Collision of Domain Walls and Brane World

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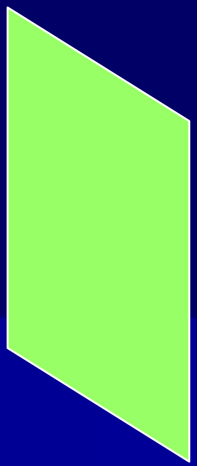
Phys. Rev. D70 (2004) 123514 , D73 (2006) 103508
hep-th/0610286, in preparation

I. Introduction

A brane: an interesting object in string theory

D3 brane : could be our universe



Some interesting cosmological senarios

Ekpyrotic (or cyclic) universe

Brane inflation (Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・)

collision of branes

A brane is usually treated as an infinitesimally thin object

To discuss “matter” on branes (e.g. reheating, localization) ,
we consider a finite thickness of brane.

a brane = a domain wall

II. collision of domain walls in 5D Minkowski space

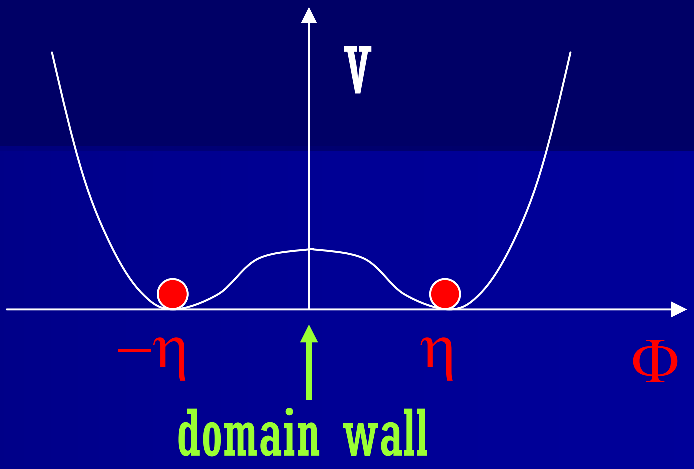
Y. Takamizu & KM: Phys.Rev. D70 (2004) 123514

5D scalar field Φ

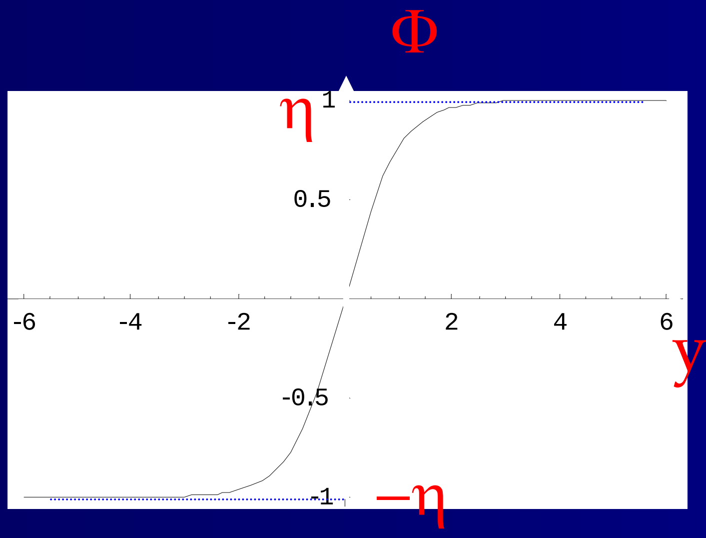
$$\Box\Phi = \frac{\partial V}{\partial \Phi}$$

