Brane annihilation in curved space-time

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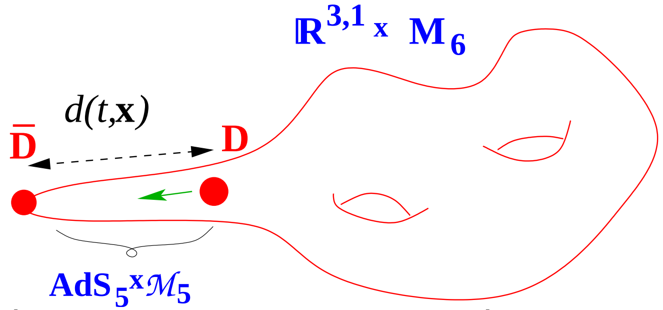
Outline of the Talk

1. Quick review of brane reheating
2. Brane annihilation in flat space-time
3. Brane decay in AdS
4. Closed and open string emission
5. Brane decay in non-critical strings
6. Lessons for brane inflation setups

A popular scenario of brane inflation

✔ Natural setting of string cosmology: flux compactification of type II string theory, with stabilized moduli

➥ generically warped throats develop

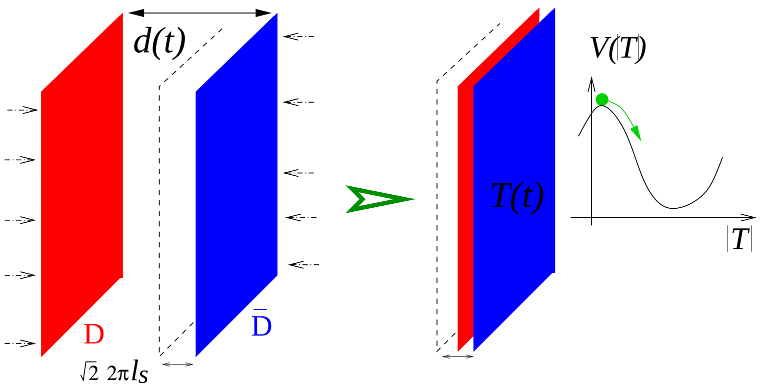


✔ AdS_5 geometry, capped both in the UV (compact 6-manifold) and in the IR (tip of the throat) *[Giddings, Kachru, Polchinski '03]*

✔ D-brane/ anti D-brane pair in the throat: Coulombian attraction redshifted by AdS_5 metric ➥ slow-roll inflation *(inflaton $d(t,\mathbf{x})$)* *[Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi '03]*

① *Brane Reheating*

✔ End of inflation: D-$\bar{\text{D}}$ annihilation ➥ open string tachyon for $d^2 < 8\pi^2\ell_s^2$



★String theory realization of hybrid inflation

✔ Tachyon condensation: involves all the massive string modes $(m > {}^1\!/\ell_s)$

➥ string corrections important

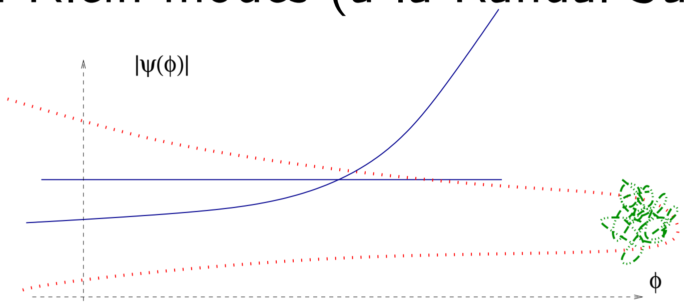
★One can use exact tree-level string computations *[Sen '02]*

