Resonant Relaxation of Stars around a supermassive BH

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Fluctuations and dissipations

Brownian movement theory and stochastic diffusion

Fluctuation-Dissipation Theorem

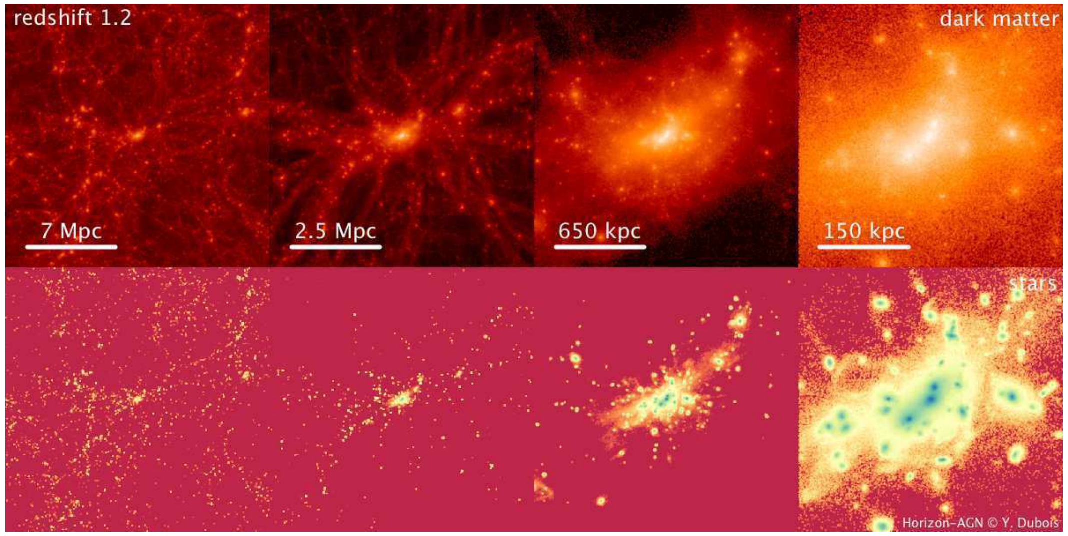
Diffusion rate ⇄ **Power-spectrum of the fluctuations**

Stars in galaxies undergo the same process.
But, gravity is a **long-range interacting force**
+ To diffuse, stars have to **resonate**, otherwise they follow the **mean field.**
+ All fluctuations are amplified by **collective effects.**

How can one describe orbital distortions on cosmic timescales ?

Gravity structures matter on all scales

1 Mpc



External perturbations and **cosmic environment**

+ Near-Field cosmology
+ Cusp-transition
+ Dynamical friction

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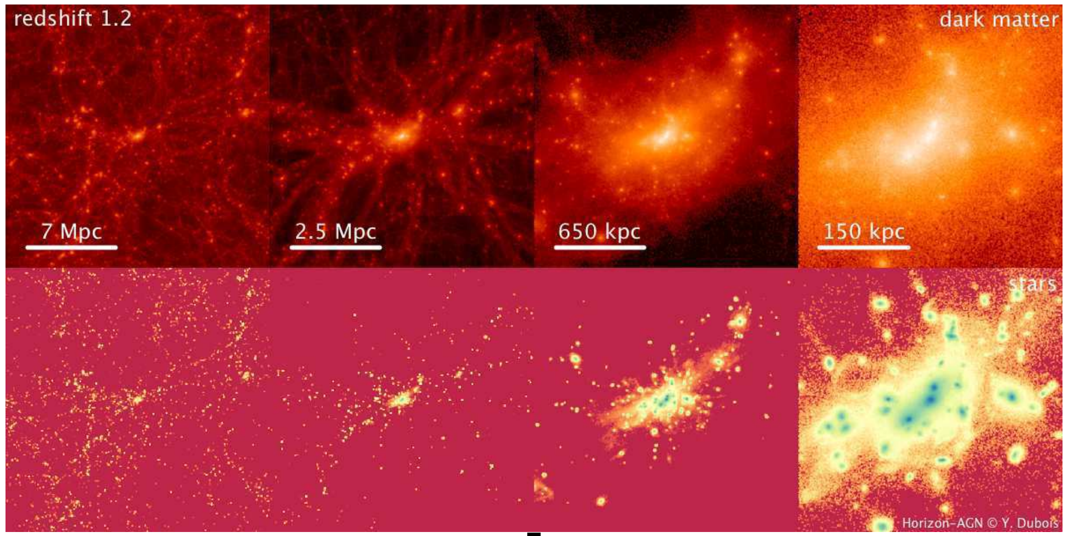

10 kpc

External perturbations and **cosmic environment**

+ Near-Field cosmology
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Evolution of **galactic discs** and the Milky Way

+ Galactic Archeology
+ Radial Migration
+ Metallicity gradients
+ Thickening

GAIA

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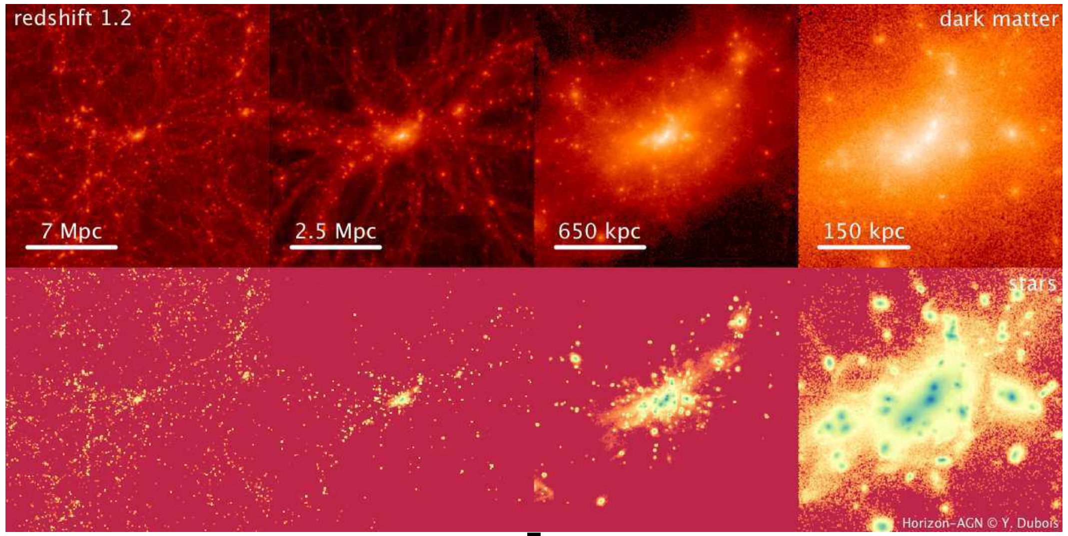

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GAIA

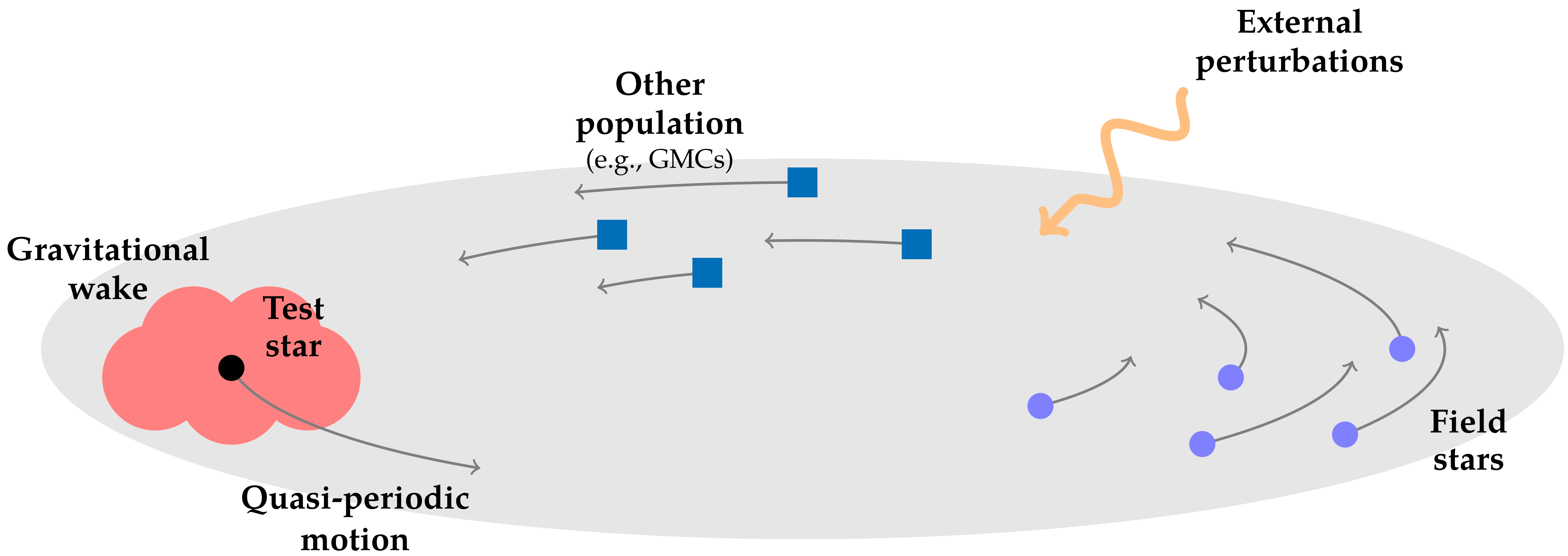
1 pc

Evolution of **galactic centers** and SgrA*

+ Stellar capture rates
+ Measure of the BH spin
+ Gravitational waves sources
+ Tests of general relativity

Gravity

Evolution on cosmic timescales

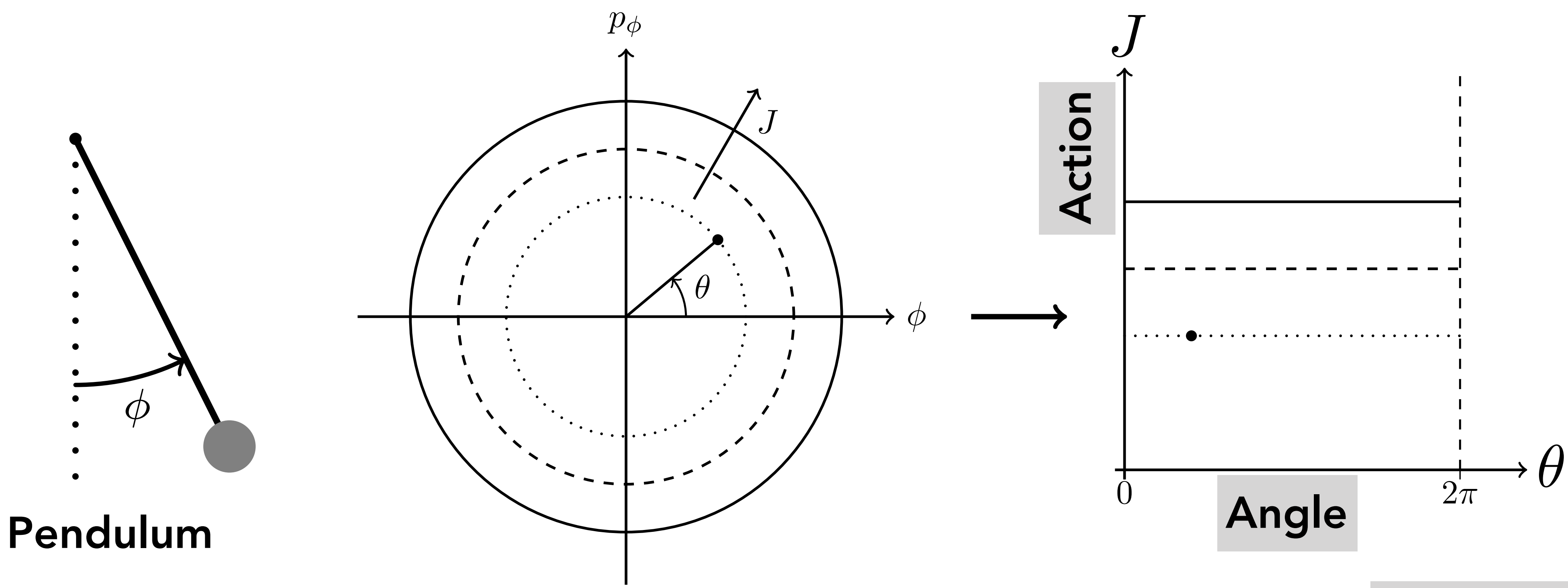


Galaxies are:

- + **Inhomogeneous** (complex trajectories) — *Angle-action coordinates*
- + **Relaxed** (equilibrium states) — *Quasi-stationary states*
- + **Resonant** (orbital frequencies) — *Fast timescale vs. cosmic timescale*
- + **Degenerate** (in some regions) — *Frequency commensurability*

Inhomogeneous systems

+ **Label** orbits with **integrals of motion**



+ **Angle-Action coordinates**

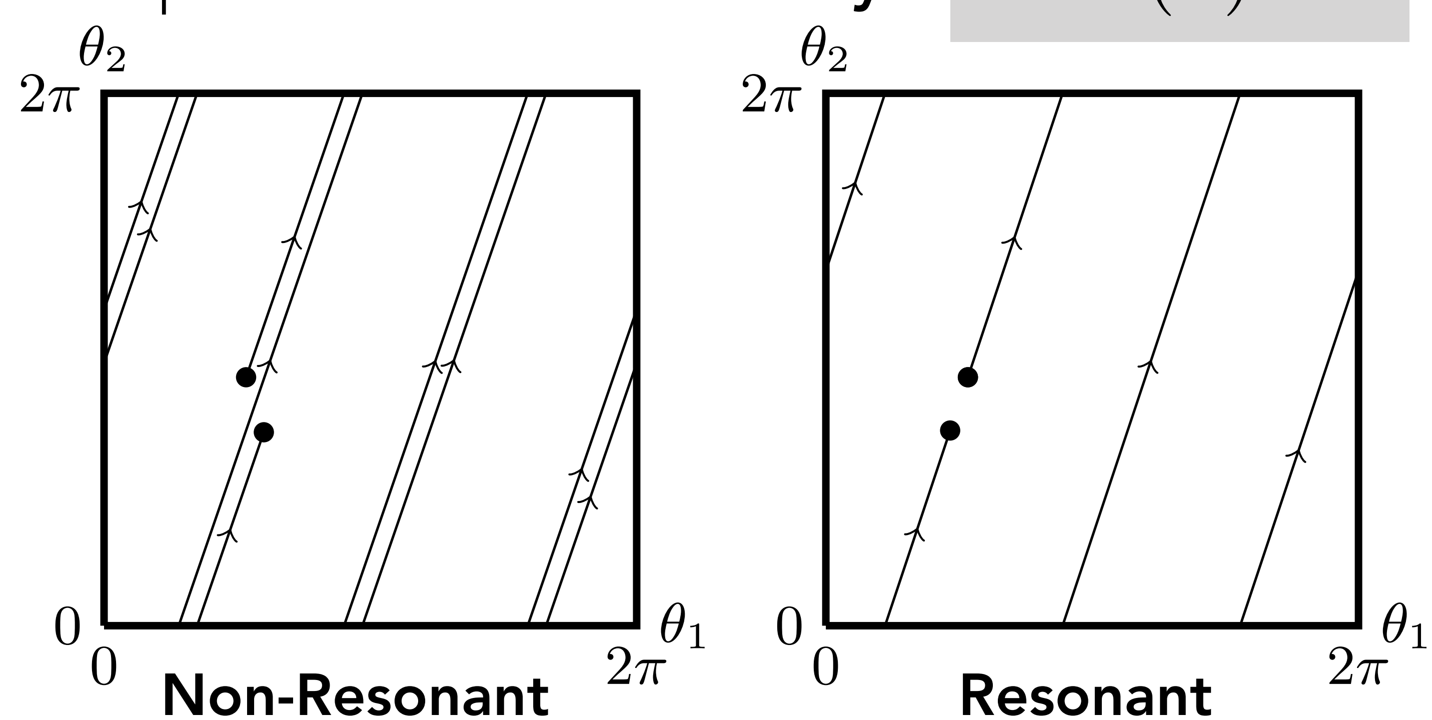
$$\begin{cases} \boldsymbol{\theta}(t) = \boldsymbol{\theta}_0 + t\,\boldsymbol{\Omega}(\mathbf{J}) \\ \mathbf{J}(t) = \mathrm{cst.} \end{cases}$$

Trajectories become **straight lines**

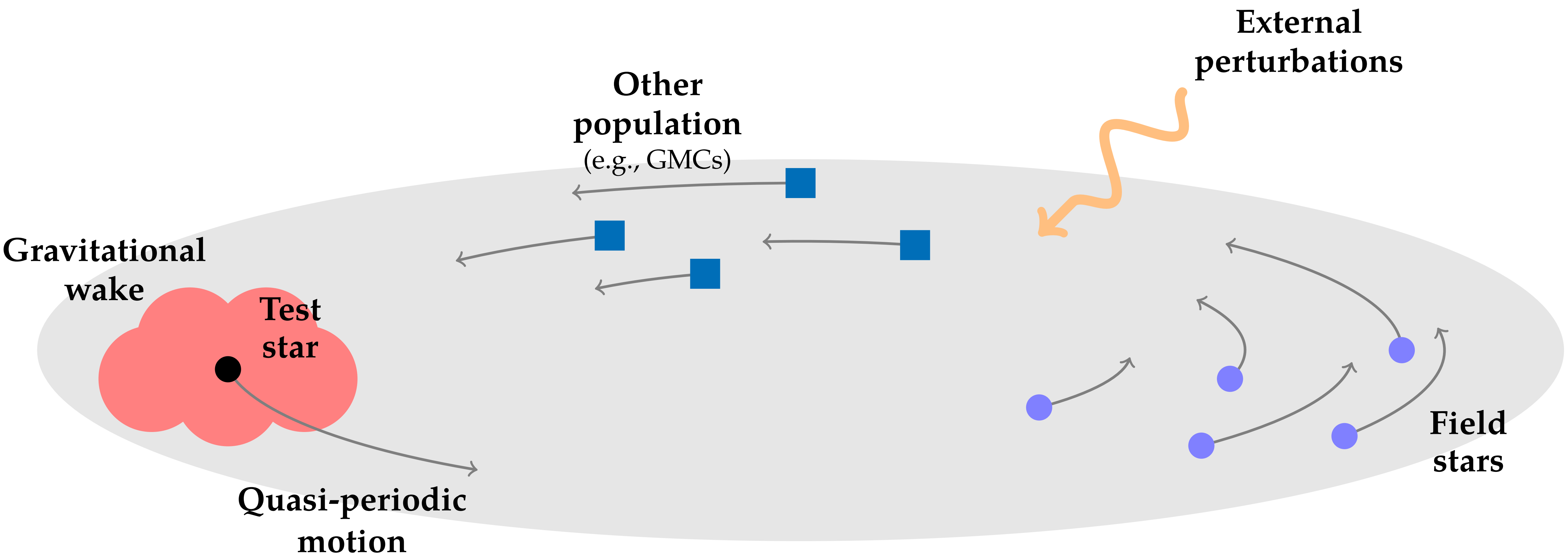
+ **Relaxation**

$$\xrightarrow{(\mathrm{few})\, t_{\mathrm{cross}}} \quad F = F(\mathbf{J}, t)$$

+ Frequencies' **commensurability** : $\mathbf{n} \cdot \boldsymbol{\Omega}(\mathbf{J}) = 0$



Evolution on cosmic timescales



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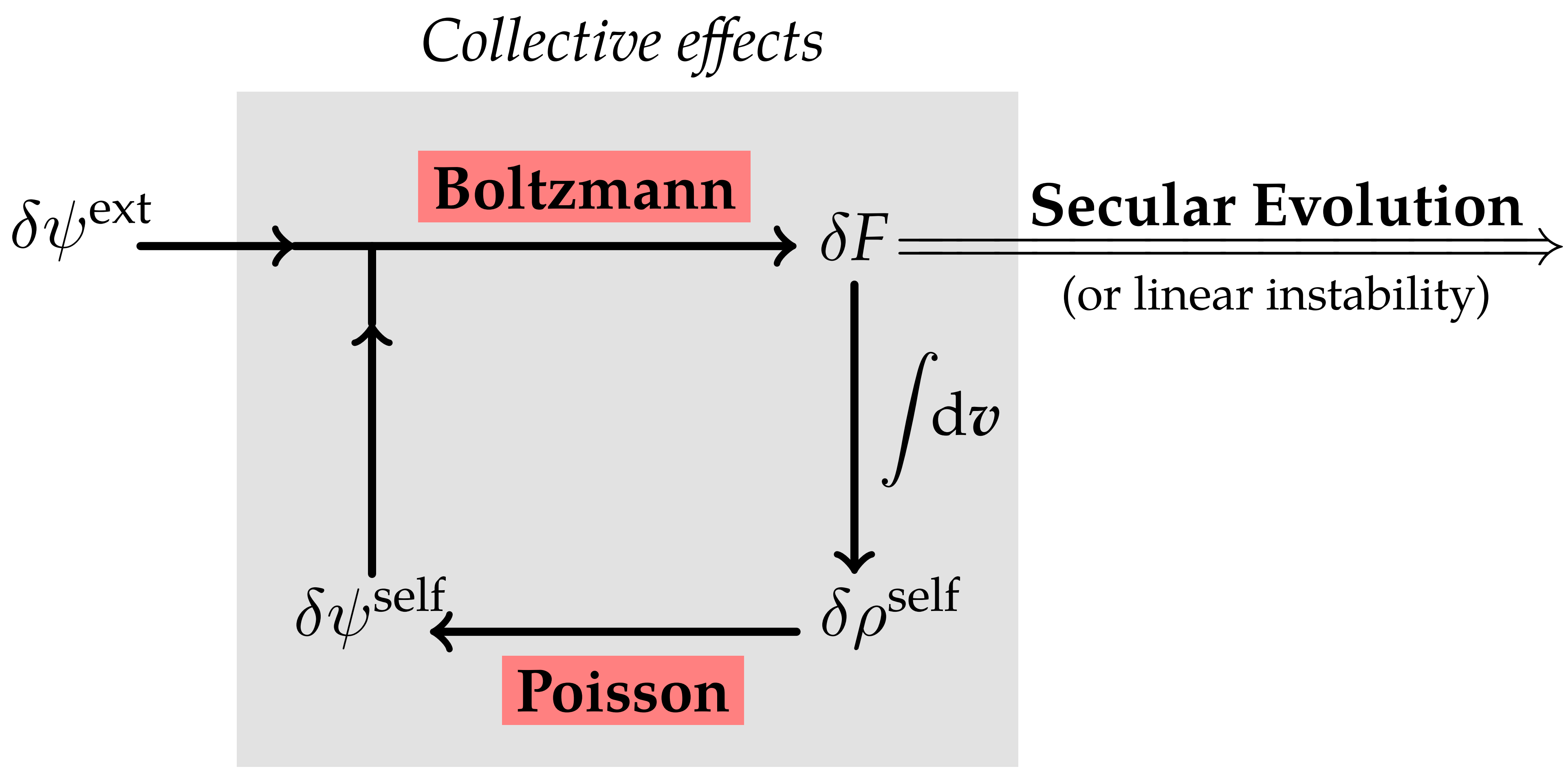
+ **Resonant** (orbital frequencies) — *Fast timescale vs. cosmic timescale*
+ **Degenerate** (in some regions) — *Frequency commensurability*
+ **Self-gravitating** (amplification of perturbations) — *Linear response theory*
+ **Discrete** (finite-N effects)
+ **Perturbed** (effects of the environment) — *Nature vs. Nurture*

