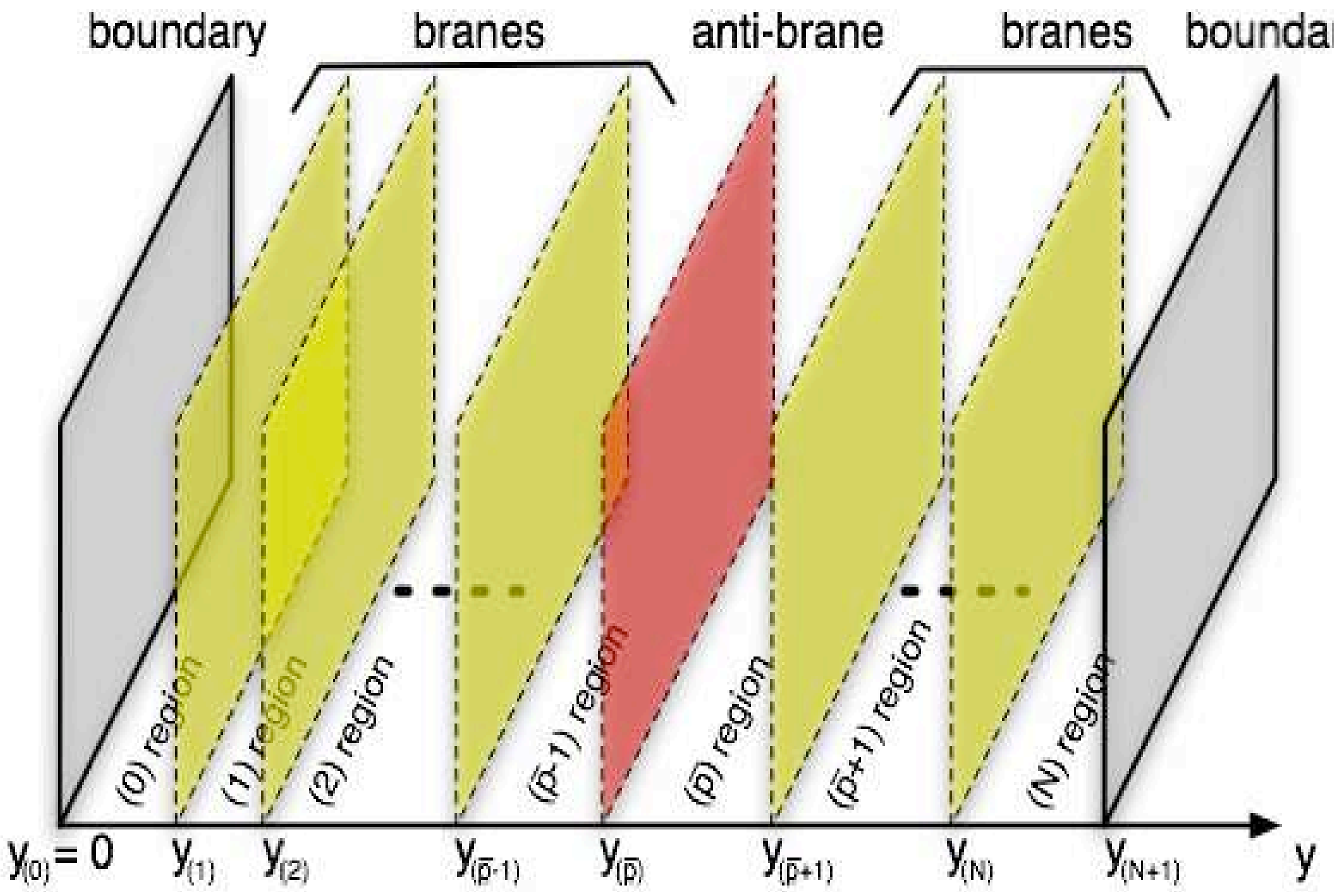
Anti-Branes in Heterotic M-theory

James Gray - IAP and APC Paris.

