# Time-delay Interferometry with unsynchronized clocks as part of an independent L0-L1 pipeline for LISA

Journée LISA à l'Observatoire, 10.12.2021

Olaf Hartwig for the theory and metrology LISA group,
SYRTE, Paris Observatory
With contributions from J.-B. Bayle (JPL) & M. Staab (AEI)

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Auxiliary data
S/C orbitographie

Raw data not usable for
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## L1 Data

TDI time series expressed in BCRS
S/C ephemerides in BCRS
…

Input to astrophysical data analysis

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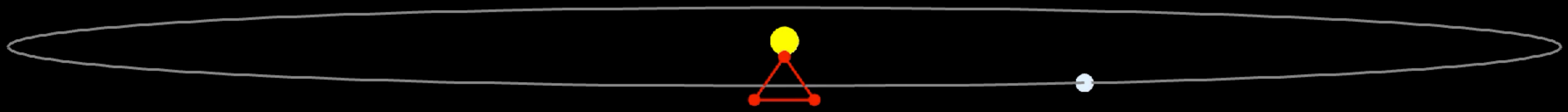
  - Increased robustness of LISA data analysis infrastructure
  - An increase in the number of LISA scientists with L0 to L1 know-how.

# Time-delay Interferometry

# Why TDI?



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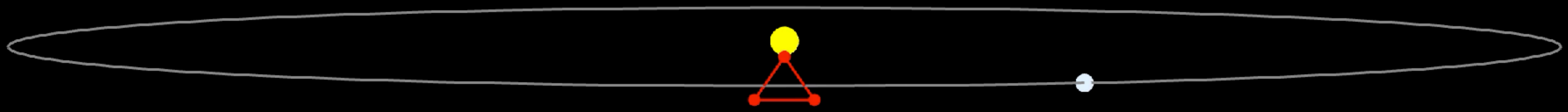

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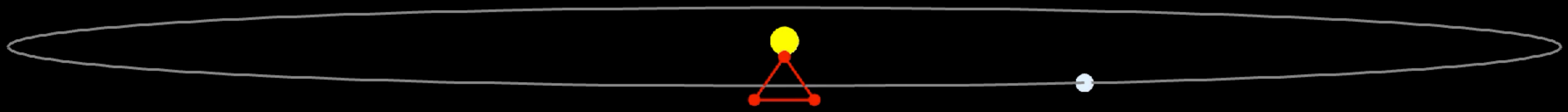

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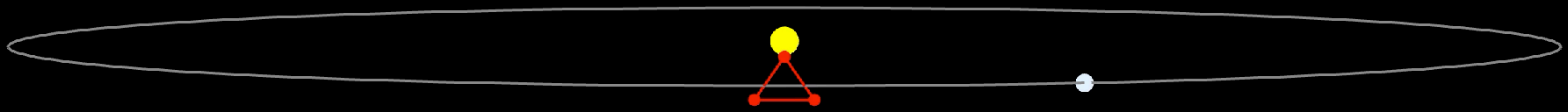

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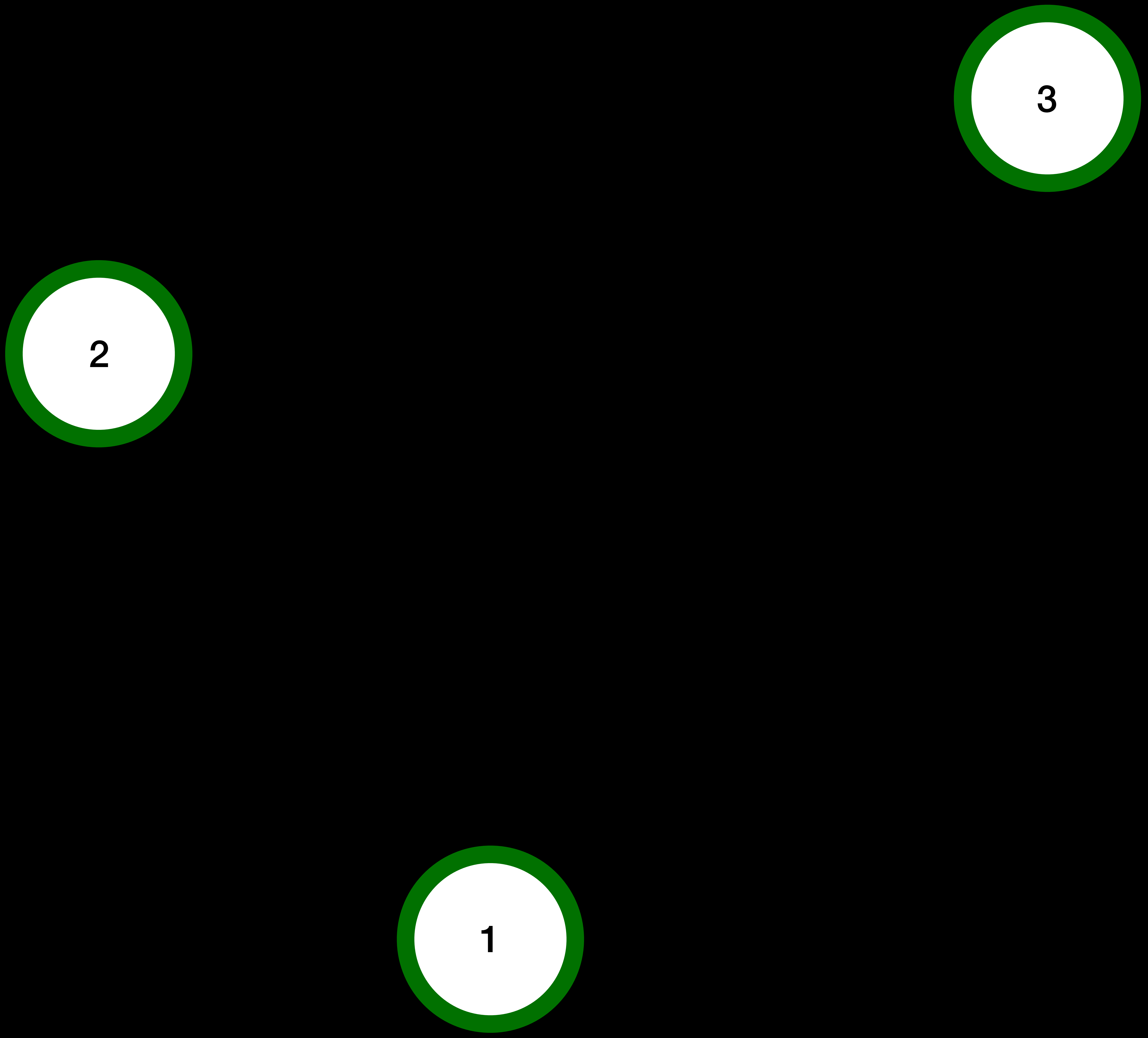
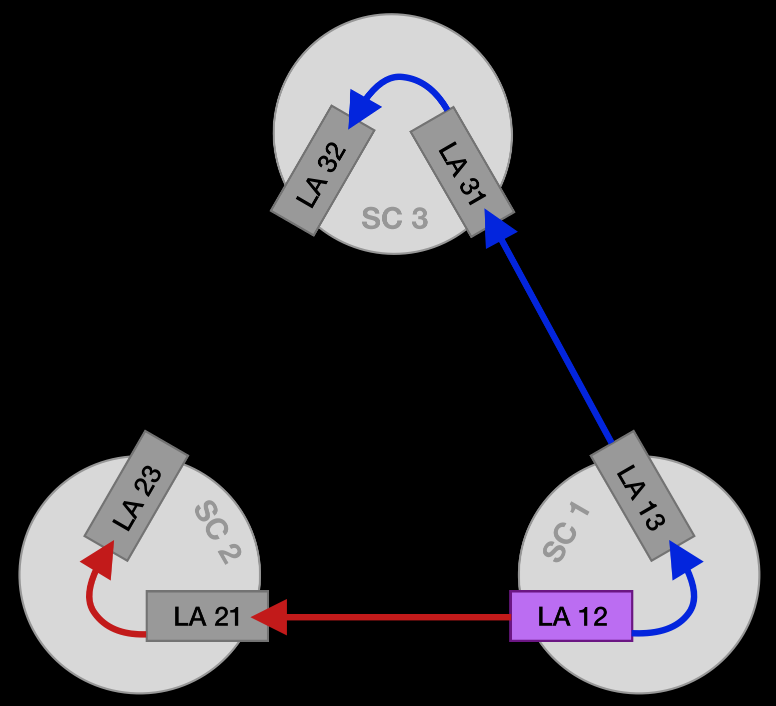
  - Armlength mismatches ±1% over a year → laser frequency noise does not cancel
  - Relative velocities ≈ 10 m/s → Doppler shifts ≈ 10 MHz
    → MHz beatnotes, clock noise couples