Collective effects and perturbations

Self-gravitating amplification

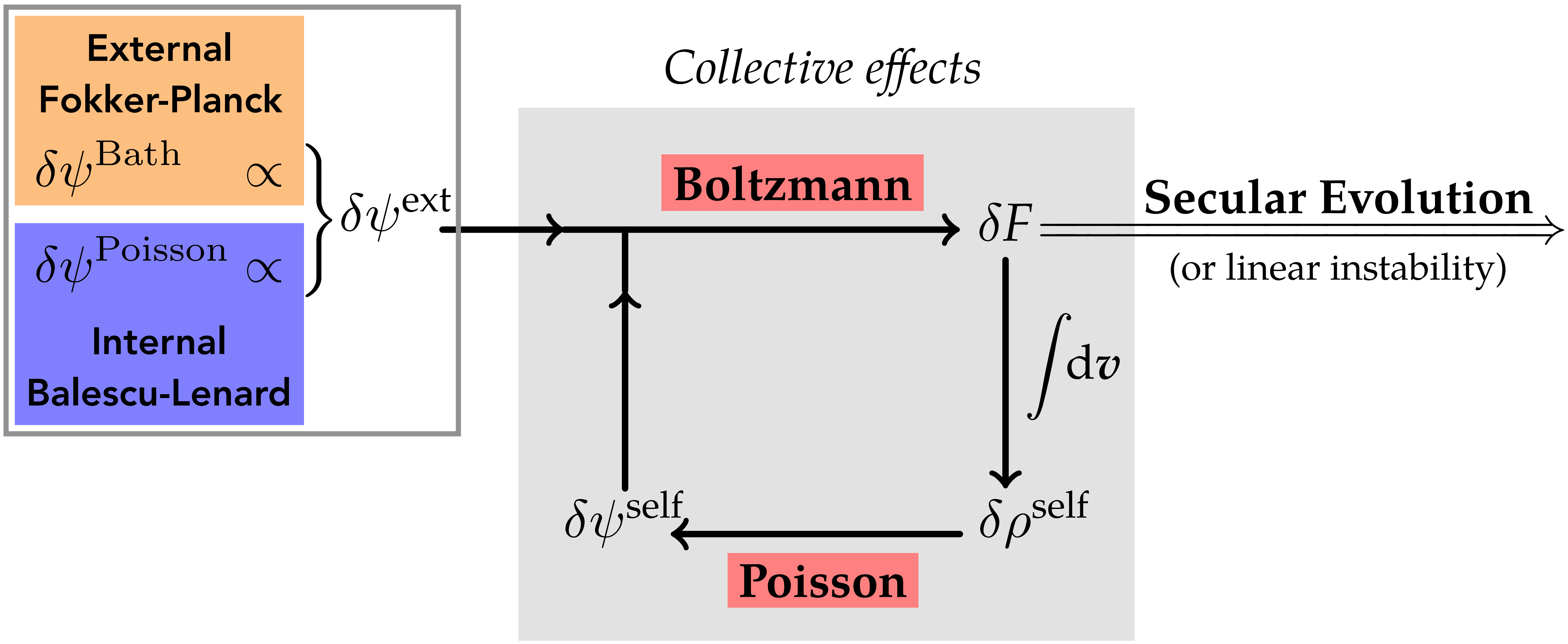


Gravitational polarisation essential to

+ *Cause dynamical instabilities*
+ *Induce **dynamical friction** and **mass segregation***
+ ***Accelerate/Slow down** secular evolution*

Collective effects and perturbations

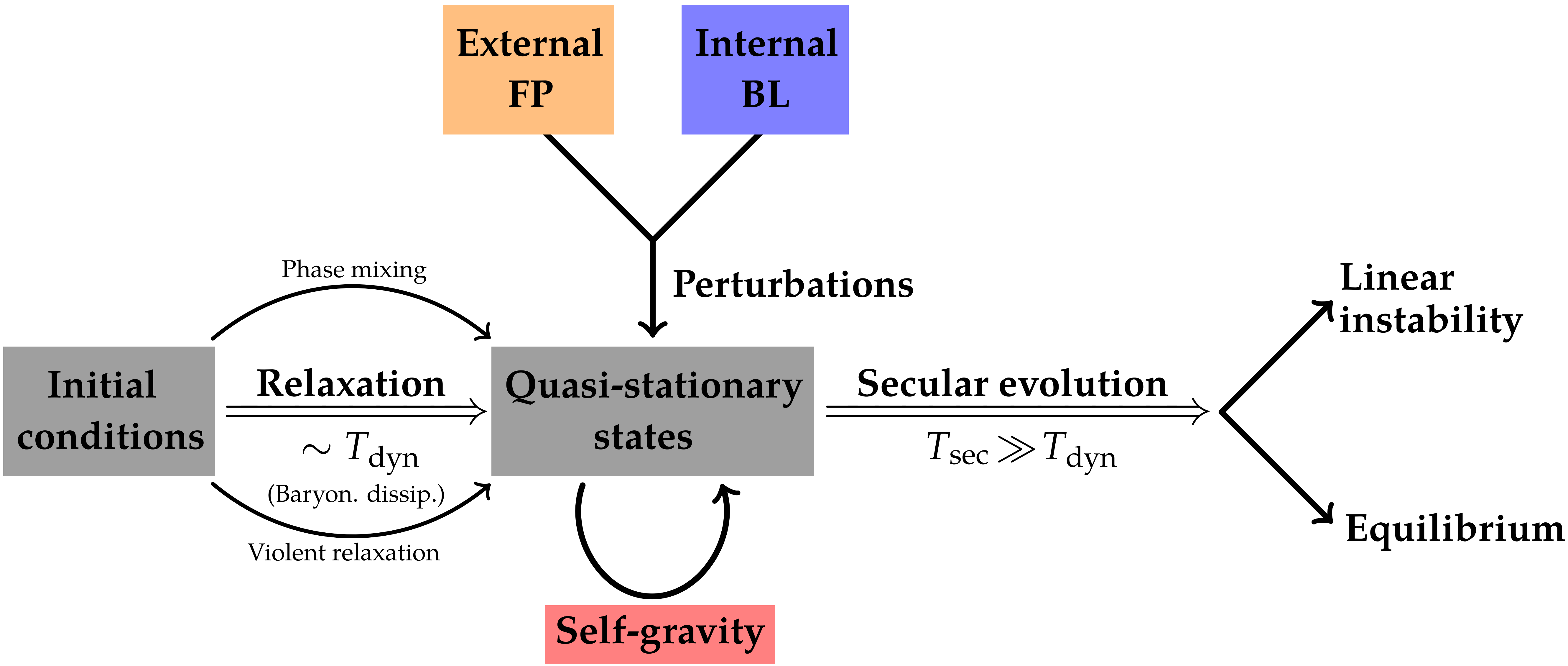
Self-gravitating amplification



Gravitational polarisation essential to

+ *Cause dynamical instabilities*
+ *Induce* ***dynamical friction*** *and* ***mass segregation***
+ ***Accelerate/Slow down*** *secular evolution*

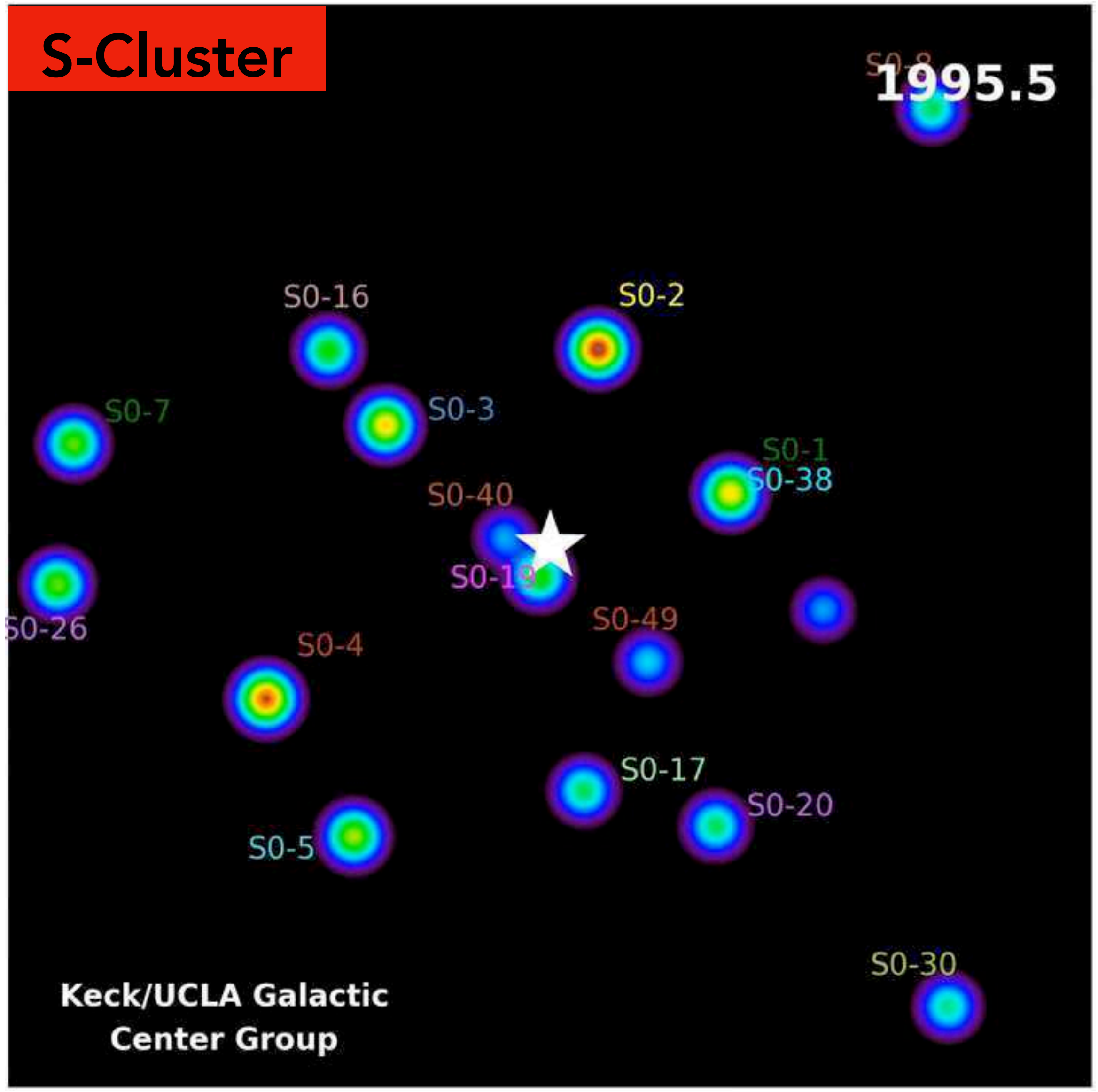
Typical fate of a self-gravitating system



Objective : Describe the **long-term dynamics of self-gravitating systems** (e.g. galactic centers)
Method : New quasi-linear approaches, coming from **kinetic theory**

+ *Self-consistent equation accounting for the roles of* ***self-gravity*** *and* ***resonances***
+ *Offers new physical insights (****phase transitions****,* ***equilibrium states****)*
+ ***Complementary*** *to numerical simulations*

The case of galactic centers



S-Cluster of ***SgrA****
Densest *stellar system of the galaxy*
Dynamics dominated by the ***central black hole***

What is the diet of a **supermassive black hole**?

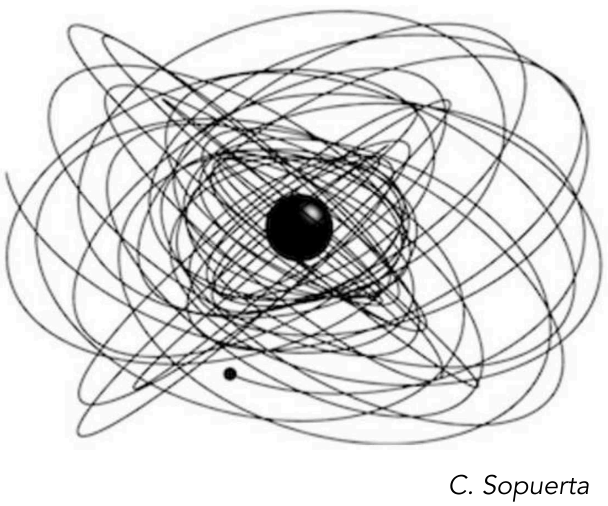
Stellar diffusion in galactic centers

- + ***Origin and structure*** *of SgrA**
- + *Relaxation in* ***eccentricity****,* ***orientation***

Sources of **gravitational waves**

- + *BHs-binary mergers*
- + *TDE, EMRIs*

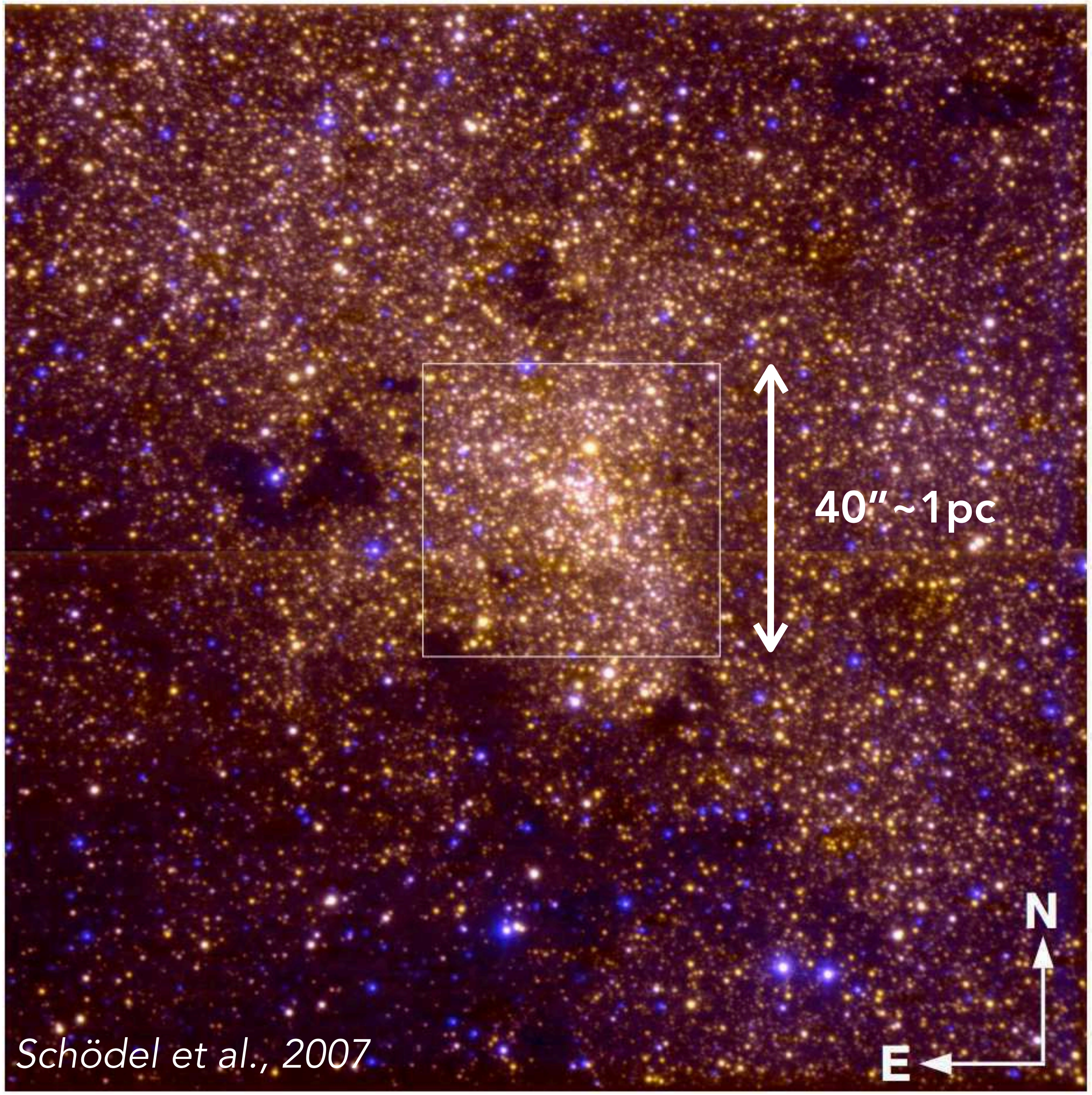
Tidal Disruption Event

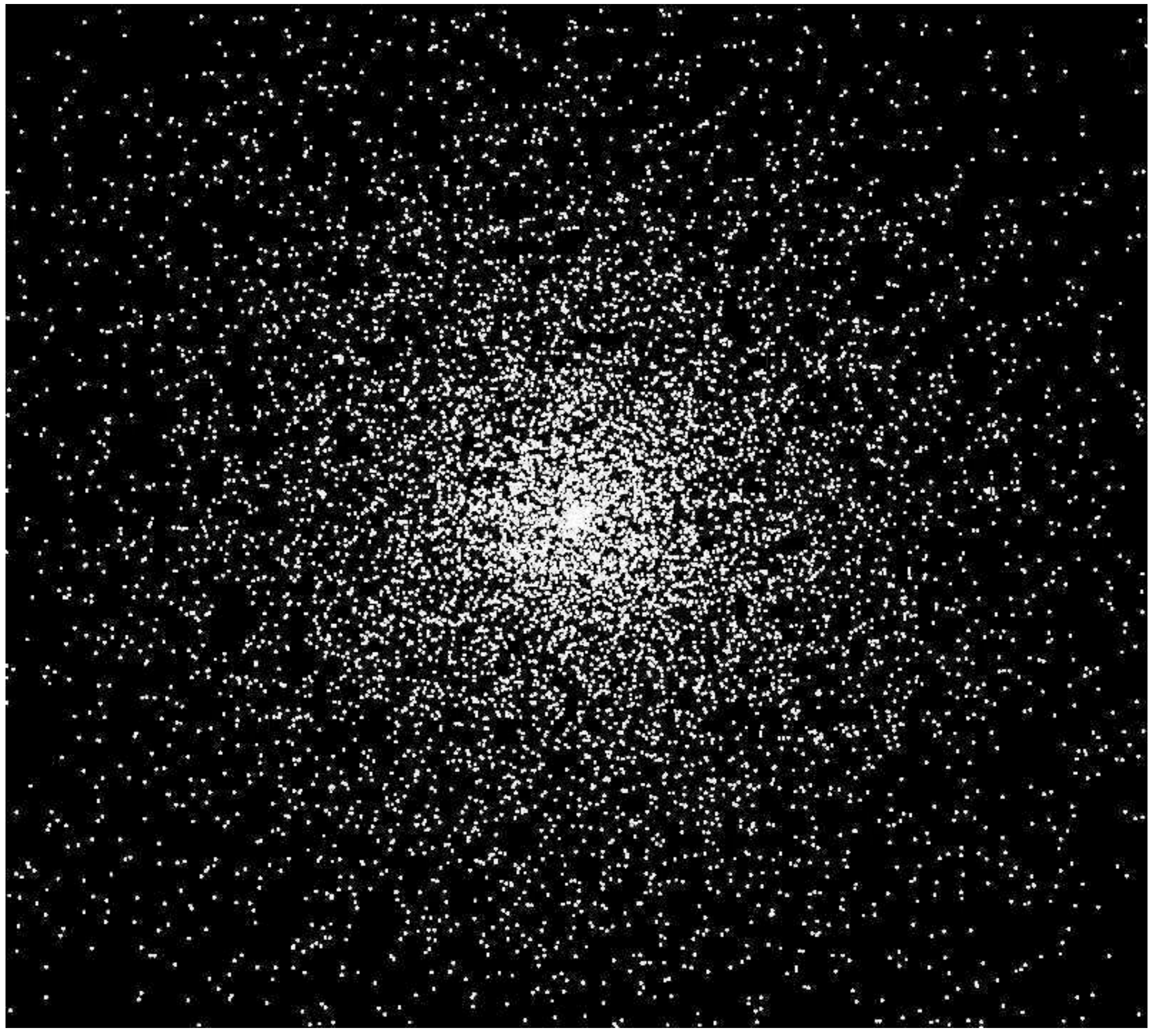


Extreme Mass Ratio Inspiral

What is the long-term dynamics of stars in these very dense systems?

Galactic centers are extremely dense





VLT observations

N-body simulations *(B. Bar-Or)*

Perfect "lab" to investigate the **statistical physics** of a stellar system

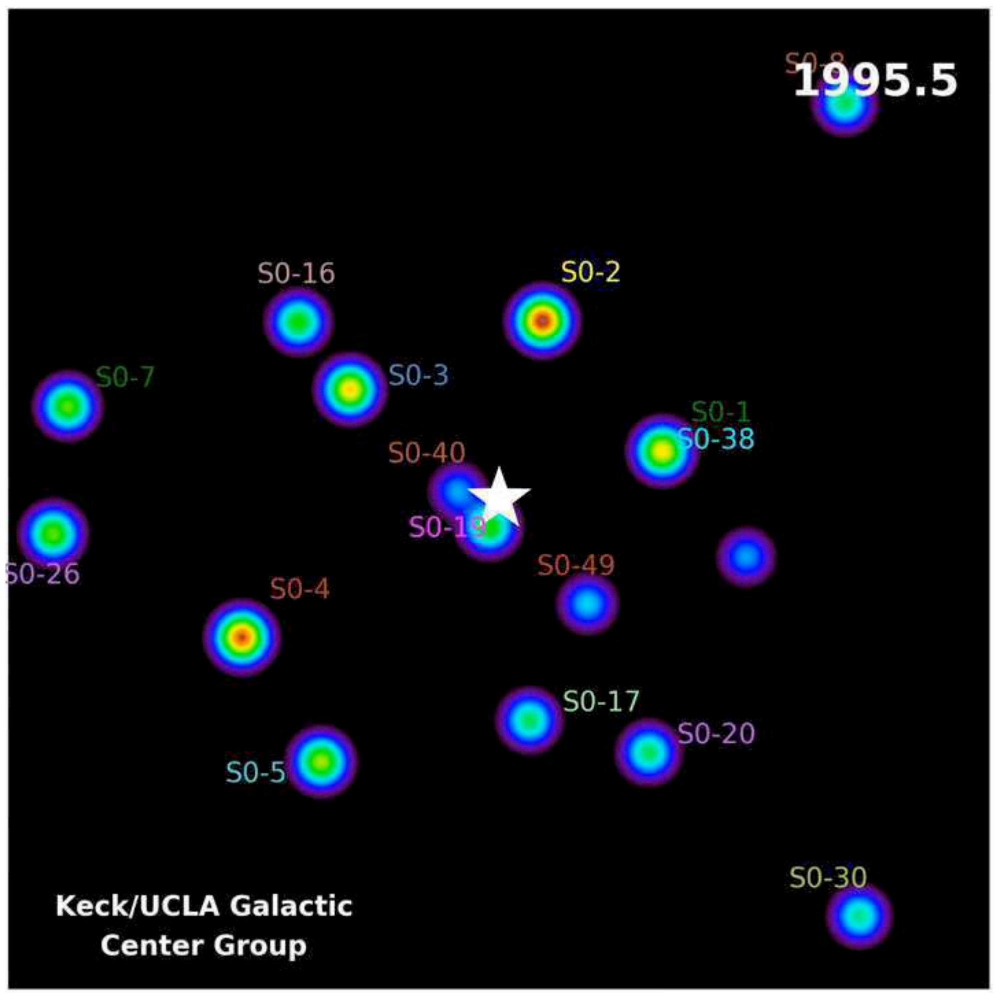
Galactic centers are degenerate

Potential dominated by the SMBH:

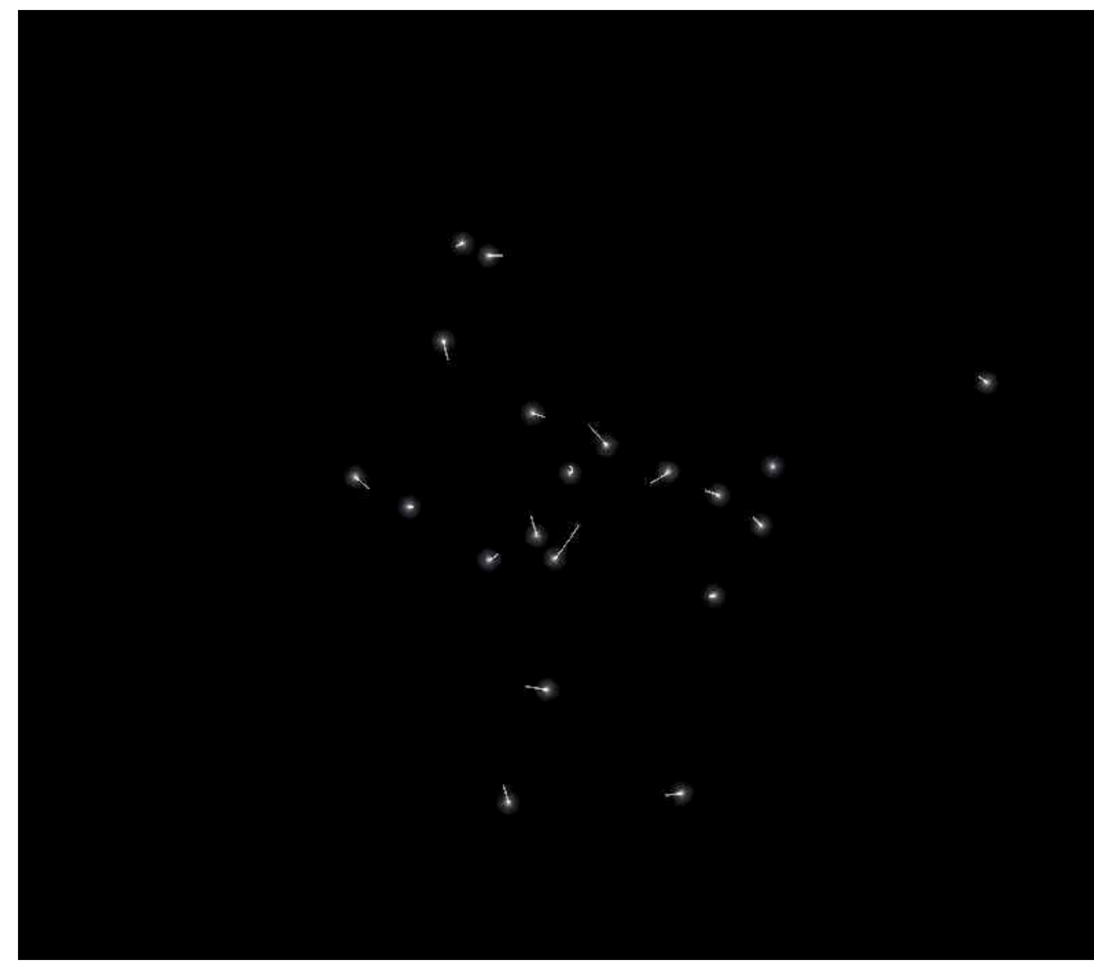
+ Keplerian orbits are **closed**

$$\varepsilon = M_\star / M_\bullet \ll 1$$

Dynamical degeneracy: $\forall \mathbf{J} \, , \; \mathbf{n} \cdot \mathbf{\Omega}_{\mathrm{Kep}}(\mathbf{J}) = 0$