Based on work with André Lukas and Burt Ovrut.

5D Heterotic M-theory including anti-branes.



What I say today will be valid for a single anti-brane, as shown, but generalizes trivially to an arbitrary number of these objects.

A Toy Model of the Warping in Heterotic.

$$S \sim \int d^5x \left[\partial_\alpha \Phi \partial^\alpha \Phi - \delta(y)\, S_{(0)} \Phi - \delta(y - \pi\rho)\, S_{(N+1)} \Phi - \sum_{p=1}^{N} \delta(y - y_{(p)}) S_{(p)} \Phi \right]$$

Bulk equation of motion: $\Box_5 \Phi = 0$

At the boundaries: $D_y \Phi|_{y=0} = -S_{(0)}$, $D_y \Phi|_{y=\pi\rho} = +S_{(N+1)}$

At the branes: $-D_y \Phi|_{y=y_{(p)}+} + D_y \Phi|_{y=y_{(p)}-} = S_{(p)}$

Perform the split: $\Phi = \phi_0(x^\mu) + \phi(x^\mu + y)$

$$\int_0^{\pi\rho} \phi \, dy = 0$$

Bulk equation becomes: $\Box_4 \phi_0 + \Box_4 \phi + D_y^2 \phi = 0$

Integrate the bulk equations across the orbifold and use boundary conditions:-

$$\Box_4 \phi_0 + \sum_i S^i = 0$$

Look at the case where warping is weak and 4d derivatives are small and substitute this back into the bulk equation:-

$$D_y^2 \phi = \sum_p S_{(p)}.$$

So in the end we have a system of equations for the warping:-

Bulk :

$$D_y^2 \phi = \sum_p S_{(p)}$$

Boundaries:-

$$D_y \phi|_{y=0} = -S_{(0)} \ , \qquad D_y \phi|_{y=\pi\rho} = +S_{(N+1)}$$

Branes:-

$$-D_y \phi|_{y=y_{(p)}+} + D_y \phi|_{y=y_{(p)}-} = S_{(p)}$$

E.G. 1: The supersymmetric vacuum

- Sources S are the tensions of the branes.
- Sum of the charges on the compact space is zero.
- Objects are BPS so tension = charge.
- Therefore $\sum S = 0$ and the bulk equation becomes $D_y^2 \phi = 0$.

The warping in the supersymmetric vacuum is linear in y.

E.G. 2: Warping due to matter fluctuations

- The sources S are now the kinetic and potential terms for brane and boundary localized fields.
- Therefore $\sum S \neq 0$ (matter on different objects is independent).
- So in this case bulk equation is $D_y^2 \phi = \sum S$.

The warping due to matter field fluctuations is quadratic. This is typical of any change away from the pure tension vacuum in heterotic M-theory.

E.G. 3: Anti-branes in heterotic M-theory.

- Sources S are the tensions of the branes and anti-branes.
- Sum of the charges on the compact space is zero.
- For an anti-brane charge = − tension.
- So tensions do not sum to zero and we have in the bulk $D_y^2\phi = \sum S$.

Thus the warping due to the anti-brane is quadratic in y.

Heterotic M-theory in five dimensions:

The bulk theory:

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{4}G_{kl}(b)\partial b^k\partial b^l + \frac{1}{2}G_{kl}(b)\mathcal{F}^k_{\alpha\beta}\mathcal{F}^{l\alpha\beta} + \frac{1}{4}V^{-2}(\partial V)^2 + \lambda(d_{ijk}b^ib^jb^k - 6)\right.$$
$$+\frac{1}{4}\mathcal{K}_{a\bar{b}}(\mathfrak{z})\partial\mathfrak{z}^a\partial\bar{\mathfrak{z}}^{\bar{b}} - V^{-1}(\tilde{\mathcal{X}}_{A\alpha} - \bar{M}_{AB}(\mathfrak{z})\mathcal{X}^B_\alpha)([\Im(M(\mathfrak{z}))]^{-1})^{AC}(\tilde{\mathcal{X}}^\alpha_C - M_{CD}(\mathfrak{z})\mathcal{X}^{D\alpha})$$
$$\left.+\frac{1}{4!}V^2G_{\alpha\beta\gamma\delta}G^{\alpha\beta\gamma\delta} + m^2V^{-2}G^{kl}(b)\hat{\beta}_k\hat{\beta}_l\right]$$
$$-\frac{1}{2\kappa_5^2}\int\left(\frac{2}{3}d_{klm}\mathcal{A}^k\wedge\mathcal{F}^l\wedge\mathcal{F}^m + 2G\wedge((\xi^A\tilde{\mathcal{X}}_A - \tilde{\xi}_A\mathcal{X}^A) - 2m\hat{\beta}_k\mathcal{A}^k)\right)$$

- V :Volume of the Calabi-Yau.
- b^k : Shape of the Calabi-Yau.

Boundary theories:

$$-\int d^5x\; \delta(y)\sqrt{-h_{(0)}}\;\left[\frac{m}{\kappa_5^2}V^{-1}b^k\tau_k^{(0)}+\frac{1}{16\pi\alpha_{\rm GUT}}V{\rm tr}({\rm F}^2_{(0)})+{\rm G}_{(0){\rm IJ}}{\rm D}_\mu{\rm C}^{\rm Ix}_{(0)}{\rm D}^\mu\bar{\rm C}^{\rm J}_{(0){\rm x}}\right.$$
$$\left.+V^{-1}G^{IJ}_{(0)}\frac{\partial W_{(0)}}{\partial C^{Ix}_{(0)}}\frac{\partial\bar{W}_{(0)}}{\partial\bar{C}^J_{(0)x}}+{\rm tr}(D^2_{(0)})\right]$$

and similarly on the other boundary

Brane theories:

$$-\frac{1}{2\kappa_5^2}\int d^5x\;\left\{\sum_{p=1}^{N}(\delta(y-y_{(p)})+\delta(y+y_{(p)}))\sqrt{-h_{(p)}}\;\left[mV^{-1}\tau_k^{(p)}b^k+\frac{2m(n^k_{(p)}\tau_k^{(p)})^2}{V(\tau_l^{(p)}b^l)}j_{(p)\mu}j^\mu_{(p)}\right.\right.$$
$$\left.\left.+[\Im\Pi]_{(p)uw}E^u_{(p)\mu\nu}E^{w\mu\nu}_{(p)}\right]-4m\hat{C}_{(p)}\wedge\tau_k^{(p)}d(n^k_{(p)}s_{(p)})-2[\Re\Pi]_{(p)uw}E^u_{(p)}\wedge E^w_{(p)}\right\}$$

where $$j_{(p)\mu}=\frac{\beta_k^{(p)}}{n^l_{(p)}\beta_l^{(p)}}(d(n^k_{(p)}s_{(p)})-\hat{\mathcal{A}}^k_{(p)})_\mu\;.$$

These actions are supplemented by some Bianchi identities:

$$\begin{aligned}
(dG)_{y\mu\nu\gamma\rho} &= -4\kappa_5^2(J^{(0)}_{4\mu\nu\gamma\rho}\delta(y) + J^{(N+1)}_{4\mu\nu\gamma\rho}\delta(y-\pi\rho)) \\
(d\mathcal{F}^k)_{y\mu\nu} &= -4\kappa_5^2(J^{(0)k}_{2\mu\nu}\delta(y) + J^{(N+1)k}_{2\mu\nu}\delta(y-\pi\rho)) \\
(d\mathcal{X}^A\mathcal{G}_A - d\tilde{\mathcal{X}}_B\mathcal{Z}^B)_{y\mu} &= -4\kappa_5^2(J^{(0)}_{1\mu}\delta(y) + J^{(N+1)}_{1\mu}\delta(y-\pi\rho))
\end{aligned}$$