# Laser noise cancellation in LISA



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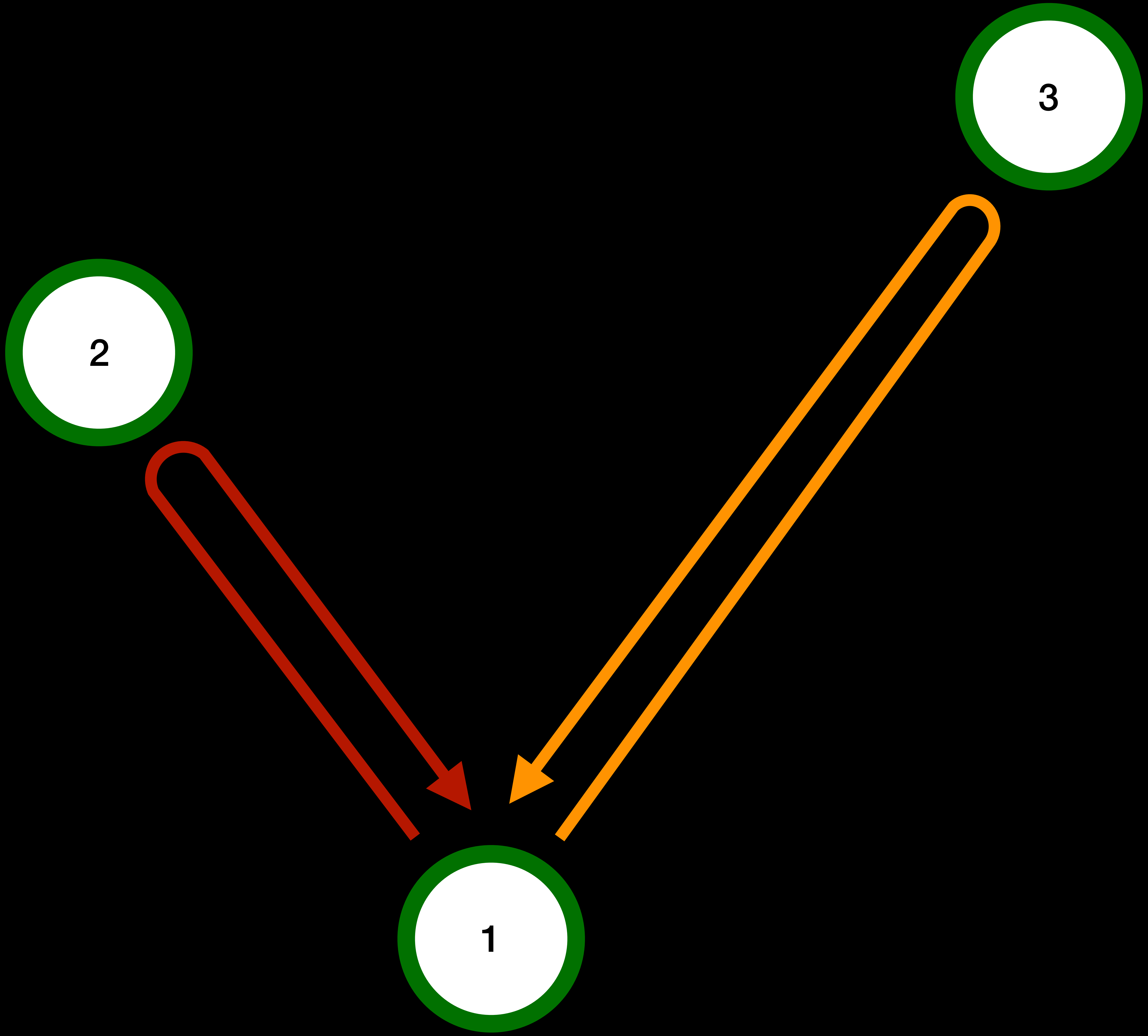


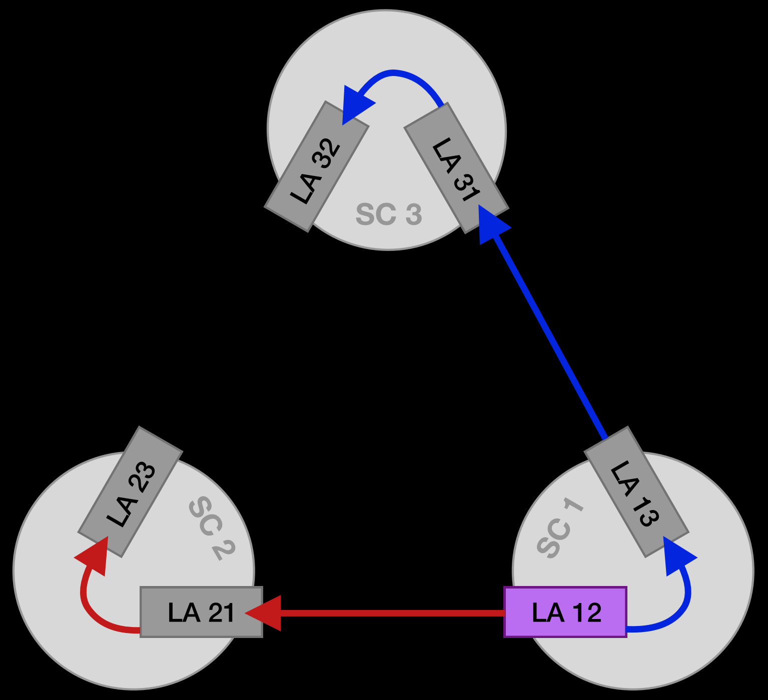

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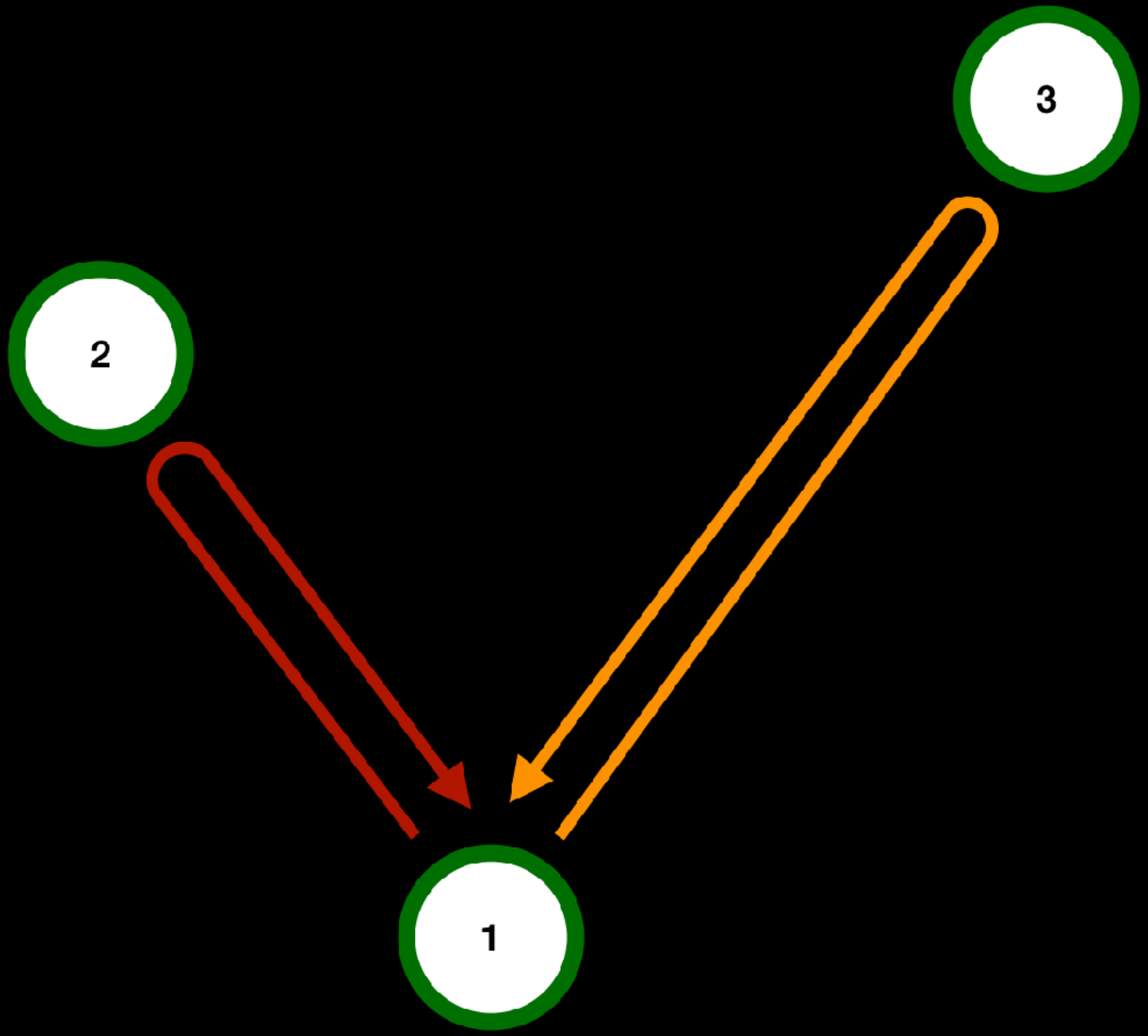