KECK observations

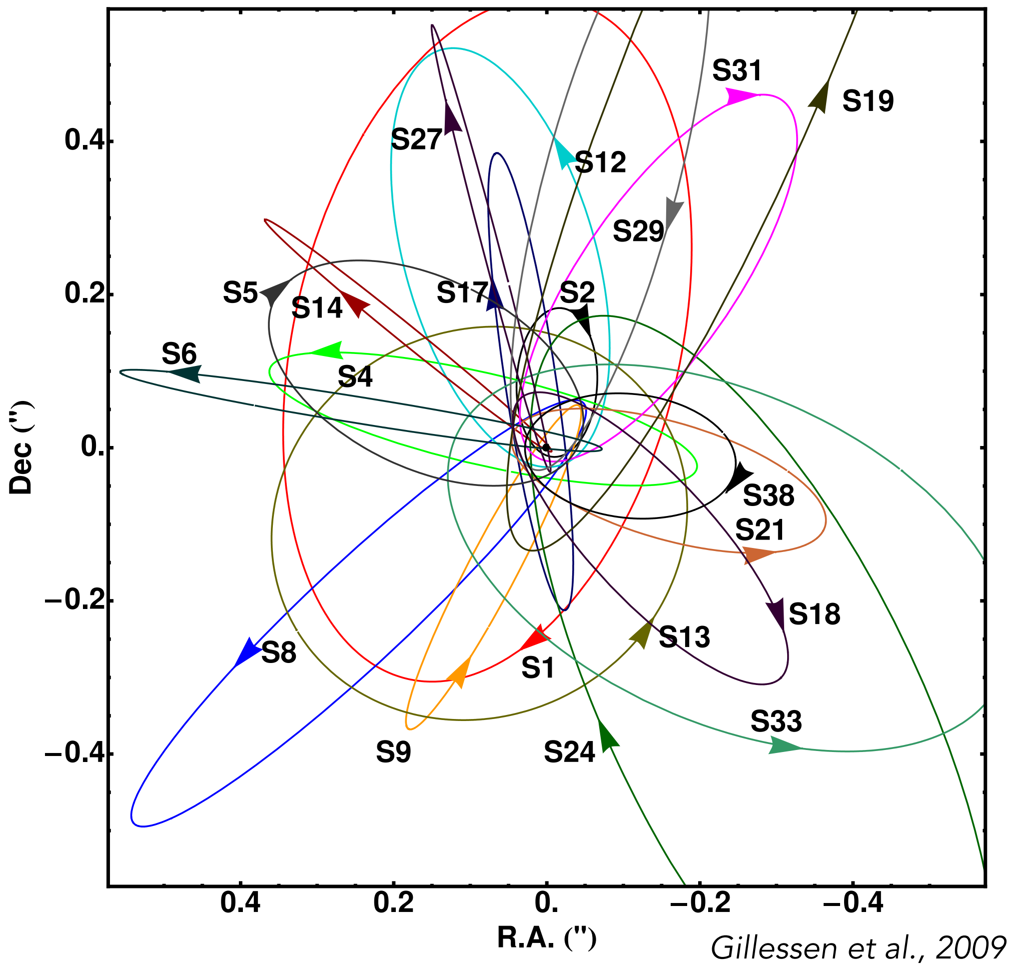


N-body simulations *(B. Bar-Or)*

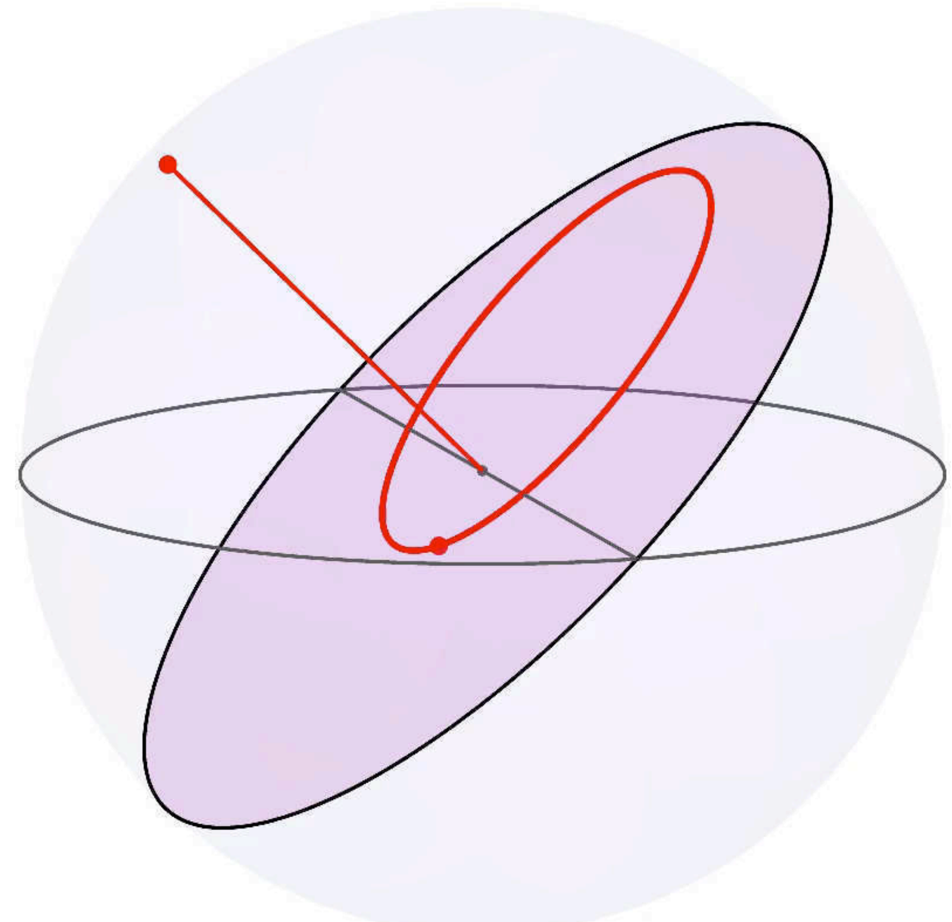
Orbit-average: stars are replaced by Keplerian wires

Describing Keplerian wires

Natural **Angle-Action** coordinates: **Delaunay Variables, i.e. orbital elements**



Gillessen et al., 2009



Degenerate coordinates

$$\mathbf{J} = (I\,,\, L\,,\, L_z)$$
$$\boldsymbol{\theta} = (M\,,\, \omega\,,\, \Omega)$$
$$\boldsymbol{\Omega} = (\Omega_{\rm Kep}\,,\, 0\,,\, 0)$$

Wires described by five numbers

+ **Shape** (a, e)
+ **Phase** ω
+ **Orientation** $\hat{\mathbf{L}}$

Wires dynamics

Orbit Average

$$J_{\text{fast}} = I(a)$$ **adiabatically conserved**

Wires may **precess constructively**:

+ In-plane precessions

- Spherical cluster mass
- 1PN relativistic **Schwarzschild precession**

$$\dot{\omega} = \Omega_{\text{prec}} \;\; ; \quad \hat{\mathbf{L}} = \text{cst} \,.$$

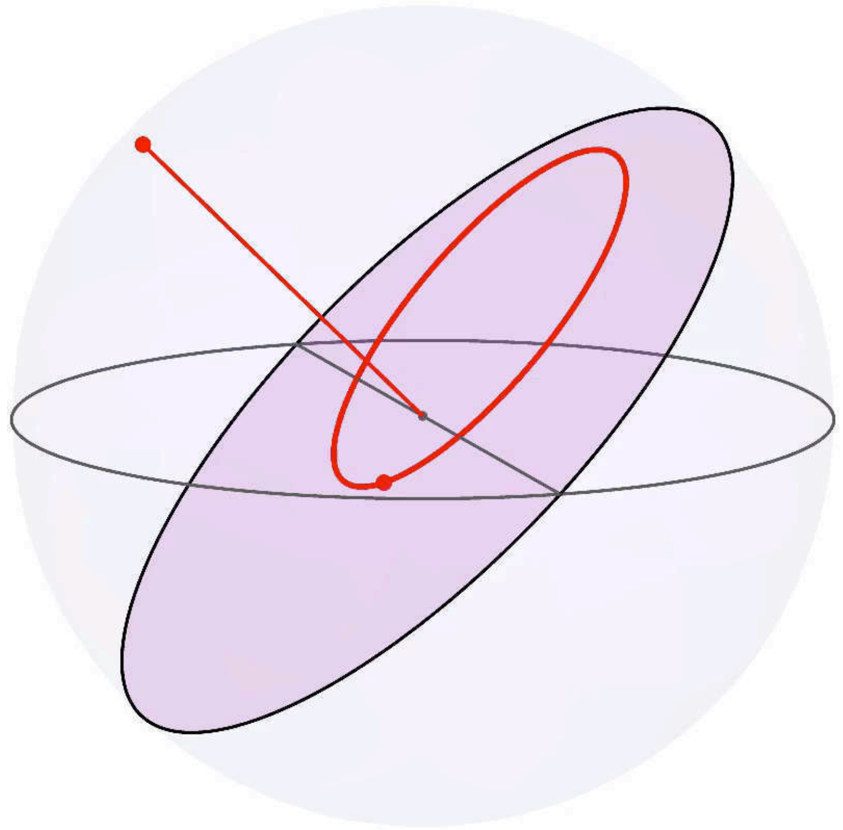
+ Out-of-plane precessions

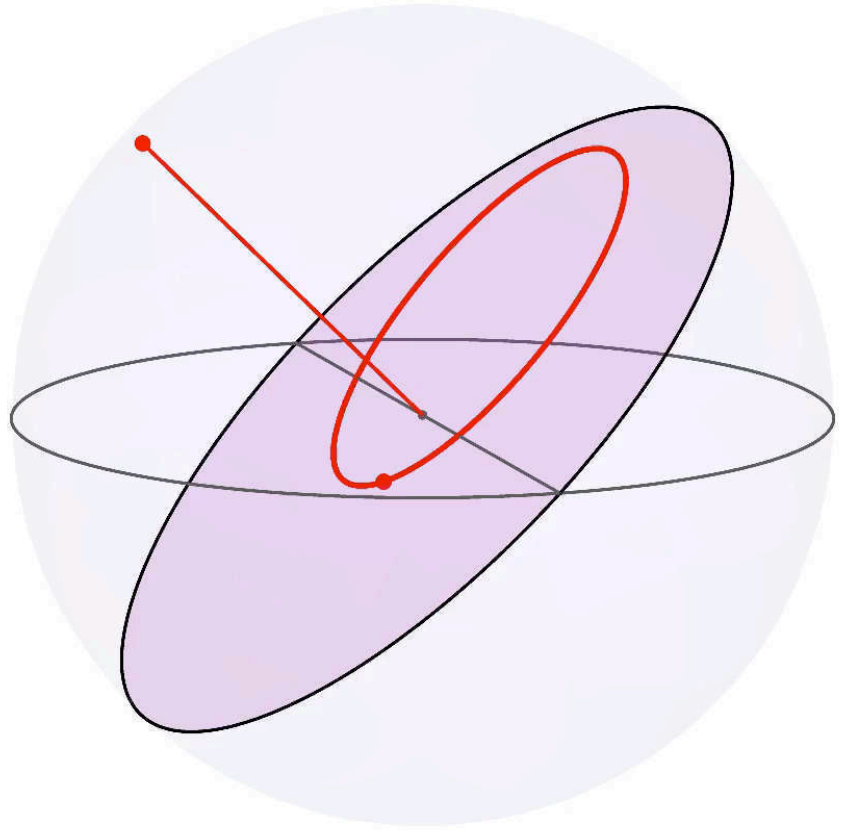
- Triaxial cluster mass
- 1.5PN relativistic **Lense-Thirring precession**

$$\dot{\hat{\mathbf{L}}} = \boldsymbol{\Omega}_{\text{prec}} \;\; ; \quad L = \text{cst} \,.$$

Wires may also **jitter stochastically**

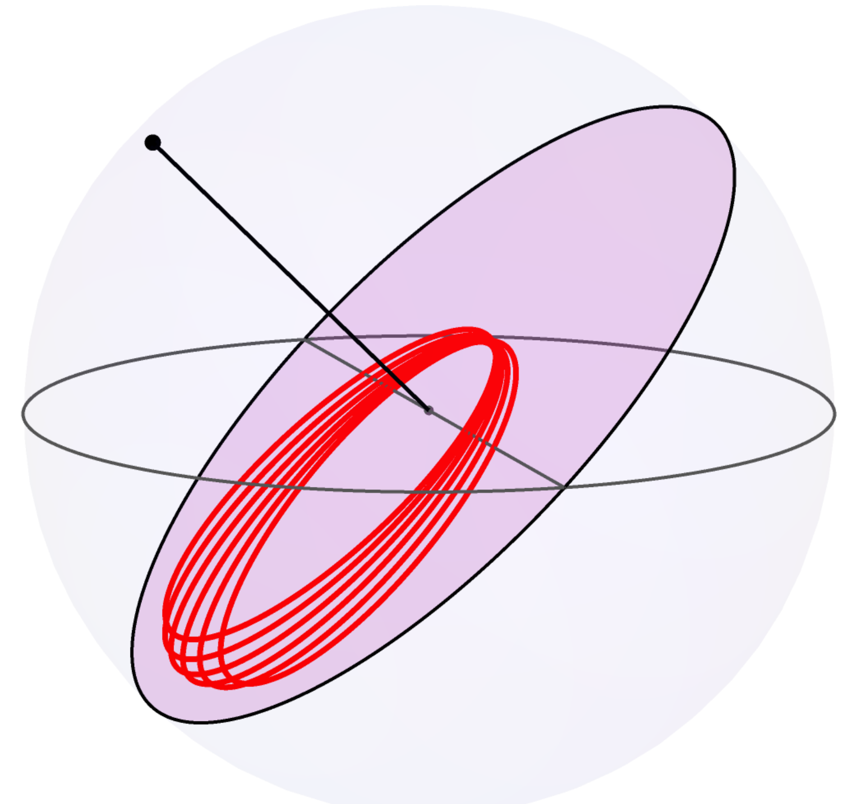
- Finite-N effects $$\dot{\hat{\mathbf{L}}} = \eta(t)$$





Long-term dynamics of wires

In-plane precessions (L, ω)



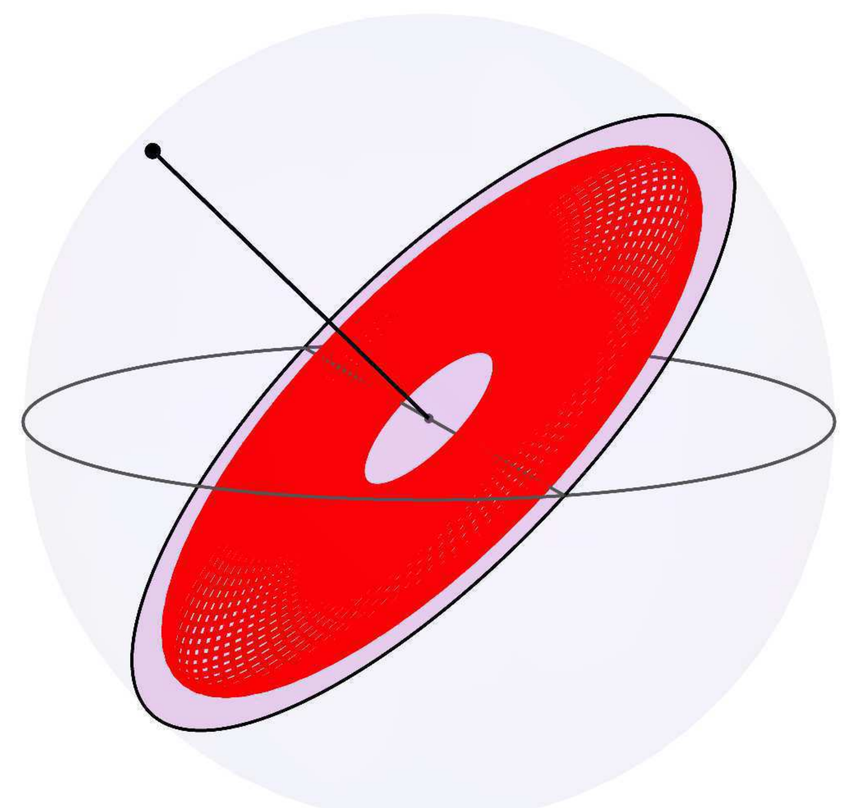
Constructive mean field motion

$$\Omega^{\mathrm{prec}} = \Omega^{\mathrm{prec}}_{\mathrm{self}} + \Omega^{\mathrm{prec}}_{\mathrm{rel}} + \Omega^{\mathrm{prec}}_{\mathrm{ext}}$$

Long-term diffusion of $L = L(e)$

Scalar Resonant Relaxation

Out-of-plane precessions $\hat{\mathbf{L}}$



No mean field motion

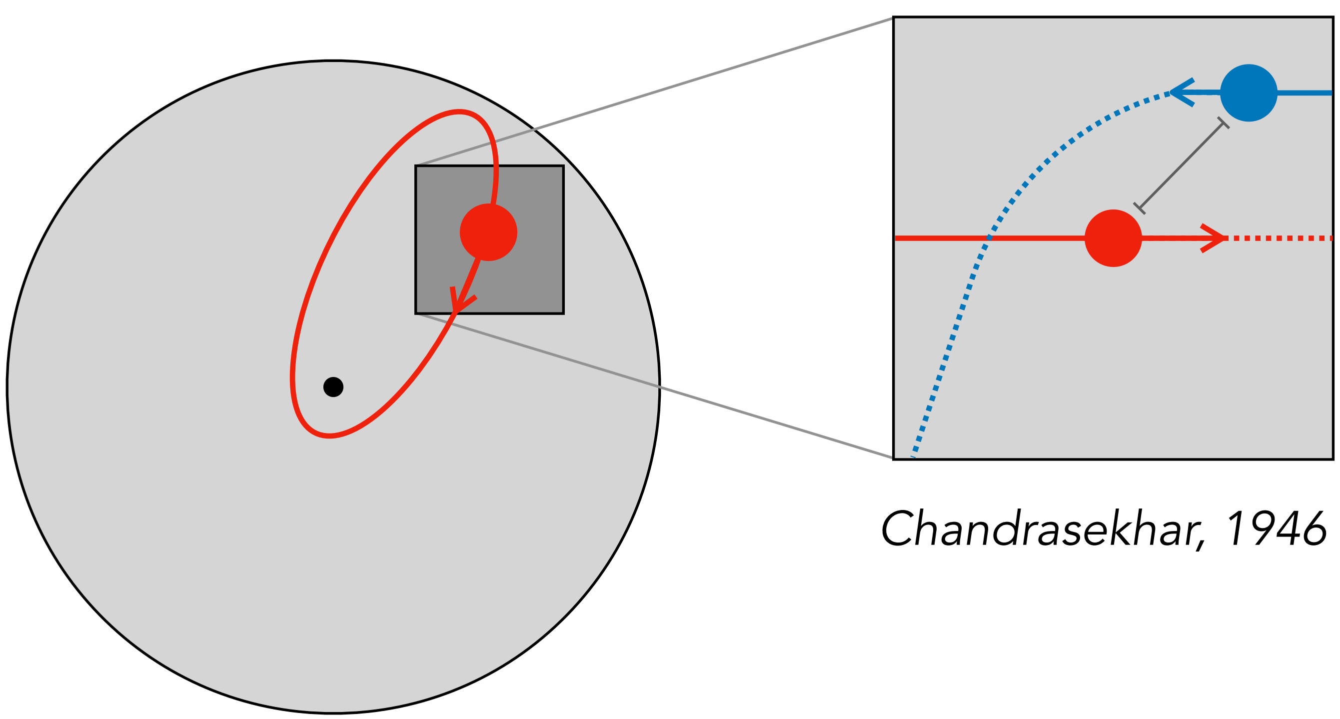
$$\langle \Omega^{\mathrm{prec}} \rangle = 0$$

Random walk on the sphere of $\hat{\mathbf{L}}$

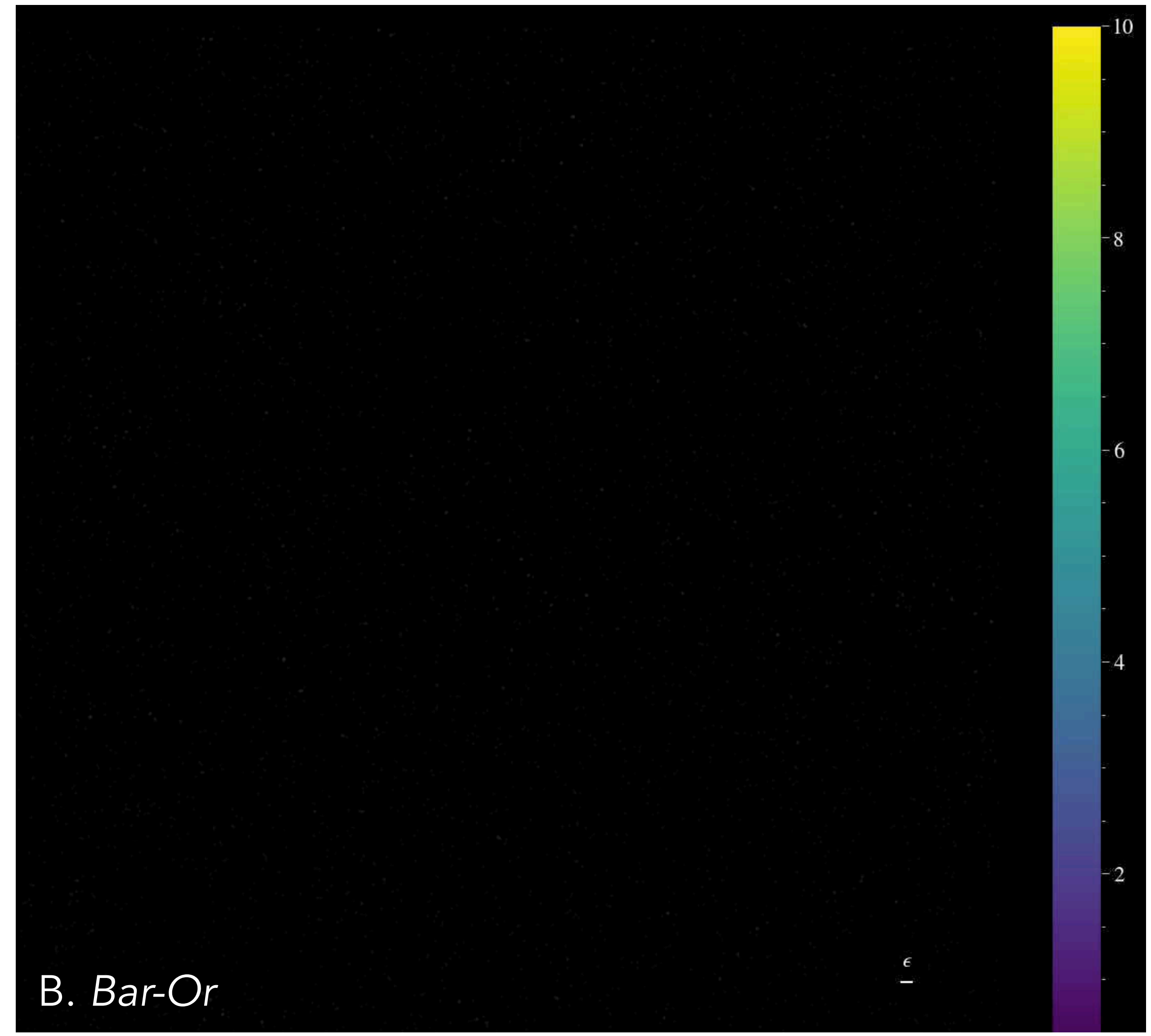
Vector Resonant Relaxation

Non-Resonant Relaxation

Orbital distortions sourced by instantaneous **kicks and deflections**



Chandrasekhar, 1946



+ **Local**, **uncorrelated** and **non-resonant** encounters, i.e. slowest dynamics