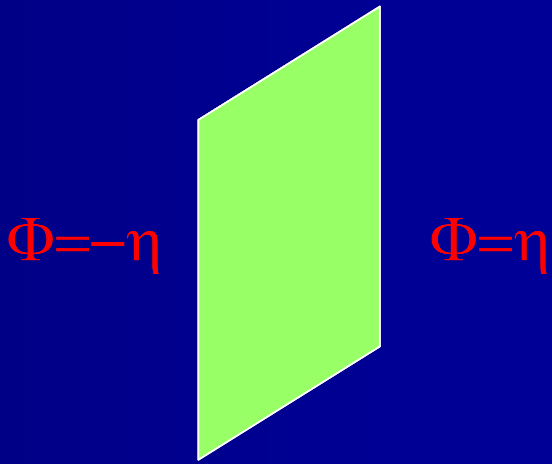
potential

$$V = \frac{\lambda}{4}\left(\Phi^2 - \eta^2\right)^2$$



domain wall solution $\Phi = \eta \tanh\left(\frac{y}{D}\right)$



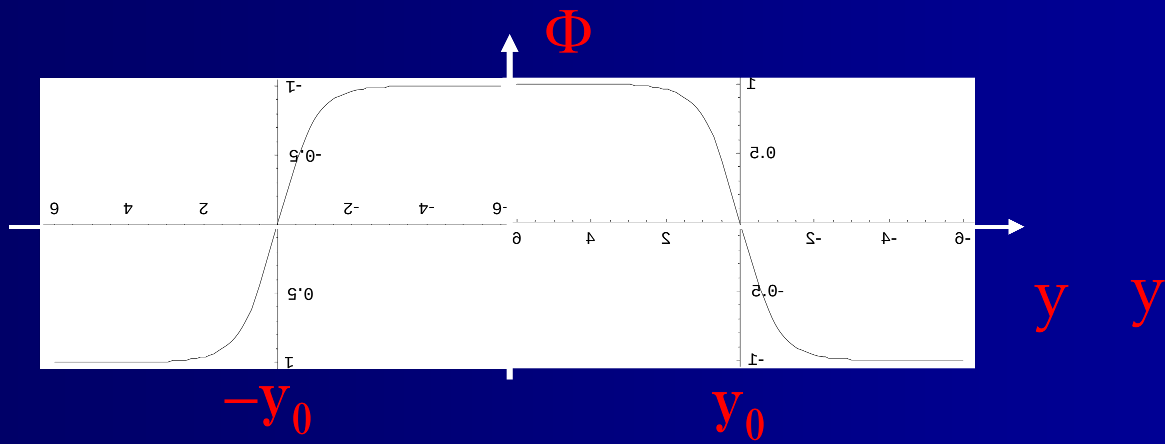


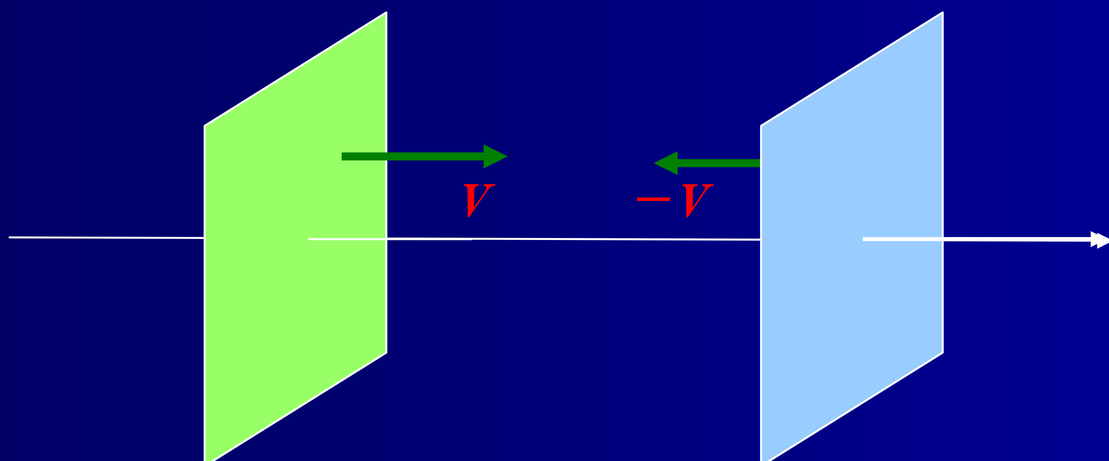
Collision of two domain walls

unit : $\eta = 1$

Two boosted domain walls with velocities v and $-v$

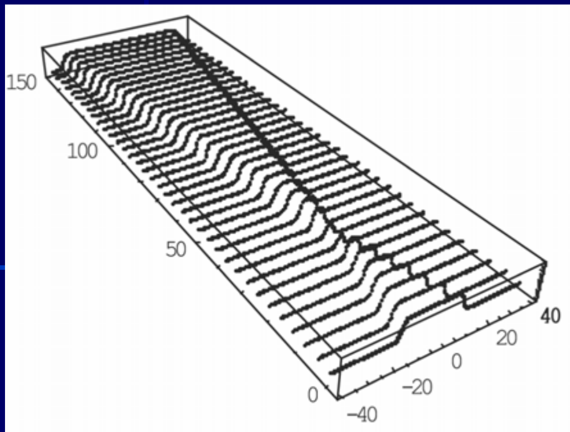
initial condition $\Phi(y,0) = \Phi_v(y+y_0,0) - \Phi_{-v}(y-y_0,0) - 1$

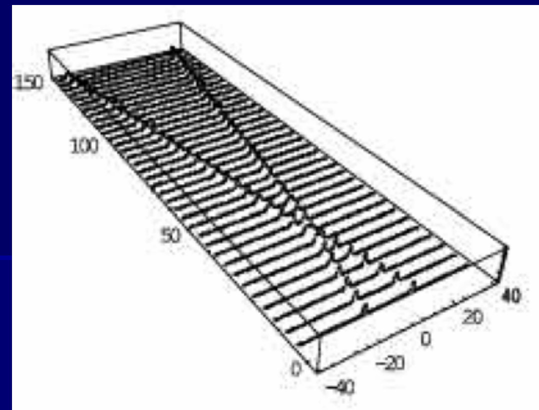




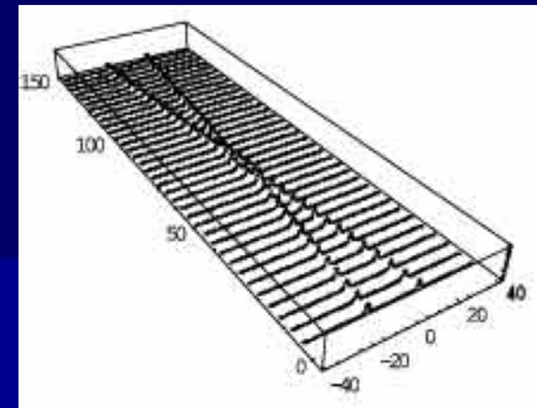
$\Phi(t, y)$

$$\rho_\Phi = \frac{1}{2}\left(\dot{\Phi}^2 + \Phi'^2\right) + \frac{\lambda}{4}\left(\Phi^2 - \eta^2\right)^2$$



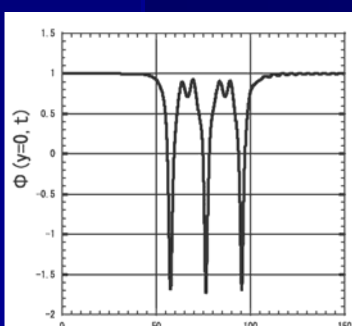


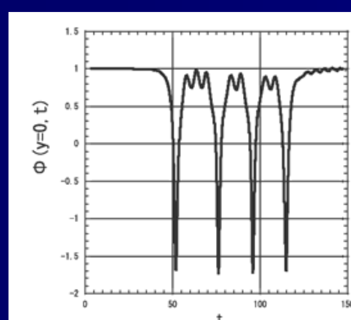
collide once **v=0.4**

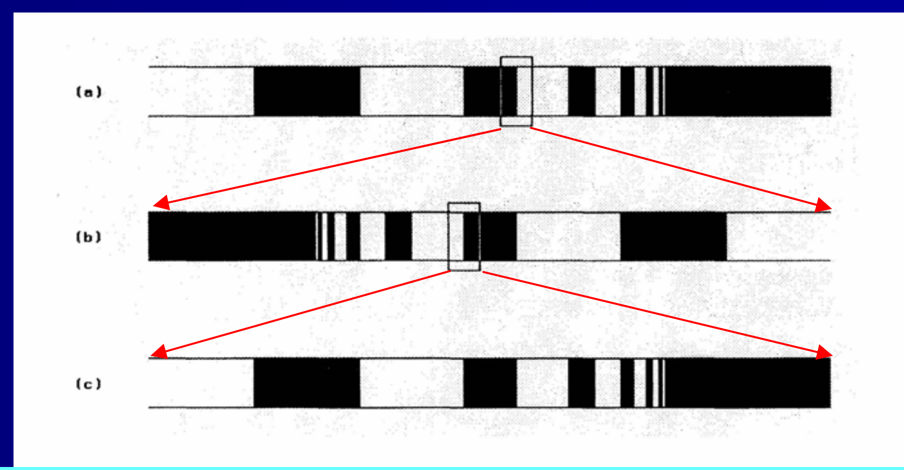


collide twice **v=0.2**

The number of bounces highly depends on the initial velocity.







N-bounce sols.

: a fractal structure

Anninos, Oliveira and Matzner, PRD 44 (1991) 1147

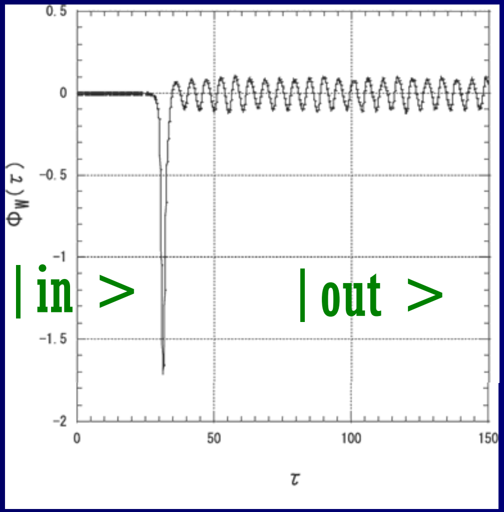
III. Reheating by Collision of Branes

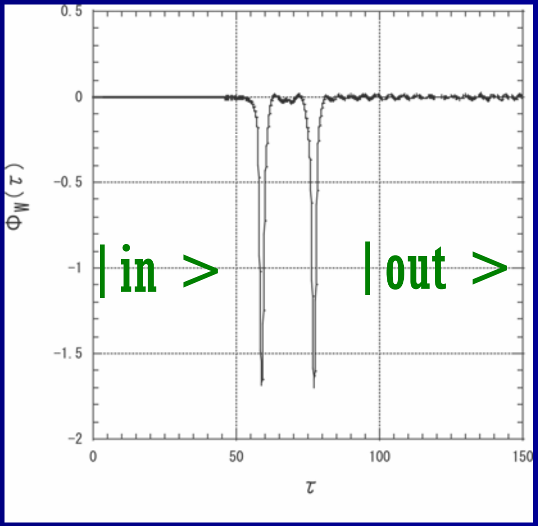
a scalar field σ confined on a brane, which is coupled to Φ

$$L_{\rm int} = \frac{1}{2} g^2 \Phi_W^2(\tau) \sigma^2$$

$\Phi_W(\tau)$: evaluated on a domain wall (a brane)

