Transplanckian String Collisions and the information paradox

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Has string theory solved the information paradox?

• BH-entropy and counting of states agree for extremal BHs (Strominger-Vafa, ..)
• Spectra from quasi-extremal BH decay follow Hawking iff one traces over initial brane configuration (= density matrix)

Questions (see e.g. D. Amati, hep-th/0612061):
1. What happens if one starts from a pure state? Fails at weak coupling, may work at strong coupling.
2. Are there corrections to a pure thermal spectrum?
3. How does this extend to more conventional (Kerr) BHs?
1. The string-black hole correspondence curve
2. Transplanckian string collisions: why and how.
   2.1 MGO vs ACV approach to the problem
   2.2 Three scales/regimes in trans-planckian string collisions
      I) $b > R, l_s$ Easy
      II) $R > b, l_s$ Hard
      III) $l_s > R, b$ Easy again?
   2.3 Approaching gravitational collapse from region III
   2.4 A unitary S-matrix with precocious black-hole-like behaviour
3. Conclusions
The string-black hole correspondence curve
String vs Black-Hole entropy

\[ h = c = \text{numerical factors } = 1 \]
\[ M_s, l_s = \text{string mass, length scales} \]

Tree-level string entropy

Counting states (FV, BM (’69), HW (’70))

\[ S_{st} = \frac{M}{M_s} = \frac{L}{l_s} \]

\[ = \text{No. of string bits in the total string length} \]

NB: no coupling, no \( G \) appears!
Black-Hole entropy

\[ S_{BH} = M R_S = (R_S/L_P)^2 \sim M^2 \]

\((GM = R_S, 1/T_{BH} = dS/dM = R_S/h)\)

to be contrasted with previous

\[ S_{st} = M/M_s = L/l_s \]

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\[ S_{st}/S_{BH} > 1 \ @ \ small \ M, \quad S_{st}/S_{BH} < 1 \ @ \ large \ M \]

Where do the two entropies meet? Obviously at

\[ R_S = l_s \ i.e. \ at \ T_{BH} = M_s! \]

“string holes” = states satisfying this entropy matching condition
Using string unification @ the string scale,

\[(L_P/l_s)^2 = g_s^2 \sim \alpha_{GUT}\]

entropy matching occurs for

\[M = M_{sh} = g_s^{-2} M_s = g_s^{-1} M_P\]

and the common value of \(S_{st}\) and \(S_{BH}\) is simply

\[S_{sh} = g_s^{-2} \sim \alpha_{GUT}^{-1}\]

In string theory \(g_s^2\) is actually a field, the dilaton. Its value is free in perturbation theory

Consider the \((M, g_s^2)\) plane
The correspondence curve (critical collapse?)

\[ M/M_s \]

\[ R_s > l_s \]

Black Holes (= Strings? )

\[ R_s < l_s \]

Strings ≠ BH

"string hole" curve many properties match here

Safe conclusion since these strings are larger than \( R_s \)

Much more difficult to establish except for extremal case

\[ g_s^2 \]
Collapse @ fixed $M$. Gravitational binding can increase (log of) density of states from linear to quadratic in the physical mass.
Turning string entropy into BH entropy

String (naïve)

Black hole

\[ M = g_s^{-2} \]

\[ M_s = M_{sh} \]
Evaporation at fixed $g_s$ or how to turn a BH into a string (Bowick, Smolin, 1987)

Is singularity at the end of evaporation avoided thanks to $l_s$?
String $S$-matrix at $E \gg M_p$

Super-planckian-energy collisions of light particles within superstring theory. Why care?

Theoretical Motivations

I) As a gedanken experiment
   - To reproduce $\text{GR expectations}$ at large distances
   - To probe how ST modifies $\text{GR}$ at short distances

II) Information paradox
“Phenomenological” Motivations

Signatures of string/quantum gravity @ colliders:

- In KK models with large extra dimensions;
- In brane-world scenarios; in general:
- If we can lower the true QG scale down to the TeV

NB. Future colliders at best marginal for producing BHs!
Two complementary approaches (> 1987):

B) 't-Hooft; Muzinich & Soldate; ACV (>1987);
    Verlinde & Verlinde; Kabat & Ortiz; FPVV;...
    de Haro; Arcioni; 't-Hooft; ... ('90s-'05)

The two approaches are very different. Yet they agree incredibly well in the (small) region of phase space where both can be justified.

I will limit myself to describing B) and, in particular, the work of ACV (the only one, besides A) that considers the problem within string theory)
Calculation (GM, 1987–’88) of elastic string scattering at very high energy and fixed scattering angle $\theta$ ($h+1 =$ number of exchanged gravitons):

$$A_{el} \sim (g_s)^{2+2h} \exp \left( -\frac{\alpha' s f(\theta)}{1 + h} \right)$$

The amplitude is exponentially suppressed but the suppression decreases as we increase the number of exchanged gravitons. A resummation was performed by Mende and Ooguri (see below)
Amati, Ciafaloni, GV (ACV) et al.