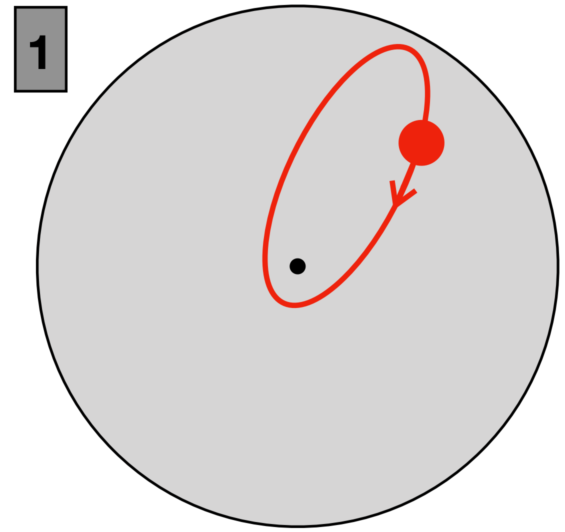
+ Immune to orbit-average and **adiabatic invariance**: $\dot{a} = \eta(t)$

Timescales are highly hierarchical

1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Omega_{\mathrm{Kep}}$$



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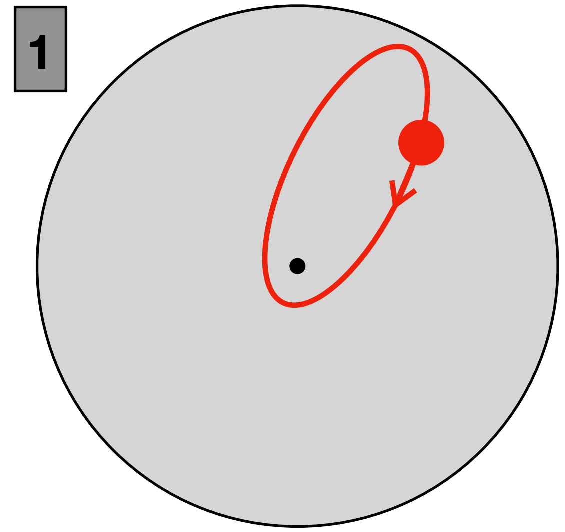
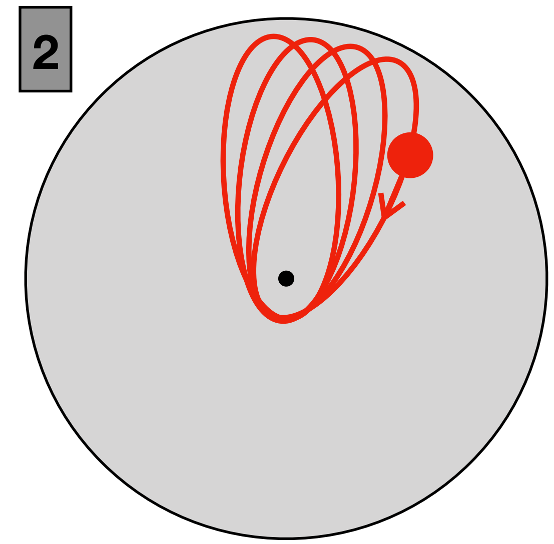
2. Precession time

In-plane precession (mass + relativity)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \Omega_{\mathrm{prec}}$$





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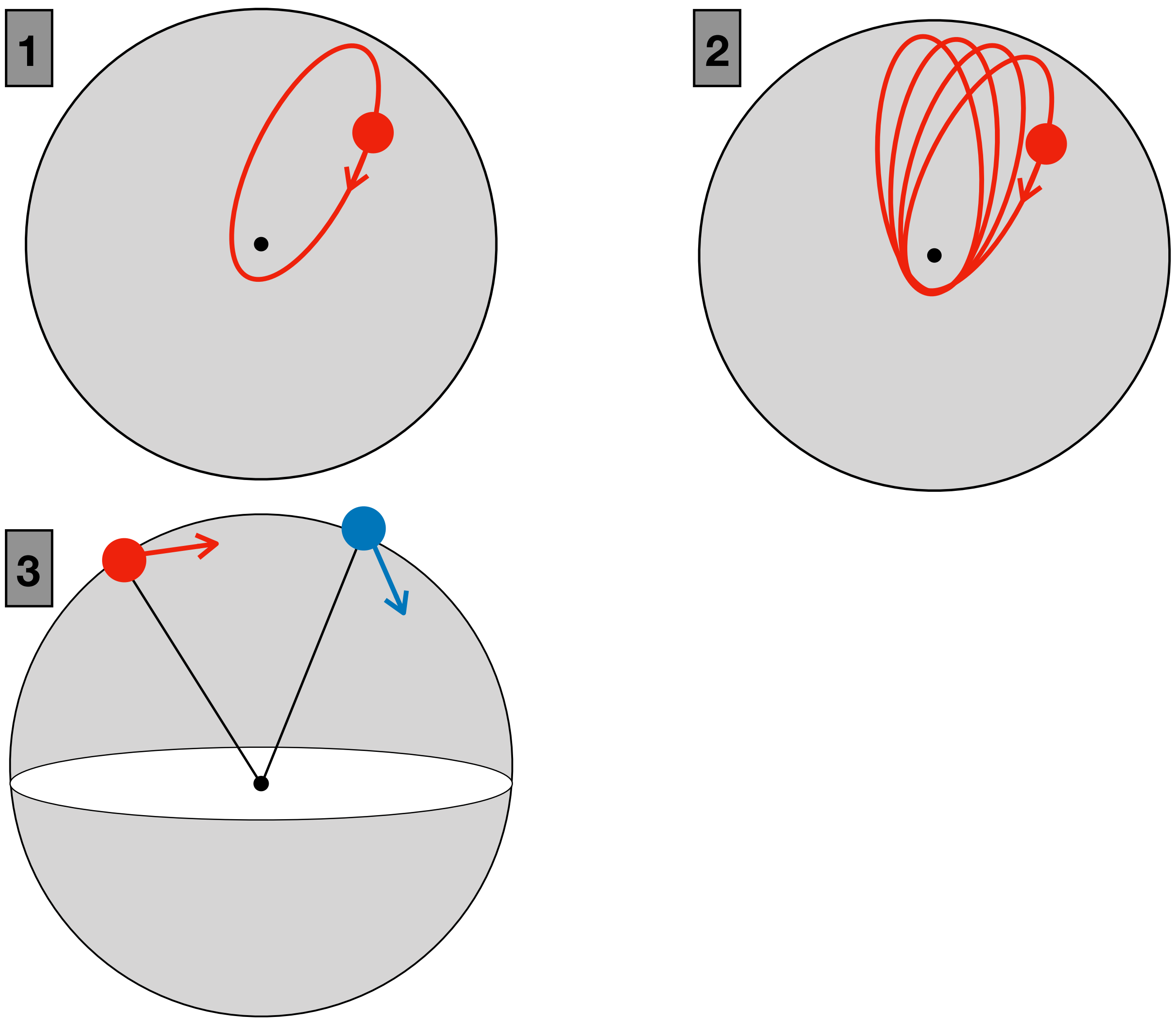
3. Vector Resonant Relaxation

Non-spherical torque coupling

$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$



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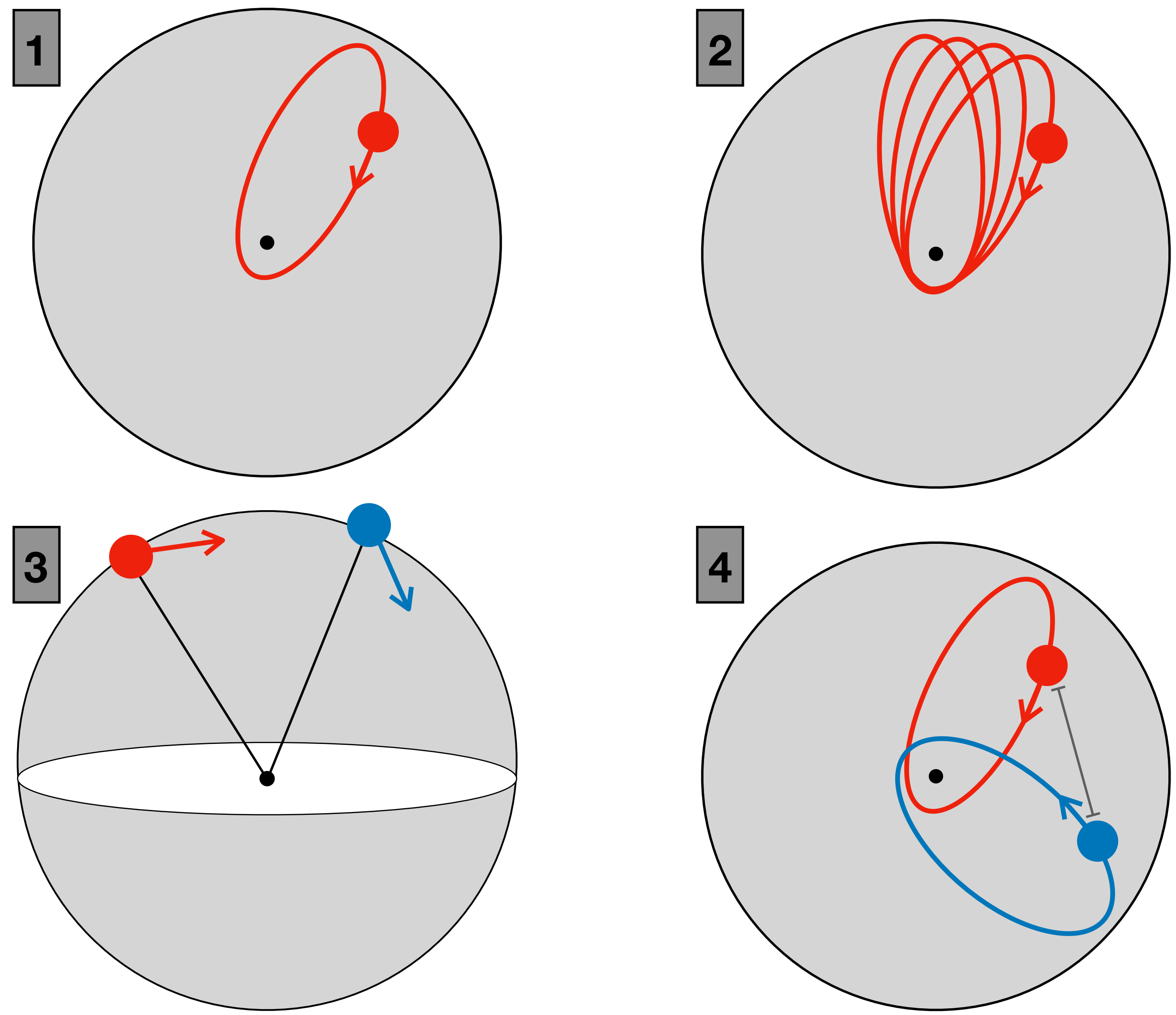
Non-spherical torque coupling

$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$

4. Scalar Resonant Relaxation
Resonant coupling on precessions

$$\frac{\mathrm{d}\,|\,\mathbf{L}\,|}{\mathrm{d}t} = \eta(\,|\,\mathbf{L}\,|\,, t)$$



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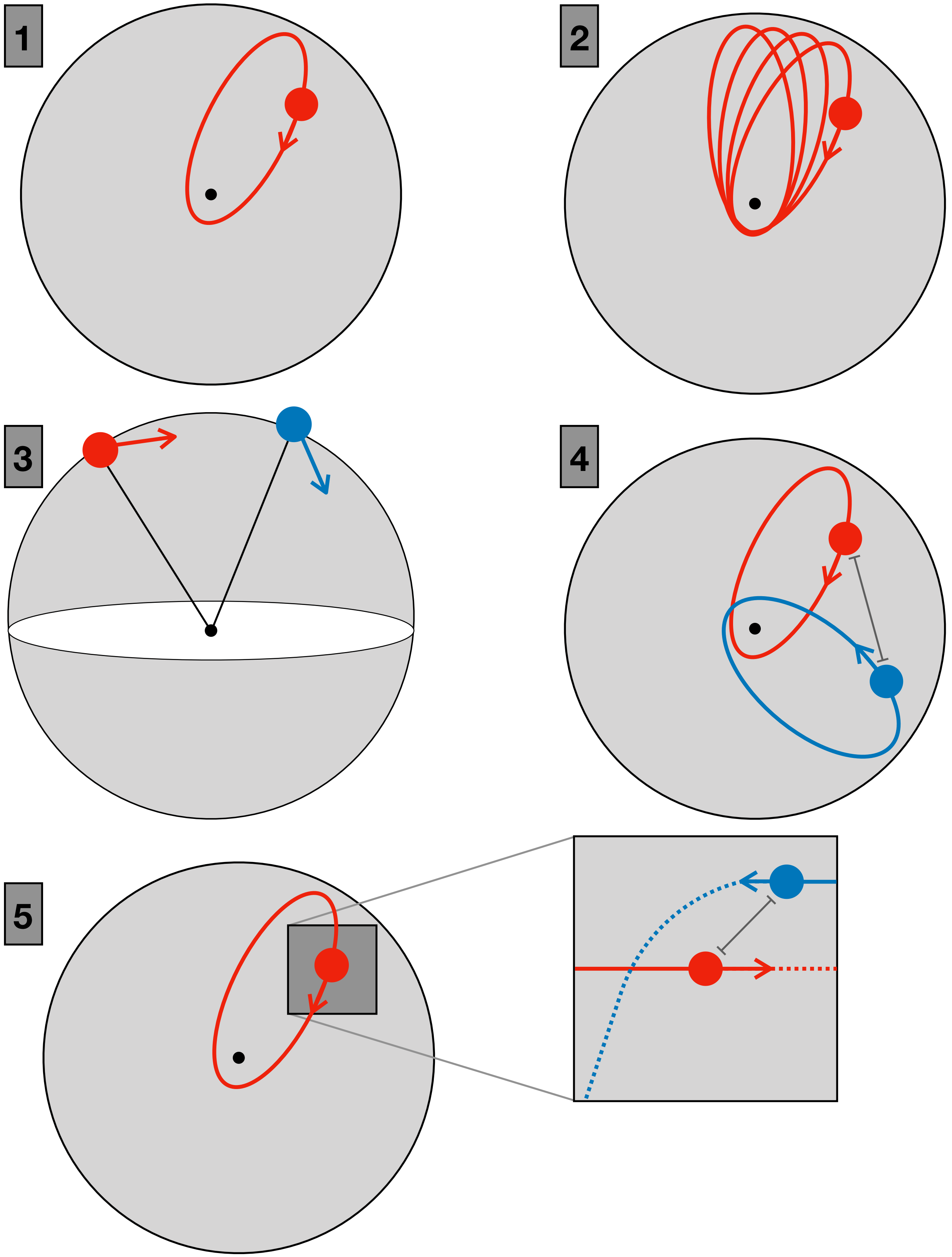
Resonant coupling on precessions

$$\frac{\mathrm{d}\,|\mathbf{L}|}{\mathrm{d}t} = \eta(|\mathbf{L}|, t)$$

5. Non-Resonant Relaxation
Local two-body encounters

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \eta(a, t)$$



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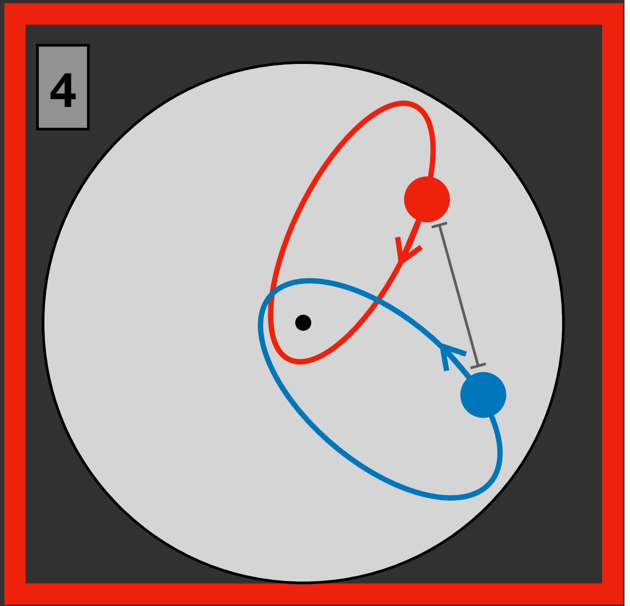
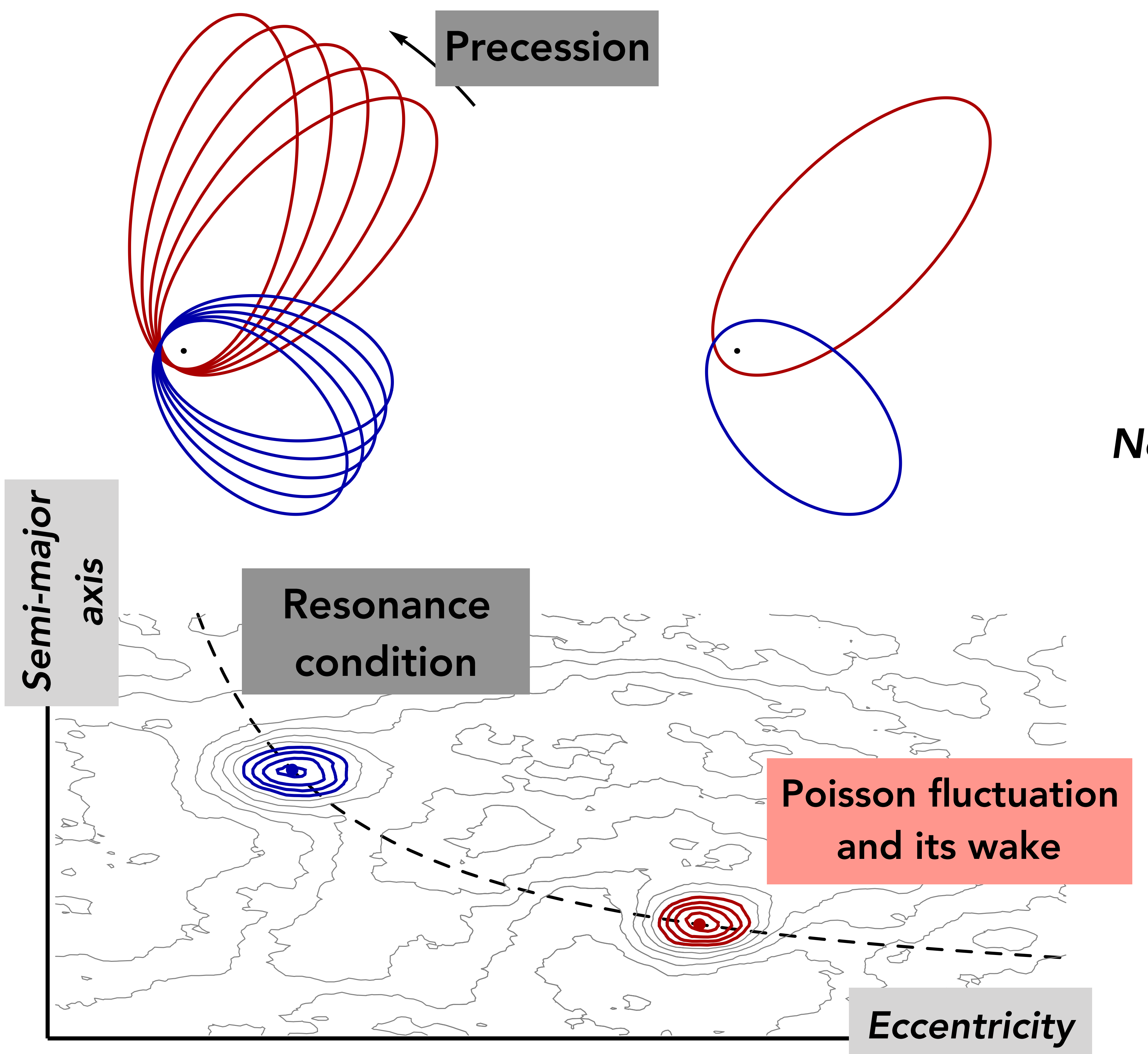


Non-local resonances



Non-local resonances
between wires

The (degenerate) Balescu-Lenard equation

The master equation of **scalar resonant relaxation**

$$\frac{\partial F(L,a,t)}{\partial t} = \frac{1}{2}\frac{\partial}{\partial L}\left[L\, D_{LL}(L,a)\,\frac{\partial}{\partial L}\frac{F(L,a,t)}{L}\right]$$

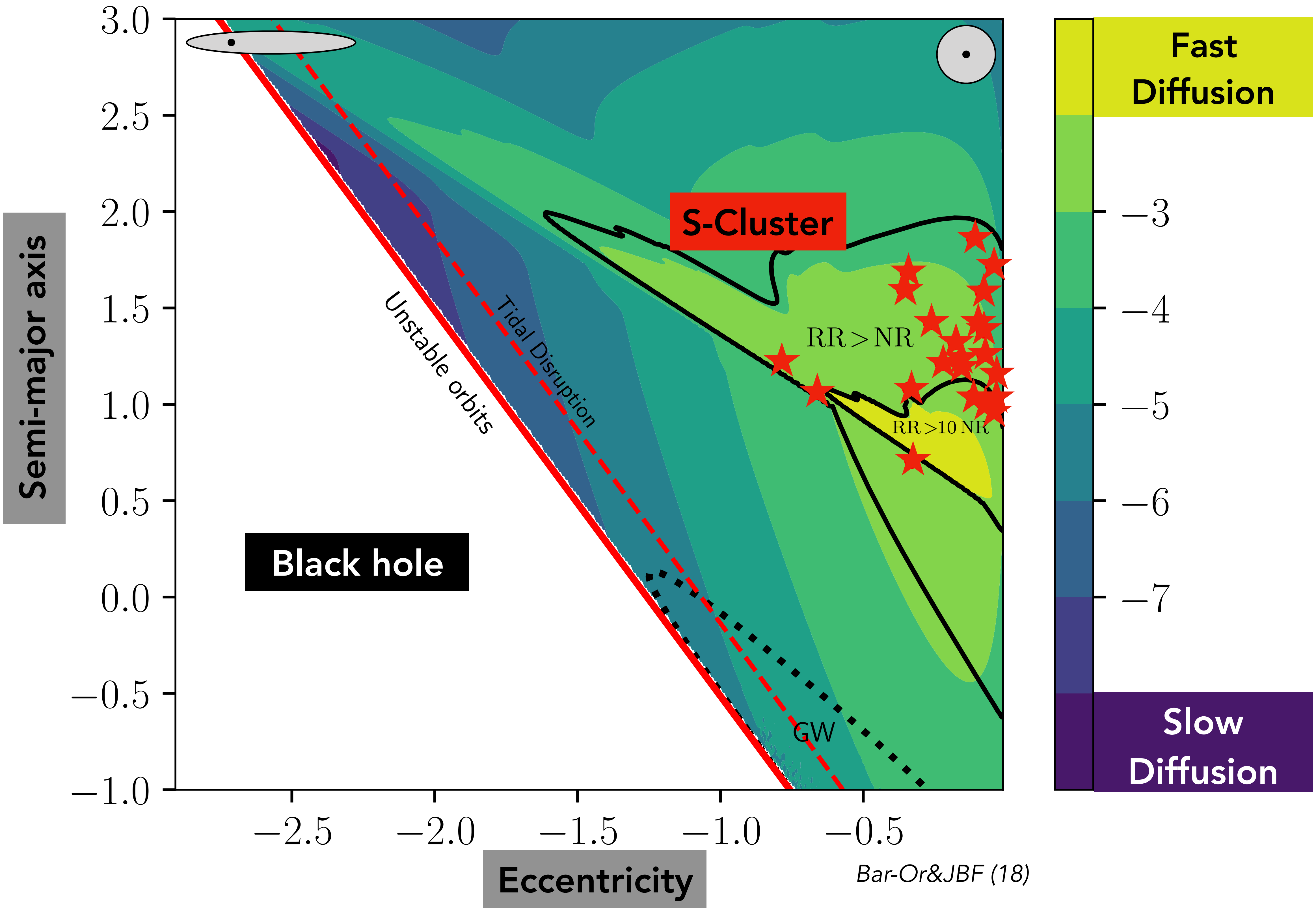
Anisotropic diffusion coefficients

$$D_{LL}(L,a) \propto \frac{1}{N_\star}\sum_{n,n'} n^2 \int \mathrm{d}L'\mathrm{d}a'\, \delta_{\mathrm{D}}(n\Omega^{\mathrm{prec}}(L,a) - n'\Omega^{\mathrm{prec}}(L',a')) \times \left|A_{nn'}(L,a,L',a')\right|^2 F^{\mathrm{Cluster}}(L',a',t)$$