τ : a proper time on a domain wall



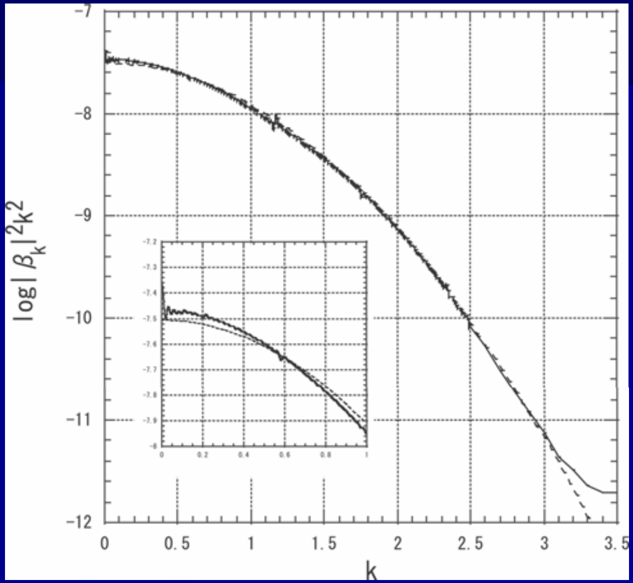


quantization of a scalar field σ with a "time-dependent mass"

⇒ particle creation

$\bar{g}$	υ	λ	D	N_b	n	ρ
0.01	0.4	1.0	1.414	1	3.69×10^{-7}	2.05×10^{-7}
		10	0.447		1.16×10^{-7}	2.05×10^{-7}
	0.2	1.0	1.414	2	7.19×10^{-7}	3.90×10^{-7}
		10	0.447		2.26×10^{-7}	3.91×10^{-7}
0.1	0.4	1.0	1.414	1	3.57×10^{-3}	2.01×10^{-3}
		10	0.447		1.16×10^{-3}	2.05×10^{-3}
	0.2	1.0	1.414	2	6.65×10^{-3}	3.81×10^{-3}
		10	0.447		2.24×10^{-3}	3.88×10^{-3}

spectrum: gaussian



⇒ quantum creation of particles

$$\rho = 20g^4 N_b, \quad n = 25Dg^4 N_b$$

$$\Rightarrow m_\eta \sim 10^{15}[\mathrm{GeV}]\ N_b^{-1/4}\left(\frac{\bar{g}}{10^{-5}}\right)^{-1}\left(\frac{T_R}{10^{10}\mathrm{GeV}}\right)$$

:mass scale of domain wall

enough reheating !

III. Fermion Localization at Collision

G. Gibbons, KM &Y. Takamizu : hep-th/0610286

5D four-component fermion Ψ coupled to the scalar field Φ

$$\Gamma^M D_M \Psi + g\Phi\Psi = 0$$

$$D_M = \partial_M + \frac{1}{4}\omega_{\hat{A}\hat{B}M}\Gamma^{\hat{A}\hat{B}}$$

$$\Gamma^M = e^M_{\hat{A}}\Gamma^{\hat{A}}$$

$$\Gamma^{\hat{A}\hat{B}} = \Gamma^{[\hat{A}}\Gamma^{\hat{B}]}$$

$$\{\Gamma^{\hat{A}}, \Gamma^{\hat{B}}\} = 2\eta^{\hat{A}\hat{B}}$$

Two chiral states

$$\Psi_- = \frac{1}{2}\left(1 - \Gamma^{\hat{5}}\right)\Psi$$

$$\Psi_+ = \frac{1}{2}\left(1 + \Gamma^{\hat{5}}\right)\Psi$$

$$\Psi_+ = \begin{pmatrix} \psi_+ \\ \psi_+ \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \psi_- \\ -\psi_- \end{pmatrix}$$

In Minkowski background

$$-\partial_5\Psi_- + \Gamma^\mu\partial_\mu\Psi_+ + g\Phi\Psi_- = 0$$

$$\partial_5\Psi_+ + \Gamma^\mu\partial_\mu\Psi_- + g\Phi\Psi_+ = 0$$

Localization on a brane

Jackiw-Rebbi (76), Rubakov-Shaposhnikov(83)
Randjbar-Daemi-Shaposhnikov(00)
Bajc-Gabadadze(00), Kehagias-Tamvakis (01)

Static domain wall

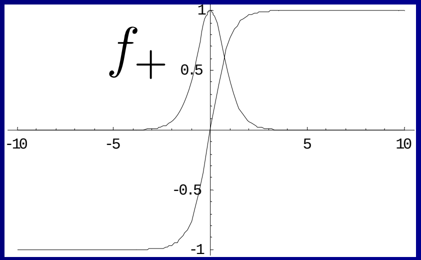
$$\Phi = \epsilon\eta \tanh\left(\frac{z}{D}\right) \qquad \epsilon = 1 : \text{kink} \qquad \epsilon = -1 : \text{antikink}$$

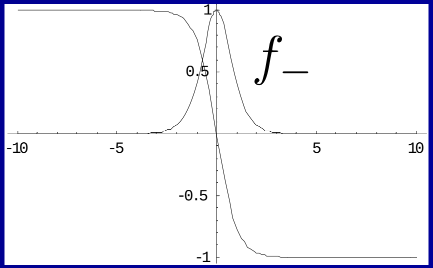
Assume massless fermion on the brane

$$\Gamma^\mu \partial_\mu \psi_+ = 0 \qquad \Psi_+(x,z) = \psi_+(x) f_+(z)$$

$$\Gamma^\mu \partial_\mu \psi_- = 0 \qquad \Psi_-(x,z) = \psi_-(x) f_-(z)$$

$$-\partial_5 f_- + g\Phi f_- = 0$$
$$\partial_5 f_+ + g\Phi f_+ = 0$$





$\epsilon = 1$ positive-chirality: localized $\qquad f_+ \propto \dfrac{1}{(\cosh(z/D))^{gD}}$