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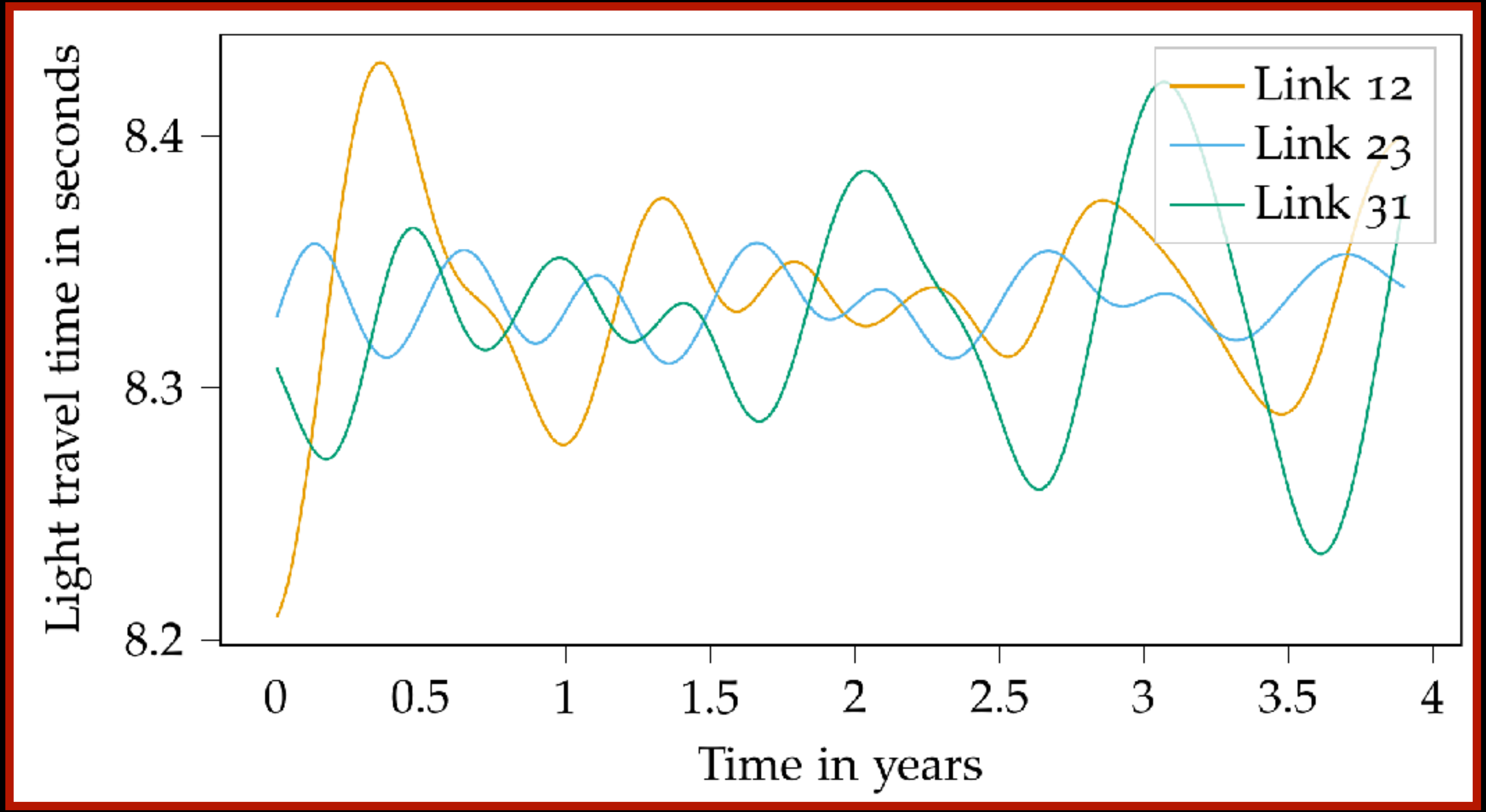
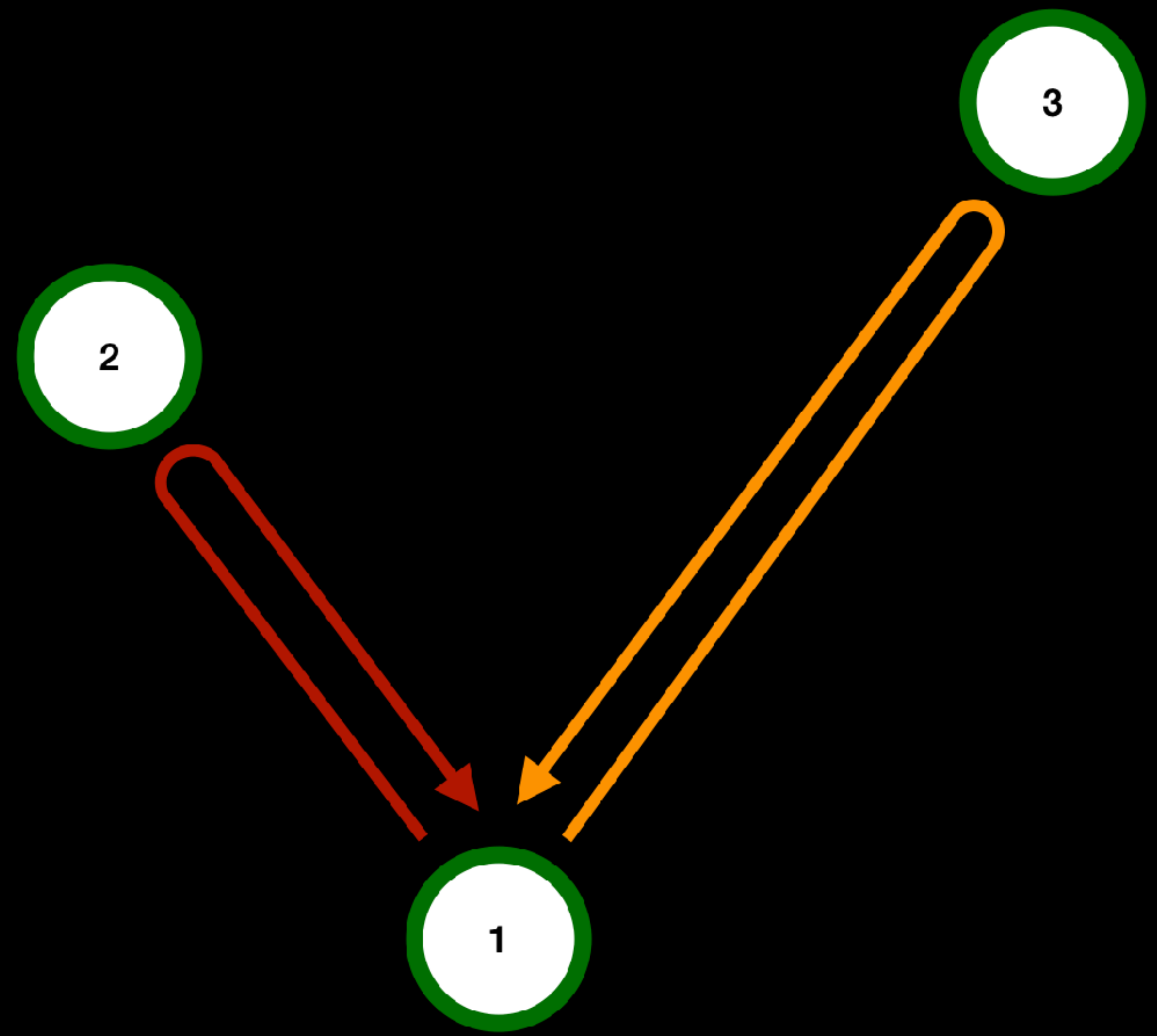
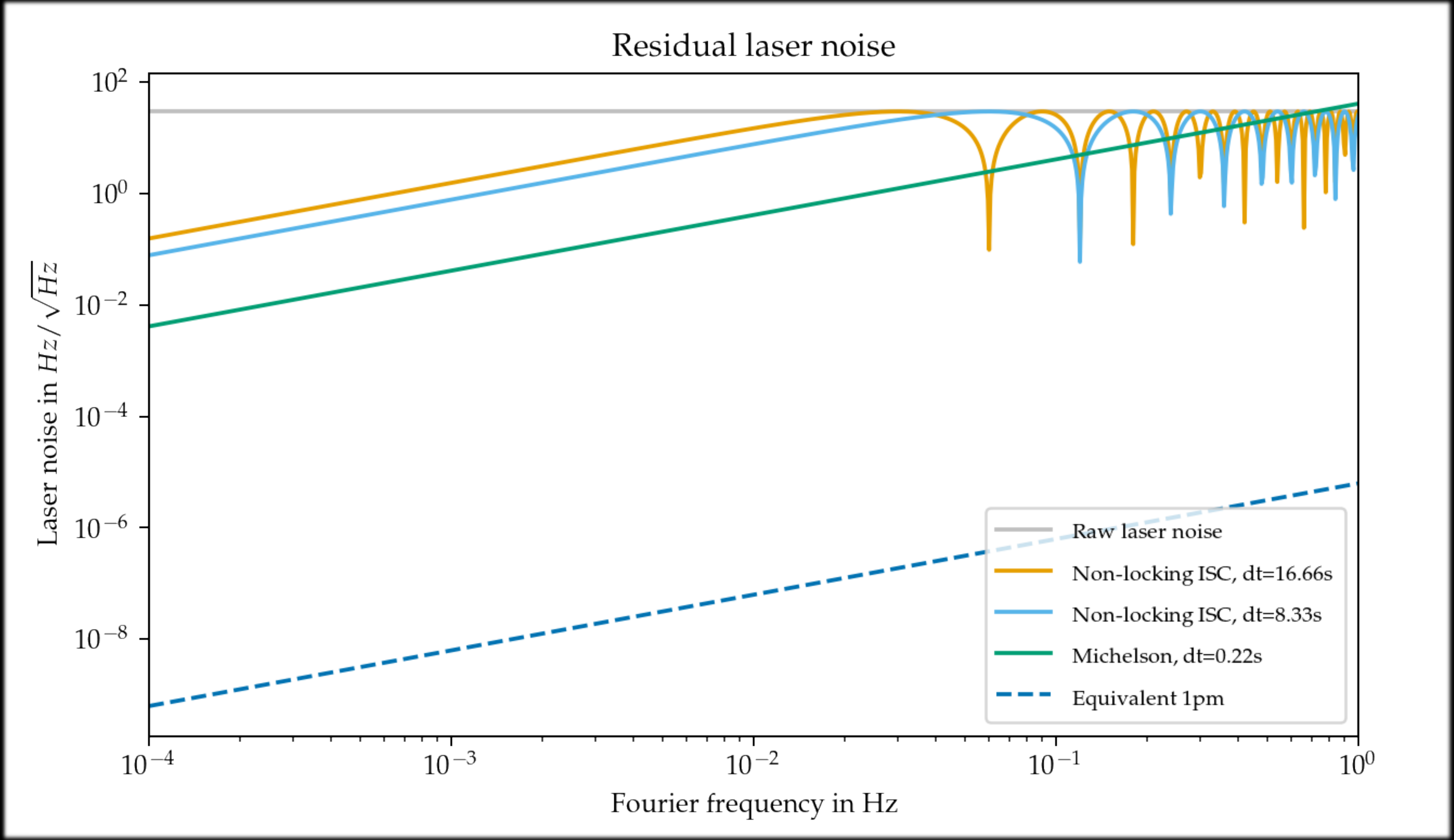