Some properties

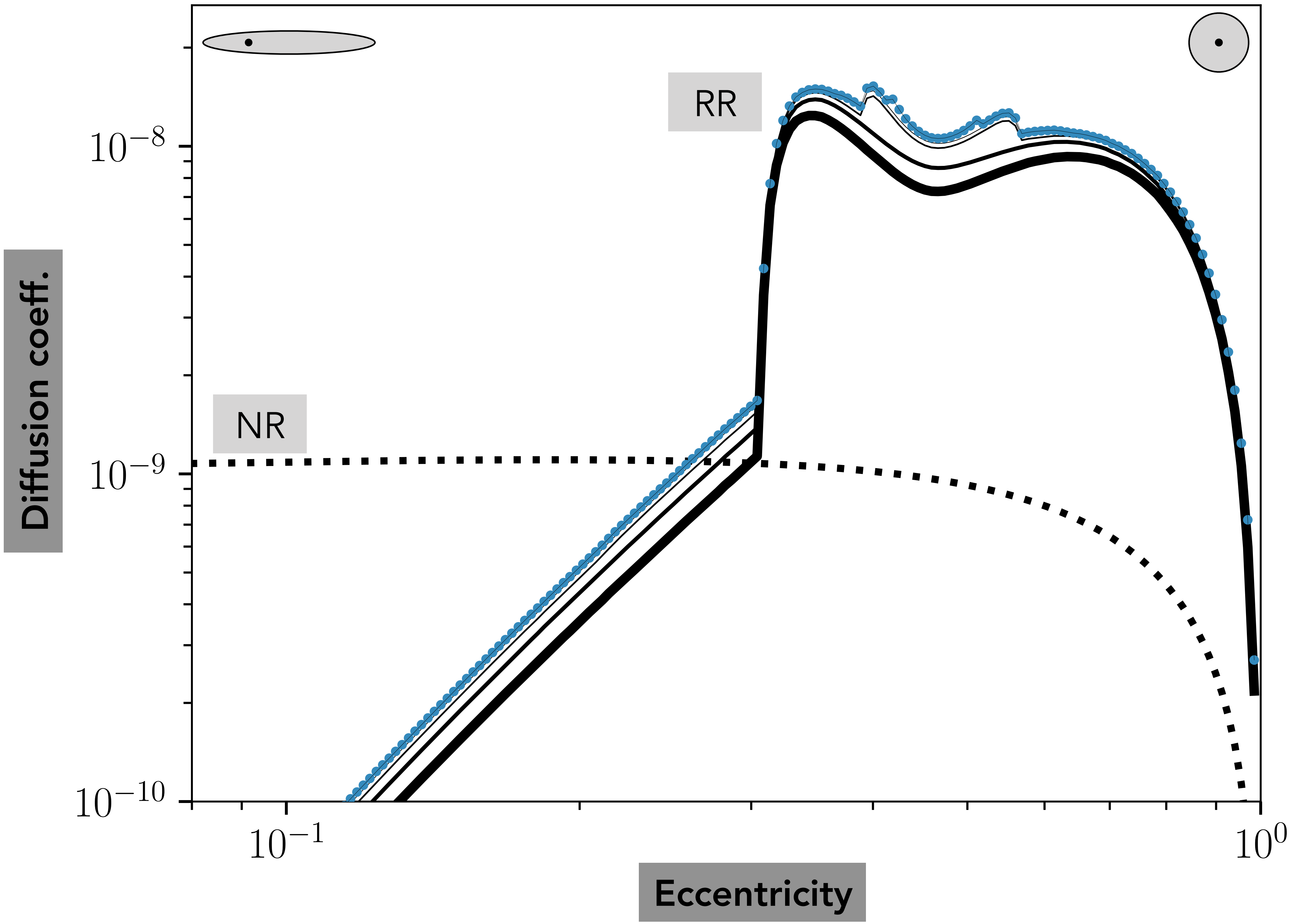
- $F(L,a,t)$ *Orbital distortion*
- $\partial/\partial L$ *Adiabatic invariance*
- $D_{LL}(L,a)$ *Anisotropic diffusion*
- $1/N_\star$ *Finite-N effects*
- n *Resonance numbers*
- $\int \mathrm{d}L'\mathrm{d}a'$ *Scan of orbital space*
- $\delta_{\mathrm{D}}(n\Omega^{\mathrm{prec}} - n'\Omega^{\mathrm{prec}\,\prime})$ *Resonance condition*
- $\left|A_{nn'}(L,a,L',a')\right|^2$ *Coupling coefficients*

Scalar Resonant Relaxation in Galactic Nuclei



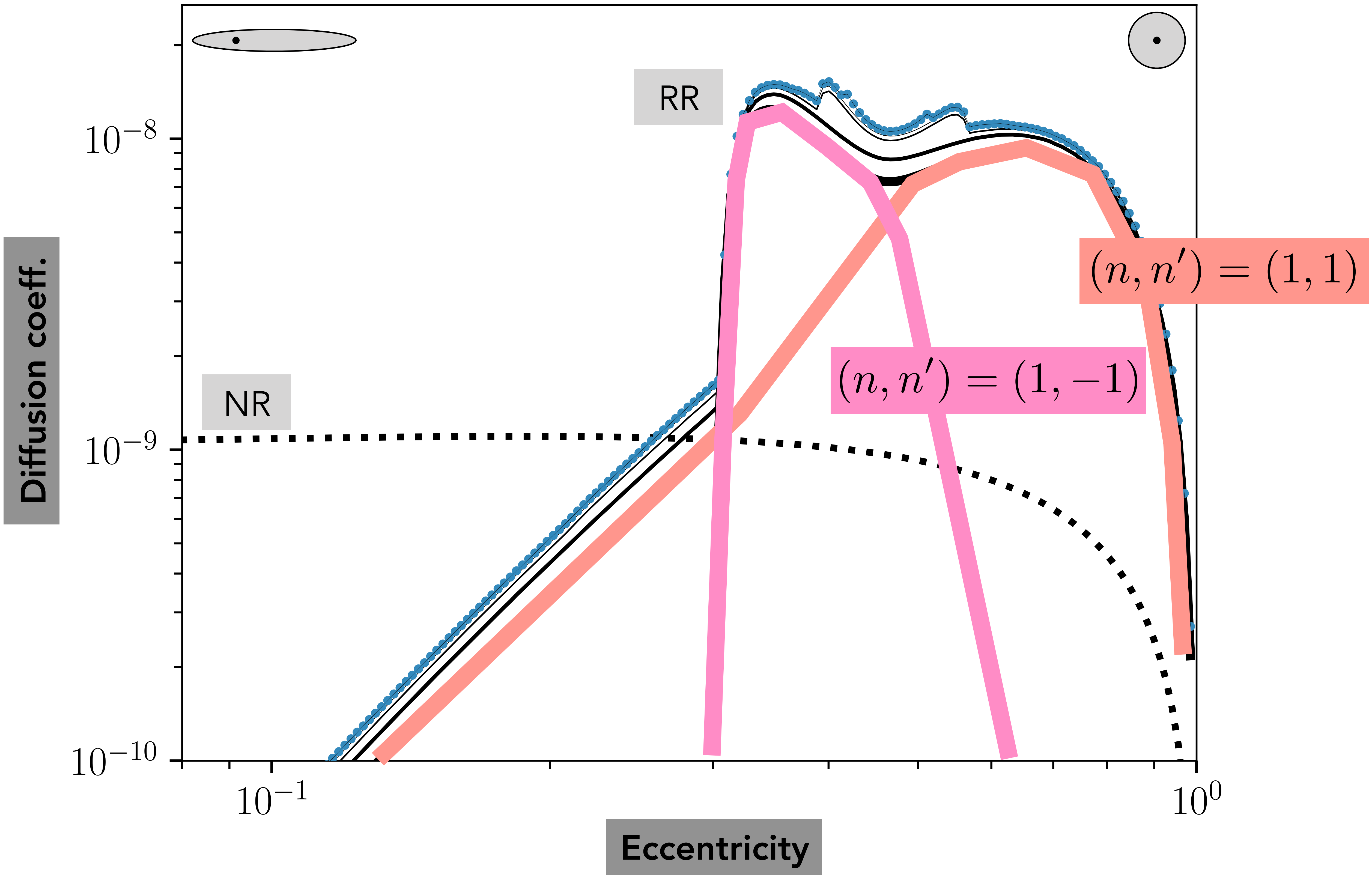
Diffusion coefficients in ***orbital space***

The diffusion coefficients of eccentricity



Non-local resonances

$$\delta_{\mathrm{D}}(n\Omega_{\mathrm{prec}}(L) - n'\Omega_{\mathrm{prec}}(L'))$$



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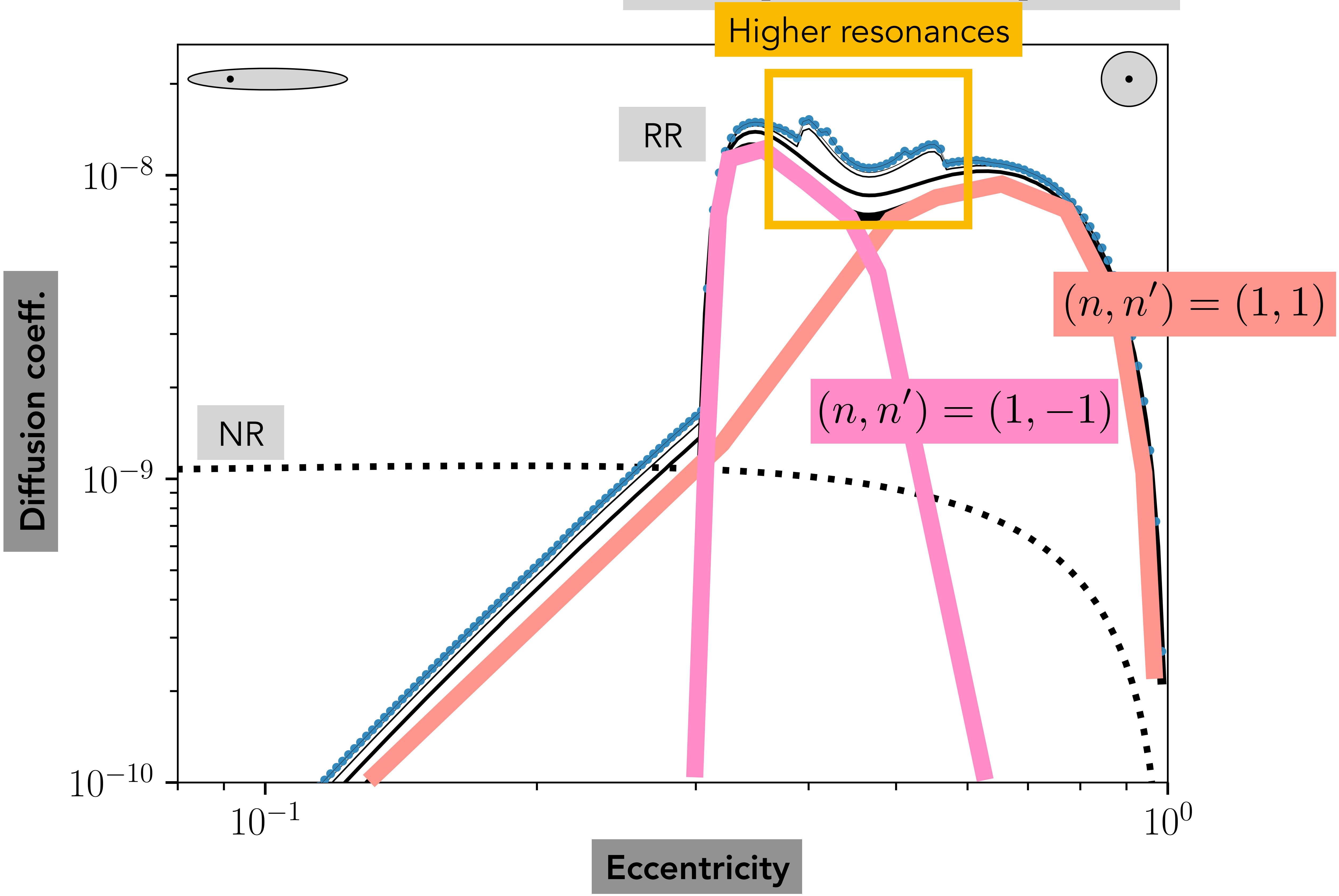
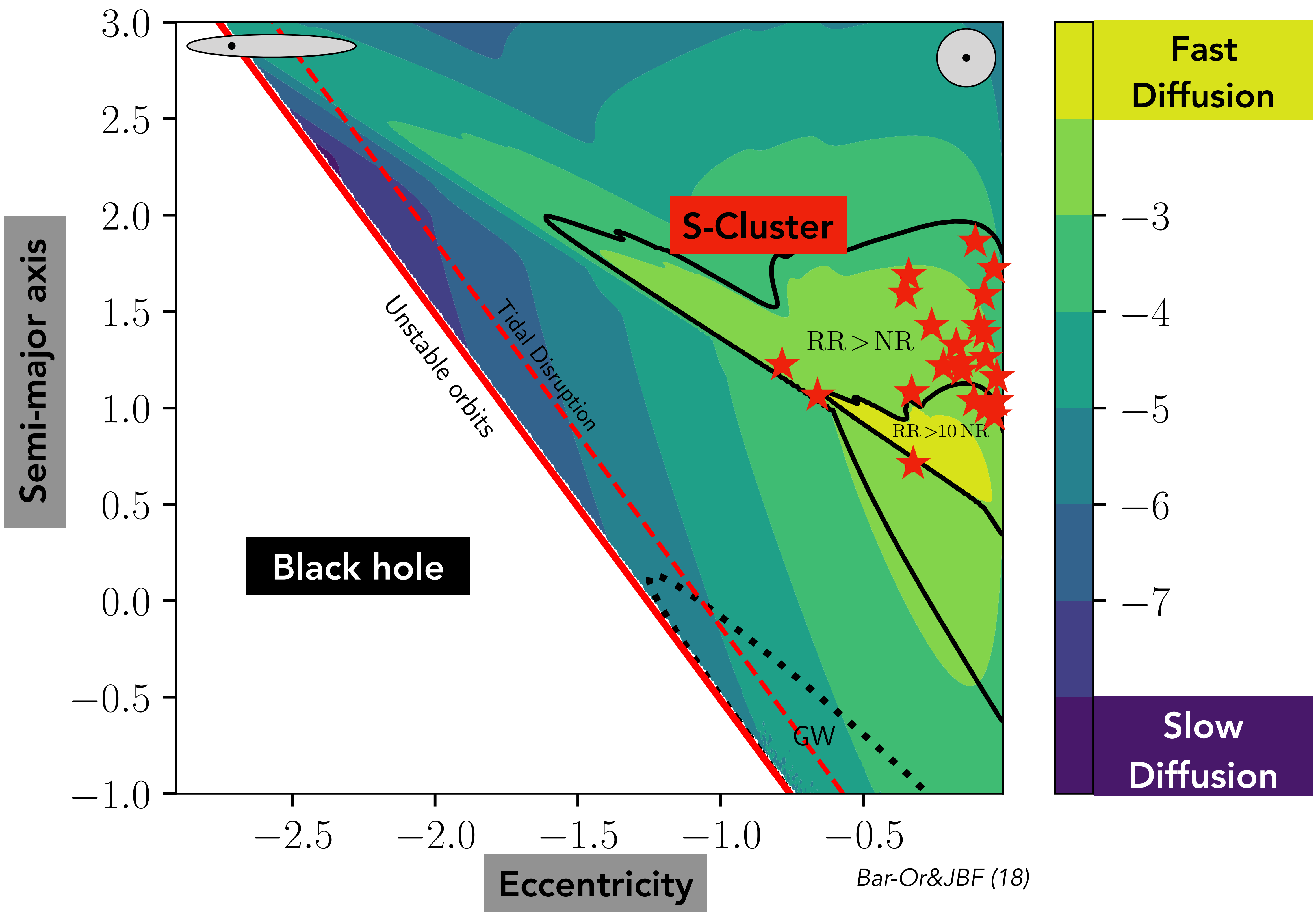


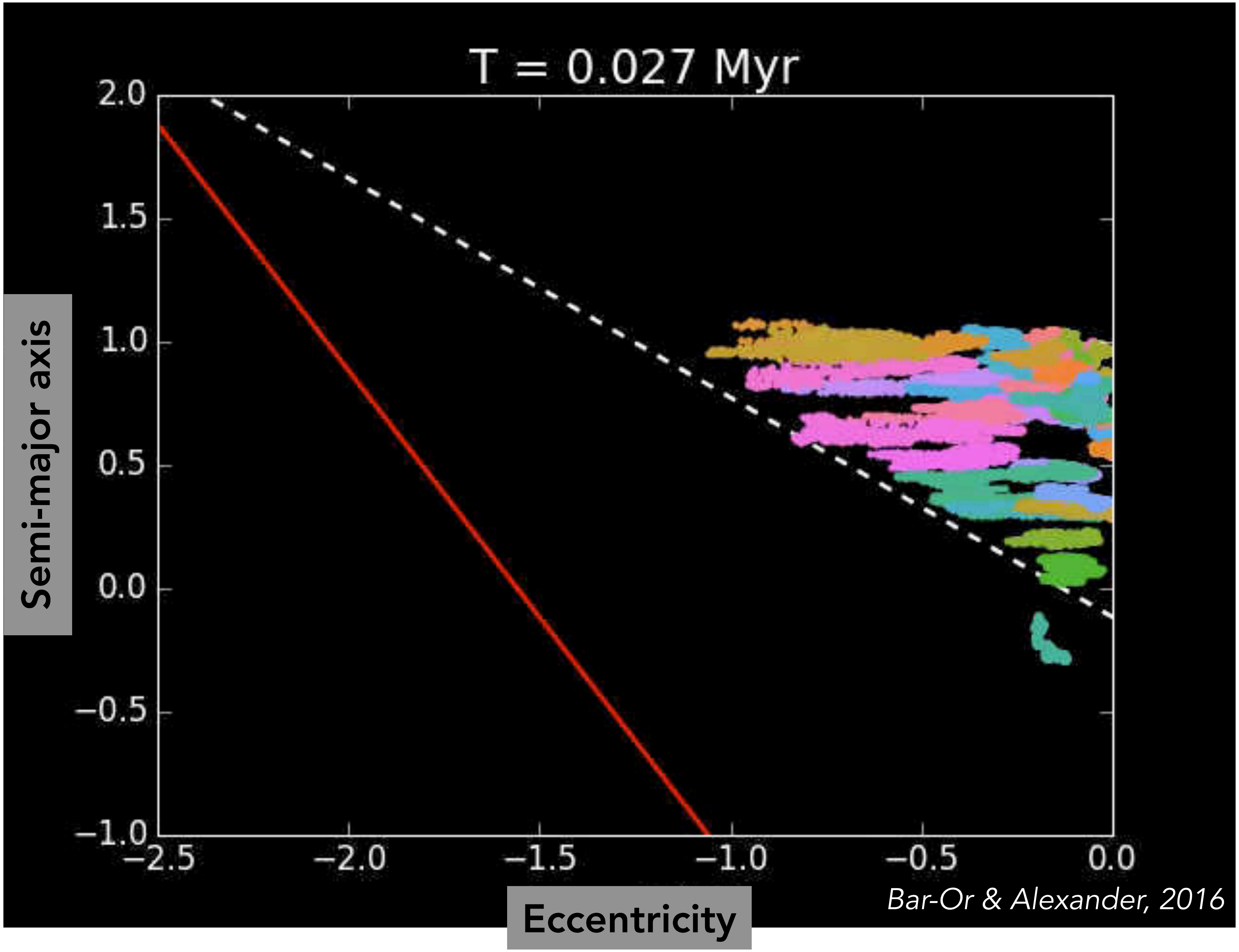
Scalar Resonant Relaxation in Galactic Nuclei



Bar-Or&JBF (18)

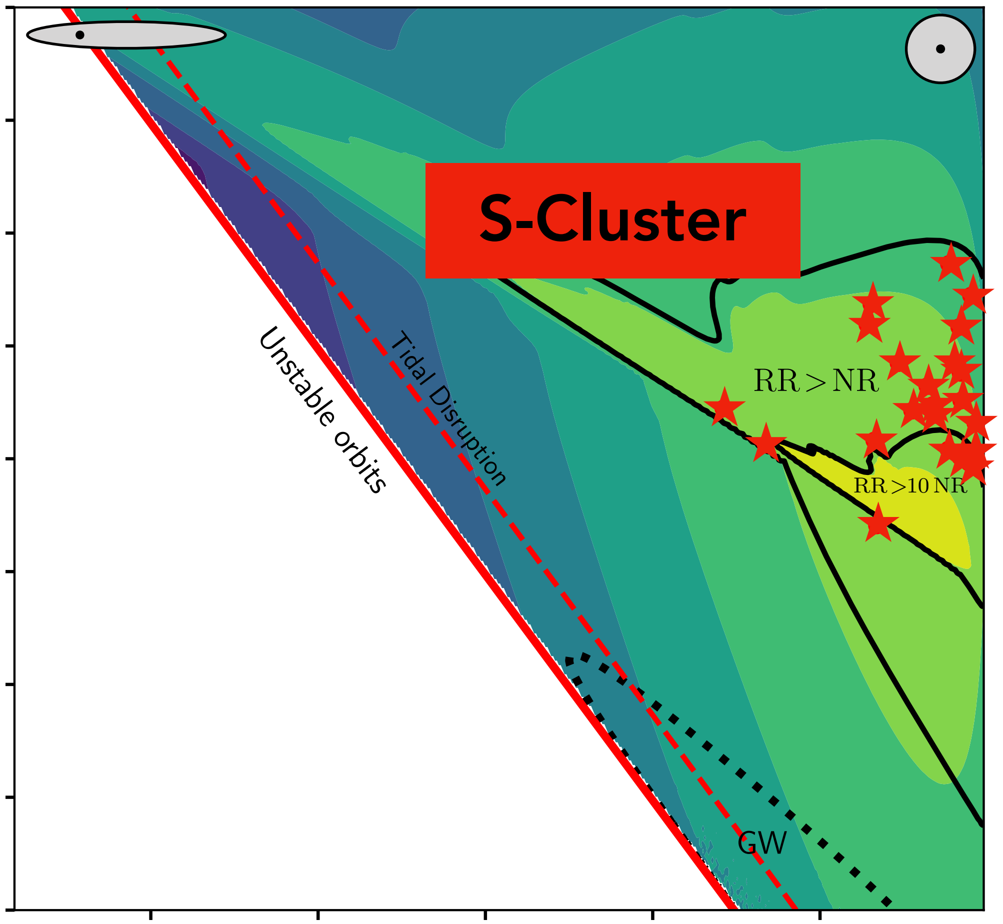
Diffusion coefficients in ***orbital space***

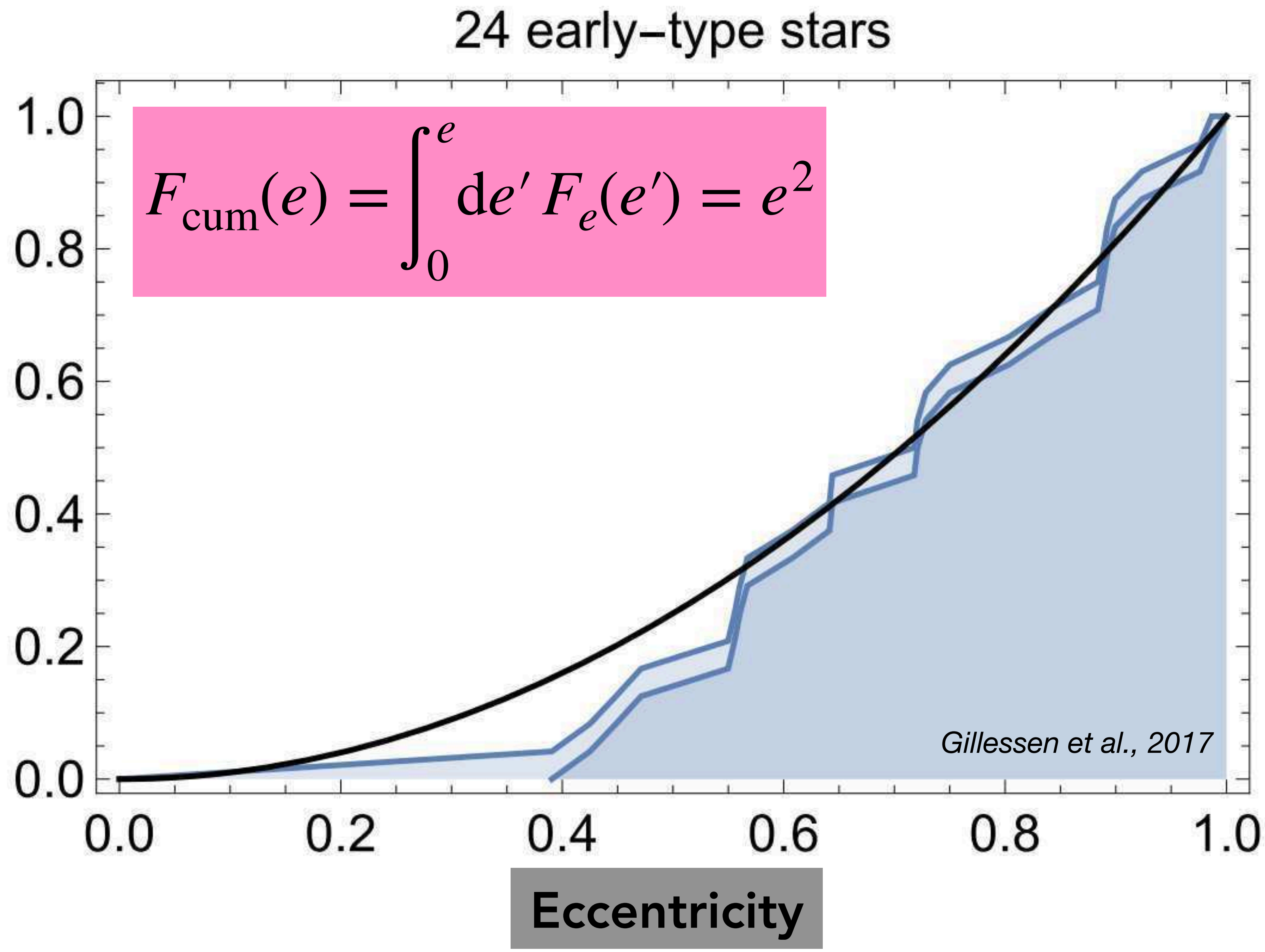
Scalar Resonant Relaxation in Galactic Nuclei