$\epsilon = -1$ negative-chirality: localized $\qquad f_- \propto \dfrac{1}{(\cosh(z/D))^{gD}}$

normalized wave function

$$f_\pm(z) = \left[\frac{\Gamma(gD+\frac{1}{2})}{2\sqrt{\pi}D\Gamma(gD)}\right]^{1/2}\left[\cosh\left(\frac{z}{D}\right)\right]^{-gD}$$

wave function of localized fermions on a kink and on an antikink

$$\Psi^{(\mathrm{K})}(x,z) = \begin{pmatrix} \overset{(4)}{\psi}_+(x) f_+(z) \\ \overset{(4)}{\psi}_+(x) f_+(z) \end{pmatrix} \qquad \Psi^{(\mathrm{A})}(x,z) = \begin{pmatrix} \overset{(4)}{\psi}_-(x) f_-(z) \\ -\overset{(4)}{\psi}_-(x) f_-(z) \end{pmatrix}$$

annihilation operators

$$a_{\mathrm{K}} = \langle \Psi^{(\mathrm{K})}, \Psi \rangle \quad \text{and} \quad a_{\mathrm{A}} = \langle \Psi^{(\mathrm{A})}, \Psi \rangle$$

Time-dependent Background

Ansatz 1: 3-space is flat

$$\psi_- = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\vec{x}} \psi_-(t, z : \vec{k})$$

$$\psi_+ = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\vec{x}} \psi_+(t, z : \vec{k})$$

$$(\partial_5 + g\Phi)\psi_+ - (i\partial_0 + \vec{k}\vec{\sigma})\psi_- = 0$$

$$(\partial_5 - g\Phi)\psi_- + (i\partial_0 - \vec{k}\vec{\sigma})\psi_+ = 0$$

Ansatz 2: Low energy state $\vec{k} \approx 0$

k : u-d mixing

$$\psi_+ = \begin{pmatrix} \psi_{+u} \\ \psi_{+d} \end{pmatrix} \qquad \psi_- = \begin{pmatrix} \psi_{-u} \\ \psi_{-d} \end{pmatrix}$$

u: up
d: down

$$i\partial_0\psi_{-u} = (\partial_5 + g\Phi)\psi_{+u}$$

$$i\partial_0\psi_{+u} = (-\partial_5 + g\Phi)\psi_{-u}$$

and u→d

time-dependence
: chirality mixing

$$\Psi = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \psi_+(z,t) + \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \psi_-(z,t)$$

up state

wave function on a moving kink

$$\psi_+^{(\mathrm{K})}(z,t;v) = \sqrt{\frac{\gamma+1}{2}}\tilde{\psi}^{(\mathrm{K})}\left(\gamma(z-vt)\right)$$

$$\psi_-^{(\mathrm{K})}(z,t;v) = i\frac{\gamma v}{\gamma+1}\sqrt{\frac{\gamma+1}{2}}\tilde{\psi}^{(\mathrm{K})}\left(\gamma(z-vt)\right)$$

wave function on a moving antikink

$$\psi_-^{(\mathrm{A})}(z,t;v) = \sqrt{\frac{\gamma+1}{2}}\tilde{\psi}^{(\mathrm{A})}\left(\gamma(z-vt)\right)$$

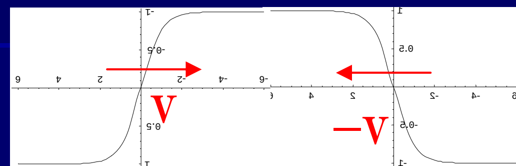
$$\psi_+^{(\mathrm{A})}(z,t;v) = -i\frac{\gamma v}{\gamma+1}\sqrt{\frac{\gamma+1}{2}}\tilde{\psi}^{(\mathrm{A})}\left(\gamma(z-vt)\right)$$

Fermion Localization on Colliding Branes

$\Phi(t, z)$ **: colliding two domain walls (Sec. II)**

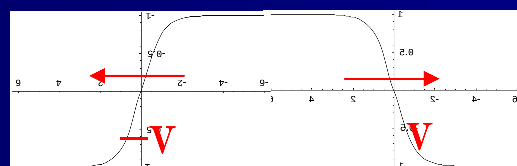
Fermion wave functions

Before collision



$$\hat{\Psi} = \Psi_{\rm in}^{\rm (K)}(x, z; v) a_{\rm K} + \Psi_{\rm in}^{\rm (A)}(x, z; -v) a_{\rm A} + \Psi_{\rm in}^{\rm (B)}(x, z) a_{\rm B}$$

After collision



$$\hat{\Psi} = \Psi_{\rm out}^{\rm (K)}(x, z; -v) b_{\rm K} + \Psi_{\rm out}^{\rm (A)}(x, z; v) b_{\rm A} + \Psi_{\rm out}^{\rm (B)}(x, z) b_{\rm B}$$

Mode mixing by domain wall collision

$$\Psi_{\rm in}^{\rm (K)}(x, z; v) \sim \alpha_{\rm K} \Psi_{\rm out}^{\rm (K)}(x, z; -v) + \beta_{\rm K} \Psi_{\rm out}^{\rm (A)}(x, z; v) + \gamma_{\rm K} \Psi_{\rm out}^{\rm (B)}(x, z)$$

$$\Psi_{\rm in}^{\rm (A)}(x, z; -v) \sim \alpha_{\rm A} \Psi_{\rm out}^{\rm (A)}(x, z; v) + \beta_{\rm A} \Psi_{\rm out}^{\rm (K)}(x, z; -v) + \gamma_{\rm A} \Psi_{\rm out}^{\rm (B)}(x, z)$$

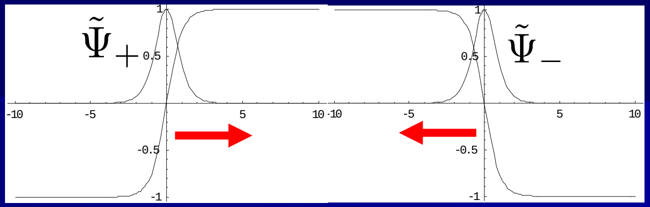
Bogoliubov transformation

$$b_{\rm K} = \alpha_{\rm K} a_{\rm K} + \beta_{\rm A} a_{\rm A} \qquad b_{\rm A} = \alpha_{\rm A} a_{\rm A} + \beta_{\rm K} a_{\rm K}$$

Two cases :

(1) same amount of fermion on each brane

Initial state



$$|\mathrm{KA}\rangle = a_{\mathrm{K}}^{\dagger} a_{\mathrm{A}}^{\dagger} |0\rangle$$

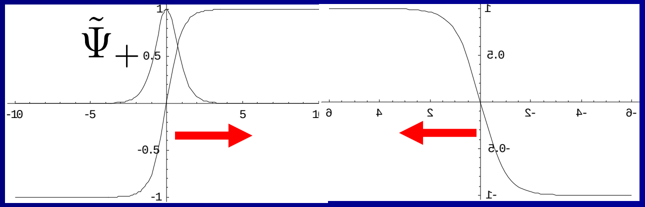
after collision

$$\langle N_{\mathrm{K}} \rangle \equiv \langle \mathrm{KA} | b_{\mathrm{K}}^{\dagger} b_{\mathrm{K}} | \mathrm{KA} \rangle = |\alpha_{\mathrm{K}}|^2 + |\beta_{\mathrm{A}}|^2$$

$$\langle N_{\mathrm{A}} \rangle \equiv \langle \mathrm{KA} | b_{\mathrm{A}}^{\dagger} b_{\mathrm{A}} | \mathrm{KA} \rangle = |\alpha_{\mathrm{A}}|^2 + |\beta_{\mathrm{K}}|^2$$

