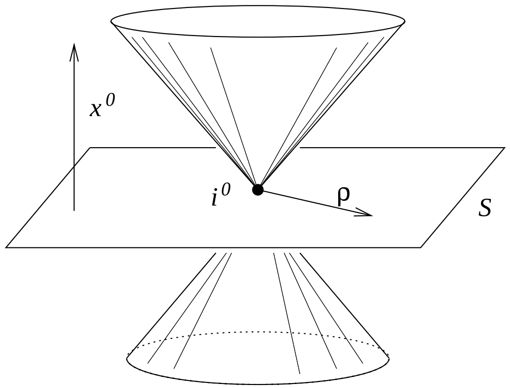
A view of spacetime near spatial infinity

Juan A. Valiente Kroon,
School of Mathematical Sciences,
Queen Mary, University of London,
United Kingdom.

November 24th, 2006.

The i^0 problem

- There is a lack of general results about the evolution of data near spatial infinity.

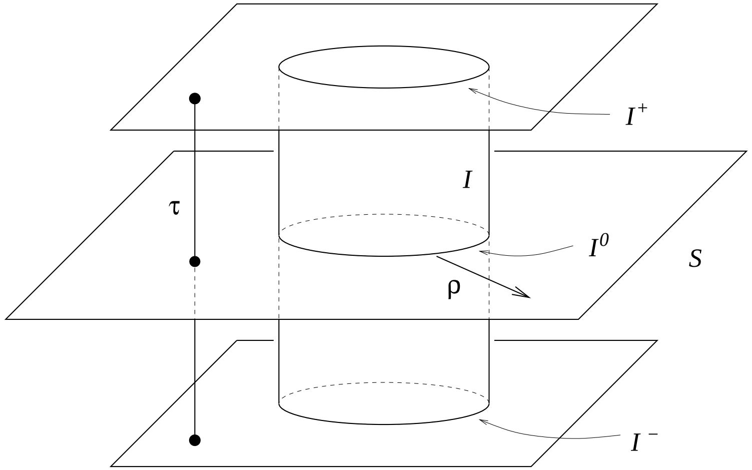


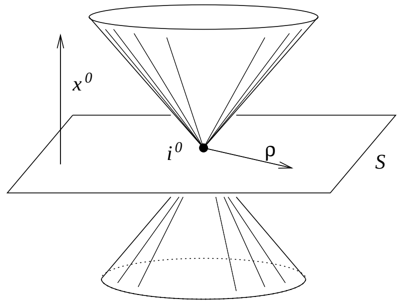
- One of the difficulties of the analysis lies in the fact that on an initial hypersurface $\mathcal{S}$, the rescaled conformal Weyl tensor behaves like:

$$d^{\mu}{}_{\nu\lambda\rho} = \Omega C^{\mu}{}_{\nu\lambda\rho} = O(r^{-3}) \text{ as } r \to 0.$$

- In order to overcome this difficulty, one has to resolve the structure contained in the point i^0.

Blow-up of i^0 into the cylinder at spatial infinity[a]





The conformal factor is given by:

$$\Omega = f(\rho, \theta, \varphi)\left(1 - \tau^2\right),$$

where
$$f(\rho, \theta, \varphi) = \rho + O(\rho^2),$$

is given in terms of initial data on $\mathcal{S}$.

---

[a] H. Friedrich. *Gravitational fields near spacelike and null infinity*. J. Geom. Phys. **24**, 83-163 (1998).

For suitable classes of initial data $(\mathcal{S}, h_{\alpha\beta}, \chi_{\alpha\beta})$ —e.g.

- time symmetric data ($\chi_{\alpha\beta} = 0$) with smooth conformal metric,
- time asymmetric ($\chi_{\alpha\beta} \neq 0$), conformally flat data,
- stationary data, and ...

the standard Cauchy problem can be reformulated as a

regular finite initial value problem for the conformal field equations.

Features:

- the data and equations are regular on a manifold with boundary;
- spacelike and null infinity have a finite representation with their structure and location known a priori.