Monte-Carlo *realisation of the diffusion coefficients*

Scalar Resonant Relaxation can affect the S-stars



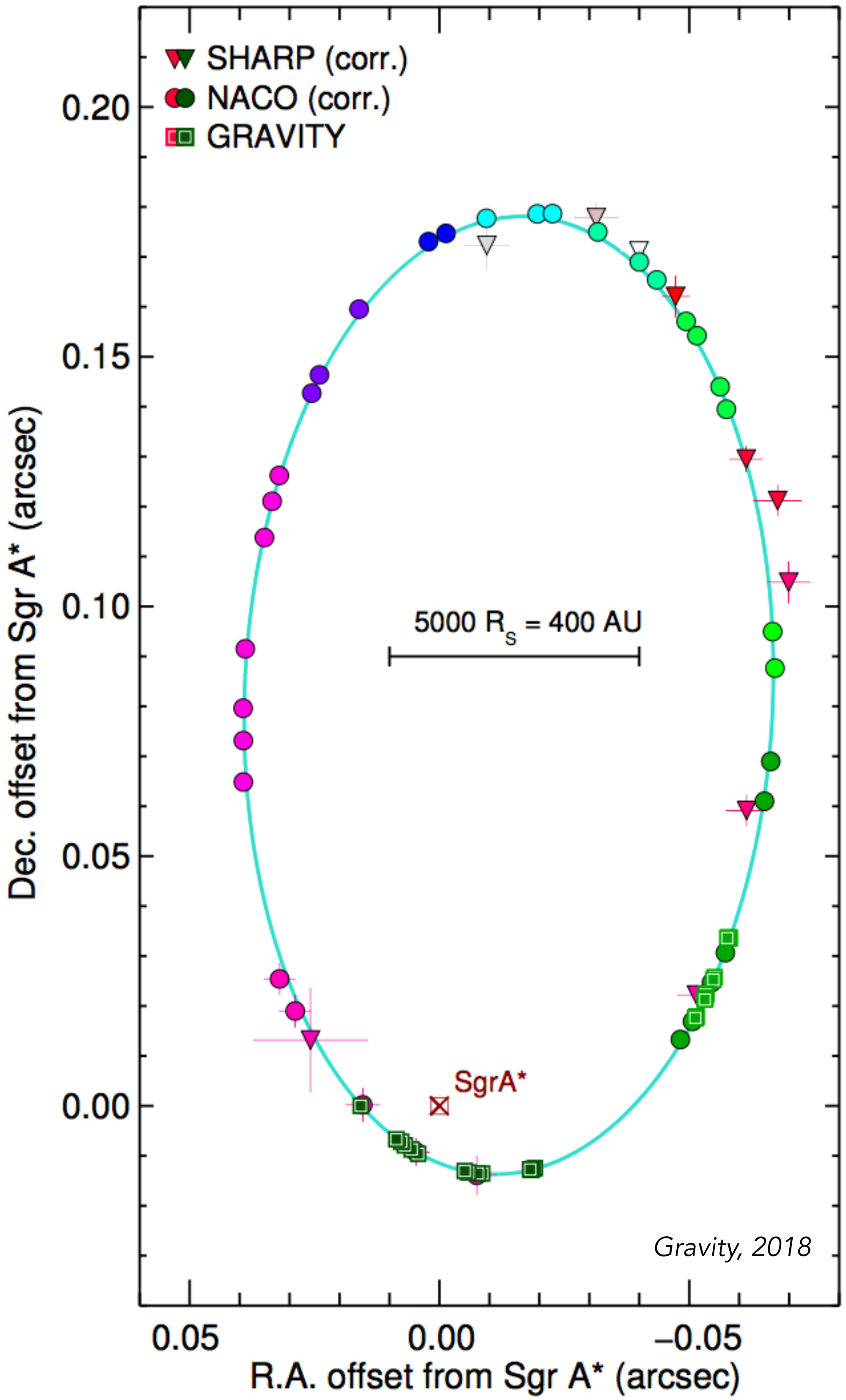


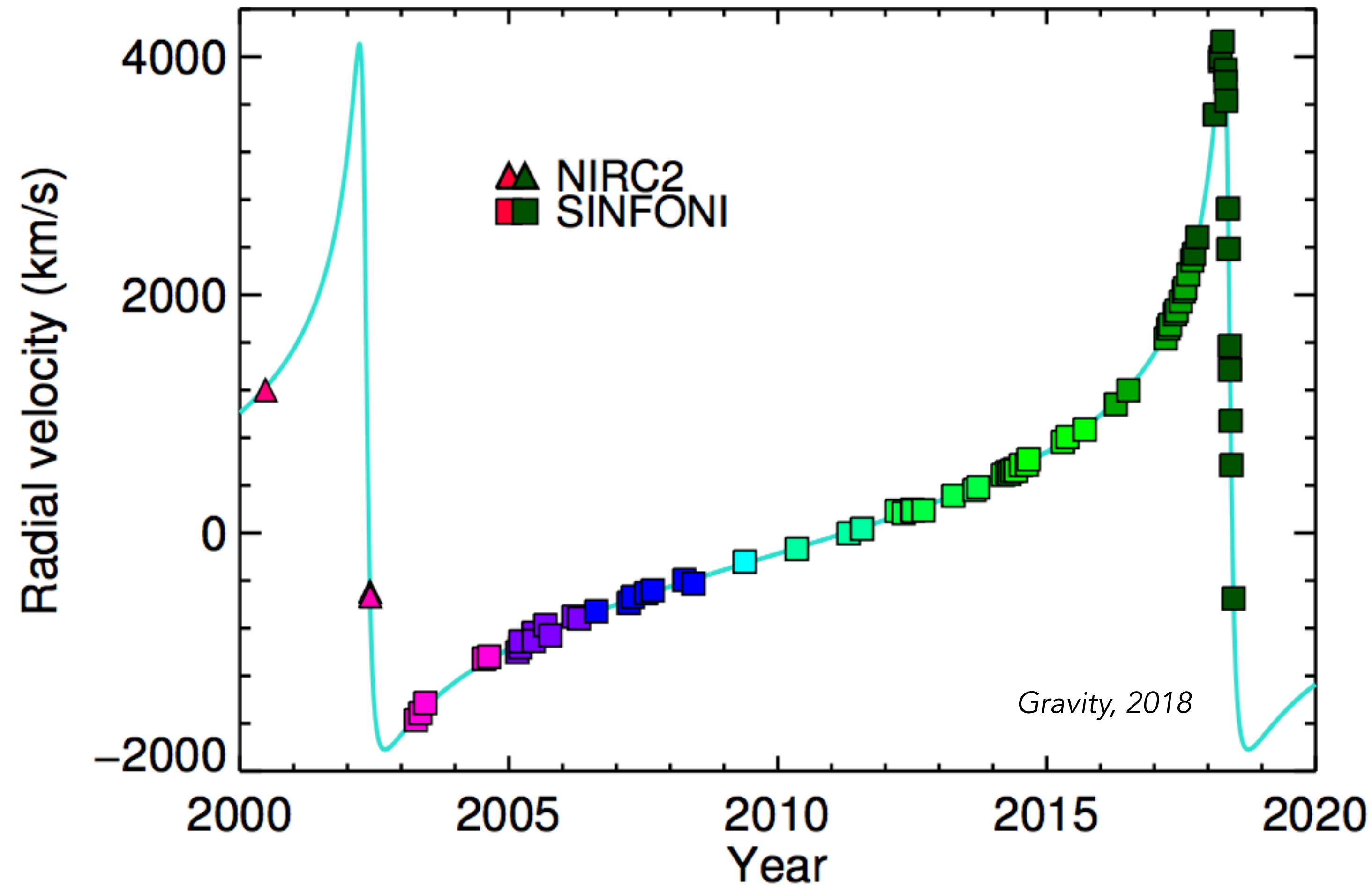
Thermal equilibrium

$$\frac{\partial F_L(L,t)}{\partial t} = \frac{1}{2}\frac{\partial}{\partial L}\left[L\, D_{LL}(L)\frac{\partial}{\partial L}\left[\frac{F_L(L)}{L}\right]\right] \Longrightarrow F_L(L) \propto L \leftrightarrow F_{\mathrm{cum}}(e) = e^2$$

Do these stars have had the time to relax in eccentricities?

More constraints from S2





+ High-precision measurement of S2's orbit, from *Gravity*, in particular at pericentre passage

+ Possible **distortions** of the Keplerian orbits due to
  - **Relativistic** precessions
  - Precession from the **enclosed mass**

$\rho(r) \propto r^{-\gamma}$ **Cusp profile** $\dfrac{\langle m^2 \rangle}{\langle m \rangle^2}$ **Mass spectrum**

Timescales are highly hierarchical

1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Omega_{\mathrm{Kep}}$$

2. Precession time

In-plane precession (mass + relativity)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \Omega_{\mathrm{prec}}$$

3. Vector Resonant Relaxation

Non-spherical torque coupling

$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$

4. Scalar Resonant Relaxation

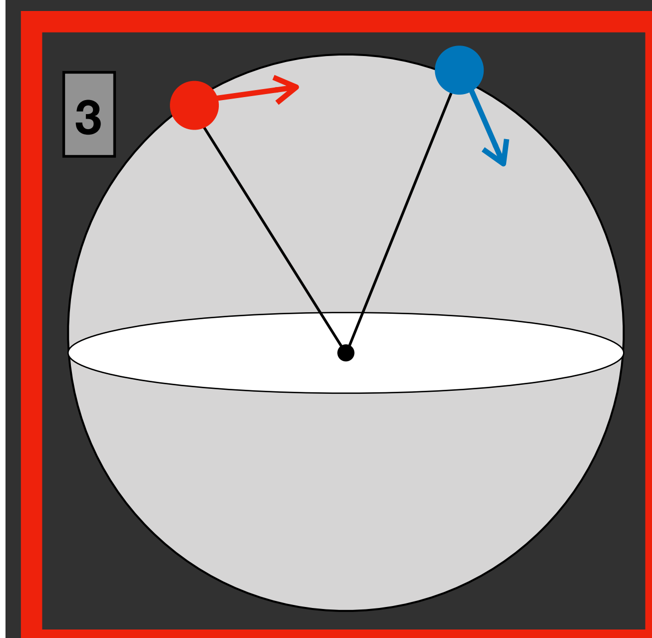
Resonant coupling on precessions

$$\frac{\mathrm{d}\,|\mathbf{L}|}{\mathrm{d}t} = \eta(|\mathbf{L}|, t)$$

5. Non-Resonant Relaxation

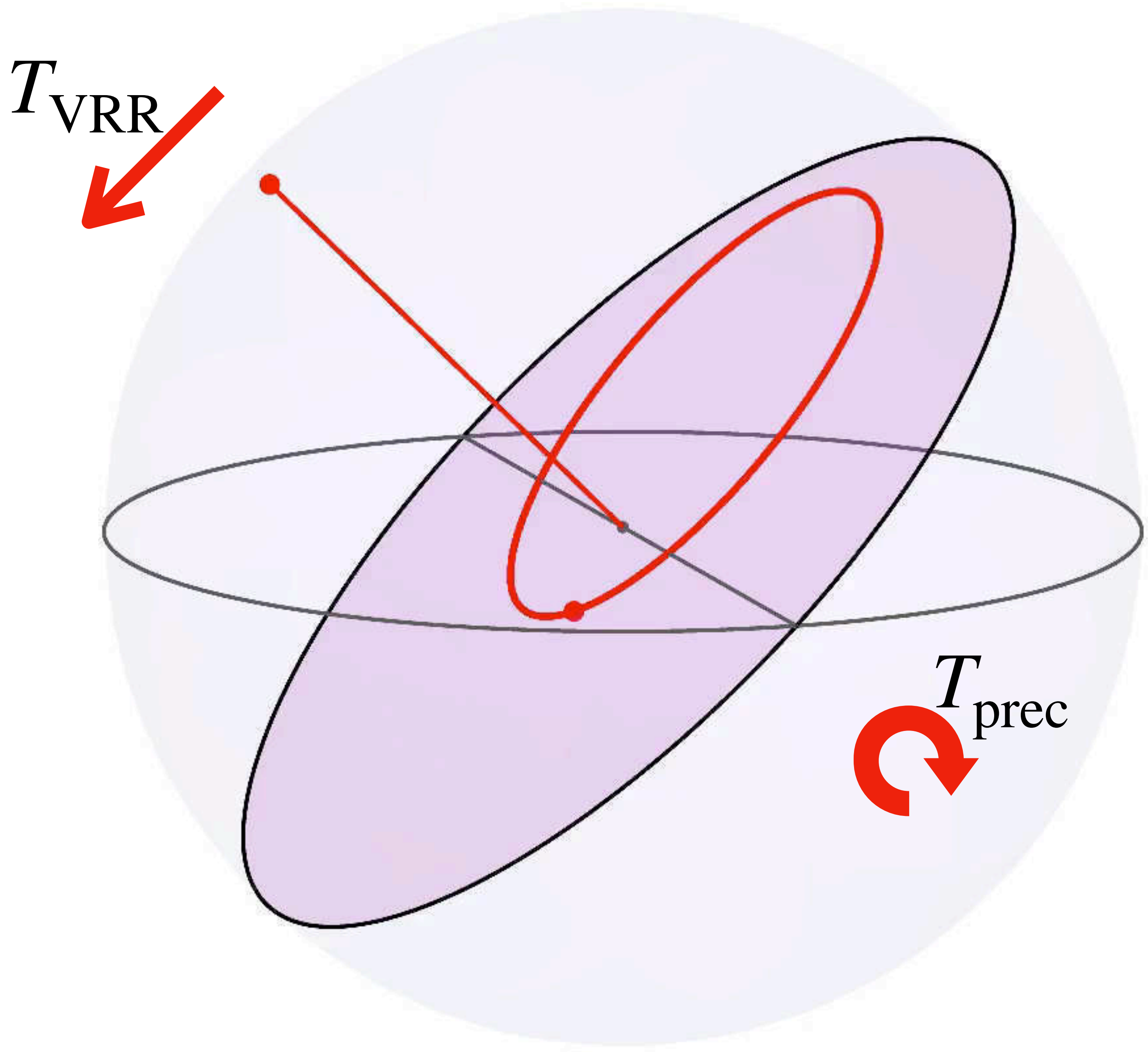
Local two-body encounters

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \eta(a, t)$$



Vector Resonant Relaxation

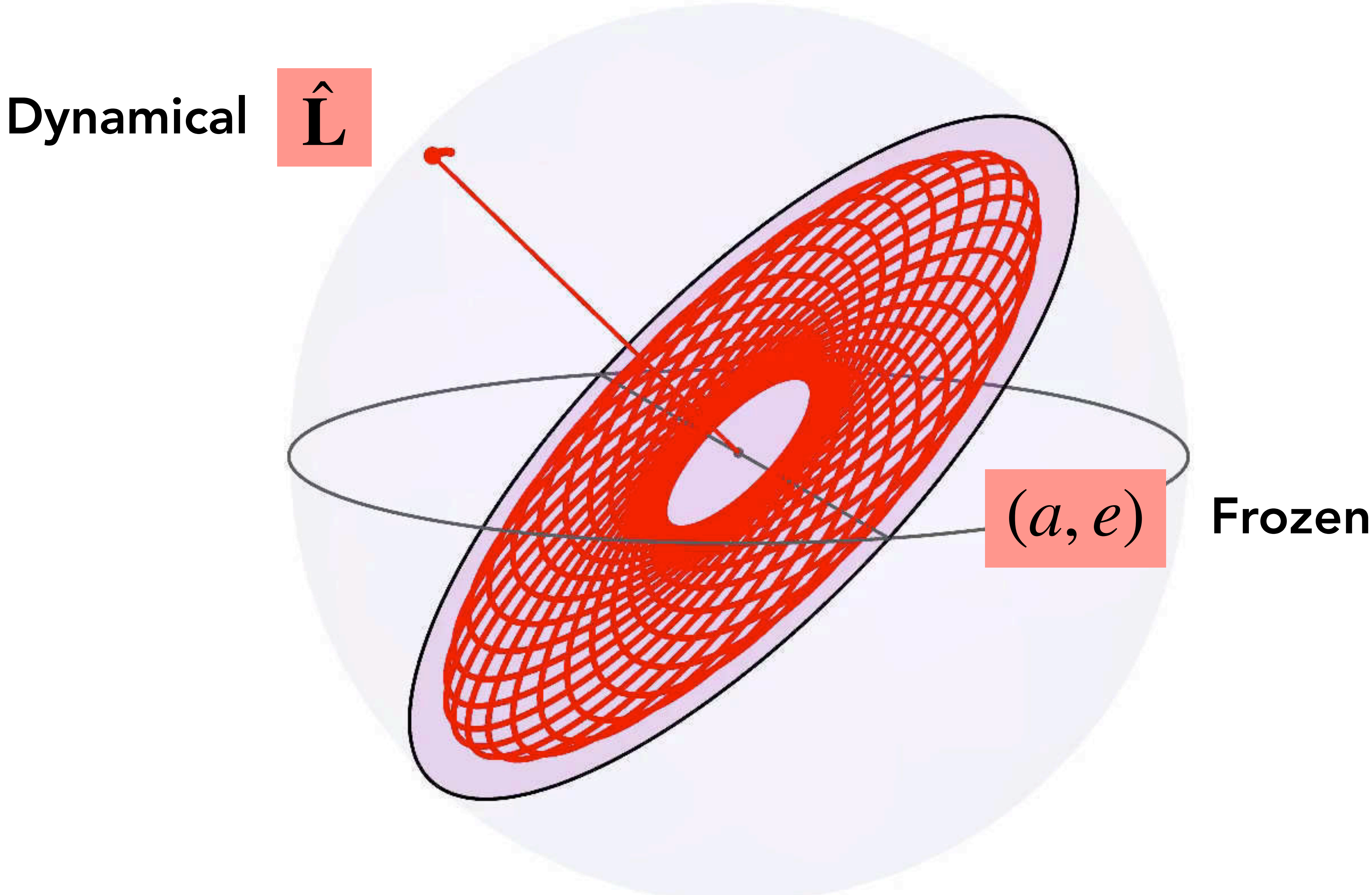
The dynamics of **Keplerian wires**



Since $T_{\mathrm{prec}} \ll T_{\mathrm{VRR}}$, we can perform a **second orbit-average**

Vector Resonant Relaxation

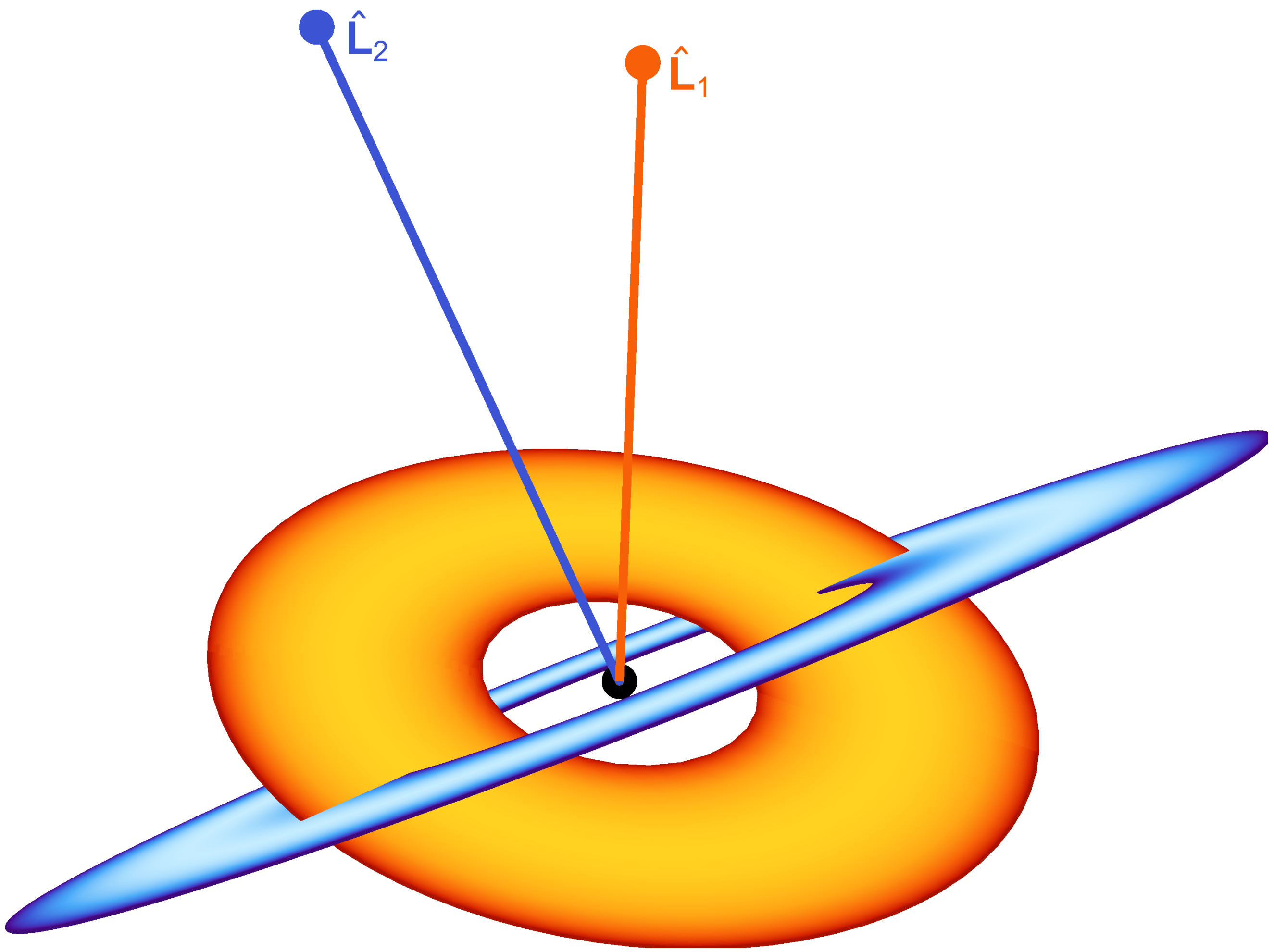
Orbit average: Wire $\Rightarrow$ Annuli



What is the dynamics of a set of **long-range coupled annuli**?