(2) one brane is empty

Initial state



$$|\mathrm{K0}\rangle = a_{\mathrm{K}}^{\dagger} |0\rangle$$

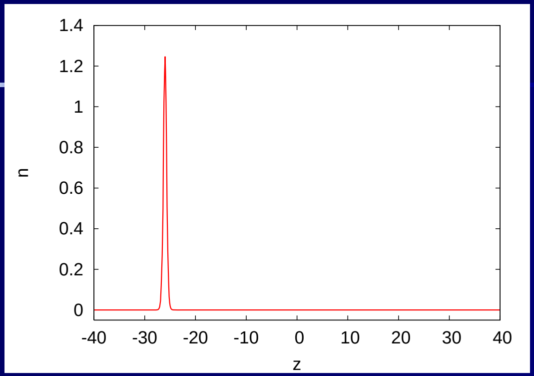
after collision

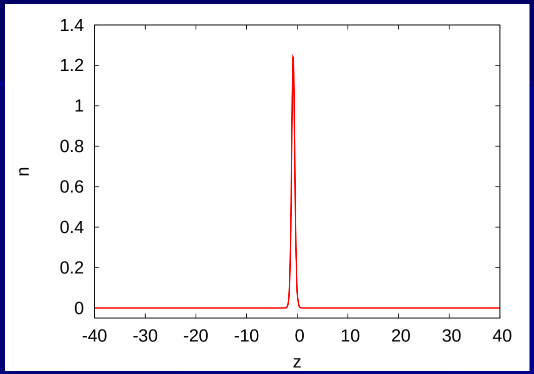
$$\langle N_{\mathrm{K}} \rangle \equiv \langle \mathrm{K0} | b_{\mathrm{K}}^{\dagger} b_{\mathrm{K}} | \mathrm{K0} \rangle = |\alpha_{\mathrm{K}}|^2$$

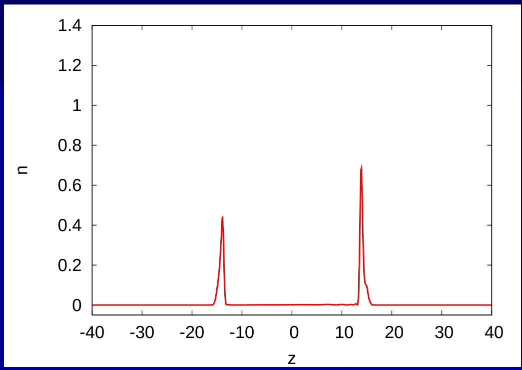
$$\langle N_{\mathrm{A}} \rangle \equiv \langle \mathrm{K0} | b_{\mathrm{A}}^{\dagger} b_{\mathrm{A}} | \mathrm{K0} \rangle = |\beta_{\mathrm{K}}|^2$$

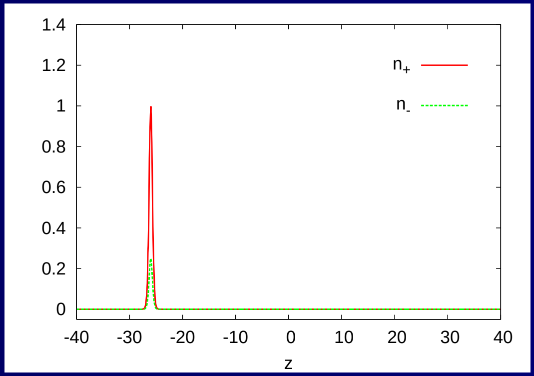
Bogoliubov coefficients by solving the Dirac eqs.

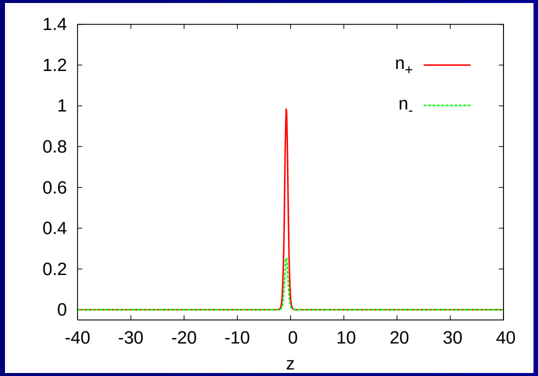
(1) v=0.8

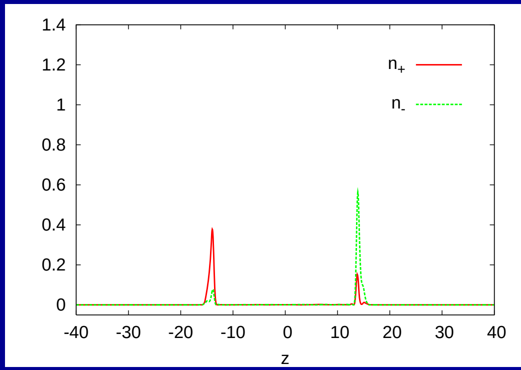












Fermions transfer to the vacuum brane

(2) v=0.4

Bogoliubov coefficients

v	$g=2$			$g=2.5$		
	$\|\alpha_{\mathrm{K}}\|^2$	$\|\beta_{\mathrm{K}}\|^2$	$\|\gamma_{\mathrm{K}}\|^2$	$\|\alpha_{\mathrm{K}}\|^2$	$\|\beta_{\mathrm{K}}\|^2$	$\|\gamma_{\mathrm{K}}\|^2$
0.3	0.94	0.056	0.004	0.47	0.53	0.00
0.4	0.87	0.12	0.01	0.57	0.40	0.03
0.6	0.69	0.30	0.01	0.78	0.17	0.05
0.8	0.42	0.55	0.03	0.88	0.02	0.10

The number of fermions are conserved as a whole $|\alpha_K|^2 + |\beta_K|^2 \approx 1$