➥ one gets a non-relativistic "tachyon dust" of massive closed strings

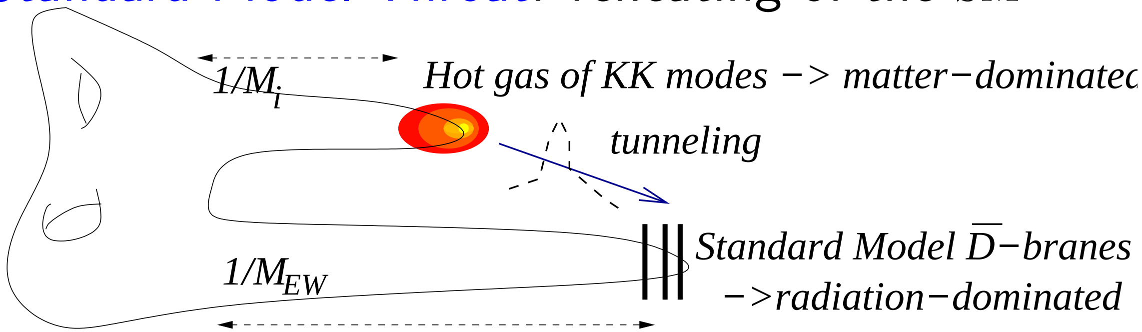
✔ Reheating of the standard model

[Barnaby, Burgess, Cline '04]

★Fast decay in Kaluza-Klein modes (à la Randal-Sundrum)



✔ Tunneling to the Standard Model Throat: reheating of the SM

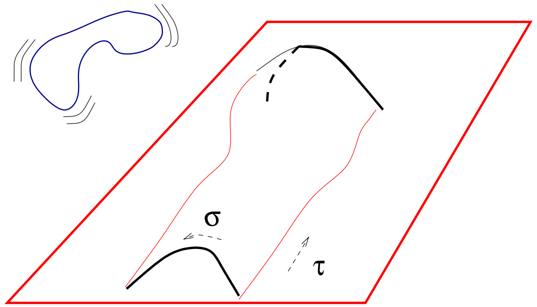


★In all these computations, $\ell_s^{\text{local}} \gg \ell_s$ due to the gravitationnal redshift of the AdS metric

$$\mathrm{d}s^2 = \mathrm{d}\phi^2 + e^{2\phi}\mathrm{d}x^\mu \mathrm{d}x_\mu \quad \Rightarrow \quad \ell_s(\phi_0) = e^{2\phi_0}\ell_s$$

② Brane Annihilation: Flat Space-Time

✔ Decay of an unstable D-brane: *equivalent to coincident D-$\bar{\text{D}}$ pair with no relative velocity (using $(-)^{F_L}$ orbifold)* ➥ solvable worldsheet string model *[Sen '02]*



$$\delta S = \lambda \int \mathrm{d}\tau \, \exp\{{X_0(\tau)}/{\ell_s}\}$$ *Wick rotation of boundary Liouville*

✔ Couplings to closed strings *(grav. sector)* $\langle V_E \rangle_\lambda = (\pi\lambda)^{-iE} \frac{\pi}{\sinh \pi E}$

➥ time-dependent source for all closed string modes

★Closed strings production *(coherent state)*

Number of emitted strings *(tree-level)*: $\mathcal{N} = \int \frac{\mathrm{d}E}{2E} \, \rho(E) \, |\langle V_E \rangle_\lambda|^2$ *[Lambert, Liu, Maldacena '03]*

✔ Density of closed strings oscillators $\rho(N)$

➥ exponentially growing *(cf. Hagedorn transition at high temperature)*

★In flat space-time, $\rho(N) \sim N^{\alpha} e^{+4\pi\sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✔ Amplitude $\boxed{\mathcal{N} \sim \int \mathrm{d}E \; E^{2\alpha-1} \; e^{2\pi E} \sinh^{-2}(\pi E)}$

➥ divergent for D0-branes $(\alpha = 0)$ *(D3-branes: instable to inhomogeneous decay)*

★Divergence signals breakdown of string perturbation theory

➥ Large gravitational back-reaction from the brane decay!