Vector Resonant Relaxation

Random walk of the stars' orientations



Pairwise coupling between two annuli

+ **Long-range** Hamiltonian system

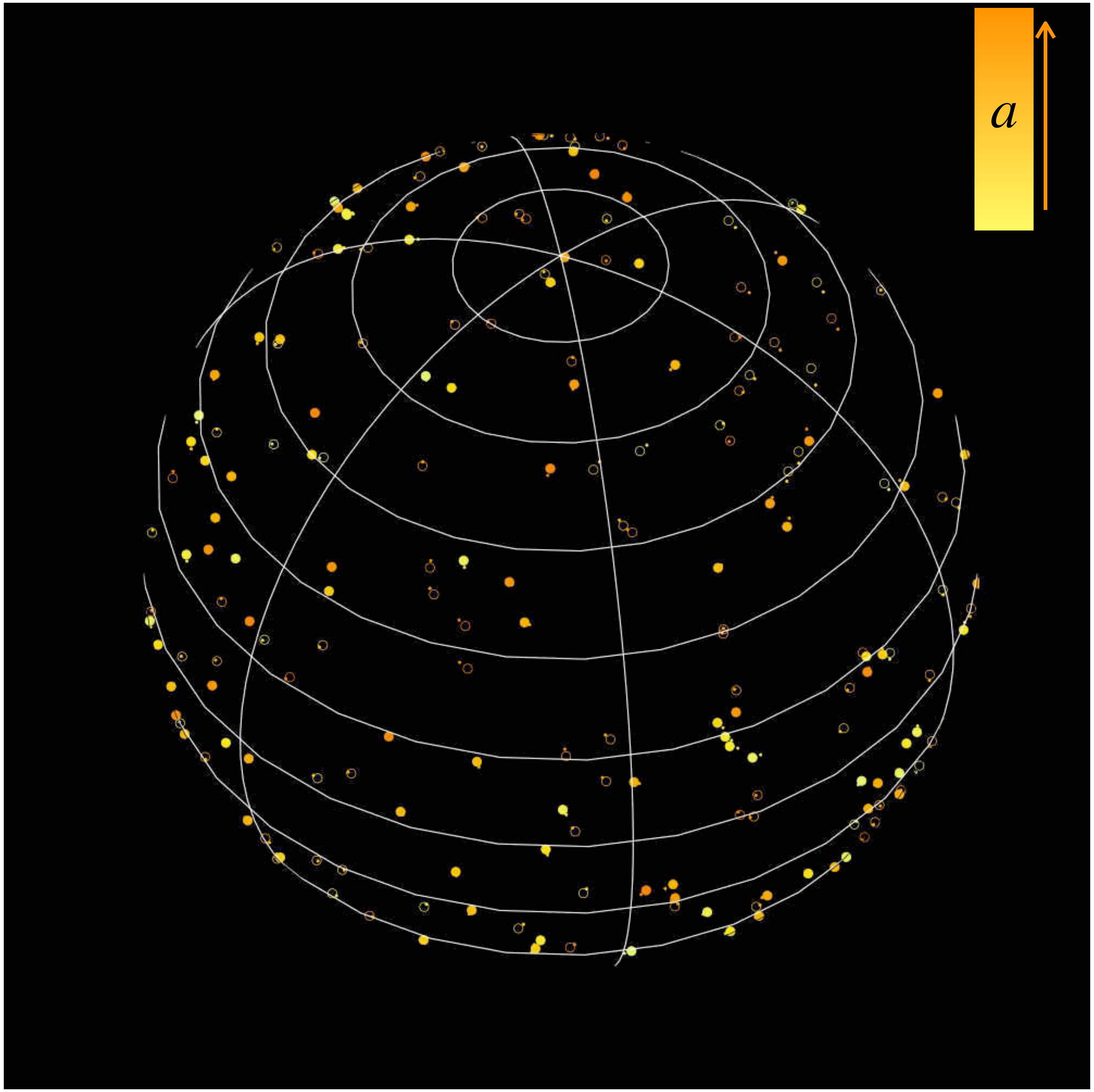
$$H = \sum_{i<j} A(a_i, e_i, a_j, e_j)\, U(\hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j)$$

+ Dynamical variables - **orientations**: $\hat{\mathbf{L}}$

+ Some properties

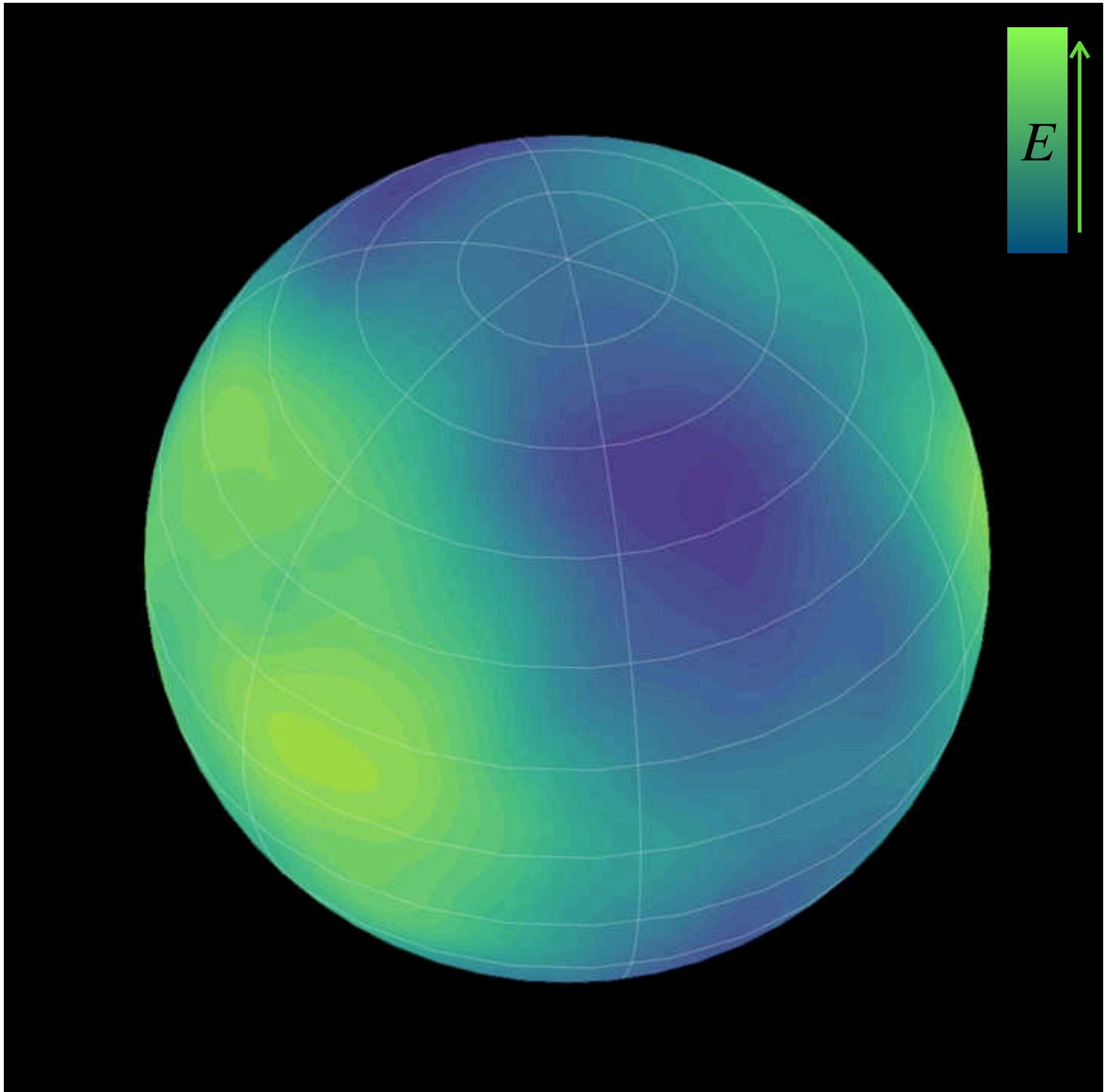
  - No **kinetic energy**
  - Vanishing **mean field** $\langle H \rangle = 0$
  - Additional "labels" (a, e)
  - **Rotational invariance** $\hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j$

Vector Resonant Relaxation



+ Motion coherent on large scales
  - **Long-range interacting system**
+ Motion smooth on short times
  - **Time-correlated noise**
+ Particles have "preferred friends"
  - **Parametric coupling** (a, e)
+ System in statistical equilibrium
  - **Time stationarity** $(t - t')$
  - **Rotation invariance** $(\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}')$

Vector Resonant Relaxation



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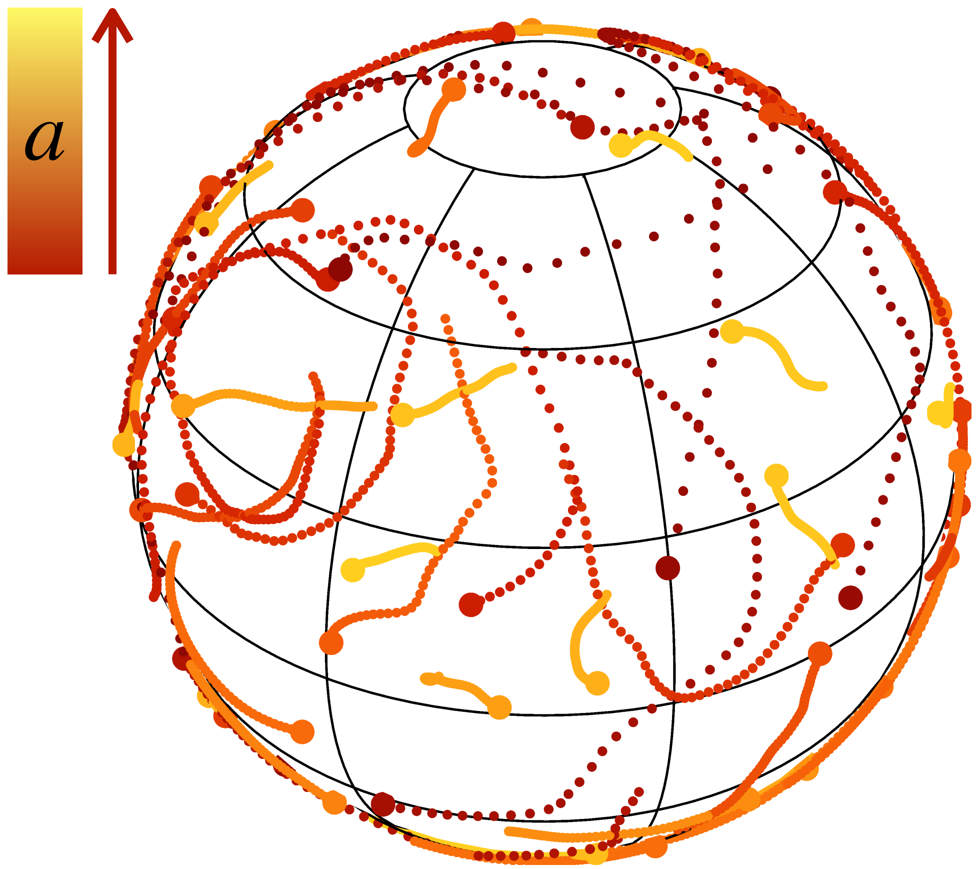
Self-consistency requirement

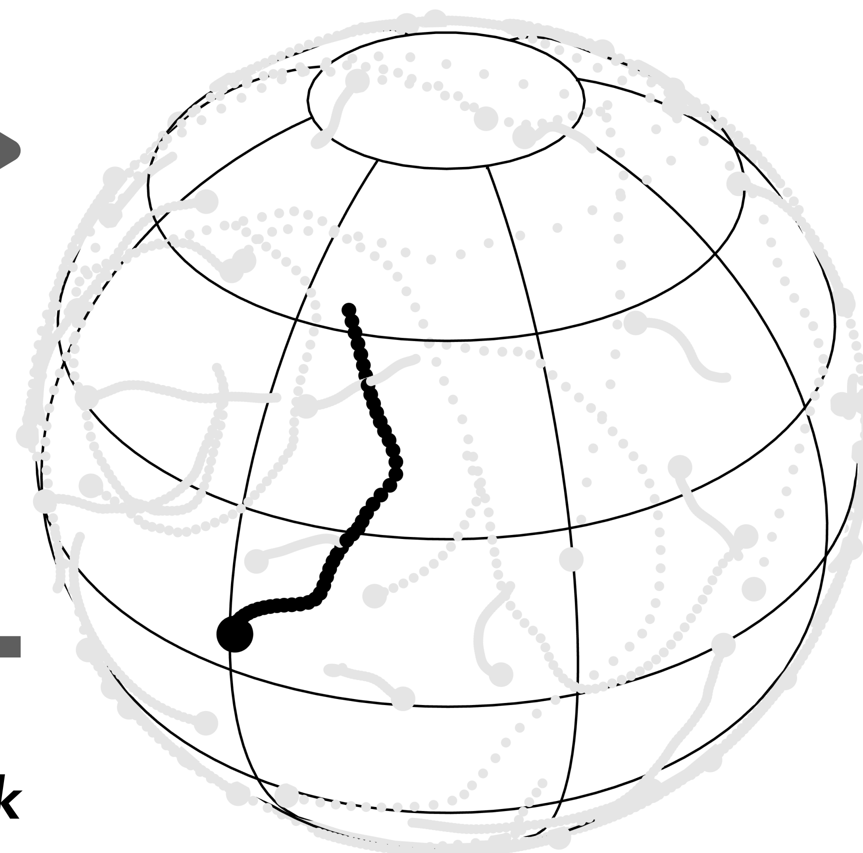
Bath of particles



Imposes a noisy (correlated) ***potential***

Undergoes a (correlated) ***random walk***

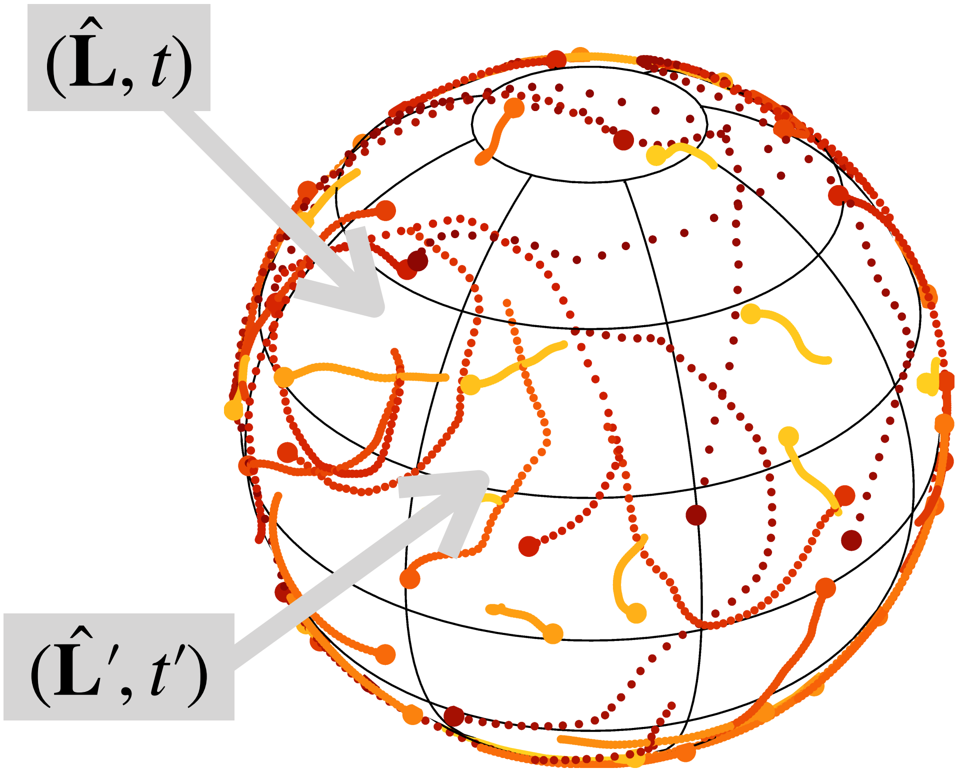
Test particle

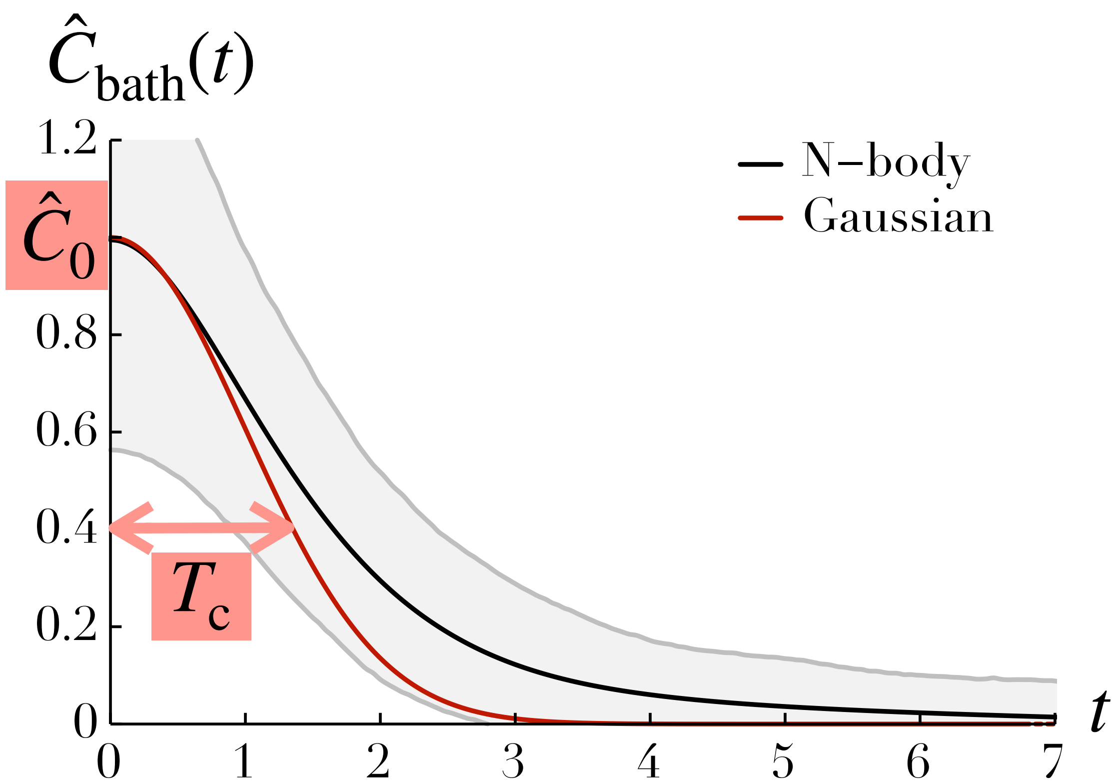


$$\hat{C}_{\mathrm{bath}} = \left\langle \eta(\hat{\mathbf{L}}, t)\, \eta(\hat{\mathbf{L}}', t') \right\rangle$$

$$\hat{C}_{\mathrm{test}} = \left\langle \hat{\mathbf{L}}_{\mathrm{test}}(t) \cdot \hat{\mathbf{L}}_{\mathrm{test}}(0) \right\rangle$$

Characterising the bath noise $\hat{C}_{\mathrm{bath}} = \left\langle \eta(\hat{\mathbf{L}}, t)\, \eta(\hat{\mathbf{L}}', t') \right\rangle$





+ The **state of the bath** is fully characterised by

$$\varphi_{\mathrm{bath}}(\hat{\mathbf{L}}, t) = \frac{1}{N} \sum_{i=1}^{N} \delta_{\mathrm{D}}(\hat{\mathbf{L}} - \hat{\mathbf{L}}_i(t))$$

+ System's (quadratic) **evolution equation**

$$\frac{\partial \varphi_{\mathrm{bath}}(t)}{\partial t} = Q\, \varphi_{\mathrm{bath}}(t)\, \varphi_{\mathrm{bath}}(t)$$

+ Good news
  - At t=0, particles are **statistically decorrelated**
  - Very constraining **spherical symmetries**

+ **Initial time statistics**

$$\left\langle \hat{C}_{\mathrm{bath}}(t=0) \right\rangle$$

Initial amplitude

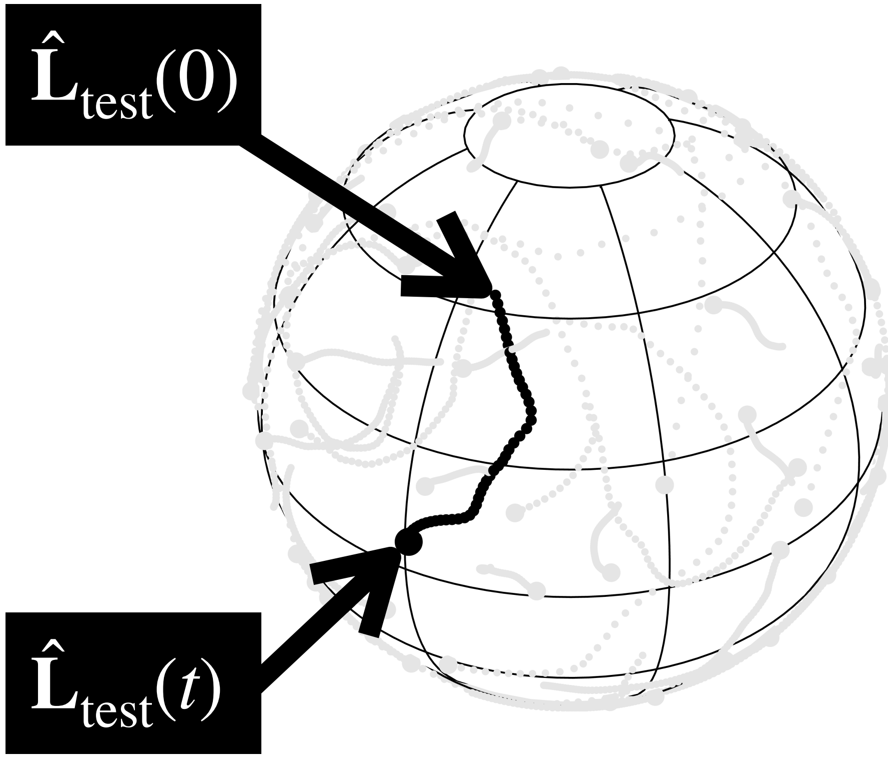
$$\left\langle \frac{\mathrm{d}^2 \hat{C}_{\mathrm{bath}}}{\mathrm{d}t^2} \bigg|_{t=0} \right\rangle$$

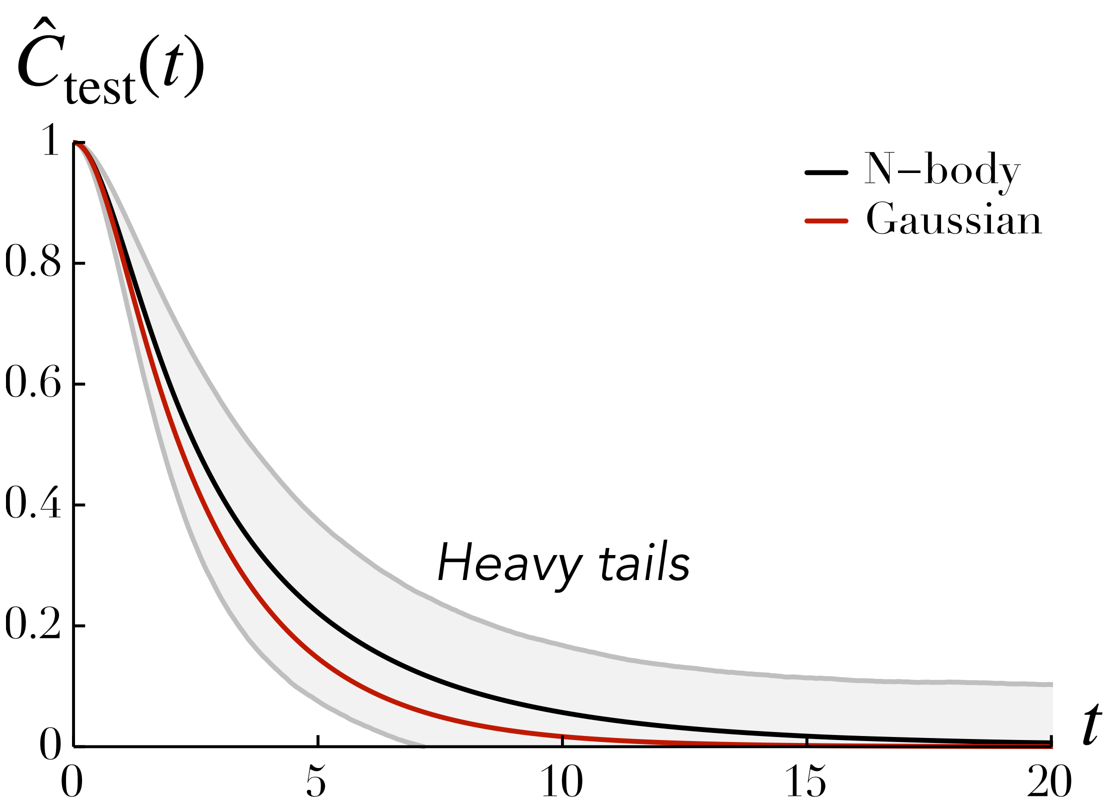
Coherence time

+ (Natural) **Gaussian** Ansatz

$$\hat{C}_{\mathrm{bath}}(t) = \hat{C}_0\, \mathrm{e}^{-(t/T_c)^2}$$

Characterising the random walk $\hat{C}_{\rm test} = \left\langle \hat{\mathbf{L}}_{\rm test}(t) \cdot \hat{\mathbf{L}}_{\rm test}(0) \right\rangle$





+ Location of the **test particle** characterised by

$$\varphi_{\rm test}(\hat{\mathbf{L}}, t) = \delta_{\rm D}(\hat{\mathbf{L}} - \hat{\mathbf{L}}_{\rm test}(t))$$

+ (Linear) **time-dependent** evolution equation

$$\frac{\partial \varphi_{\rm test}(t)}{\partial t} = \eta_{\rm bath}(t)\, \varphi_{\rm test}(t)$$

+ Good news
  - Noise is treated as **external**
  - Very constraining **spherical symmetry**