A few percent of fermions escape to bulk space $|\gamma_K|^2 \ll 1$

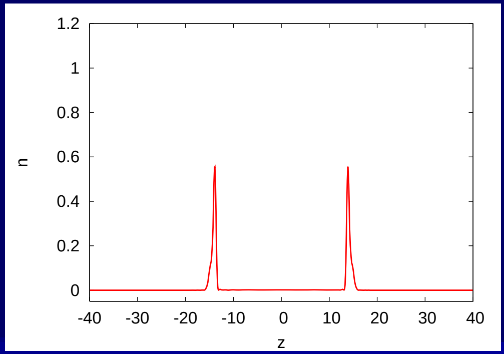
Because of the left-right symmetry,

$$|\alpha_K|^2 = |\alpha_A|^2 \qquad |\beta_K|^2 = |\beta_A|^2$$

(1) collision of two fermion branes

$$\langle N_{\rm K}\rangle = |\alpha_{\rm K}|^2 + |\beta_{\rm A}|^2 \approx 1$$

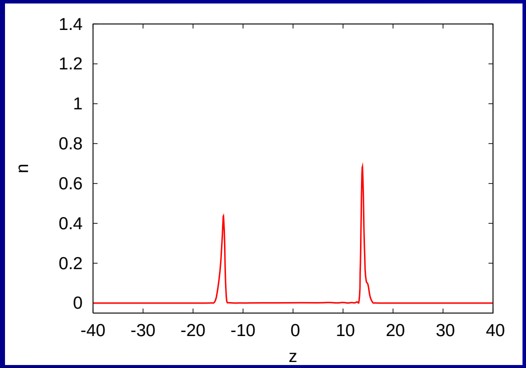
$$\langle N_{\rm A}\rangle = |\alpha_{\rm A}|^2 + |\beta_{\rm K}|^2 \approx 1$$



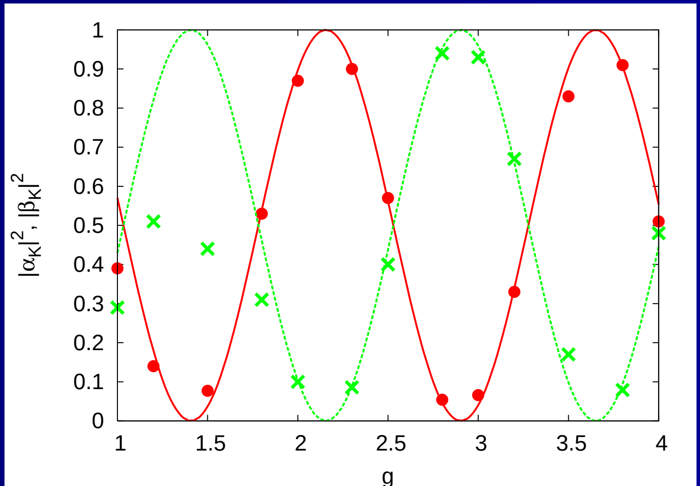
(2) collision of fermion-vacuum branes

$$\langle N_{\rm K}\rangle = |\alpha_{\rm K}|^2$$

$$\langle N_{\rm A}\rangle = |\beta_{\rm K}|^2$$



g-dependence



$$|\alpha_K|^2, |\beta_K|^2 = \frac{1}{2}\left[1 \pm \sin(3\sqrt{2}\,\varepsilon g/\sqrt{\lambda} + C_{\alpha,\beta}(v))\right]$$

$$\varepsilon = \pm 1$$

The amount of fermions on each wall

depends sensitively on v and $\frac{g}{\sqrt{\lambda}}$

Some remarks:

(1) $g < 2/D$: the localization of fermions on a domain wall is not sufficient. $g = 1, v = 0.8$ $|\alpha_K|^2 + |\beta_A|^2 = 1.28$

(2) If we change the incident velocity very little, the number of bounces changes. This causes a drastic change of final distribution of fermions on each wall.

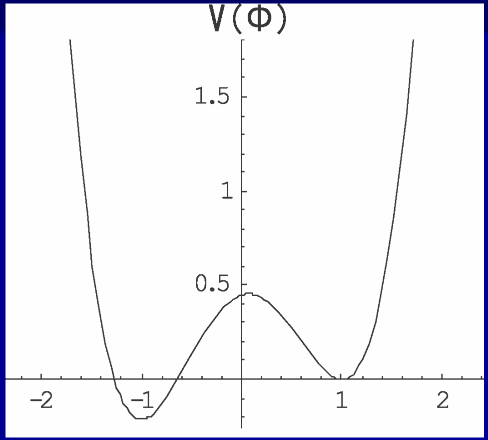
IV. collision of domain walls in AdS space

Y. Takamizu & KM: Phys.Rev. D73 (2006) 103508

BPS domain wall solution

$$V(\Phi) = \left(\frac{\partial W}{\partial \Phi}\right)^2 - \frac{8}{3}\kappa_5^2 W^2$$

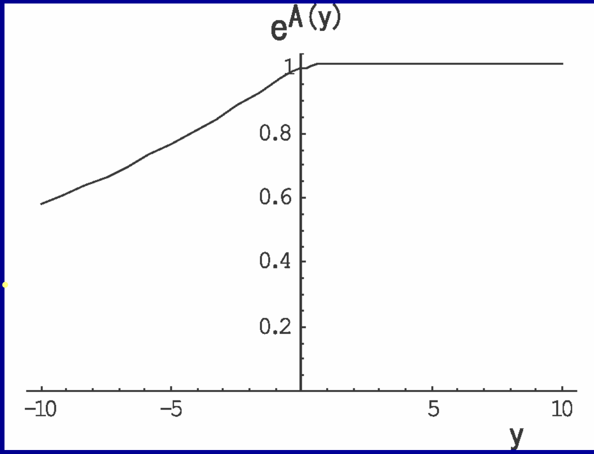
$$W \equiv \frac{1}{D}\left(\Phi - \frac{1}{3}\Phi^3 - \frac{2}{3}\right)$$ **superpotential**



$$\Phi_K(y) = \tanh\left(\frac{y}{D}\right)$$

$$ds^2 = e^{2A_K(y)}(-dt^2 + d\mathbf{x}^2) + \mathbf{dy^2}$$

$$A_K(y) = -\frac{4}{9}\kappa_5^2\left\{\ln\left[\cosh\left(\frac{y}{D}\right)\right] + \frac{\tanh^2(y/D)}{4} - \frac{y}{D}\right\}.$$