# Residual laser noise in LISA



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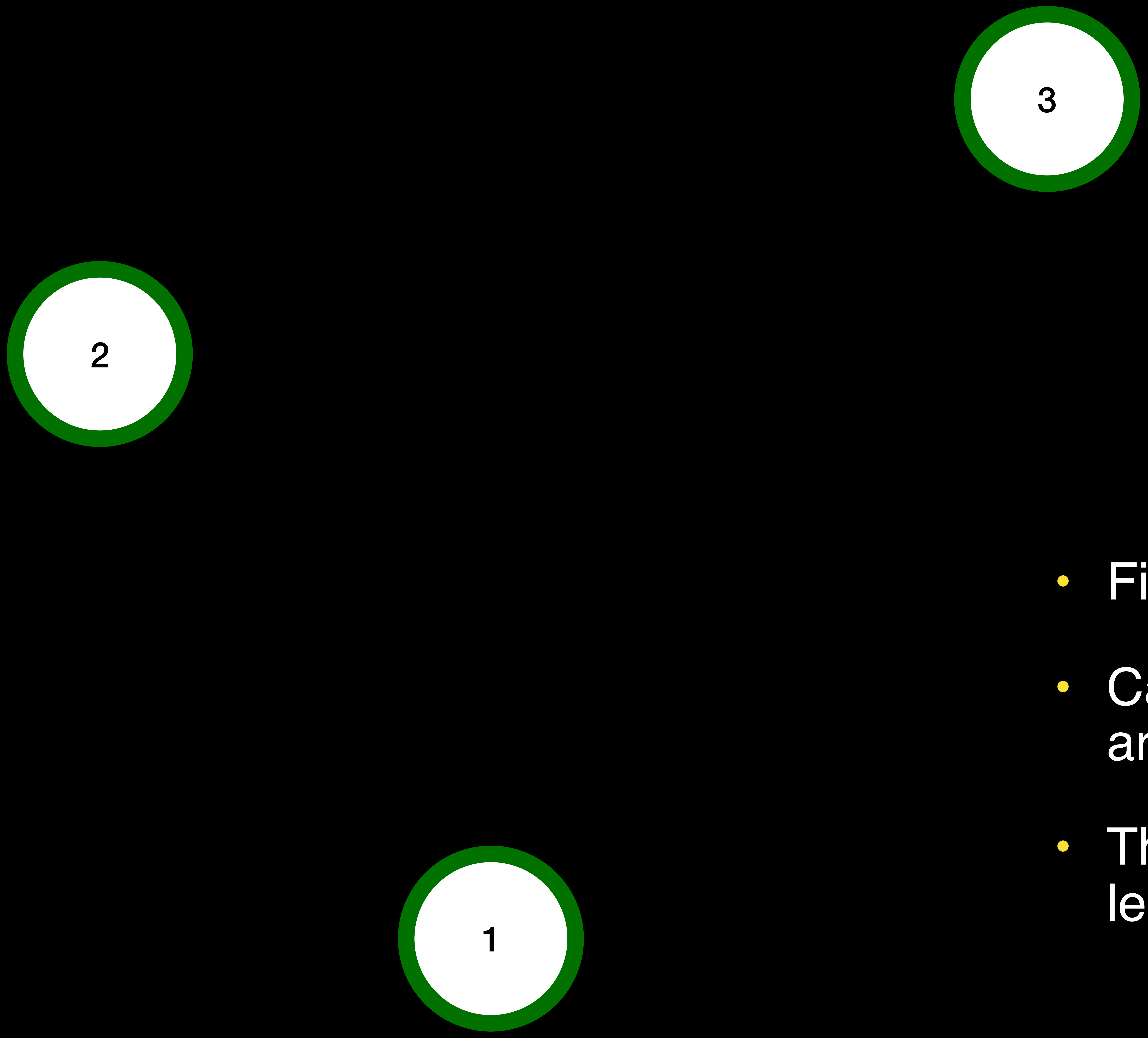


- First proposed in [Tinto99]
- Cancel laser noise by constructing equal arm interferometer in post-processing
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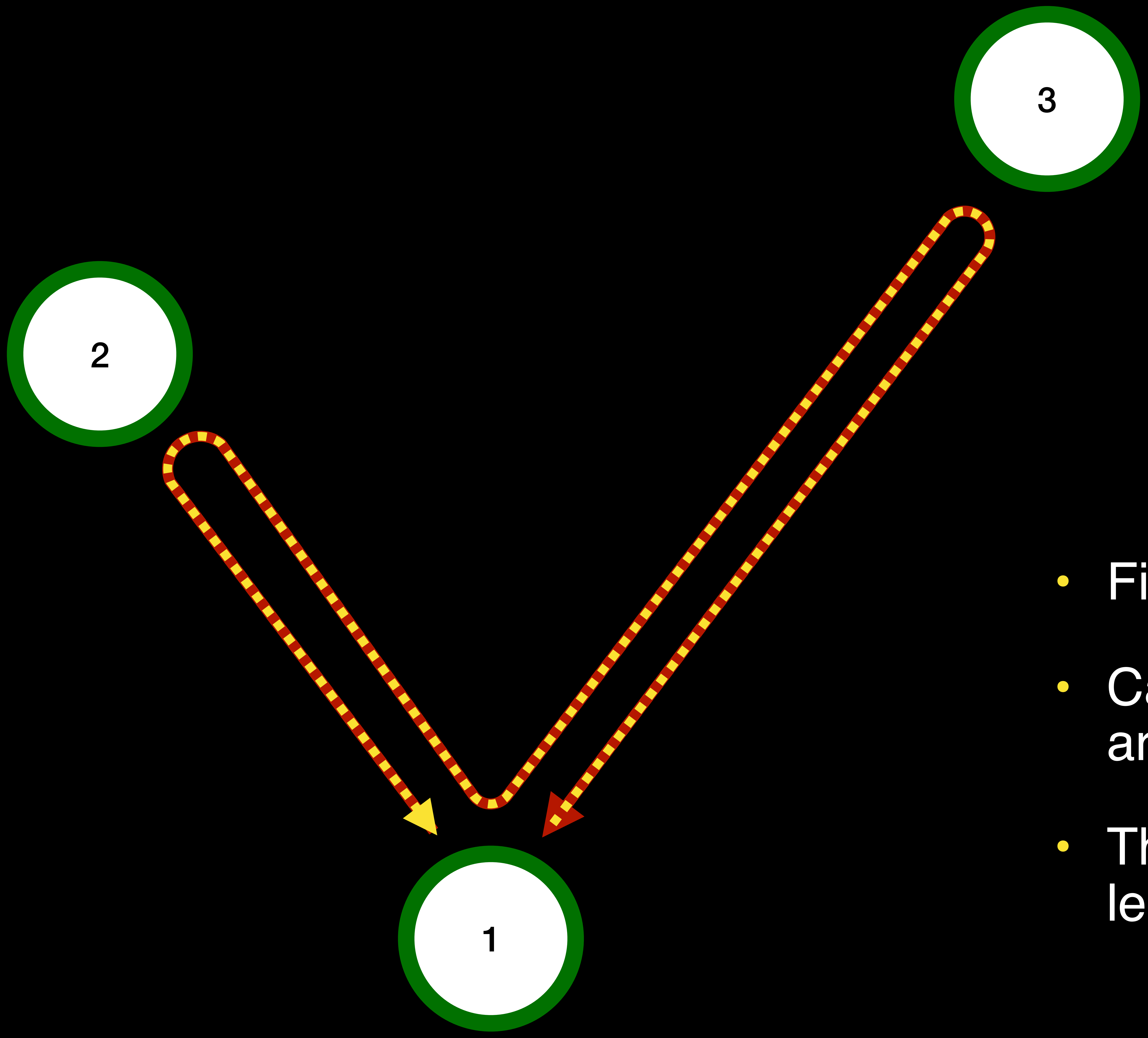