About the initial data:

- Construct maximal initial data $(\tilde{h}_{\alpha\beta}, \tilde{\chi}_{\alpha\beta})$ by means of the conformal Ansatz:

$$\tilde{h}_{\alpha\beta} = \vartheta^4 h_{\alpha\beta}, \qquad \tilde{\chi}_{\alpha\beta} = \vartheta^{-2}\psi_{\alpha\beta},$$

so that the constraint equations reduce to:

$$D^\alpha \psi_{\alpha\beta} = 0,$$
$$\left(D^\alpha D_\alpha - \frac{1}{8}r\right)\vartheta = \frac{1}{8}\psi_{\alpha\beta}\psi^{\alpha\beta}\vartheta^{-7}.$$

- Consider conformally flat initial data:

$$h_{\alpha\beta} = \vartheta^4 \delta_{\alpha\beta}.$$

- To solve the momentum constraint write:

$$\psi_{\alpha\beta} = \psi^A_{\alpha\beta} + \psi^J_{\alpha\beta} + \psi^Q_{\alpha\beta} + \psi^\lambda_{\alpha\beta},$$

where

$$\psi^A_{\alpha\beta} = \frac{A}{|x|^3}\left(3n_\alpha n_\beta - \delta_{\alpha\beta}\right),$$

$$\psi^J_{\alpha\beta} = \frac{3}{|x|^3}\left(n_\beta \epsilon_{\gamma\alpha\rho} J^\rho n^\gamma + n_\alpha \epsilon_{\rho\beta\gamma} J^\gamma n^\rho\right),$$

$$\psi^Q_{\alpha\beta} = \frac{3}{2|x|^2}\left(Q_\alpha n_\beta + Q_\beta n_\alpha - (\delta_{\alpha\beta} - n_\alpha n_\beta) Q^\gamma n_\gamma\right)$$

$$\psi^\lambda_{\alpha\beta} = O(1/|x|) \qquad \text{(higher multipoles).}$$

- The term $\psi^{\lambda}_{\alpha\beta}$ is calculated out of a smooth complex function λ.
- If

$$\lambda = \lambda^{\flat}/\rho + \lambda^{\natural}$$

with $\lambda^{\flat}$, $\lambda^{\natural}$ smooth, then the conformal factor ϑ admits the parametrisation

$$\vartheta = \frac{1}{\rho} + W$$

with $W(i) = m/2$ and expandible in powers of ρ solely [a].

---

[a] S Dain & H Friedrich, *Asymptotically flat initial data with prescribed regularity at infinity* Comm. Math. Phys. **222**, 569 (2001)

For later use, we define the tensor

$$C^R_{\alpha\beta} = D_\gamma \chi^R_{\delta(\alpha} \epsilon^{\gamma\delta}{}_{\beta)},$$

where $\chi^R_{\alpha\beta} = \theta^{-4}\psi^R_{\alpha\beta}$ is the part of the second fundamental form arising from the real part of λ.

For later use, we define the tensor

$$C^R_{\alpha\beta} = D_\gamma \chi^R_{\delta(\alpha} \epsilon^{\gamma\delta}{}_{\beta)},$$

where $\chi^R_{\alpha\beta} = \theta^{-4}\psi^R_{\alpha\beta}$ is the part of the second fundamental form arising from the real part of λ.

- $C^R_{\alpha\beta}$ can be thought of as the magnetic part of the Weyl tensor arising from $\mathrm{Re}(\lambda)$.