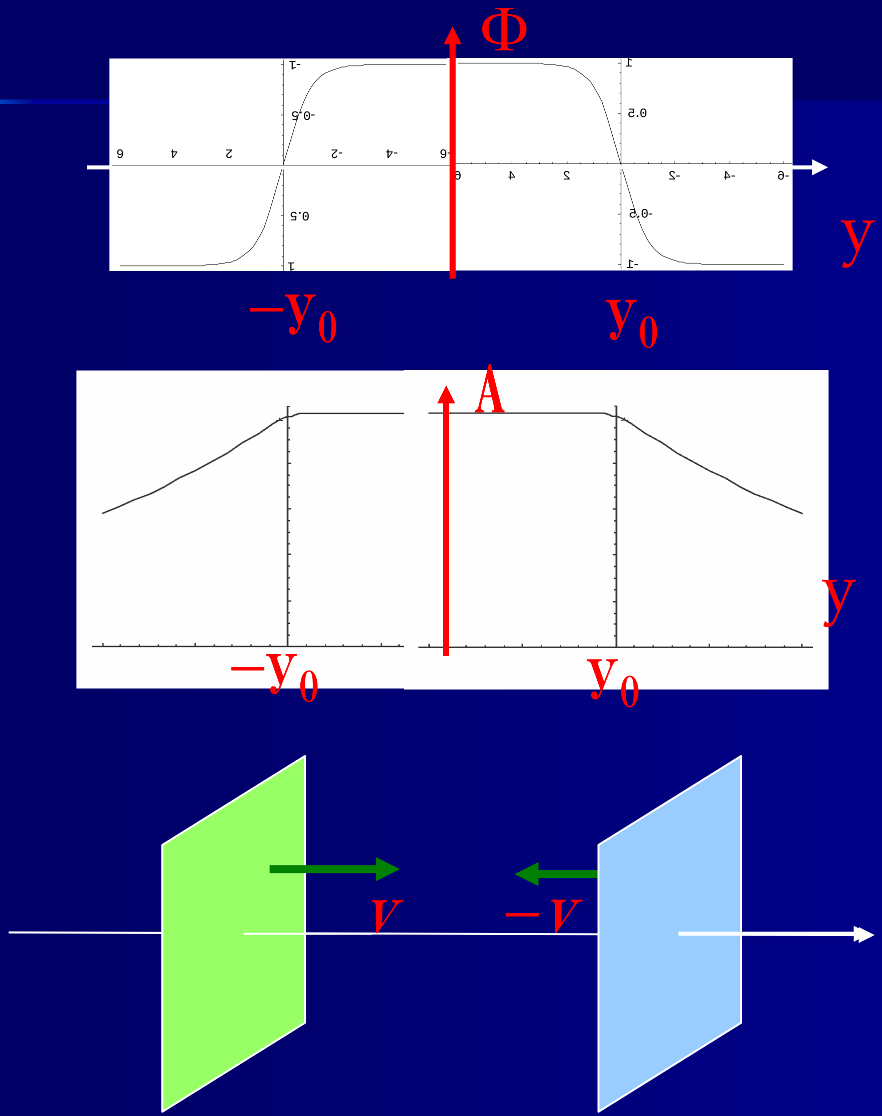


Eto-Sakai, PRD68(2003)125001

Arai et al., PLB556 (2003) 192-202

Initial setting

Two domain walls in asymptotocally AdS background



scalar field

metric

Lorentz boost

Dynamics

metric form

$$ds^2 = e^{2A(t,z)}(-dt^2 + dz^2) + e^{2B(t,z)}d\mathbf{x}^2$$

Dynamical equations

$$\ddot{A} = A'' + 3\dot{B}^2 - 3B'^2 - \kappa_5^2(\dot{\Phi}^2 - \Phi'^2 + \frac{1}{3}e^{2A}V(\Phi))$$

$$\ddot{B} = B'' - 3\dot{B}^2 + 3B'^2 + \frac{2}{3}\kappa_5^2 e^{2A}V(\Phi)$$

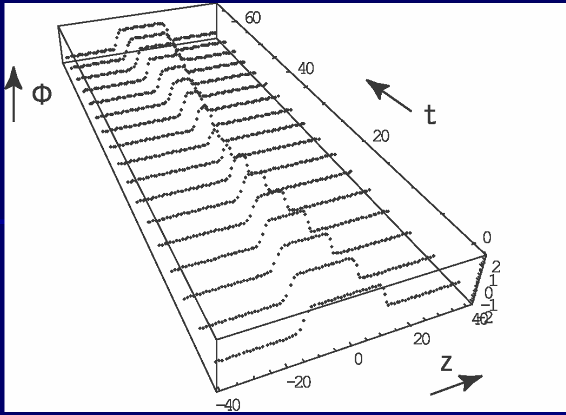
$$\ddot{\Phi} = \Phi'' - 3\dot{B}\dot{\Phi} + 3B'\Phi' - \frac{1}{2}e^{2A}V'(\Phi)\,,$$

Constraint equations

$$\dot{B}B' - A'\dot{B} - \dot{A}B' + \dot{B}' = -\frac{2}{3}\kappa_5^2\dot{\Phi}\Phi'$$

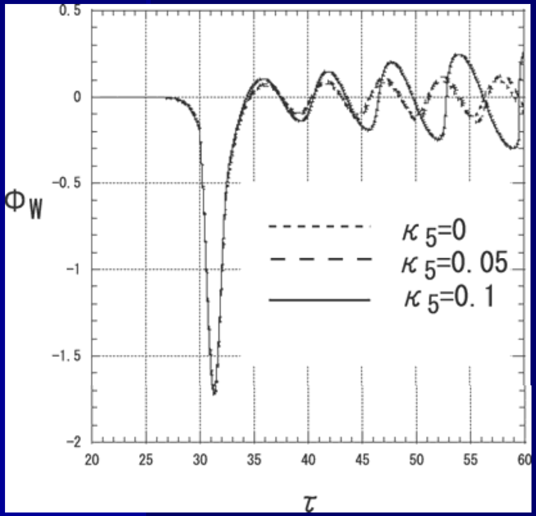
$$2B'^2 + B'' - A'B' - \dot{A}\dot{B} - \dot{B}^2 = -\frac{1}{3}\kappa_5^2(\dot{\Phi}^2 + \Phi'^2 + e^{2A}V(\Phi))$$

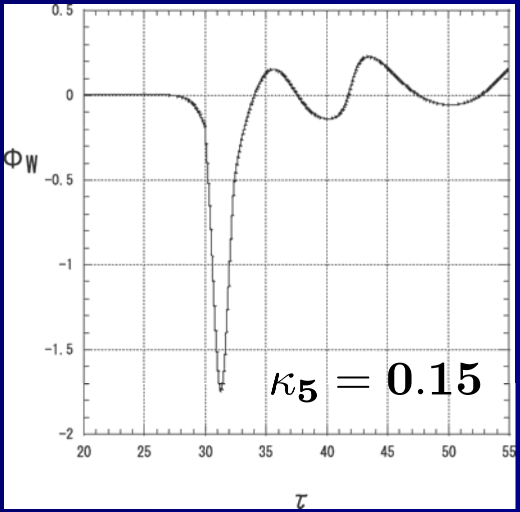
We recover the same results for weak gravity limit ($\kappa_5 << 1$)