Where the magnetic sources here are determined by the matter and gauge field fluctuations.

$$\begin{aligned}
J^{(p)}_{4\mu\nu\gamma\rho} &= \frac{1}{16\pi\alpha_{\rm \ GUT}}\mathrm{tr}(F_{(p)}\wedge F_{(p)})_{\mu\nu\gamma\rho} \\
J^{(p)k}_{2\mu\nu} &= -i\sum_{I,J}\Gamma^k_{(p)IJ}(D_\mu C^{Ix}_{(p)}D_\nu\bar{C}^J_{(p)x} - D_\mu\bar{C}^I_{(p)x}D_\nu C^{Jx}_{(p)}) \\
J^{(p)}_{1\mu} &= \frac{e^{-\mathcal{K}}}{2V}\sum_{I,J,K}\lambda_{IJK}f^{(IJK)}_{xyz}C^{Ix}_{(p)}C^{Jy}_{(p)}D_\mu C^{Kz}_{(p)}
\end{aligned}$$

So we just follow a very similar procedure to that shown in the toy model:

- We need a metric ansatz:

$$
\begin{aligned}
ds_5^2 &= a^2(y, x^\mu) g_{4\mu\nu} dx^\mu dx^\nu + b^2(y, x^\mu) dy^2 \\
V &= V(y, x^\mu) \\
b^k &= b^k(y, x^\mu) \ .
\end{aligned}
$$

- We need embeddings for the branes (appears in the induced metric etc.).

$$
X^\mu = \sigma^\mu \qquad\qquad Y = y_{(p)}(\sigma^\mu)
$$

Solve for the warping as before:

$$\frac{a_{(p)}}{a_0} = 1 - \epsilon_0 \frac{b_0}{3V_0} b_0^k \left[h_{(p)k} - \delta_k \left(z^2 - \frac{1}{3} \right) \right]$$

$$\frac{V_{(p)}}{V_0} = 1 - 2\epsilon_0 \frac{b_0}{V_0} b_0^k \left[h_{(p)k} - \delta_k \left(z^2 - \frac{1}{3} \right) \right]$$

$$b_{(p)}^k = b_0^k + 2\epsilon_0 \frac{b_0}{V_0} \left[\left(h_{(p)}^k - \frac{1}{3} h_{(p)l} b_0^k b_0^l \right) - \left(\delta^k - \frac{1}{3} \delta_l b_0^k b_0^l \right) \left(z^2 - \frac{1}{3} \right) \right]$$

Here y has been rescaled to give z and h is a linear function in z.

$$h_{(p)k}(z) = \sum_{q=0}^{p} \tau_k^{(q)} (z - z_{(q)}) - \frac{1}{2} \sum_{q=0}^{N+1} \tau_k^{(q)} z_{(q)} (z_{(q)} - 2) - \delta_k$$

Points to notice:

- The warping is quadratic as promised. As we turn the anti-brane into a brane ($\delta \longrightarrow 0$) then it goes back to being linear.