The conformal propagation equations near spatial infinity:

- The unknowns are given by the components of the frame, connection, and Ricci tensor

$$v = (c^{\mu}_{AB}, \Gamma_{ABCD}, \Phi_{ABCD}),$$

and the components of the Weyl spinor

$$\phi = (\phi_0, \phi_1, \phi_2, \phi_3, \phi_4).$$

- The evolution equations are given by:

$$\partial_\tau v = Kv + Q(v,v) + L\phi,$$
$$A^0 \partial_\tau \phi + A^\alpha \partial_\alpha \phi = B(\Gamma_{ABCD})\phi,$$

- The matrix associated to the ∂_τ term in the Bianchi propagation equations is given by:

$$A^0 = \sqrt{2}\mathrm{diag}(1-\tau, 1, 1, 1, 1+\tau).$$

  - Thus, the equations degenerate at the sets where null infinity touches spatial infinity:

$$I^\pm = \{\rho = 0, \tau = \pm 1\}$$

  - Standard methods of symmetric hyperbolic systems cannot be used to analyse the equations near $I^\pm$.

Transport equations on I

- The procedure by which i^0 is replaced by I leads to an unfolding of the evolution process near spatial infinity which permits an analysis to arbitrary order and in all detail.

- Consistent with our choice of initial data assume that the field quantities admit the following *Taylor like expansions:*

$$v_j \sim \sum_{p\geq 0} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

Transport equations on I

- The procedure by which i^0 is replaced by I leads to an unfolding of the evolution process near spatial infinity which permits an analysis to arbitrary order and in all detail.
- Consistent with our choice of initial data assume that the field quantities admit the following *Taylor like expansions:*

$$v_j \sim \sum_{p\geq 0} \frac{1}{p!} v_j^{(p)}(\tau, \theta, \varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau, \theta, \varphi)\rho^p.$$

- In order to determine the coefficients $v_j^{(p)}$ and $\phi_j^{(p)}$ exploit the fact that the cylinder I is a total characteristic of the propagation equations:
  - The equations reduce to an interior system on I.

- Exploiting the total characteristic one can obtain a hierarchy of interior equations for the coefficients in the expansions:

$$\partial_\tau v^{(p)} = K v^{(p)} + Q(v^{(0)}, v^{(p)}) + Q(v^{(p)}, v^{(0)}) + \sum_{j=1}^{p-1} \left(Q(v^{(j)}, v^{(p-j)}) + L^{(j)} \phi^{(p-j)} \right) + L^{(p)} \phi^{(0)},$$

$$A^{0,(0)} \partial_\tau \phi^{(p)} + A^{C,(p)} \partial_C \phi^{(p)} = B(\Gamma^{(0)}_{ABCD}) \phi^{(p)} + \sum_{j=1}^{p} \begin{pmatrix} p \\ j \end{pmatrix} \left(B(\Gamma^{(j)}_{ABCD}) \phi^{(p-j)} - A^{\mu,(j)} \partial_\mu \phi^{(p-j)} \right),$$

which can be solved recursively —the equations are linear and decoupled.

- $v^{(p)}_j$ and $\phi^{(p)}_j$ are completely determined by the expansions of the initial data on $\mathcal{S}$ near spatial infinity.

- Thus, one can relate properties of the initial data with the asymptotic behaviour of the spacetime near null and spatial infinities.

Obstructions to the smoothness of null infinity:

Due to the degeneracy of the Bianchi propagation equations at the critical sets $I^{\pm}$, any hint of non-smoothness is bound to arise first in the coefficients $\phi^{(p)}$.

Obstructions to the smoothness of null infinity:

Due to the degeneracy of the Bianchi propagation equations at the critical sets $I^{\pm}$, any hint of non-smoothness is bound to arise first in the coefficients $\phi^{(p)}$.

- Decompose $\phi^{(p)}$ in spherical harmonics:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \; {}_{j-2}Y_{lm}$$

Obstructions to the smoothness of null infinity:

Due to the degeneracy of the Bianchi propagation equations at the critical sets $I^{\pm}$, any hint of non-smoothness is bound to arise first in the coefficients $\phi^{(p)}$.

- Decompose $\phi^{(p)}$ in spherical harmonics:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \ {}_{j-2}Y_{lm}$$

- A first analysis of the equations at the level of the linearised Bianchi equations —spin 2 zero-rest-mass field— reveals that the coefficients

$$a_{j;p,p,m}(\tau) \longleftrightarrow {}_{j-2}Y_{pm}, \qquad m = -p, \ldots, p$$

develop a certain type of logarithmic singularities at $\tau = \pm 1$.

