{H}^{-1}(0)/R, \quad (15)$$

is straight forward, and moreover the quotient M , i.e *the space of classical histories satisfying the equations of motion or equivalently the space of physically distinct classical initial conditions* inherits a symplectic form. Thus provides a 'natural' measure on space of initial conditions as suggested by Gibbons Hawking and Stewart *.

*The use of the Liouville measure in a different context had earlier been suggested by Henneaux

The word ‘natural’ is being used here in the sense that the construction of the measure requires no more additional elements other than are present already in the equations of motion *. It contains no arbitrary cut-offs †. Moreover, by its construction, the measure is invariant under any additional canonical symmetries of the system.

The quotient M is that space of physically distinct initial conditions, in other words two sets of Cauchy data describing the same space-time, but taken at different times are identified. Thus $\{M, \omega\}$ may be thought of as the **multiverse**.

Actually in our example $P = T^(Q)$

†Actually we will need an IR cutoff

It is important to realise that the multiverse so defined carries no information about the direction of time nor any preferred instant of time. Many discussions of the plausibility or otherwise of certain initial conditions make explicit, or more dangerously implicit, assumptions about either or both.

This initially seeming abstract construction may be made more intuitive using **the concept of flux**.

In general the symplectic form is a covariant second rank anti-symmetric tensor field with components in an arbitrary coordinate system

$$\omega_{\mu\nu} = -\omega_{\nu\mu}$$

where the indices μ, ν take $2n$ values. The symplectic form is closed,

$$d\omega = 0.$$

This is equivalent to

$$\partial_{[\mu}\omega_{\nu\tau]} = 0.$$

Moreover if \mathcal{H} is the Hamiltonian, then Hamilton's equations are

$$V^\mu = \omega^{\mu\nu} \partial_\nu \mathcal{H}$$

where $\omega^{\mu\nu}$ is the inverse of $\omega_{\mu\nu}$ and $V^\mu = \frac{dx^\mu}{dt}$ is tangent to the flow.

so we may re-write this as

$$\omega_{\mu\nu} V^\nu = \partial_\mu \mathcal{H}$$

It implies that

$$V^\mu \partial_\mu \mathcal{H} = 0$$

This means that V^μ lies in the level sets of the Hamiltonian $\mathcal{H} = \text{constant}$.

Now let us choose coordinates such that

$$x^{2n} = \mathcal{H}.$$

The closure condition, restricted to spatial indices is

$$\partial_{[i}\omega_{jk]} = 0.$$

we have

$$V^{2n} = 0,$$

Thus $V^i\omega_{ij} = 0$.

It is simplest to see what this means in the example $n = 2$, for which $i = 1, 2, 3$. Define a “magnetic field” by

$$B_i = \epsilon_{ijk}\omega_{ij}$$

then \mathbf{B} is divergence free

$$\partial_i B_i = 0.$$

Moreover

$$\epsilon_{ijk} B_j V_k = 0.$$

thus $\mathbf{B} \times \mathbf{V} = 0$, or \mathbf{V} is parallel to \mathbf{B} .

Now consider a bunch of trajectories , i.e a subset of the multiverse M which lie in the Constraint manifold $\mathcal{H} = 0$.

Cut them with a transverse Cauchy surface Σ . The flux of the magnetic field through Σ

$$\int_{\Sigma} \mathbf{B} \cdot d\Sigma \quad (16)$$

counts the number of universes in the bunch and is independent of which surface Σ we use, provided only that Σ intersects each trajectory once and only once

This is the measure advocated by Gibbons, Hawking and Stewart.

The argument in arbitrary dimension goes as follows.

Let

$$B_i = \epsilon_{ipqrs\dots tu} \omega_{pq} \omega_{rs} \dots \omega_{tu} \quad (17)$$

Then

$$\partial_i B_i = 0. \quad (18)$$

Thus

$$\omega_{[pq} \omega_{rs} \dots \omega_{tu]} \propto \epsilon_{ipqrs\dots tu} B_i \quad (19)$$

and so

$$V_p \epsilon_{ipqrs\dots tu} B_i = 0, \quad (20)$$

which implies that

$$V_{[i}B_{j]} = 0, \quad (21)$$

that is, B_i is parallel to V_i .