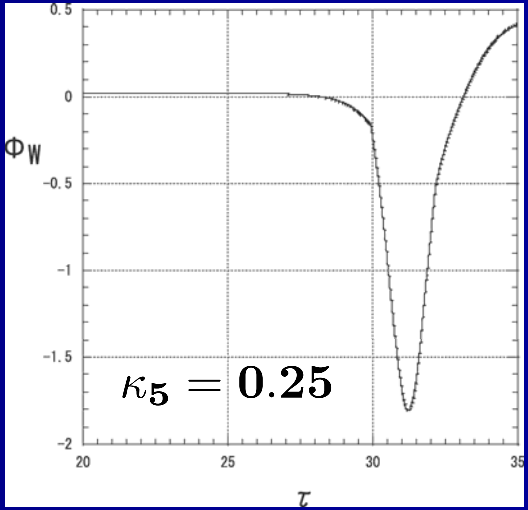


Effect of gravity

Stability



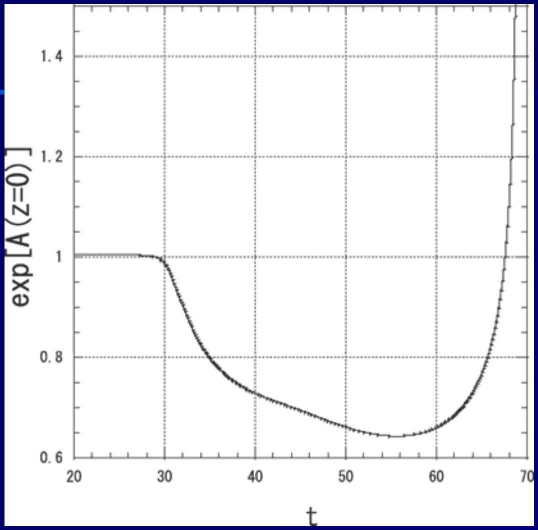




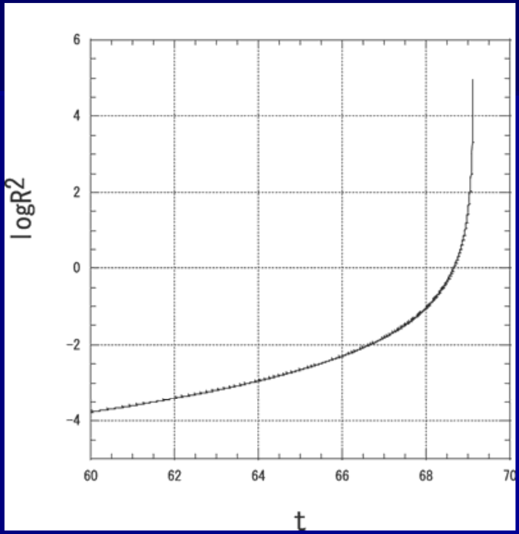
Φ becomes unstable

Spacetime evolves into a singularity

metric e^A



curvature invariant $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

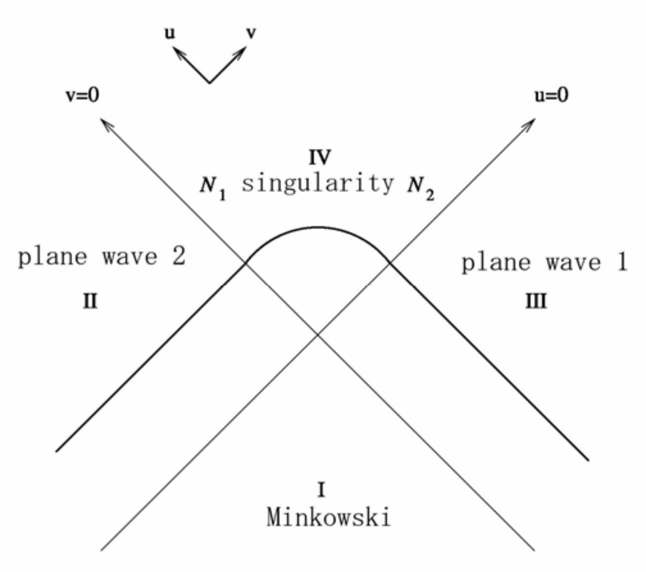


singularity

cf. plane wave collision

Khan-Penrose: Nature 229 (1971) 185

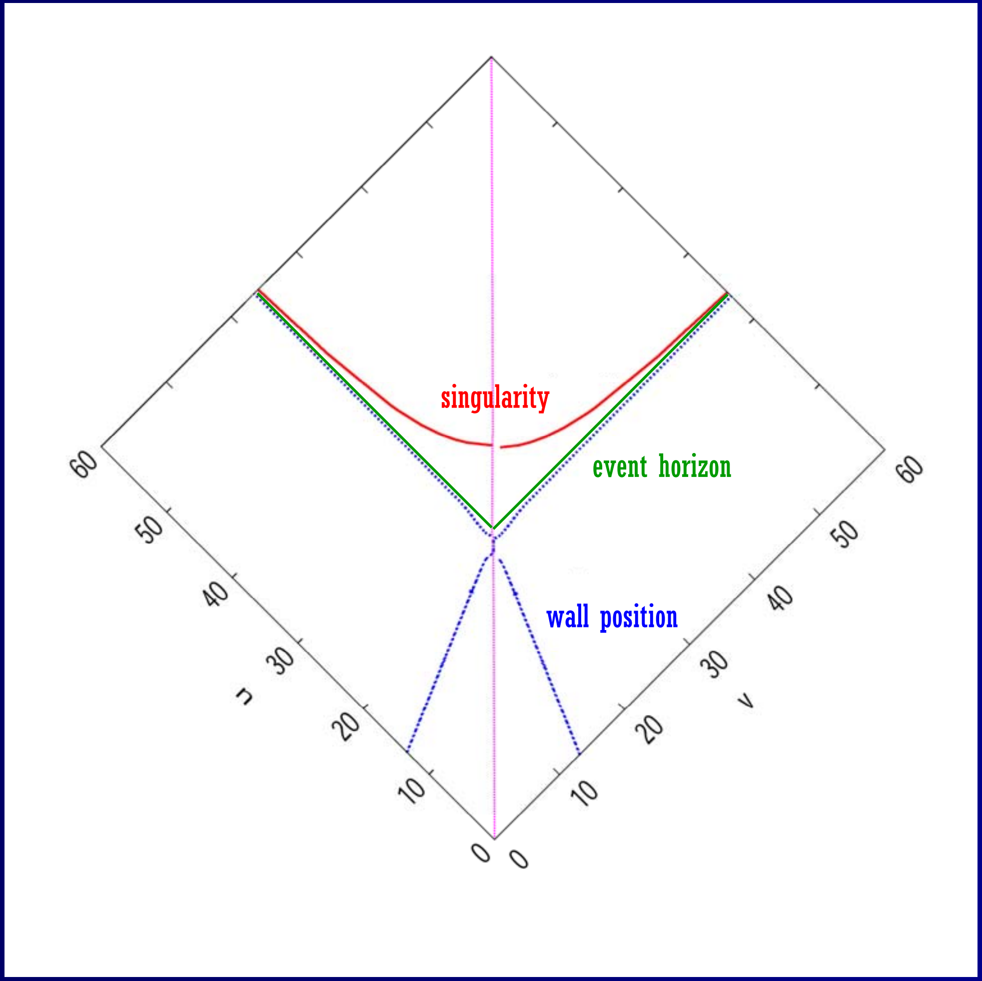
F.J. Tipler : PRD 22 (1980) 2929



spacelike singularity

Takamizu, Kudoh, KM, in preparation

Domain walls after collision are moving outside event horizon



"We" do not see a singularity

V. Summary

We discuss collision of domain walls (branes)

In 5D Minkowski background

We find a bounce (or a few bounces) of domain walls.

We study particle production at the collision.

⇒ **reheating of the universe**

We analyze localization of fermions on branes.

⇒
- **localized after collision**
- **transfer to vacuum brane**
- **v and g-dependence**

Including gravitational effects

We study dynamics of spacetime with asymptotically AdS

⇒
- **formation of singularity**
- **event horizon appears**

Remarks

It may be interesting to see what happens on fermion distribution when gravity is included.

One may look for the origin of matter (baryon asymmetry) in a braneworld scenario.

Our analysis is based on field theory (supergravity).
It may be more important to study collision of branes based on superstring or M-theory.