- More precisely,

$$a_{j;p,p,m}(\tau) = A_p(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1-\tau) + B_p(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1+\tau) + (\text{polynom in } \tau)$$

for $p = 2, 3, \ldots$.

  - A_p and B_p depend on $\mathrm{Re}(\lambda)$ only.

- More precisely,

$$a_{j;p,p,m}(\tau) = A_p(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1-\tau) + B_p(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1+\tau) + (\text{polynom in } \tau)$$

  for $p = 2, 3, \ldots$.
  - A_p and B_p depend on $\text{Re}(\lambda)$ only.

- These singularities can be preluded by imposing a certain regularity condition at the initial hypersurface:

$$\mathscr{C}(D_{\gamma_p} \cdots D_{\gamma_1} C^R_{\alpha\beta})(i) = 0,$$

  for $p = 0, \ldots, 5$, where $\mathscr{C}$ denotes the symmetric tracefree part.

Further obstructions to the smoothness of null infinity:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \; {}_{j-2}Y_{lm}$$

- Even if the regularity condition

$$\mathscr{C}(D_{\gamma_p} \cdots D_{\gamma_1} C^R_{\alpha\beta})(i) = 0,$$

is satisfied, there are logarithmic singularities in the coefficients $a_{j;p,l,m}$ for $p \geq 5$ at the critical sets $I^{\pm}$.

Further obstructions to the smoothness of null infinity:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \; {}_{j-2}Y_{lm}$$

- Even if the regularity condition

$$\mathscr{C}(D_{\gamma_p} \cdots D_{\gamma_1} C^R_{\alpha\beta})(i) = 0,$$

 is satisfied, there are logarithmic singularities in the coefficients $a_{j;p,l,m}$ for $p \geq 5$ at the critical sets $I^{\pm}$.

- Associated with these singularities is a hierarchy of obstructions $\Upsilon^{\pm}_{p;l,m}$ where a clear pattern is recognizable:

Further obstructions to the smoothness of null infinity:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \ {}_{j-2}Y_{lm}$$

- Even if the regularity condition

$$\mathscr{C}(D_{\gamma_p} \cdots D_{\gamma_1} C^R_{\alpha\beta})(i) = 0,$$

  is satisfied, there are logarithmic singularities in the coefficients $a_{j;p,l,m}$ for $p \geq 5$ at the critical sets $I^{\pm}$.

- Associated with these singularities is a hierarchy of obstructions $\Upsilon^{\pm}_{p;l,m}$ where a clear pattern is recognizable:
  - If $\Upsilon^{\pm}_{p,l,m} = 0$ for given p, l, m then a certain subset of the logarithmic singularities is not present.
  - The obstructions are expressible in terms of the initial data.

- For $0 \leq p \leq 4$ the coefficients $a_{j,p;m,l}$ are polynomials in τ.
- For $p \geq 5$ the coefficients contain —generically— terms of the form:

$$(1-\tau)^{m_1}\ln(1-\tau), \quad (1+\tau)^{m_2}\ln(1+\tau).$$

  - In particular, for $p = 5$, one has quadrupolar obstructions (harmonics ${}_{j-2}Y_{2m}$) of the form:

$$\Upsilon^{+}_{5;2,m} = \Upsilon^{-}_{5;2,m} = m \times (\text{quadrupole}) + (\text{dipole})^2 + J^2,$$

    the obstructions are of a time symmetric nature.

Assume that $\Upsilon^{\pm}_{5;2,m} = 0$.

- For $p = 6$ the structure of the obstructions is much more involved:

Assume that $\Upsilon^{\pm}_{5;2,m} = 0$.