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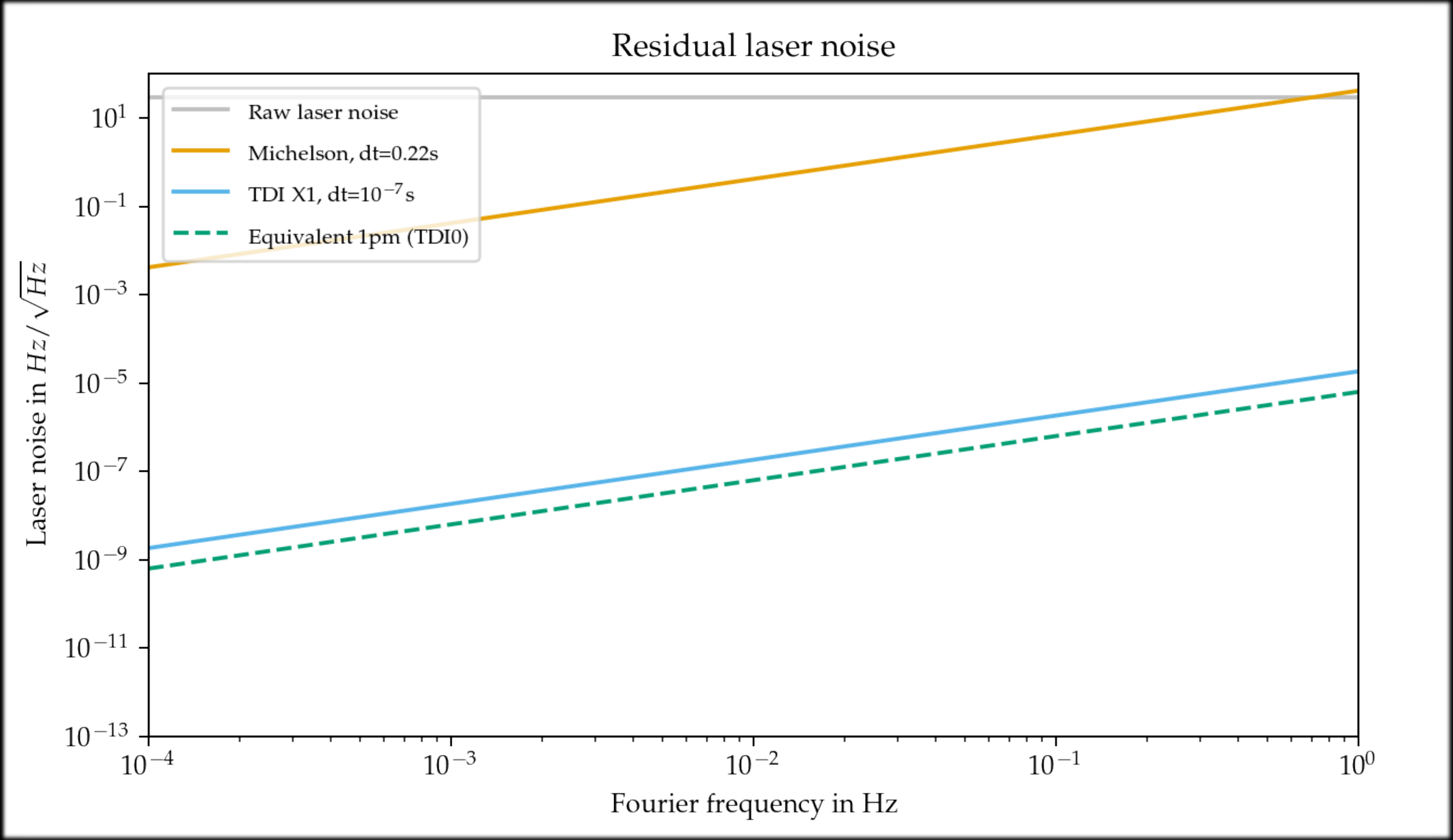

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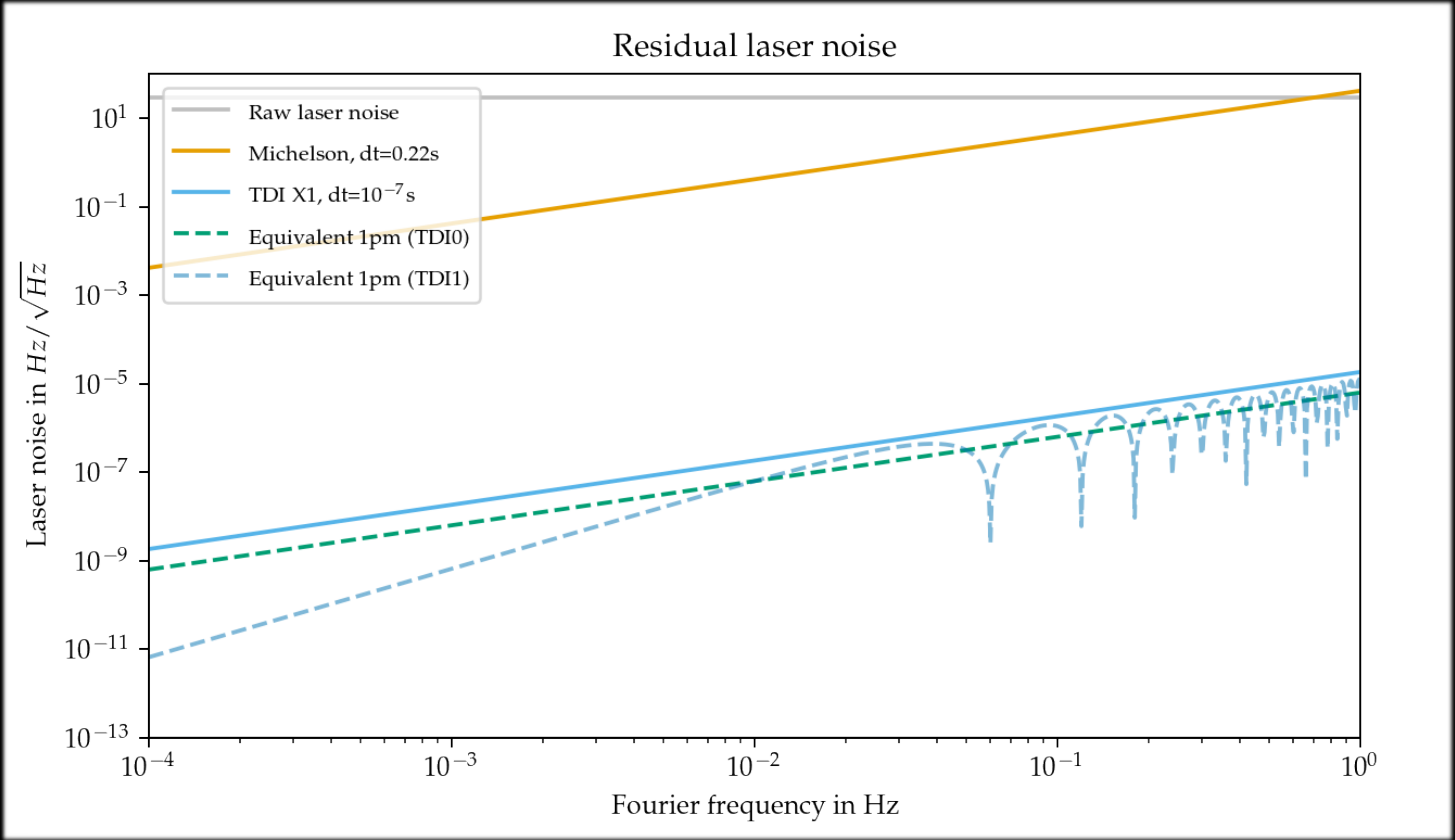