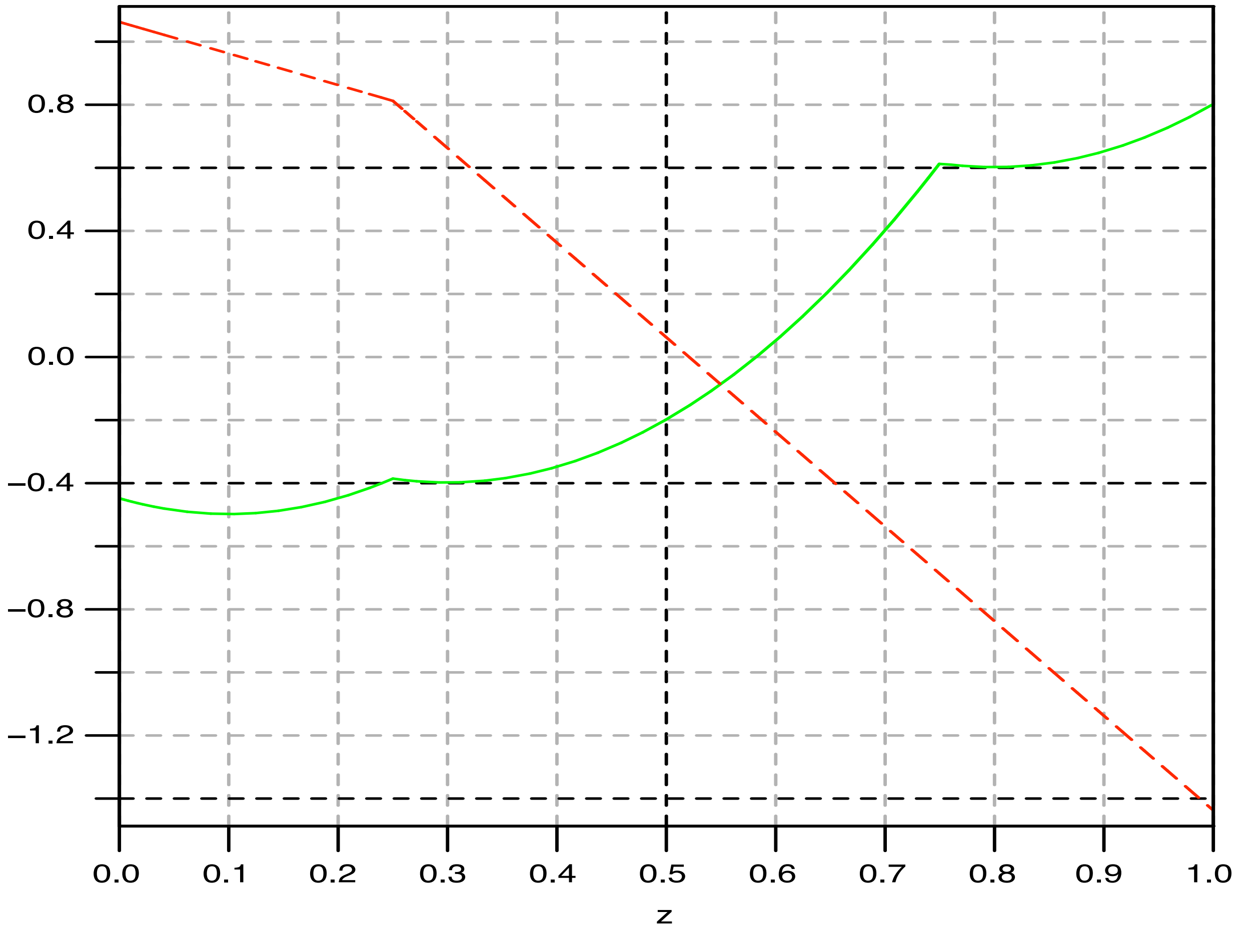
- The orbifold average of the z dependent parts are zero.

- The warpings are all controlled by the parameter:

$$\epsilon_S = \epsilon_0 \frac{b_0}{V_0}$$

Four dimensional heterotic M-theory is constructed as an expansion in this quantity.

An example:



- Red line is a supersymmetric case with one brane.
- The green line is what happens if you add an anti-brane.

Results: The four dimensional effective theory.

- We can use these warpings to systematically derive the four dimensional effective theory by dimensional reduction.
- Today I will present parts of the bosonic action. I will start with zeroth and first order in ϵ_S and then move on to some terms at second order.