- For $p = 6$ the structure of the obstructions is much more involved:
  - Harmonics Y_{2m}:

$$\Upsilon^{+}_{6;2,m} = (\text{dipole})^2 + (A+1)J^2, \qquad \Upsilon^{-}_{6;2,m} = (\text{dipole})^2 + (A-1)J^2,$$

so the obstructions are time asymmetric!!!

Assume that $\Upsilon^{\pm}_{5;2,m} = 0$.

- For $p = 6$ the structure of the obstructions is much more involved:
  - Harmonics Y_{2m}:

    $$\Upsilon^{+}_{6;2,m} = (\text{dipole})^2 + (A+1)J^2, \qquad \Upsilon^{-}_{6;2,m} = (\text{dipole})^2 + (A-1)J^2,$$

    so the obstructions are time asymmetric!!!
  - Harmonics Y_{3m}:

    $$\Upsilon^{+}_{6;3,m} = \Upsilon^{-}_{6;3,m} = (\text{Octupolar object}),$$

    which is time symmetric.

Assume that $\Upsilon^{\pm}_{5;2,m} = 0$.

- For $p = 6$ the structure of the obstructions is much more involved:
  - Harmonics Y_{2m}:

$$\Upsilon^{+}_{6;2,m} = (\text{dipole})^2 + (A+1)J^2, \qquad \Upsilon^{-}_{6;2,m} = (\text{dipole})^2 + (A-1)J^2,$$

    so the obstructions are time asymmetric!!!
  - Harmonics Y_{3m}:

$$\Upsilon^{+}_{6;3,m} = \Upsilon^{-}_{6;3,m} = (\text{Octupolar object}),$$

    which is time symmetric.
- And so on...

From formal expansions to solutions

- One of the remaining outstanding hurdles in the analysis is to show existence of the soultions up to the critical sets $I^{\pm}$, and that the expansions

$$v_j \sim \sum_{p\geq 0} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

approximate suitably a solution of the conformal field equations.

From formal expansions to solutions

- One of the remaining outstanding hurdles in the analysis is to show existence of the soultions up to the critical sets $I^{\pm}$, and that the expansions

$$v_j \sim \sum_{p\geq 0} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

approximate suitably a solution of the conformal field equations.

- In particular one would like to estimate the remainders

$$\mathscr{R}_N(v) = v - \sum_{p=0}^{N} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p,$$

$$\mathscr{R}_N(\phi) = \phi - \sum_{p=0}^{N} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

From formal expansions to solutions

- One of the remaining outstanding hurdles in the analysis is to show existence of the soultions up to the critical sets $I^{\pm}$, and that the expansions

$$v_j \sim \sum_{p\geq 0} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

  approximate suitably a solution of the conformal field equations.

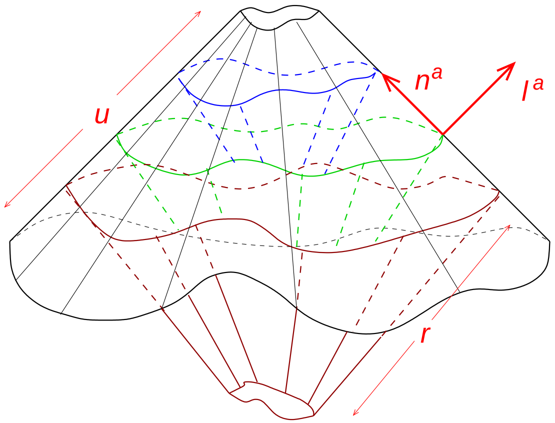
- In particular one would like to estimate the remainders

$$\mathscr{R}_N(v) = v - \sum_{p=0}^{N} \frac{1}{p!} v_j^{(p)}(\tau,\theta,\varphi)\rho^p,$$

$$\mathscr{R}_N(\phi) = \phi - \sum_{p=0}^{N} \frac{1}{p!} \phi_j^{(p)}(\tau,\theta,\varphi)\rho^p.$$

- In what follows, we shall assume this can be done.

How does this translate into the NP gauge?