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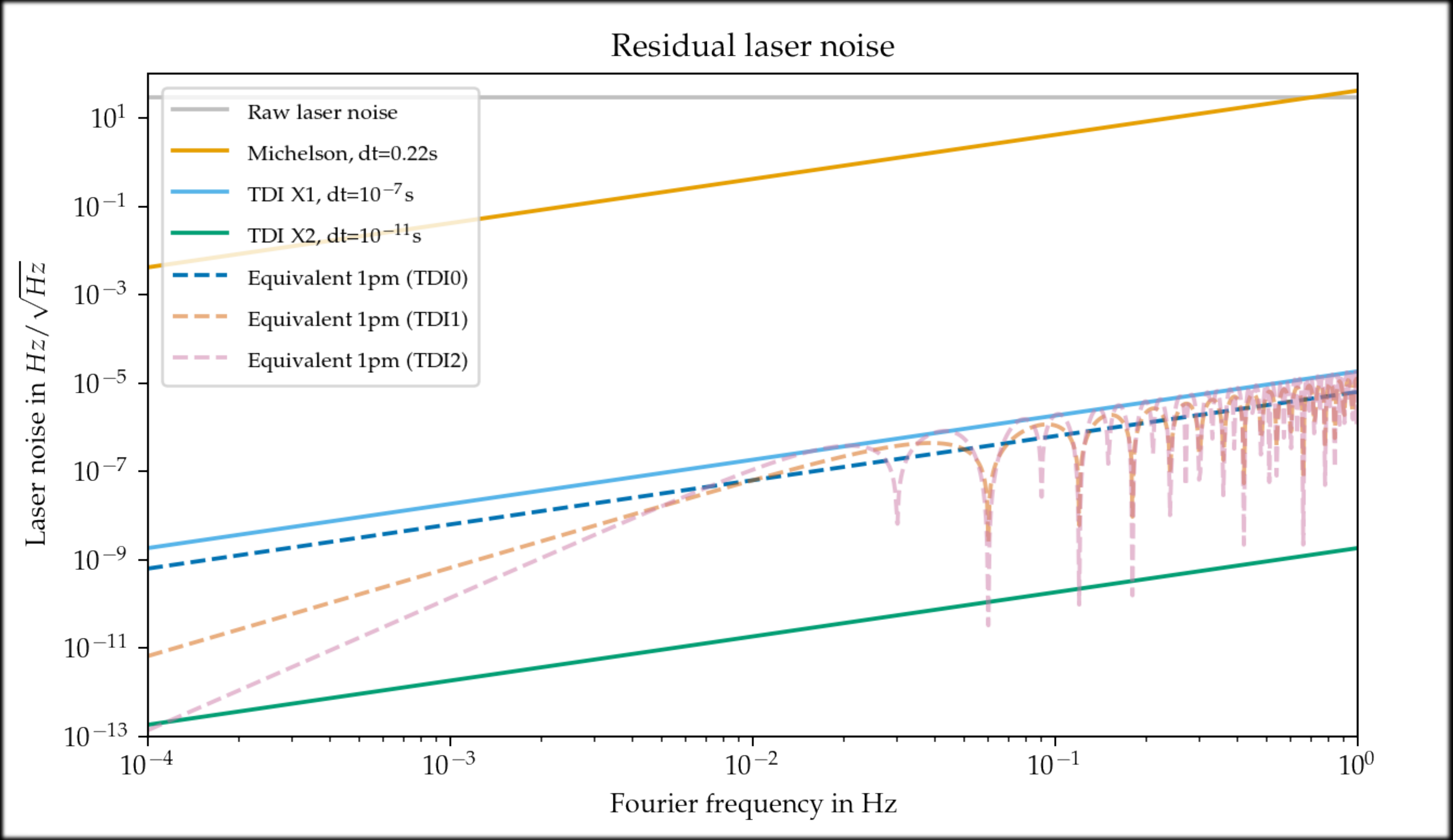
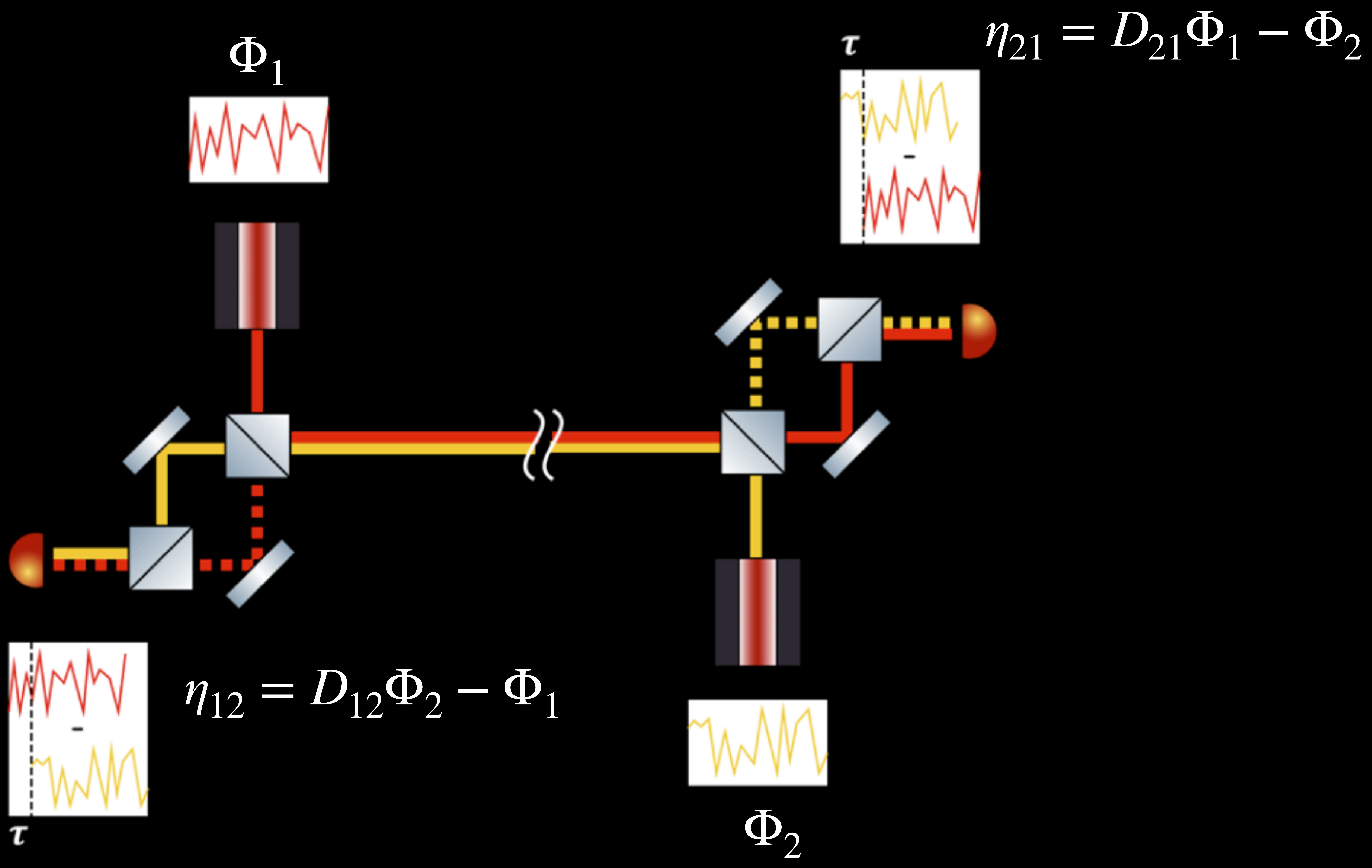