Consider a single minimally-coupled scalar field ϕ with potential $V(\phi)$ in a homogeneous and isotropic (FRW) Universe with line element

$$-N^2 dt^2 + a^2(t) \gamma_{ij} dx^i dx^j, \quad (22)$$

where γ_{ij} is a metric on a space of constant (three-dimensional) scalar curvature $k = 0$ or ± 1 . In units in which $(8\pi G) = 1$, the Einstein-scalar action is

$$\mathcal{S} = \int dt N \left(-3a(N^{-2}a'^2 - k) + \frac{1}{2}a^3 N^{-2} \phi'^2 - a^3 V(\phi) \right), \quad (23)$$

where primes denote t derivatives.

Varying the action with respect to the lapse function N yields the usual Friedmann equation

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{k}{a^2}, \quad (24)$$

where dots denote proper time derivatives, with $d\tau = Ndt$, and $H = \dot{a}/a$ is the expansion rate or Hubble parameter. Varying with respect to ϕ yields the scalar field equation

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}. \quad (25)$$

Taking the time derivative of (24) and using (25) then yields

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 + \frac{k}{a^2}. \quad (26)$$

Finally, varying with respect to a yields a linear combination of (24) and (26).

Equation (26) will be of particular interest to our later discussion. For $k \leq 0$, the Hubble parameter H never increases, so no classical trajectory can cross an $H=\text{constant}$ hypersurface more than once.

The canonical momenta conjugate to a , ϕ and N are

$$p_a = -6a\dot{a} = -6a^2H, \quad p_\phi = a^3\dot{\phi}, \quad p_N = 0, \quad (27)$$

and the Hamiltonian is

$$\mathcal{H} = N\left(-\frac{p_a^2}{12a} + \frac{1}{2} \frac{p_\phi^2}{a^3} + a^3V(\phi) - 3ak\right), \quad (28)$$

which vanishes by the equation of motion for p_N . We can use this to eliminate one of the four canonical variables a, p_a, ϕ, p_ϕ . In view of the monotonically decreasing property of H , mentioned earlier, and because $V(\phi)$ is, in general, complicated, we choose to eliminate p_ϕ , obtaining

$$p_\phi = \pm \sqrt{-\frac{1}{6}p_a^2a^2 - 2a^6V(\phi) + 6a^4k}. \quad (29)$$

If $a = e^\lambda$, routine calculations give

$$V^i = (\dot{\phi}, \dot{H}, \dot{\lambda}) = (\pm\sqrt{6H^2 - 2V + \frac{6k}{a^2}}, V - 3H^2 - \frac{2k}{a^2}, H). \quad (30)$$

$$\omega = e^{3\lambda}(-6d\lambda \wedge dH \pm 3\frac{6H^2 - 2V + 4ke^{-2\lambda}}{\sqrt{6H^2 - 2V + 6ke^{-2\lambda}}}d\phi \wedge d\lambda \quad (31)$$

$$\pm \frac{6H}{\sqrt{6H^2 - 2V + 6ke^{-2\lambda}}}d\phi \wedge dH). \quad (32)$$

$$\mathbf{B} = \frac{\pm 3e^{3\lambda}}{\sqrt{6H^2 - 2V + 6ke^{-2\lambda}}}\mathbf{V} \quad (33)$$

Note that we are not using Darboux-coordinates, but a physically more convenient choice.



Gaston Darboux

As before, the equations of motion imply that

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 + \frac{k}{a^2} \quad (34)$$

Thus if $k \leq 0$, a good Cauchy surface is $H = \text{constant}$, The measure is

$$\int 3e^{3\lambda} \frac{6H^2 - 2V + 4ke^{-2\lambda}}{\sqrt{6H^2 - 2V + 6ke^{-2\lambda}}} d\phi \wedge d\lambda \quad (35)$$

The range of ϕ is usually finite but the measure clearly diverges at large λ , i.e. large scale factors. This is essentially the problem pointed out by Hawking and Page.