★mass of a D0-brane $m_{\mathrm{D0}} \propto {}^{1}/_{\ell_s g_s}$

➥ energy conservation not "built-in" the (tree-level) computation

✔ One needs a UV cutoff at $E \sim m_{\mathrm{D0}}$

★fraction of total energy in strings of mass $m \sim$ cst. (up to m_{D0})

➥ most energy in strings $m \sim m_{\mathrm{D0}}$, non-relativistic ($p \propto {}^{1}/_{\ell_s \sqrt{g_s}}$): tachyon dust

Sen's Conjecture

1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$
   ➥ all energy is converted into *tachyon dust* of massive closed strings

2. However the open string description of the process remains valid
   ➥ may be spoiled by open string pair production (more later)

3. The open string description is *holographically dual* to the closed strings description, hence is *complete*

4. One can use the tachyon low-energy effective action
   $S_{\rm T} = \int \mathrm{d}^d x \cosh(T/\sqrt{2})^{-1} \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \cdots)}$ ➥ late-time "dust"

5. Conjecture has been checked in 2D string theory

What Should be Modified?

✔ Cosmological context: $\mathrm{D}/\bar{\mathrm{D}}$ in a curved space-time *(e.g. capped* AdS_5*)*
➥ is the physics of the decay similar? (in string theory, UV-IR relation)

✔ In particular cancellation between asympt. density of closed string states & closed string emission amplitude may not be true anymore
★In CFT with minimal dimension Δ_m, $\rho(E) \sim \exp\{\sqrt{1-\Delta_m} 2\pi E\} \longrightarrow$ UV finite?

✔ Is the process still well-described by the curved background generalization of the open string tachyon effective action?

$$S_{\mathrm{T}} = \int \mathrm{d}^{p+1}x \sqrt{-g}\cosh(\tfrac{T}{\sqrt{2}})^{-1}\sqrt{-\det\{(g+B+2\pi\ell_s^2 F)_{\mu\nu} + \partial_\mu T \partial_\nu T\}} + \int W(T)\mathrm{d}T \wedge C_{[p]}$$

★In particular, if all the brane energy is not radiated into massive closed strings, the whole picture may be challenged

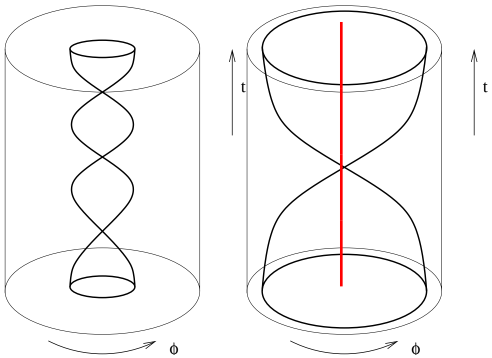
③ *Decay in Curved Space (I): Anti-de Sitter*

✔ Brane inflation setup: Approx. AdS_5 geometry

➥ However, despite recent progress AdS_5 string theory not solvable

✔ Solvable "toy model": three-dimensional AdS ➥ conformal field theory on the string worldsheet: Wess-Zumino Witten model for the group manifold SL(2,$\mathbb{R}$)

$\mathrm{d}s^2 = \ell_s^2 k \left[\mathrm{d}\rho^2 + \sinh^2\rho \mathrm{d}\phi^2 - \cosh^2\rho\ \mathrm{d}\tau^2\right]$, with a B-field $B = \ell_s^2 k \cosh 2\rho\ \mathrm{d}\tau \wedge \mathrm{d}\phi$



Two types of string modes:

short strings trapped in AdS (exponentially decreasing wave-functions)

long strings, macroscopic solutions winding w-times around ϕ

✔ Unstable D0-brane of type IIB superstrings in $AdS_3 \times \mathcal{M}_7$: localized at the origin $\rho = 0$ (infrared) ➥ decay of the brane solvable *(equivalent to D-$\bar{D}$ annihilation)*

Closed Strings Emission by the brane decay

✔ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2,\mathbb{R})$ ➥ decay described by the same boundary deformation as in flat space $\delta S = \lambda \int_{\partial\Sigma} \mathrm{d}x \; \mathbb{I} \times \exp\{\sqrt{k/2}\,\tau(x)\}$