- Work in energy-impact parameter space, $A(E,b)$ ($b \sim J/E$)
- Go to arbitrarily high $E$ while increasing $b$ correspondingly: $b > R_s(E) \sim GE$
- Go over to $A(E, q\sim \theta E)$ by FT trusting saddle p. contributions from above region
- Reach the regime of fixed $\theta \ll 1$
- Compare w/ GMO in appropriate region
At fixed $b$ we have to compute ($D=4$ when not specified)

$$\delta(E, b) = \frac{1}{(2\pi)^{D-2}} \int d^{D-2}q \frac{A_{\text{tree}}(s,t)}{4s} e^{-iqb}, \ s = E^2, \ t = -q^2$$

For the real part we get, at large $b$,

$$\text{Re}\delta \sim Gs \log b^2$$

Consequences discussed below

The graviton being “reggeized” in string theory, we also get

$$\text{Im}\delta \sim \frac{G_D}{(Y l_s)^{D-2}} s l_s^2 e^{-b^2/b_I^2}, \ b_I^2 \equiv l_s^2 Y^2, \ Y = \sqrt{\log(\alpha's)}$$

Since Im $A$ has no Coulomb pole its FT is exp.

\ll\text{small at } b \gg b_I$$
Im $A$ is due to closed strings in $s$-channel (DHS duality)

Heavy closed strings produced in $s$-ch.

Gravi-reggeon exchanged in $t$-ch.
Tree level cont.

- Tree level violates p.w. unitarity as $s$ goes transplanckian
- Tree-level too large at fixed $b$, too small at fixed $\theta$
- String loops take care of both problems!
- What do we expect from GR-type arguments?
\[ \theta \sim \left( \frac{R_S}{b} \right)^{D-3} \]

\[ \text{Corr.}^{\text{ns}} \sim \left( \frac{R_S}{b} \right)^{2(D-3)} \]

\[ \theta \sim 2\pi \]

**WITHOUT STRING THEORY**

**II COLLAPSE**
WITH STRING THEORY

corr’s to eik. ~

\[ \left( \frac{R_s}{l_s + b} \right)^{2(D-3)} \]

\[ E = E_{th} = M_s g_s^{-2} = M_{sh} \]
Accretion at fixed $g_s$ or how to turn a string into a black hole

$M / M_s$ vs. $g_s^2$

Black Holes

$R_S = l_s$

Strings

string-holes
I) **Small angle scattering**: relatively easy

II) **Large angle, collapse**: very hard, all attempts have failed so far

III) **Stringy** (easy again)

A single, compact formula covers regions I and III!
Unitary $S$-matrix in regions I and III

$$S = e^{2i\delta} e^{2i\sqrt{\text{Im}\delta}} C^\dagger e^{2i\sqrt{\text{Im}\delta}} C$$

$$[C, C^\dagger] = 1$$

$$\delta(E, b) = \frac{1}{(2\pi)^{D-2}} \int d^{D-2} q \frac{A_{\text{tree}}(s, t)}{4s} e^{-iqb}, \ s = E^2, \ t = -q^2$$

Actually $\delta$ becomes an operator, but we shall neglect this complication physically related to the «diffractive» excitation of each string by the tidal forces due to the other string.
Diffractive excitation from $b \rightarrow b + \Delta X$
Another way of “cutting” the diagram

Diffractively produced closed strings

exchanged gravi-reggeons
We will instead concentrate on the operators $C, C^+$ (appearing iff $\delta$ is not real) corresponding to the «Reggeization» and duality of graviton exchange in string theory.
NB: any number of gravi-Reggeons can be cut: AGK rules.

Heavy closed string produced

Exchanged gravi-reggeons
Recall that:

\[
\text{Im} \delta \sim \frac{G_D s l_s^2}{(Y l_s)^{D-2}} e^{-b^2/b_I^2}, \quad b_I^2 \equiv l_s^2 Y^2, \quad Y = \sqrt{\log(\alpha' s)}
\]

Thus, for \( b \gg b_I \) (Region I), we can forget about \( C, C^* \). Also:

\[
\text{Re} \delta \sim G_D s \frac{b^{4-D}}{D - 4}
\]

Going over to scattering angle \( \theta \) by FT, we find a saddle point:

\[
b_s^{D-3} = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2} \theta}
\]

i.e.

\[
\theta = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2} b^{D-3}}
\]

Corresponding **precisely** to the relation between \( b \) and \( \theta \) in an AS metric\(^*\)): clearly, fixed \( \theta \), large \( E \) probes large \( b \)