However, an important point is that if $k = 0$, then a or equivalently λ , is neither geometrically meaningful nor physically measurable.

This suggest fixing λ and integrating only over ϕ . Later will do something better. but for the time being let's pursue this idea in the simplest $k = 0$ case.

If $k = 0$, the equations of motion become

$$\dot{\phi} = \sqrt{2}\sqrt{3H^2 - V}, \quad \dot{H} = V - 3H^2, \quad (36)$$

$$\dot{\lambda} = H. \quad (37)$$

The $\phi - H$ motion decouples and one obtains an autonomous system

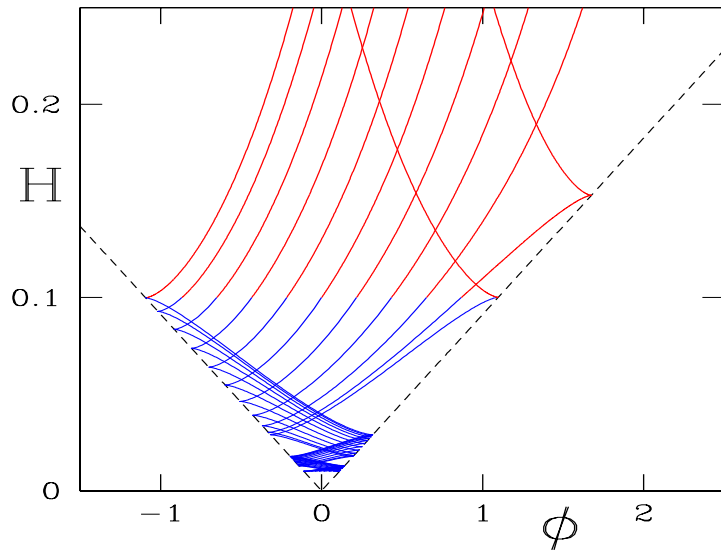
$$\sqrt{2}\frac{dH}{d\phi} = \pm\sqrt{3H^2 - V}. \quad (38)$$

If $a = e^\lambda = e^{-N}$, where N is the number of e-folds then

$$\sqrt{2}\frac{dN}{d\phi} = \frac{H}{\sqrt{3H^2 - V}}. \quad (39)$$

Slow roll is

$$H \approx \sqrt{\frac{V}{3}}\left(1 + \frac{1}{12}\left(\frac{V'}{V}\right)^2\right) \quad (40)$$



The set of trajectories in the (ϕ, H) plane for $k = 0$, and $V = \frac{1}{2}m^2\phi^2$ with $m^2 = .05$ in reduced Planck units. The measure surface is taken at $H = 0.1$, and the trajectories plotted are equally spaced in ϕ on that surface. Only the trajectories with positive $dH/d\phi$ are shown: those with negative $dH/d\phi$ are obtained by mirror reflection about $\phi = 0$.

Along a Cauchy surface $\Sigma : H = \text{constant}$ the measure is

$$\int e^{3\lambda} d\lambda d\phi \left| \frac{dH}{d\phi} \right|_{\Sigma}. \quad (41)$$

Since the number of e-folds N can be determined as a function of the initial value of ϕ along Σ , we can convert the probability distribution over ϕ to one over N .

Perturbing the formula

$$\sqrt{2} \frac{dN}{d\phi} = \frac{H}{\sqrt{3H^2 - V}} \quad (42)$$

one discovers that

$$\frac{d\delta H}{dN} = 3\delta H \quad (43)$$

which implies that N is an extremely sensitive function of the value of ϕ on Σ .