# TDI working principles

# Simplified LISA link



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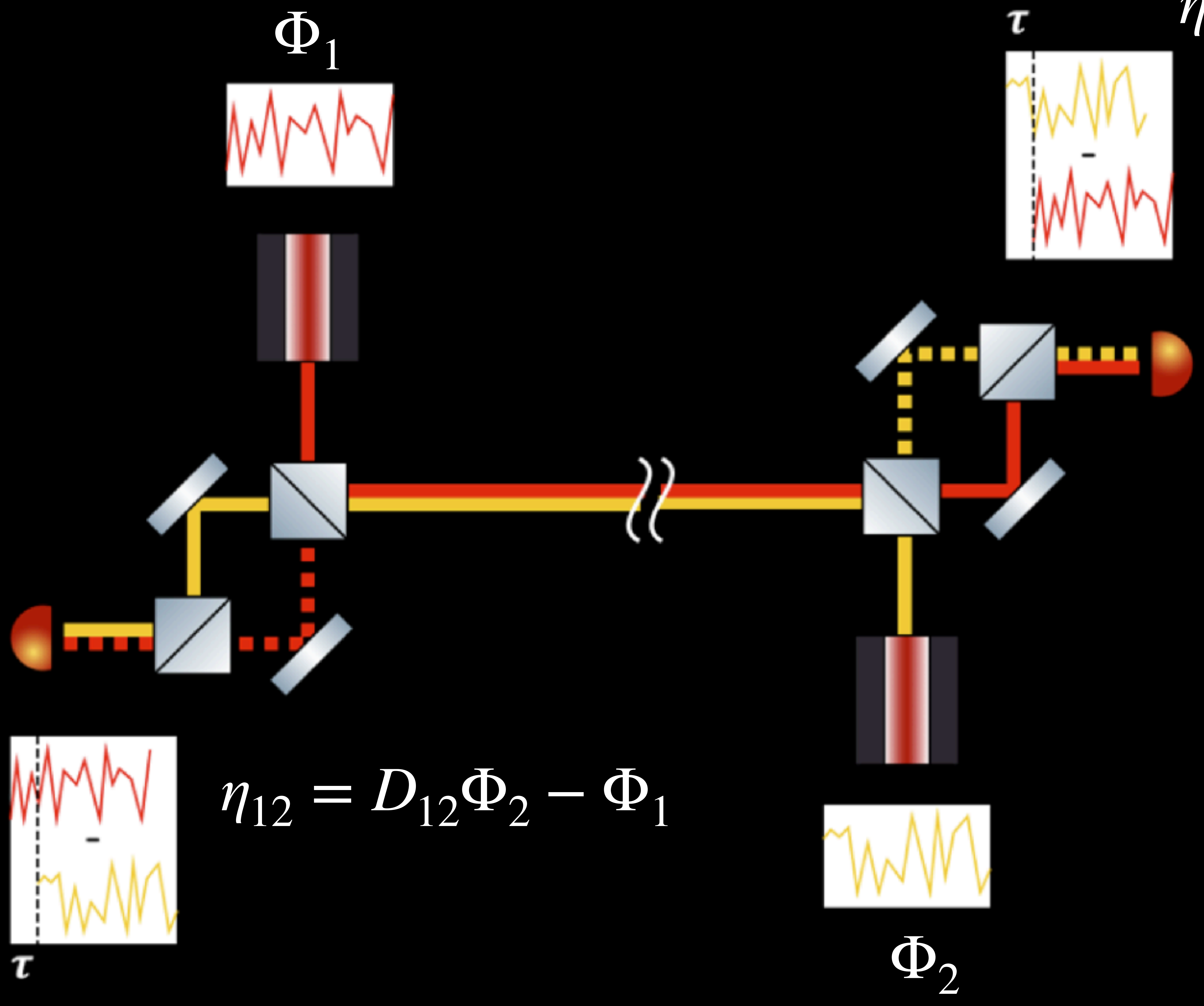
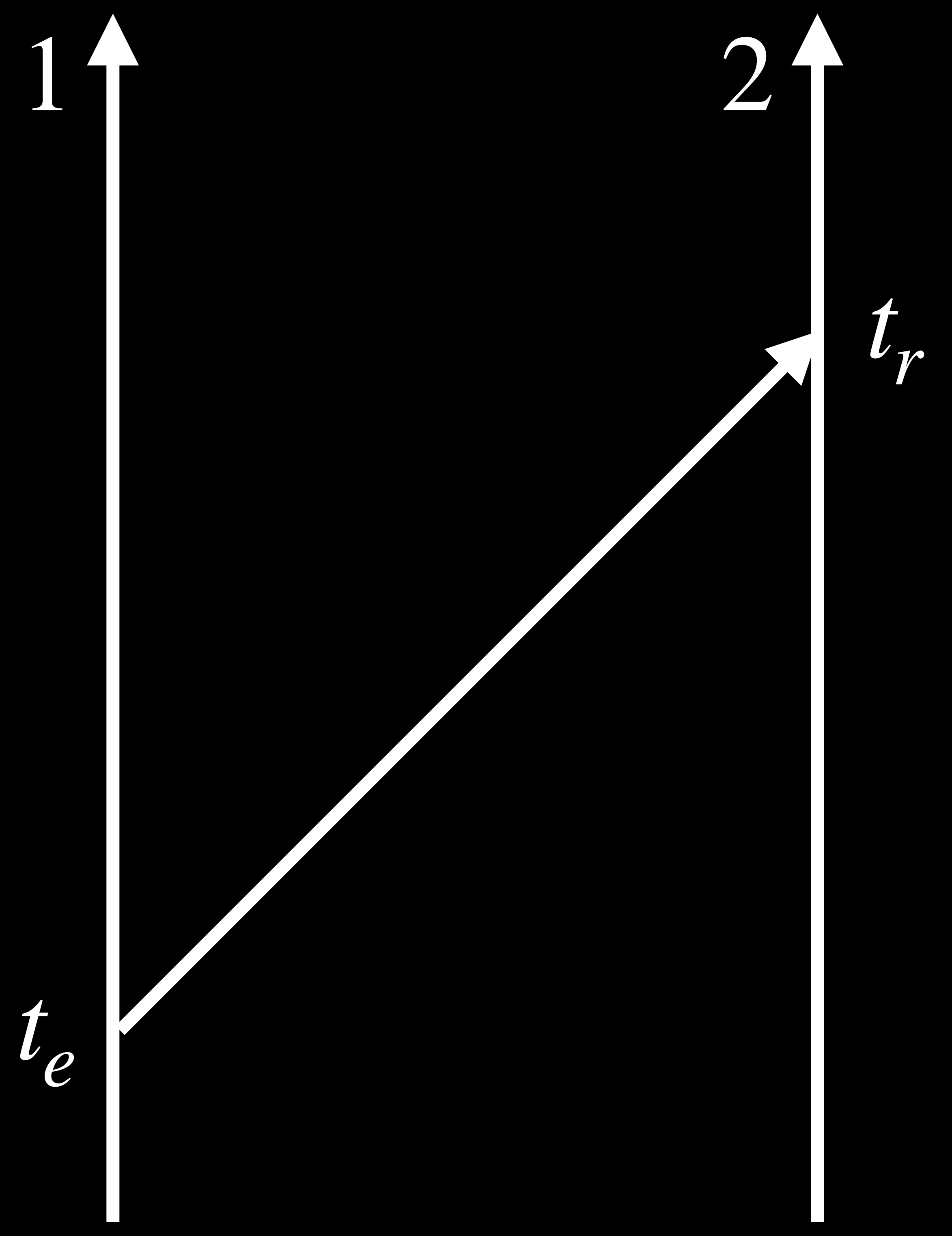