Split up the first order result into pieces which contain the sum of the tensions and pieces which do not. : $S = S_{\delta^0} + S_{\delta^1}$

$$S_{\delta^0} = S_4^{\text{moduli}} + S_4^{\text{gauge}} + S_4^{\text{matter}}$$

$$\begin{aligned} S_4^{\text{moduli}} &= -\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g_4}\,\Big[\frac{1}{2}R_4 + \frac{3}{4}(\partial\beta)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{4}e^{-2\phi}(\partial\sigma)^2 + \frac{1}{4}G_{kl}\partial b^k\partial b^l + e^{-2\beta}G_{kl}\partial\chi^k\partial\chi^l \\ &\quad + \frac{1}{4}\mathcal{K}_{a\bar{b}}(\mathfrak{z})\partial\mathfrak{z}^a\partial\bar{\mathfrak{z}}^{\bar{b}} + 2\epsilon_0\sum_{p=1}^{N}\tau_k^{(p)}z_{(p)}e^{-2\phi}\partial\sigma\partial(n_{(p)}^k\nu_{(p)}) + \frac{\epsilon_0}{2}\sum_{p=1}^{N}b^k\tau_k^{(p)}e^{\beta-\phi}(\partial z_{(p)})^2 \\ &\quad + 2\epsilon_0\sum_{p=1}^{N}\frac{\tau_l^{(p)}\tau_k^{(p)}}{\tau_m^{(p)}b^m}e^{-\phi-\beta}\left(\chi^l\chi^k(\partial z_{(p)})^2 - 2\chi^k\partial(n_{(p)}^l\nu_{(p)})\partial z_{(p)} + \partial(n_{(p)}^k\nu_{(p)})\partial(n_{(p)}^l\nu_{(p)})\right) \\ &\quad + \lambda(d_{ijk}b^ib^jb^k - 6)\Big] \end{aligned} \tag{41}$$

$$\begin{aligned} S_4^{\text{gauge}} &= -\frac{1}{16\pi\alpha_{\text{GUT}}}\int d^4x\sqrt{-g_4}\,\Big[e^{\phi}\left(\text{tr}F_{(0)}^2 + \text{tr}F_{(N+1)}^2\right) - \frac{1}{2}\sigma\epsilon_{\mu\nu\rho\gamma}\left(F_{(0)}^{\mu\nu}F_{(0)}^{\rho\gamma} + F_{(N+1)}^{\mu\nu}F_{(N+1)}^{\rho\gamma}\right) \\ &\quad + \sum_{p=1}^{N}\left([\Im\Pi]_{(p)uw}E_{(p)}^{u}E_{(p)}^{w} - \frac{1}{2}[\Re\Pi]_{(p)uw}\epsilon_{\mu\nu\rho\gamma}E_{(p)}^{u\mu\nu}E_{(p)}^{w\rho\gamma}\right)\Big] \end{aligned} \tag{42}$$

$$\begin{aligned} S_4^{\text{matter}} &= -\int d^4x\sqrt{-g_4}\sum_{p=0,N+1}\Big[\frac{1}{2}\Big(e^{-\beta}G_{(p)MN}DC_{(p)}^{Mx}D\bar{C}_{(p)x}^{N} - 2e^{-2\beta}G_{kl}\omega_{1\mu}^{(p)k}\partial^\mu\chi^l \\ &\quad + e^{-\phi-2\beta}G_{(p)}^{MN}\frac{\partial W_{(p)}}{\partial C_{(p)}^{Mx}}\frac{\partial\bar{W}_{(p)}}{\partial\bar{C}_{(p)x}^{M}} + e^{-2\beta}\text{tr}(D_{(p)}^2)\Big)\Big] \end{aligned} \tag{43}$$

Supersymmetric in form despite containing anti-brane moduli.

$$S_{\delta^1} = -\int d^4x\sqrt{-g_4}\,\mathcal{V}_1$$

$$\mathcal{V}_1 = \kappa_4^{-2}\frac{\epsilon_0}{(\pi\rho)^2}e^{-\phi-2\beta}b^k\delta_k$$