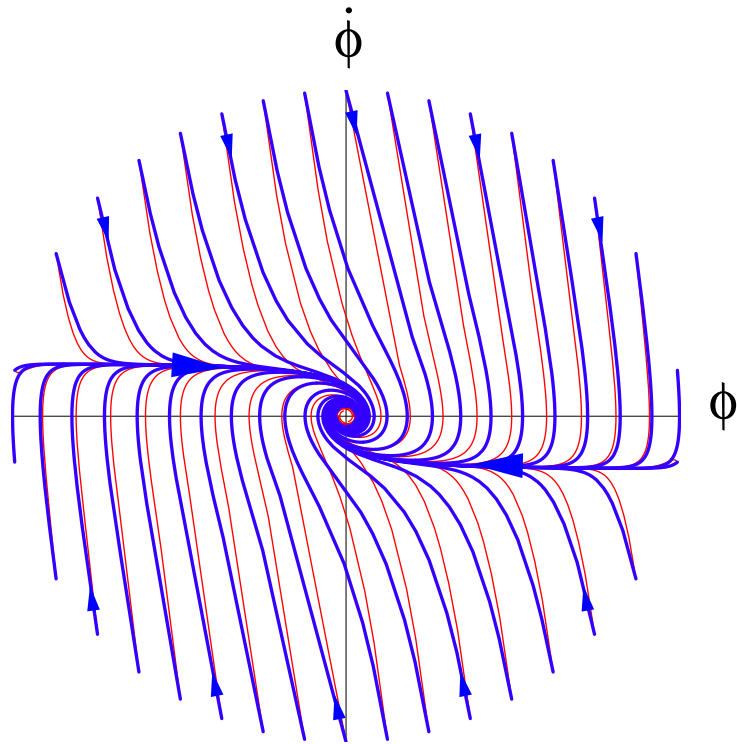
$$\delta H_{\Sigma} = \left| \frac{dH}{d\phi} \right|_{\Sigma} \delta\phi_{\Sigma} = e^{-3N} \delta H. \quad (44)$$

If we take δH to be sufficient for slow roll to break down, the measure on N becomes (setting $\lambda = 0$),

$$\int dN C(N) e^{-3N}, \quad (45)$$

where $C(N)$ is a slowly varying function of N . Thus, from this point of view, the probability of inflation is vanishing small!

This result may appear puzzling since in earlier work the fact that slow roll is an attractor to the future seemed to imply that inflation



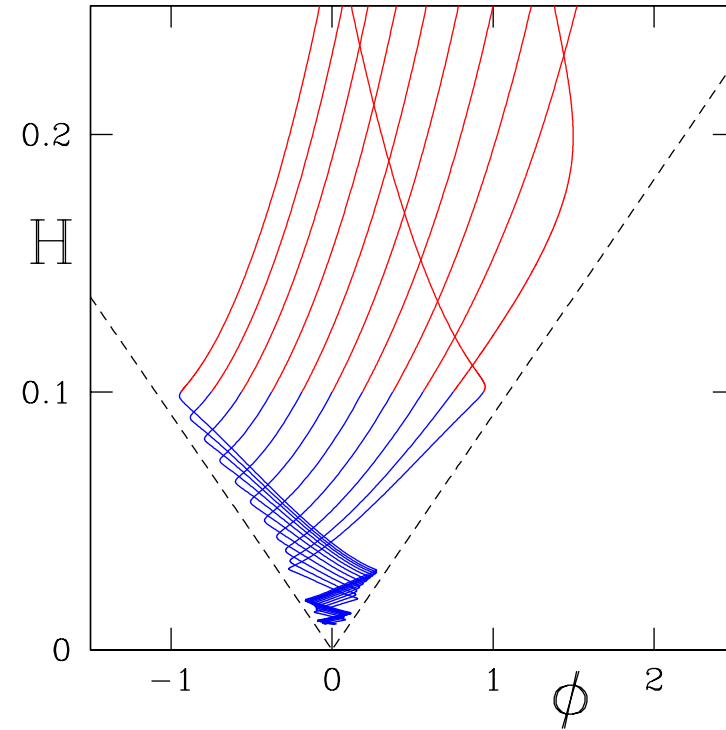
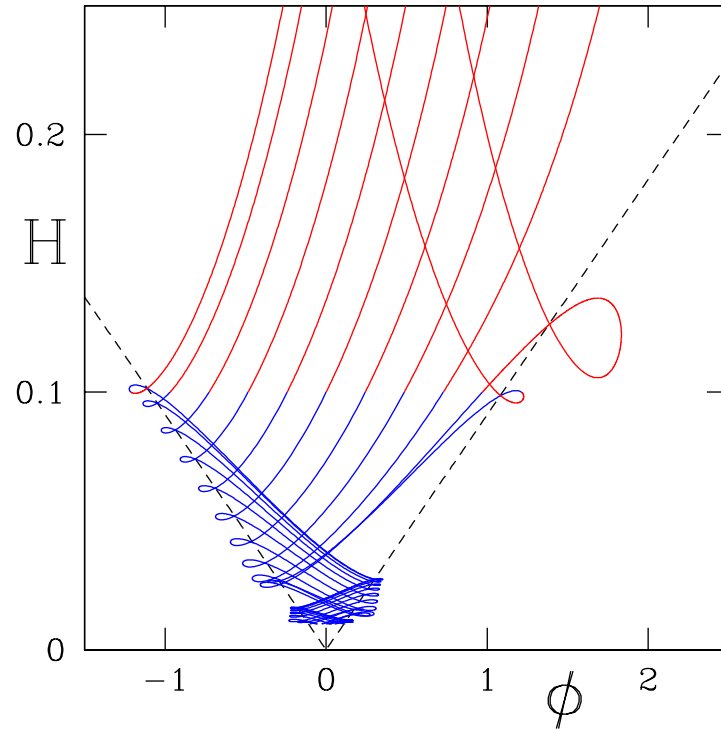
is generic