★One gets the couplings of closed string modes to the brane, e.g. for long strings with radial momentum p_ρ and winding w:

$$\left|\langle V_{p_\rho,w,E}\rangle_\lambda\right| \propto \sqrt{\frac{\sinh 2\pi p_\rho \sinh \frac{2\pi p_\rho}{k}}{\cosh 2\pi\rho + \cos\pi(E-kw)}}\;\frac{1}{|\sinh\frac{\pi E}{\sqrt{2k}}|} \quad \textit{with } E = \frac{kw}{2} + \frac{2}{w}\left[\frac{p_\rho^2+\frac{1}{4}}{k} + N + \cdots\right]$$

➥ also coupling to discrete states (i.e. localized strings)

★Total number of emitted closed strings given by the imaginary part of the annulus one-loop amplitude, using *optical theorem + open/closed channel duality*

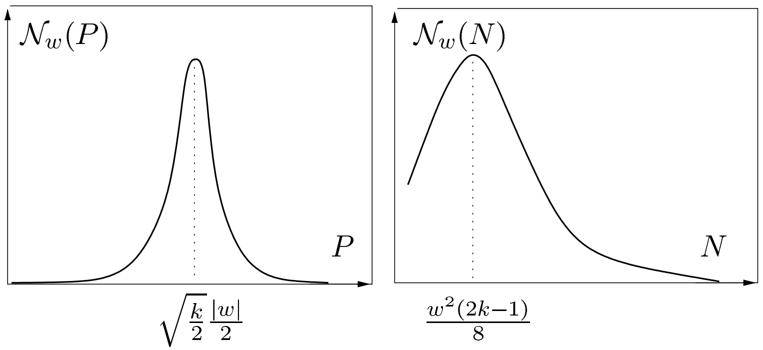
$$\mathcal{N} = \mathrm{Im}\left[\int \frac{\mathrm{d}s}{2s} \mathrm{Tr}_{\text{open}} e^{-\pi s \mathcal{H}}\right]$$

✔ As in flat space, an important input is the asymptotic density of string states

★$E \sim \frac{2N}{w}$ ➥ $\rho(E) \sim E^{\alpha} \exp\{2\pi\sqrt{(1-\frac{1}{2k})wE}\}$ (*while* $|\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}}\pi E\}$)

★Like a 2D field theory (cf. AdS_3/CFT_2)

★for *given winding* w, long strings emission is (exponentially) UV-finite!

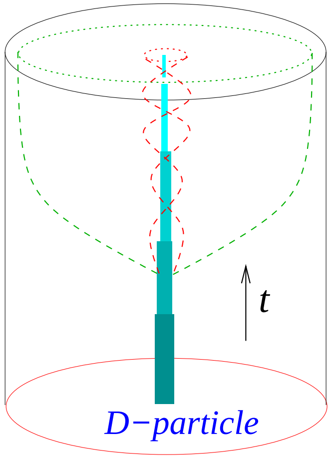


- Displacement of $\overline{p_\rho}$ due to non-perturbative corrections in ℓ_s^2 *(worldsheet instantons)*
  ➥ not seen in SUGRA limit
- For large w, $\bar{E} \sim kw$

✔ Summation over spectral flow: $\mathcal{N}_{\text{long}} \sim \sum_{w=1}^{\infty} {}^1/_w$ ➥ divergence at high energies

★Needs *non-perturbative* UV cutoff:
- $w \lesssim {}^1/g_s^2$ (NS-NS *charge conservation*)
- $w \lesssim {}^1/g_s$ (*energy conservation*)

★On the contrary, emission of *short strings* (localized strings) stays *finite*



✔ Conclusion: most of the energy converted into highly excited long strings of winding $w \sim {}^{1}/{g_s}$, expanding at speed ${}^{\mathrm{d}\rho}/{\mathrm{d}t} \sim {}^{1}/{\ell_s\sqrt{k}}$

★Closed string emission fails to be convergent because of non-perturbative effects in $\alpha' = \ell_s^2$

★Production of short strings negligible in the perturbative regime $g_s \ll 1$ (since it does not depend on the coupling constant)

✔ AdS_3/CFT_2 correspondence string theory on AdS_3 dual to a symmetric product 2D CFT ➥ dual description of tachyon decay?