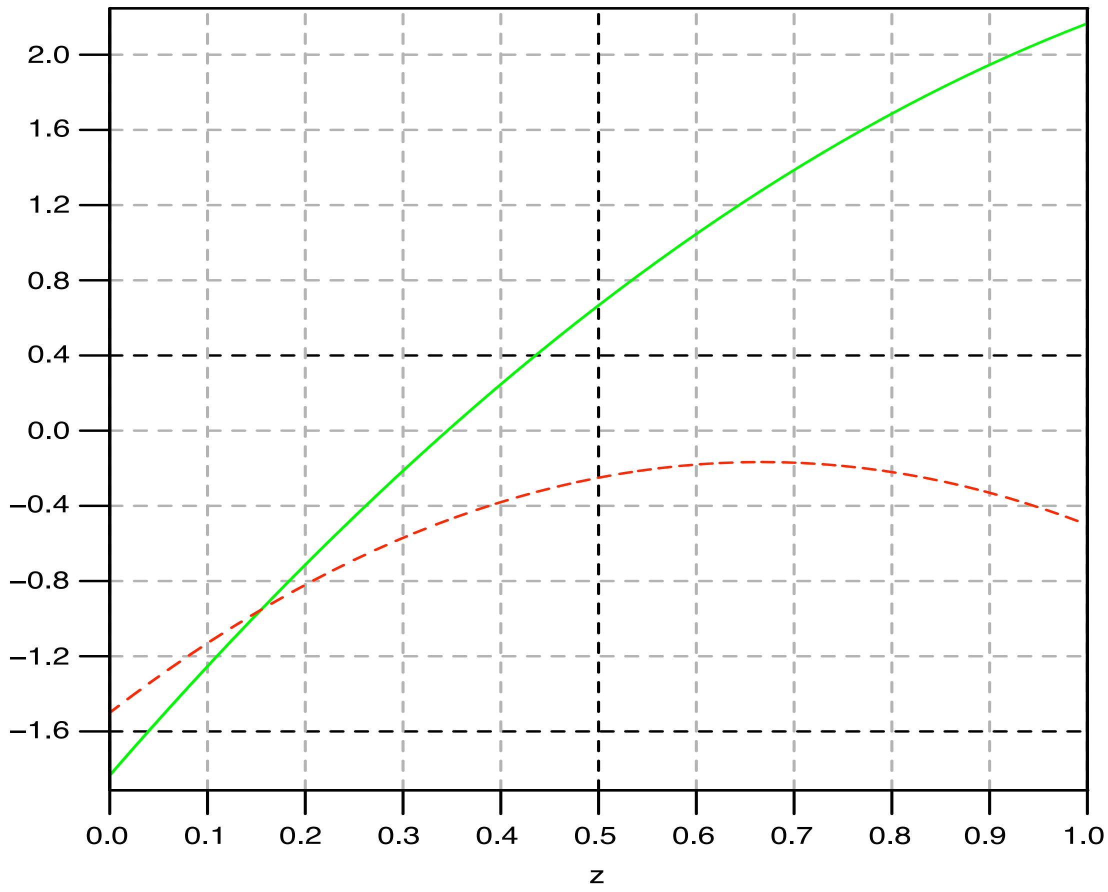
- This, up to a factor of 2, is just the energy density of the anti-brane.
- So, from the point of view of moduli stabilization, the `KKLT' procedure of just adding the anti-brane energy to the supersymmetric theory is exact to first order....
- ....up to a correction to the gauge kinetic functions which I will present in a minute.