The $\phi - \dot{\phi}$ phase plane

However slow roll is a repeller to the past, and initial conditions ensuring that in the past there was *sufficient* inflation must be extremely finely tuned.

Note that these argument looks as if they are asymmetric with respect to reversal of time. However the measure we use is independent of time orientatation. This illustrates how dangerous it is to let unacknowledged assumptions slip in.

They also seem to depend on the radius of the circle centred on the origin. Choosing a very large circle gives one result. Choosing a very small circle gives the opposite result.



If $k \neq 0$ we have

At large a the situation should not be significantly different from the $k = 0$ case. However, if $k \neq 0$, rescaling a is not an exact symmetry.

One possibility is to integrate over all scale factors smaller than a certain fixed value $a_{\max} = e^{\lambda_{\max}}$ and ignore those larger than a_{\max} , on the ground that they all give the same presently observed universe *

The integral

$$\int_{-\infty}^{\lambda_{\max}} e^{3\lambda} d\lambda \frac{6H^2 - 2V + 4ke^{-2\lambda}}{\sqrt{6H^2 - 2V + 6ke^{-2\lambda}}} d\phi \quad (46)$$

certainly converges.

*We can't integrate over all scale factors bigger than a_{\max} because this would diverge

There is a certain analogy here with state counting in elementary quantum mechanics. Given a Hamiltonian $\mathcal{H}(q, p)$, the number of states with energy less than or equal to E , $N(< E)$ (the cumulative density of states) is given semi-classically by

$$\int_{\mathcal{H} \leq E} dpdq \quad (47)$$

One then estimates the entropy as

$$S = \ln(N(< E)). \quad (48)$$

In our case the obvious guess would be

$$S \approx 3 \ln a_{\max}, \quad (49)$$

which is not completely ridiculous.

Note that it is not obviously related to the entropy of de-Sitter space-time given by $\frac{1}{4}$ the area of the cosmological horizon.

For one degree of freedom

$$\int_{\mathcal{H} \leq E} dpdq = \oint_{\mathcal{H} = E} pdq \quad (50)$$

This suggest our integration process is topological, and in some situations will be independent of the nature of the universe at small scales. In our case we are on a cotangent bundle $P = T^*Q$. and ω is exact

$$\omega = d\theta, \quad \theta = p_i dq^i, \quad (51)$$

the canonical one-form θ is *globally* defined. Thus

$$\omega^k = d(\theta \wedge \omega^{k-1}) \quad (52)$$

If $M \supset D = \{p, q | a < a_{\max}\}$, then

$\int_D \omega^{n-1} = \int_{\partial D} \theta \wedge \omega^{n-2}$ (53) where the r.h.s. is over a surface in M given by $a = a_{\max}$ *.