★Difficult since 2D CFT is singular *(unstable to fragmentation ↔ long strings emission)*

Remarks on Open String Pair Production

✔ Open string point of view: time-dependent Hamiltonian ➥ pair production
Mini-superspace limit : $\left[\partial_t^2 + \lambda e^t + \mathbf{p}^2 + N - 1\right]\psi(t) = 0$ *[Gutperle, Strominger '03]*

★String theory naturally "chooses" (from Liouville theory) the $|out\rangle$ vacuum:
$\psi \propto H^{(2)}_{-2iE}(2\sqrt{\lambda}e^{t/2}) \overset{t\to-\infty}{\sim} e^{-iEt} + R(E)e^{iEt}$ *(R(E): reflection coefficient)*

➥ Bogolioubov coefficient $\gamma = \frac{\beta_E}{\alpha_E} \leftrightarrow$ open string two-point function $\langle e^{iEt(\tau)}e^{-iEt(\tau')}\rangle$

★Tension with Sen's conjecture in flat space?
Rate of pair production $W = -\mathrm{Re}\ln\langle out|in\rangle \sim \int \mathrm{d}E\rho(E)e^{-2\pi E}$
➥ *power-law convergent* only (divergent for $D_{p>22}$ in bosonic strings)

✔ High energy behavior of open string pair production in AdS_3

★For open strings with angular momentum r, one gets (orbifold construction)

$$|R(E)| = \left| \frac{\sinh \pi(E + r/\sqrt{k}) \sinh \pi(E - r/\sqrt{k})}{\sinh^2 2\pi E} \right|$$

➡ same large E asymptotics as in flat space

★Density of states smaller ($\Delta_{\text{min}} > 0$): $\rho(E) \sim \exp\{2\pi \sqrt{1 - 1/2k}\, \ell_s E\}$

➡ open string production rate exponentially convergent for very massive open strings on the D0-brane in AdS_3

★One gets that open string perturbative string (field) theory remains a valid description *(despite the disappearance of the brane!)*

④ *Decay in Curved Space (II): Non-Critical Strings*

✔ Non-critical superstrings: superstrings in spacetime dimension $d < 10$

➥ extra ($\mathcal{N} = 2$) Liouville (super-)field ϕ

★Einstein frame: warped geometry $\mathrm{d}s^2 = \mathrm{d}r^2 + r^2(\mathrm{d}x^\mu \mathrm{d}x_\mu + \mathrm{d}s^2(\mathcal{M}))$

★Corresponds to string theory near genuine CY singularities

✔ Mass gap $\boxed{\ell_s\, m > \sqrt{8-d}/2}$ in the closed string sector (δ-normalizable states)

➥ lower density of states $\rho(E) \sim \exp\{2\pi\sqrt{1 - \frac{8-d}{16}}E\}$ *(higher Hagedorn temp.)*

✔ From these considerations, it has been suggested that closed string emission in non-critical string is UV-FINITE *[Karczmarek,Liu,Hong,Maldacena,Strominger]*

- would raise a puzzle: what is the leftover of the brane mass? ($\ell_s m_{\mathrm{D}} \sim {}^1/g_s^{\text{local}}$)
- would challenge Sen's conjecture ("universality" of DBI tachyonic action)

Decay of extended branes

✔ Brane extended along the dilaton gradient in $\mathcal{N}=2$ Liouville *(cf.* FZZT *brane)*

★Continuous spectrum (δ-norm) above a gap

➥ vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8}+iP)\phi(x) + p_\mu X^\mu(x) + \cdots\}$

✔ Non-BPS D-brane *(or $D/\bar{D}$ pair)*: open string tachyon of mass $\ell_s m = i\sqrt{d}/4$

★Homogeneous decay: $\delta S = \lambda\sigma^1 \oint \mathrm{d}x\; G_{-1/2}\; e^{-\sqrt{1-d/8}\,\phi(x)+\frac{\sqrt{d}}{4\ell_s}X_0(x)}$

➥ not a known conformal field theory

✔ One could instead deform the worldsheet with $\delta S = \lambda\sigma^1 \oint \mathrm{d}x\; G_{-1/2}\, \mathbb{I}\times e^{\frac{X_0(x)}{\sqrt{2}\ell_s}}$