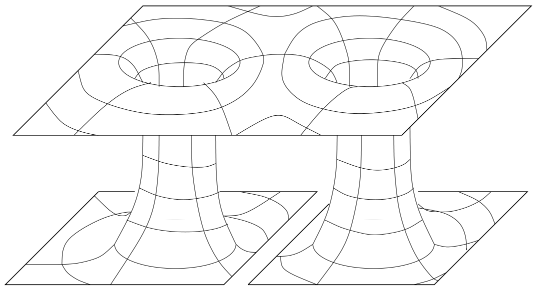
$$\widetilde{\Psi}_0 \sim \psi_0^5/r^5 + k_0 \sum_m A_m \ln r/r^5 + \cdots,$$
$$\widetilde{\Psi}_1 \sim \psi_1^4/r^4 + \cdots,$$
$$\widetilde{\Psi}_2 \sim \psi_2^3/r^3 + \cdots,$$
$$\widetilde{\Psi}_3 \sim \psi_3^2/r^2 + \cdots,$$
$$\widetilde{\Psi}_4 \sim \psi_4^1/r + \cdots.$$

for initial data for which

$$\mathscr{C}(D_\gamma C^R_{\alpha\beta})(i) \neq 0.$$

- The spacetime cannot be *stationary* if $\Upsilon^+_{5;2,m} \neq 0$ —stationary spacetimes do not contain logarithms in their asymptotic expansions.

An example: *Brill-Lindquist data*



$$\widetilde{h}_{\alpha\beta} = \left(1 + \frac{m_1}{2|\vec{x} - \vec{x}_1|} + \frac{m_2}{2|\vec{x} - \vec{x}_2|}\right)^4 \delta_{\alpha\beta},$$

$$\widetilde{\chi}_{\alpha\beta} = 0.$$

- In this case one finds,

$$\widetilde{\Psi}_0 = \psi_0^5 r^{-5} + \cdots + k_0 \Upsilon \ln r / r^8 + \cdots$$

$$\widetilde{\Psi}_1 = \psi_1^4 r^{-4} + \cdots + k_1 \Upsilon \ln r / r^8 + \cdots$$

$$\widetilde{\Psi}_2 = O(r^{-3})$$

$$\vdots$$

where $\Upsilon = m_1 m_2 |\vec{x}_1 - \vec{x}_2|^2$.

- Similar behaviour occurs for Bowen-York data!

The behaviour of the asymptotic shear near i^0

- Newman & Penrose [a] have shown that if the leading term of the coefficient σ goes to zero as one approaches i^0 along the null generators of $\mathscr{I}^+$, then there is a canonical way of selecting the Poincaré group out of the BMS group —the asymptotic symmetric group.

- This construction is tied with the possibility of defining in an ambiguous fashion angular momentum at null infinity.

[a] ET Newman & R Penrose *A note on the BMS group.* J. Math. Phys. **7**, 863 (1966).

Proposition 1. *The asymptotic shear of peeling spacetimes arising from conformally flat initial data satisfies*

$$\sigma^0 = O(1/u^2), \text{ as } u \to -\infty$$

that is, as one approaches i^0 along the generators at null infinity.

Proposition 1. *The asymptotic shear of peeling spacetimes arising from conformally flat initial data satisfies*

$$\sigma^0 = O(1/u^2), \text{ as } u \to -\infty$$

that is, as one approaches i^0 along the generators at null infinity.

- A similar result is expected to hold for nonconformally flat initial data.

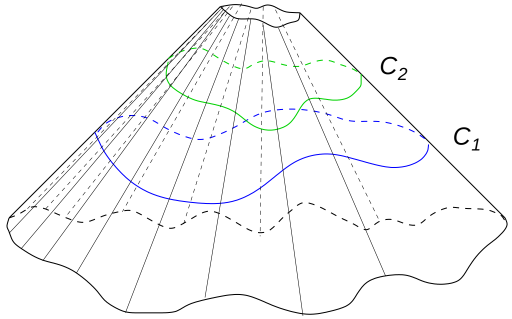
Proposition 1. *The asymptotic shear of peeling spacetimes arising from conformally flat initial data satisfies*

$$\sigma^0 = O(1/u^2), \textit{ as } u \to -\infty$$

that is, as one approaches i^0 *along the generators at null infinity.*

- A similar result is expected to hold for nonconformally flat initial data.
- In order to obtain spacetimes for which $\sigma^0 \not\to 0$ as $u \to -\infty$, one may have to consider initial data sets with linear momentum —*boosted data*.