The second order potential can also be obtained

$$\mathcal{V}_2 = \frac{1}{(\pi\rho)^2\kappa_4^2}\epsilon_0^2 e^{-\beta-2\phi} G^{kl}\delta_l \left[\sum_{p=0}^{\bar{p}-1}\tau_k^{(p)}\bar{z} - \sum_{p=\bar{p}+1}^{N+1}\tau_k^{(p)}\bar{z} - \sum_{p=0}^{\bar{p}-1}\tau_k^{(p)} z_{(p)} \right.$$

$$\left. + \sum_{p=\bar{p}+1}^{N+1}\tau_k^{(p)} z_{(p)} + \sum_{p=0}^{N+1}\tau_k^{(p)}\left(1 - z_{(p)}\right) z_{(p)} - \frac{2}{3}\delta_k \right]$$

- The first four terms are the expected coulomb forces between the branes.
- The last two terms are new. They are in no sense smaller than the coulomb terms.
- We would expect similar terms to arise in other contexts such as type II string theories.



- Note these potentials would be straight lines in the case of the naive coulomb force.

- Note these inter-brane forces are second order in ϵ_S and so extremely weak in any controlled regime of moduli space.

Another change in the 4D action - the gauge kinetic function.

$$\begin{aligned}
S_4^{\rm GKF} &= \frac{-1}{32\pi\alpha_{\rm GUT}}\int d^4x\sqrt{-g_4}\left[\left(e^{\phi}+\epsilon_0 e^{\beta}b^k\left(\sum_{p=0}^{N+1}\tau_k^{(p)}\left(z_{(p)}^2-2z_{(p)}\right)+\frac{4}{3}\delta_k\right)\right)\mathrm{tr}(F_{(0)}^2)\right.\\
&\qquad + \left(e^{\phi}+\epsilon_0 e^{\beta}b^k\left(\sum_{p=0}^{N+1}\tau_k^{(p)}z_{(p)}^2-\frac{2}{3}\delta_k\right)\right)\mathrm{tr}(F_{(N+1)}^2)\\
&\quad -\frac{1}{2}\left(\sigma+2\epsilon_0\left(\sum_{p=1}^{N}\beta_k^{(p)}\chi^k\left(z_{(p)}^2-2z_{(p)}\right)-\beta_k^{(N+1)}\chi^k+2\sum_{p=1}^{N}\tau_k^{(p)}(n_{(p)}^k\nu_{(p)})\right)\right)\epsilon_{\mu\nu\rho\sigma}F_{(0)}^{\mu\nu}F_{(0)}^{\rho\sigma}\\
&\quad \left.-\frac{1}{2}\left(\sigma+2\epsilon_0\left(\sum_{p=1}^{N}\beta_k^{(p)}\chi^k z_{(p)}^2+\beta_k^{(N+2)}\chi^k\right)\right)\epsilon_{\mu\nu\rho\sigma}F_{(N+1)}^{\mu\nu}F_{(N+1)}^{\rho\sigma}\right]. \qquad (63)
\end{aligned}$$

- This change has important physical consequences: it can change which boundary undergoes gaugino condensation for example.

A simple example.

$$S = -\frac{1}{2\kappa_4^2}\int \sqrt{-g}d^4x\left[\frac{R}{2} + \frac{3}{4}(\partial\beta)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{2}|q|e^{\beta-\phi}(\partial\bar{z})^2 + 2|q|e^{-2\beta-\phi} + e^{-\beta-2\phi}\left(6|q|q_1\bar{z} - 6|q|q_2\bar{z} + 6q^2(1-\bar{z})\bar{z} + 6|q|q_2 - \frac{4}{3}q^2\right)\right]$$

- e^{β} gives size of orbifold
- e^{ϕ} gives size of Calabi-Yau
- $\bar{z}$ is position of anti-brane
- q_1, q_2, q are charges of fixed planes and anti-brane respectively ($q_1 + q_2 + q = 0$)

Conclusions

- We have included anti-branes in the vacuum of heterotic M-theory.
- There are unexpected forces between the branes and anti-branes. These are vital in any discussion of the cosmology or stabilization of anti-branes.
- There are corrections to the gauge kinetic functions which change the potential obtained from gaugino condensation.
- Similar conclusions would be expected in other contexts involving anti-branes.