\(*\) metric produced by a pointlike relativistic particle
Region III

Let us neglect (for a moment!) Im $\delta \neq 0$, $C$ and $C^+$

$$Re\delta = -\frac{G_D s b^2}{(Y l_s)^{D-2}}$$

The saddle point condition now gives the relation:

$$\theta = G_D \rho b, \quad \rho = \frac{E}{(Y l_s)^{D-2}}$$

corresponding to deflection from an homogeneous beam of transverse size $\sim l_s$: $\theta_{max} \sim GE/l_s^{D-3}$ reached for $b \sim l_s$
Analysis of final state in Region III

Take into account $\text{Im} \delta \neq 0$. $C$ and $C^*$ are now “activated”. Recall:

$$S = e^{2i\delta} e^{2i\sqrt{\text{Im} \delta}} C^* e^{2i\sqrt{\text{Im} \delta}} C$$

The elastic amplitude, $\langle 0|S|0 \rangle$, is suppressed as $\exp(-2 \text{ Im} \delta)$:

$$\sigma_{el} \sim \exp(-4\text{Im} \delta) = \exp \left[ -\frac{G_D s l_s^2}{(Y l_s)^{D-2}} \right] \equiv \exp \left[ -\frac{s}{M_{*}^2} \right]$$

$$M_* = \sqrt{M_s M_{sh}} \sim M_s g_s^{-1}$$

$(= M_p$ in $D=4$, $M_* > M_p$ for $D>4$)

If we go to $E= E_{th}$ we find:

$$\sigma_{el} \sim \exp(-g_s^{-2}) \sim \exp(-S_{sh})$$

Amazingly: $M_*$ is just the D0-brane mass scale!
Which final states saturate unitarity?

Recall once more:

The final state, $S|0\rangle$, is a coherent state of quanta associated with $C, C^\dagger$. These quanta are just the closed strings dual to the gravi-reggeon (CGRs for “cut gravi-reggeons”) The probability of producing $n$ CGRs thus obeys a Poisson distribution with an average given by:

$$\langle N_{CGR}\rangle = 4\text{Im}\delta = \frac{G_D s l_s^2}{(Y l_s)^{D-2}} = O\left(\frac{s}{M^2_*}\right)$$
Final state via optical theorem & AGK rules
(NB: different CGRs overlap in rapidity)

Unitarity cut through 5 GRs
At this point we can compute the average energy of a final state/string associated with a single CGR:

\[ \langle E \rangle_{\text{CGR}} = \frac{\sqrt{s}}{\langle N_{\text{CGR}} \rangle} \sim M_s Y^{D-2} \left( \frac{l_s}{R_s} \right)^{D-3} \sim T_{\text{eff}} \equiv \frac{M^2}{E} = \frac{M_s^2}{g_s^2 E} \]

We have thus found that final-state energies obey a sort of «anti-scaling» law

\[ \langle E \rangle_{\text{CGR}} \sqrt{s} = M_*^2 = M_s^2 g_s^{-2} \]

This antiscaling is very unlike what we are familiar with in HEP

It is however similar to what we expect in BH physics!

In particular: For D=4, \( T_{\text{eff}} \sim T_{\text{Haw}} \) even at \( E < E_{\text{th}} \)
$M_s/g_s^2$

$M_s/g_s$

$M_s$

$E$

$E^{-1}$

$<E>_{cg}$

$r$

$E^{-1/(D-3)} \sim T_H$

window

$M_D$

$M_* = M_s/g_s$

$M_s/g_s^2$
We conclude that, at least below $E_{th}$, there is no loss of quantum coherence, but the spectra aren’t thermal either.

Above $E_{th}$ we can no-longer neglect “classical” corrections corresponding to interactions among CGRs: these will hopefully turn the Poisson distribution into an approximately Planckian one.

No reason to expect a breakdown of unitarity.
If we could prepare as initial state:

\[
|in\rangle = S^\dagger |0\rangle = e^{-2i\delta^*} e^{-2i\sqrt{Im\delta} \cdot C^\dagger} |0\rangle
\]

the final state would be just a two-particle state!
String theory pretends to be the way to combine the principles of quantum mechanics and general relativity in a consistent framework. As such it should provide answers to the physics of black holes and cosmology in regimes where quantum effects are important/dominant.

So far, most of the progress has been in the former problem as seen from an outside observer (the physics inside a black hole is similar to that of a big crunch in cosmology).

We have seen that string theory may be able to provide a microscopic, stat. mech. interpretation of black hole entropy.
- We have also been able to recast the main results of ACV in the form of an approximate, but *exactly unitary*, $S$-matrix, whose range of validity covers a large region of the kinematic energy-angular-momentum plane;
- We have found a sort of *precocious black-hole behaviour*, in particular an « anti-scaling » dependence of $<E_f>$ from $E_i$, reminiscent of the inverse relation between black-hole mass and temperature; this may have phenomenological applications in the context of the string/quantum-gravity signals expected at colliders in models with a low string/quantum-gravity scale.