★However the identity $\mathbb{I}$ is not normalizable on the extended brane in Liouville theory (measure $\propto \mathrm{d}\phi\, e^{\sqrt{4-d/2}\,\phi}$)

➥ does not represent the decay of the open string tachyon but changes the boundary conditions at $\phi\to-\infty$ (*however leads to a* UV*-finite result)*

Decay of localized branes

✔ Brane localized in the strong coupling end in $\mathcal{N}=2$ Liouville *(cf.* ZZ *brane)*

★ Discrete spectrum built on the *identity representation* of the $\mathcal{N}=2$ SCA

➥ identity $\mathbb{I}$ is a normalizable state

★ A non-BPS localized brane has an open string tachyon built on the identity

➥ decay corresponds to $\delta S = \lambda \sigma^1 \oint \mathrm{d}x \; G_{-1/2} \; e^{X_0(x)/\sqrt{2}\ell_s}$

✔ One-point function in the rolling tachyon background:

$$\langle V_{p_\phi \, E \, \mathbf{p} \, s} \rangle_\lambda = e^{i\mathbf{p}\cdot\hat{\mathbf{x}}} \; \frac{\sinh \frac{2\pi p_\phi}{Q} \sinh Q\pi p_\phi}{\cosh \frac{\pi p_\phi}{Q} + \cos \pi s} \; \frac{(\pi\lambda)^{2iE}}{\sinh \pi E}$$

★ Gives closed strings production

$$\mathcal{N} \sim \int \mathrm{d}E \; \mathrm{d}p_\phi \; \mathrm{d}\mathbf{p} \; \sum_N \rho(N) \; \left| \langle V_{p_\phi \, E \, \mathbf{p} \, s} \rangle_\lambda \right|^2 \; \delta(E^2 - p_\phi^2 - 2N - \mathbf{p}^2 + d/8)$$

➥ $\rho(N)$ smaller than in flat space, but $\int \mathrm{d}p_\phi$ gives UV divergent production

⑤ *Application to brane inflation*

✔ In both examples of "throat geometries" studied above: despite the lower asymptotic density of states

★All the brane mass converted into massive closed strings

★However, the decay products may be very different (e.g. *long strings*)

✔ Inflationary throat in brane inflation models

★Capped AdS_5 ➥ AdS_5 results valid up to energy scale $\sim 10^2/\ell_s^{\text{local}}$ *(warping)*

★$\mathrm{AdS}_5 \times \mathrm{S}^5$ string theory can be described by supercoset + pure spinor ghost CFT w. non-trivial cohomology

★BF bound $\leftrightarrow$ lower perturbative high-energy density of states w.r.t. flat space?? (⚠complicated cohomology)

➥ at higher energies, black holes $\leftrightarrow$ free YM degrees of freedom

★In AdS_5, no *long strings* to facilitate conversion of the brane energy into closed strings modes (*giant magnons, dual giant gravitons...* cannot do the job!)

✔ One can try to use AdS_5/CFT_4 correspondence

★non-BPS D0-brane $\leftrightarrow$ $U(N)$ sphalerons *[Drukker, Gross, Itzhaki]*

★Time-dependent solution of YM $\leftrightarrow$ tachyon decay *[Peeters,Zamalkar]*

★However, perturbative YM $\leftrightarrow$ strongly curved AdS_5

➥ difficult to use in this non-BPS sector

✔ One expects that $D/\bar{D}$ annihilation in inflationary throat converts all the energy into closed strings modes, however little is known about the decay products

Conclusions

- Brane annihilation ➥ involves all the tower of string modes
- Non-perturbative α' effects & asympt. density of states are crucial ingredients
- String theory clever enough to convert all brane mass into closed strings
- However, perturbative string theory leaves many issues open (backreaction)
- Sen's conjecture seems universal ➥ DBI approach

- Warped geometries brings down this phenomenon to observable scales

- Brane inflation scenarii ➥ may have an imprint in cosmological data

- The tachyon itself may lead to inflation *[Gibbons'03, Cremades Quevedo Sinha'05]*

- Dynamics of the decay of the massive string modes not well understood