*This looks rather Chern-Simons like

Extension to Perfect Fluids (wk.with T Damour)

$$X = -\nabla^2. \quad (54)$$

$$L = L(X) \quad P = L \quad \rho = 2XL_X - L. \quad (55)$$

e.g. radiation *

$$L = X^4 = (\nabla\phi)^4 \quad (56)$$

Constructing the Hamiltonian is now straightforward the measure is related to the entropy of the fluid!

(in fact the entropy current is a Noether current for $\phi \rightarrow \phi + \text{constant}$)

*conformally invariant theory

Initial Conditions for Fluctuations

If η is conformal time

$$\eta = \int_{\eta_i}^0 \frac{dt}{a(t)} \quad (57)$$

ones sets

$$\eta - \chi = \tan U \quad \eta + \chi = \tan V \quad (58)$$

The origin of coordinates is $U = V$

If $a(t_i) \rightarrow 0$, then there is an initial null hypersurface H^-

$$U = -\frac{\pi}{2}. \quad (59)$$

In the exact de-Sitter metric this is a smooth null Cauchy hypersurface, on which Gibbons and Hawking imposed a positive frequency condition defining the de-Sitter invariant (Euclidean or Bunch-Davies) vacuum state.

If $a(t)$ does not go to zero exactly exponentially the surface H^- is singular. One may say that it is on this surface that boundary conditions for inflation must be imposed. For example the **Lorentzian No Boundary Proposal**[†] of Friedman Gott, etc says that **what went before is the same as what went after**.

This particular proposal gives rise to a non-time-orientable and hence quantum mechanically inconsistent universe

[†]Or better One Boundary

BPS solutions often solve first order equations

$$\mathcal{L} = \left(\frac{1}{2}g_{ij}\dot{q}^i\dot{q}^j - V(q)\right) \quad (60)$$

with super potential

$$V = -\frac{1}{2}g^{ij}\partial_i W \partial_j W \quad (61)$$

Thus

$$g_{ij}\dot{q}^j = p_i = \pm\partial_i W, \quad (62)$$

solves both the the equations of motion and the constraint

$$= \left(\frac{1}{2}g^{ij}p_i p_j + V(q)\right) = 0 \quad (63)$$

The set of BPS orbits $p_i = \partial_i W$, defines an n dimensional **Lagrangian submanifold** $L \subset P$ which lies in the constraint manifold C on which

$$\omega|_L = dp_i \wedge dq^i = \partial_j \partial_i W dq^j \wedge dq^i = 0. \quad (64)$$

Thus BPS solutions are of measure zero in the multiverse $\{M, \omega\}^*$.

*This may have some application to attractors

Conclusions

- In this talk I have explained that there are two concepts of probability on the market: the **meta-universe** and the **multiverse** concepts.
- The former, although related to some interesting properties of causal structures, seems fatally flawed because it cannot be normalised.
- I have shown how the latter gives rise, via the symplectic structure of the gravitational phase space, to a natural measure on the space of classical solutions.

- Applied to scalar field cosmology one must impose an upper cut-off a_{\max} on the scale $a(t)$ factor to obtain finite results. Given this, there is a precise and meaningful and quantitative way in which to judge whether inflationary solutions are probable or improbable.
- Our calculations indicate that, in this sense, the probability goes like $\exp -3N$, where N is the number of e-folds.
- The cut-off, is equivalent to integrating over all scale factors less than a_{\max} .
- This integral is topological in character and is independent of the nature of the universe at small scales.

- The technique may be applied to BPS solutions (if there are any), which are found to be of zero measure.
- It is straightforward to extend the measure to incorporate perfect fluids.
- The main challenge is to extend these ideas to give a measure on initial conditions for fluctuations