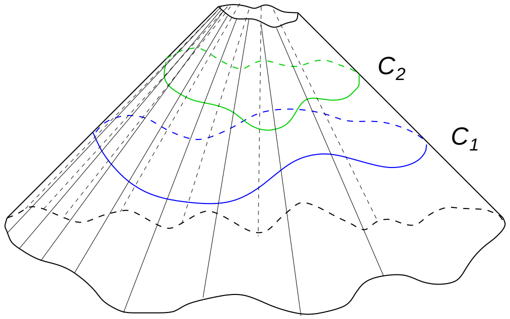
The Newman-Penrose constants



- These are a set of 5 complex absolutely conserved quantities defined on a cut of $\mathscr{I}^+$ and $\mathscr{I}^-$:

$$G_m^+ = \oint {}_2\bar{Y}_{2,m}\psi_0^6 dS, \quad G_m^- = \oint {}_2Y_{2,m}\psi_4^6 dS.$$

The Newman-Penrose constants



- These are a set of 5 complex absolutely conserved quantities defined on a cut of $\mathscr{I}^+$ and $\mathscr{I}^-$:

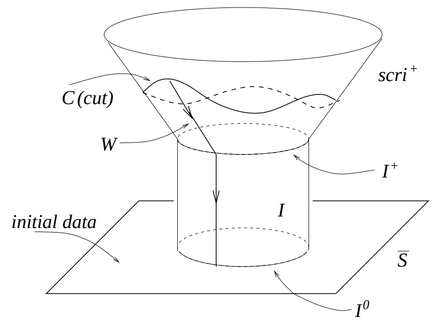
$$G_m^+ = \oint {}_2\bar{Y}_{2,m}\psi_0^6 dS, \quad G_m^- = \oint {}_2Y_{2,m}\psi_4^6 dS.$$

- If

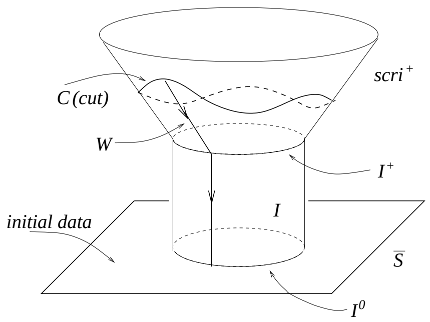
$$\mathscr{C}(D_{\gamma_2}D_{\gamma_1}C^R_{\alpha\beta})(i) \neq 0,$$

then the spacetime is regular enough so that the constants are well defined.

- The solutions of the transport equations on I can be used to write the NP constants in terms of initial data quantities.



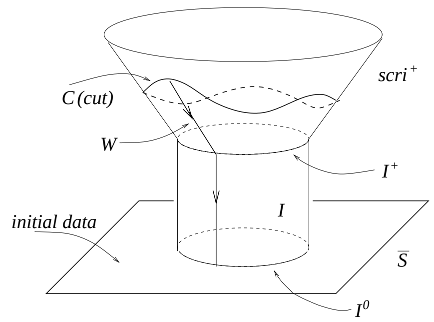
- The solutions of the transport equations on I can be used to write the NP constants in terms of initial data quantities.



Proposition 2. *For the class of data under consideration one has that*

$$G_m^+ = G_m^-.$$

- The solutions of the transport equations on I can be used to write the NP constants in terms of initial data quantities.