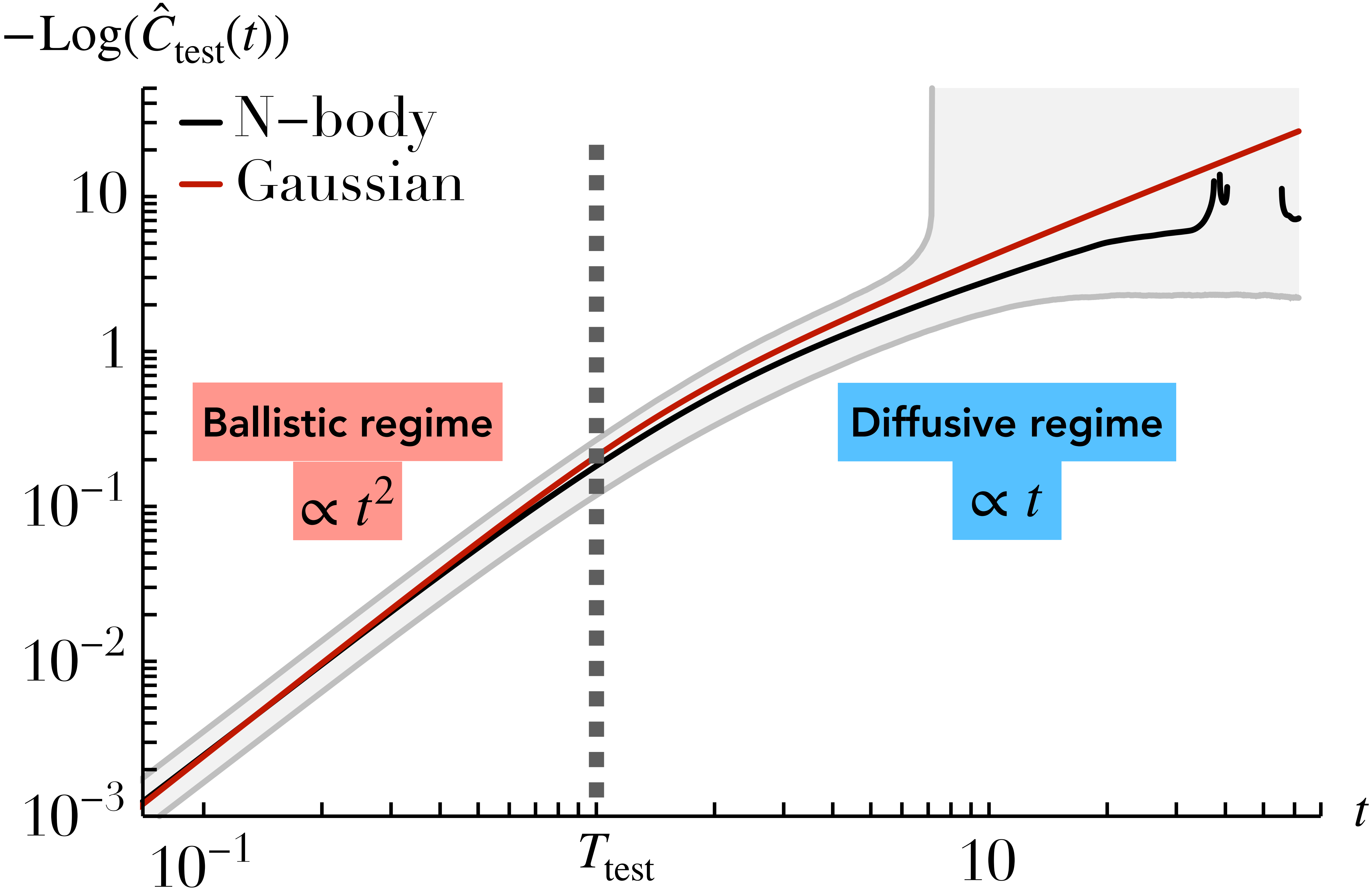
+ Motion solved using **Magnus series**

$$\varphi_{\rm test}(t) = \mathrm{e}^{\Omega(t)}\, \varphi_{\rm test}(0) \quad \text{with} \quad \Omega(t) = \int_0^t \mathrm{d}t'\, \eta_{\rm bath}(t')$$

+ Explicit expression of the **time correlation**

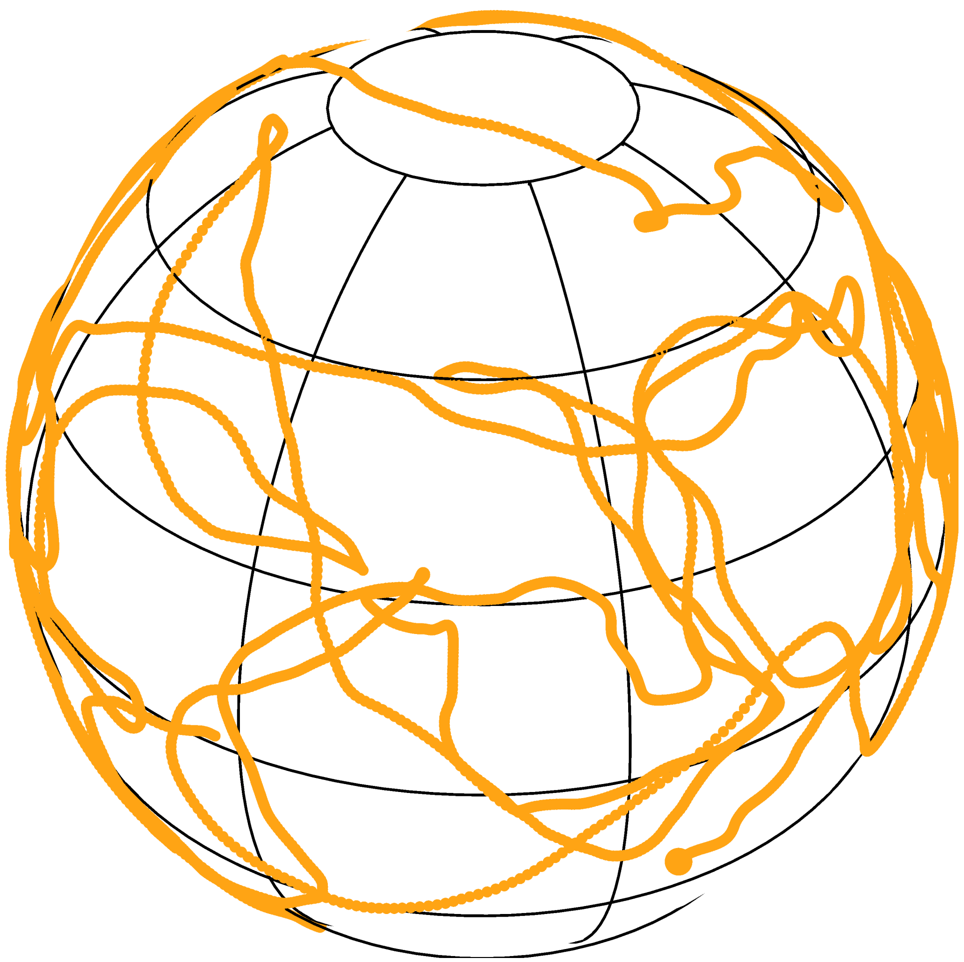
$$\hat{C}_{\rm test}(t) = \exp\left[- \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2\, \hat{C}_{\rm bath}(t_1 - t_2) \right]$$

Characterising the random walk $\hat{C}_{\mathrm{test}} = \left\langle \hat{\mathbf{L}}_{\mathrm{test}}(t) \cdot \hat{\mathbf{L}}_{\mathrm{test}}(0) \right\rangle$



Mimicking the random walk

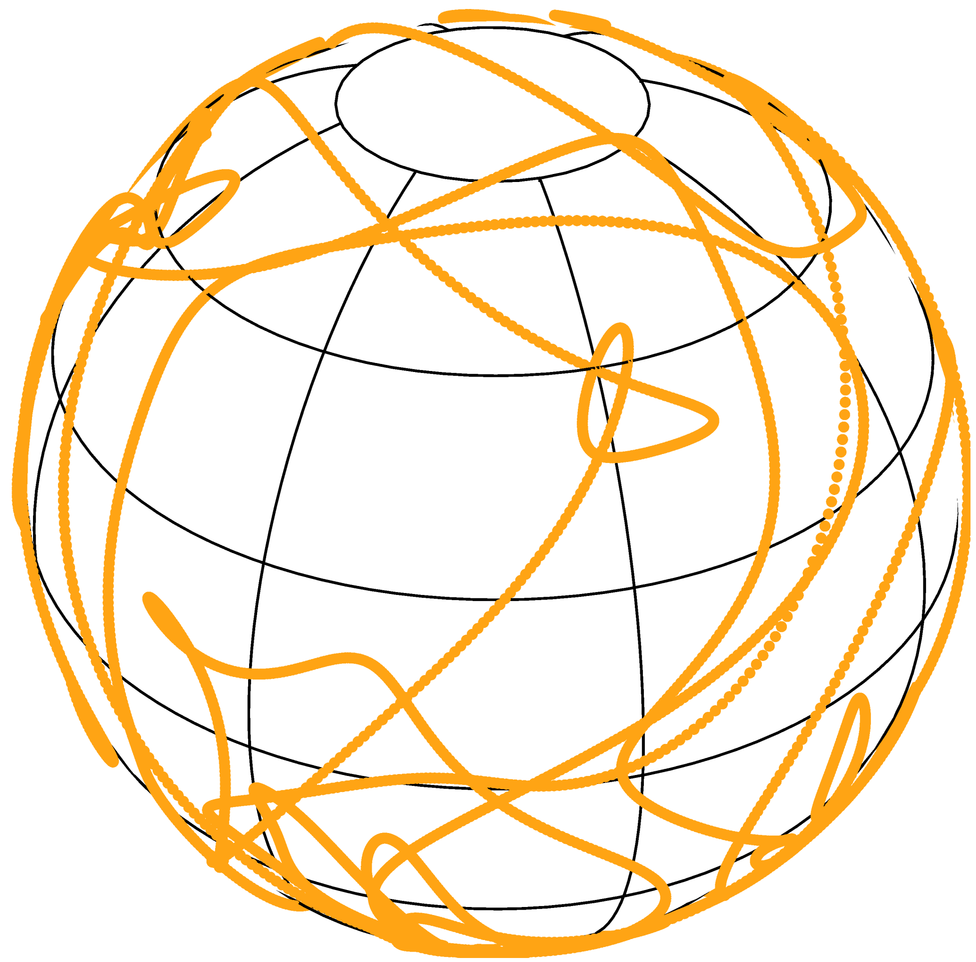
N-body simulations



$$H = \sum_{i<j}^{N} A_{ij}\, U(\hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j)$$

Full N-body problem of $\mathcal{O}(N^2)$ complexity

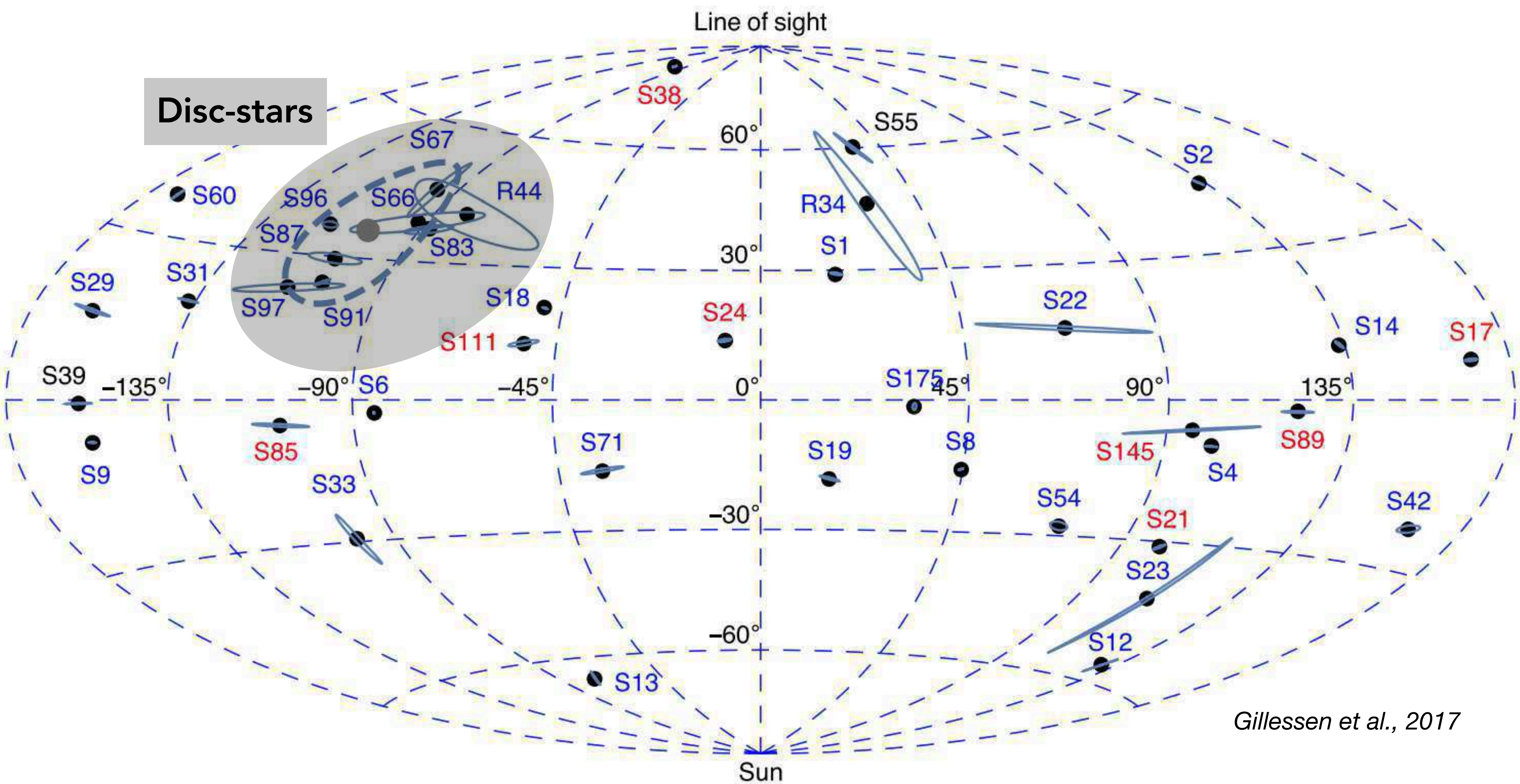
Effective model



$$\frac{\mathrm{d}\hat{\mathbf{L}}_{\mathrm{test}}}{\mathrm{d}t} = \Gamma_{\mathrm{test}}\, \boldsymbol{\eta}(t) \times \hat{\mathbf{L}}_{\mathrm{test}}(t)$$

with
$$\begin{cases} \Gamma_{\mathrm{test}} & \textit{Amplitude} \\ \langle \boldsymbol{\eta}(t)\, \boldsymbol{\eta}(t') \rangle = \mathrm{e}^{-((t-t')/T_{\mathrm{test}})^2} & \textit{Coherence time} \end{cases}$$

Vector Resonant Relaxation can affect the disc-stars

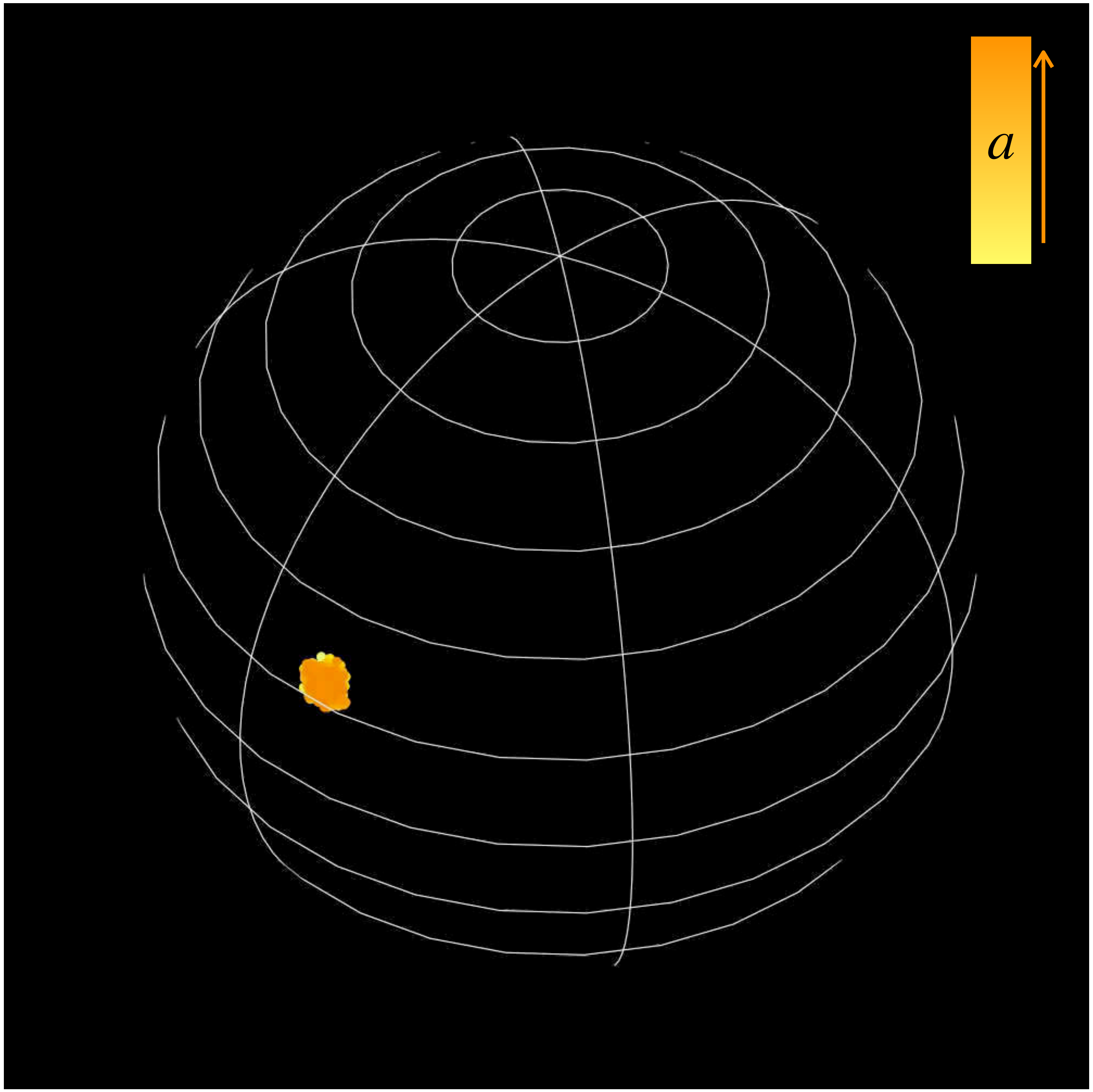


Gillessen et al., 2017

How long should these stars stay "neighbors"?

Are they young enough?

Vector Resonant Relaxation can randomize disc stars



+ How "**neighbors**" get separated

$$\frac{\mathrm{d}\hat{\mathbf{L}}_i}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}_i, t)$$

+ Evolution sourced by a **shared**, **spatially-extended** and **time-correlated** noise

$$\left\langle \eta(a_i, \hat{\mathbf{L}}_i, t)\, \eta(a_j, \hat{\mathbf{L}}_j, t') \right\rangle = C(a_i, a_j, \hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j, t - t')$$

+ Two joint sources of **separation**

- **Parametric** separation

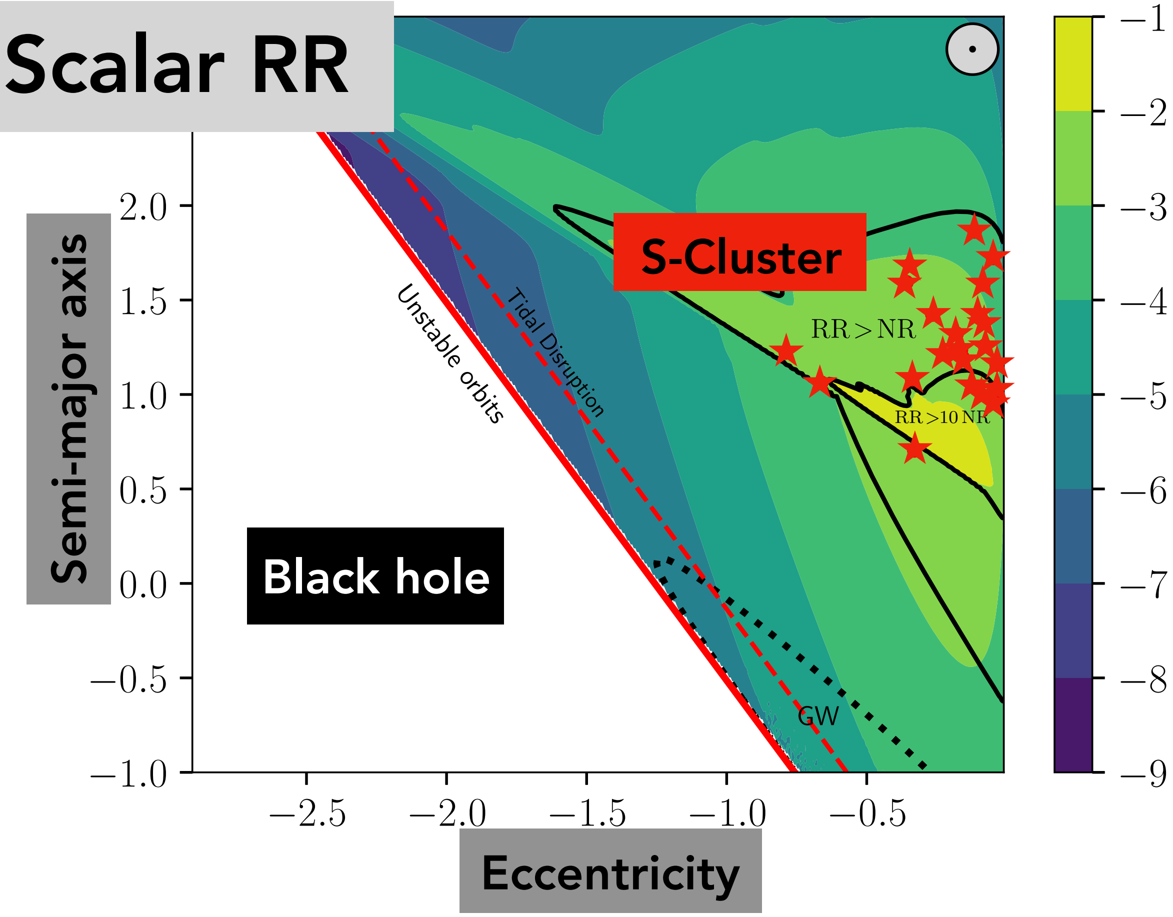
$$a_i \neq a_j$$

- **Angular** separation

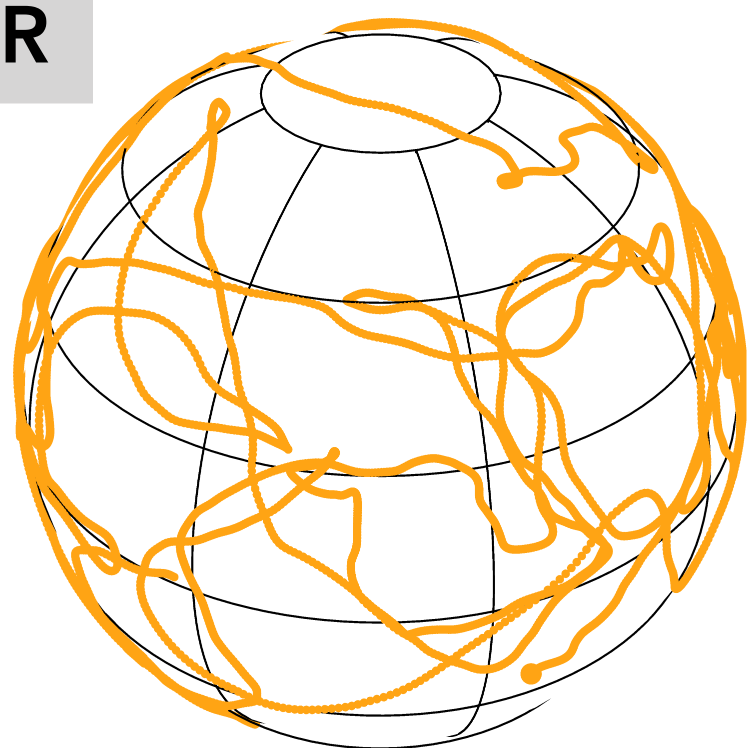
$$\hat{\mathbf{L}}_i \neq \hat{\mathbf{L}}_j$$

Resonant Relaxation in Galactic Nuclei

Scalar RR



Vector RR



Context

How to aliment a **supermassive black hole**?

Stellar diffusion in galactic centers

+ ***Origin and structure*** *of SgrA**
+ *Relaxation in* ***eccentricity***, ***orientation***

Sources of **gravitational waves**

+ *BHs-binary mergers*
+ *TDEs, EMRIs*

Novelties

+ **New kinetic equations** written and implemented
+ Confronted to **astrophysical observations**
+ Theory in a regime inaccessible to simulations

Next steps

Galactic centers

Stellar capture rates

Gravitational waves sources

Galactic discs

Galactic Archeology

Radial Migration/Thickening

Globular clusters

Effect of velocity anisotropy

Effect of rotation

Dark Matter halo

Cusp-Core transition

Environmental forcing