Proposition 2. *For the class of data under consideration one has that*

$$G^+_m = G^-_m.$$

- Roughly, one has that

$$G_m = m \times (\text{Quadrupole}) + (\text{Dipole}) + J^2 + (\text{Ang. Mom. Quad.})$$

Back to the obstructions:

- If the initial data is conformally flat (but not necessarily time symmetric), then the vanishing of the obstructions up to $p = 7$ imply:

$$\vartheta = \frac{1}{\rho} + \frac{m}{2} + O(\rho^4), \qquad \psi_{\alpha\beta} = \psi^A_{\alpha\beta} + O(1).$$

Back to the obstructions:

- If the initial data is conformally flat (but not necessarily time symmetric), then the vanishing of the obstructions up to $p = 7$ imply:

$$\vartheta = \frac{1}{\rho} + \frac{m}{2} + O(\rho^4), \qquad \psi_{\alpha\beta} = \psi^A_{\alpha\beta} + O(1).$$

  – The data is Schwarzschildean up to octupolar terms.

Back to the obstructions:

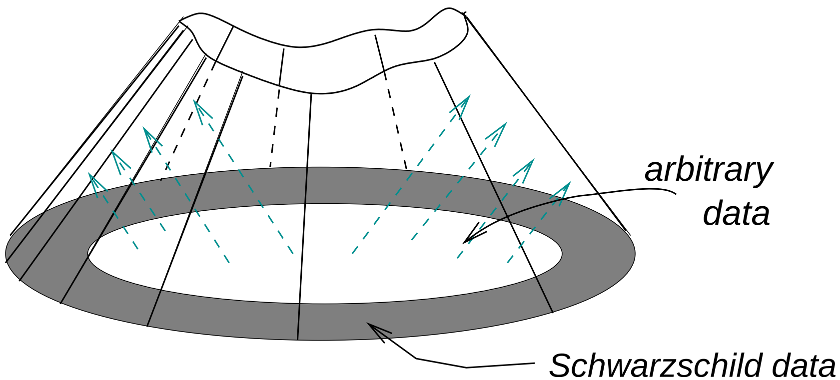
- If the initial data is conformally flat (but not necessarily time symmetric), then the vanishing of the obstructions up to $p = 7$ imply:

$$\vartheta = \frac{1}{\rho} + \frac{m}{2} + O(\rho^4), \qquad \psi_{\alpha\beta} = \psi^A_{\alpha\beta} + O(1).$$

  - The data is Schwarzschildean up to octupolar terms.
  - The only stationary data in the class of conformally flat initial data are the Schwarzschildean ones.

- In general one would expect the following to hold:

  Conjecture. *If the time development of conformally flat initial data admits a smooth conformal extension at both future and past null infinity, then the initial data is Schwarzschildean in a neighbourhood of infinity.*



Some references:

- J. A. Valiente Kroon, *A new class of obstructions to the smoothness of null infinity*, Comm. Math. Phys. **244**, 133 (2004). Also at `gr-qc/0211024`.
- J. A. Valiente Kroon, *Does asymptotic simplicity allow for radiation near spatial infinity?*, Commun.Math.Phys. **251**, 211 (2004). Also at `gr-qc/0309016`.
- J. A. Valiente Kroon, *Nonexistence of conformally flat slices in the Kerr and other stationary spacetimes*, Phys. Rev. Lett. **92**, 041101 (2004). Also at `gr-qc/0310048`.
- J. A. Valiente Kroon, *Time asymmetric spacetimes near null and spatial infinity. I. Expansions of developments of conformally flat data.* Class.Quantum Grav. **21**, 5457-5492 (2004). Also at `gr-qc/0408062`.
- J. A. Valiente Kroon, *Time asymmetric spacetimes near null and spatial infinity. II. Expansions of developments of initial data sets with non-smooth conformal metrics.* Class.Quantum Grav. **22**, 1683 (2005). Also at `gr-qc/0412045`.
- J. A Valiente Kroon, *On smoothness asymmetric null infinities.* Class.Quantum Grav. **23**, 3593 (2006). Also at `gr-qc/0605056`.