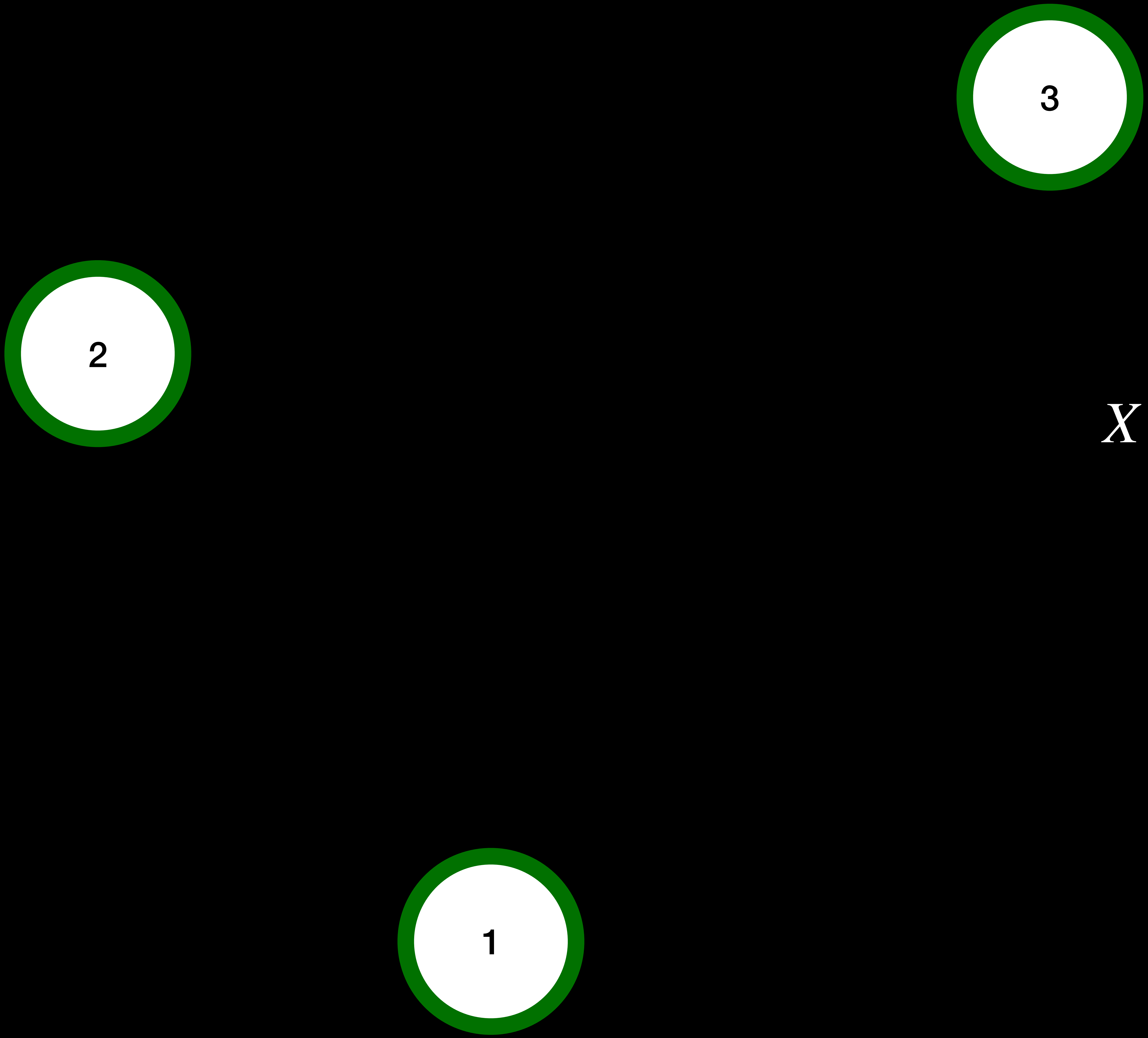

$$\eta_{21} = D_{21}\Phi_1 - \Phi_2$$

$$\eta_{12} = D_{12}\Phi_2 - \Phi_1$$




$$\Phi_1(t_e) \equiv D_{21}\Phi_1(t_r)$$
$$= \Phi_1(t_r - d_{21}(t_r))$$

# Full first generation TDI




$$X =$$

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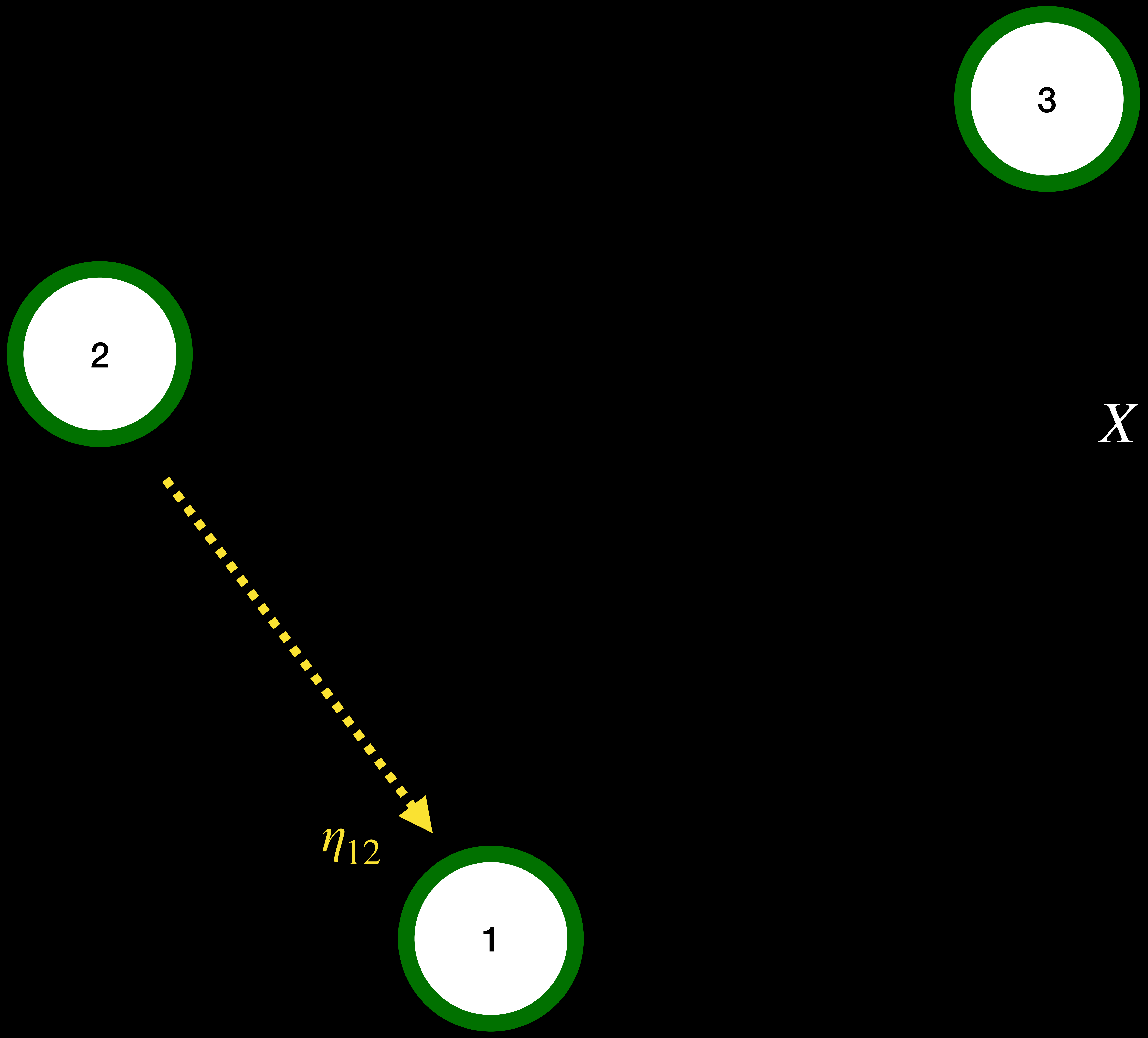


$$X = \eta_{12}$$

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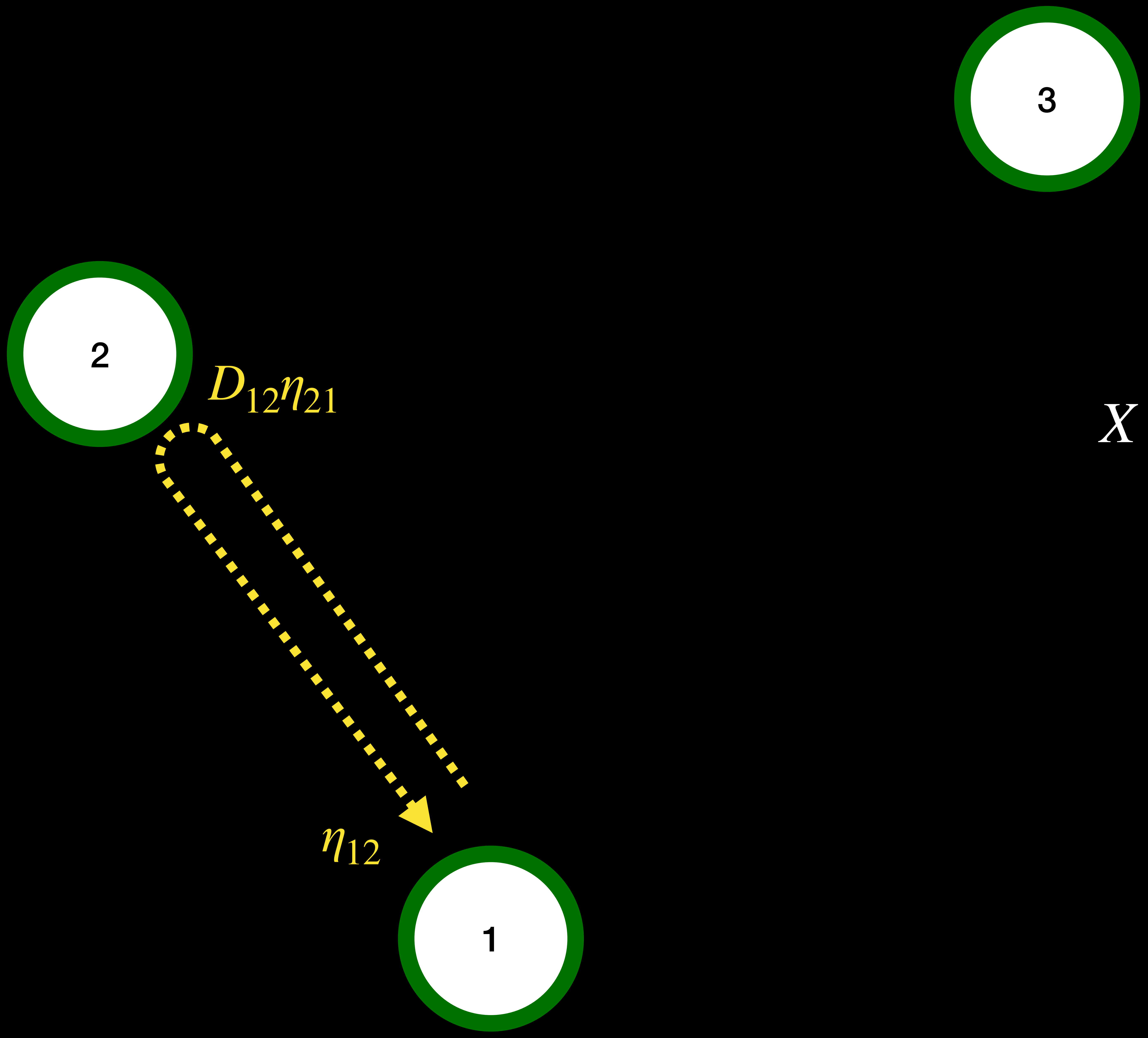


$$X = \eta_{12} + D_{12}\eta_{21}$$

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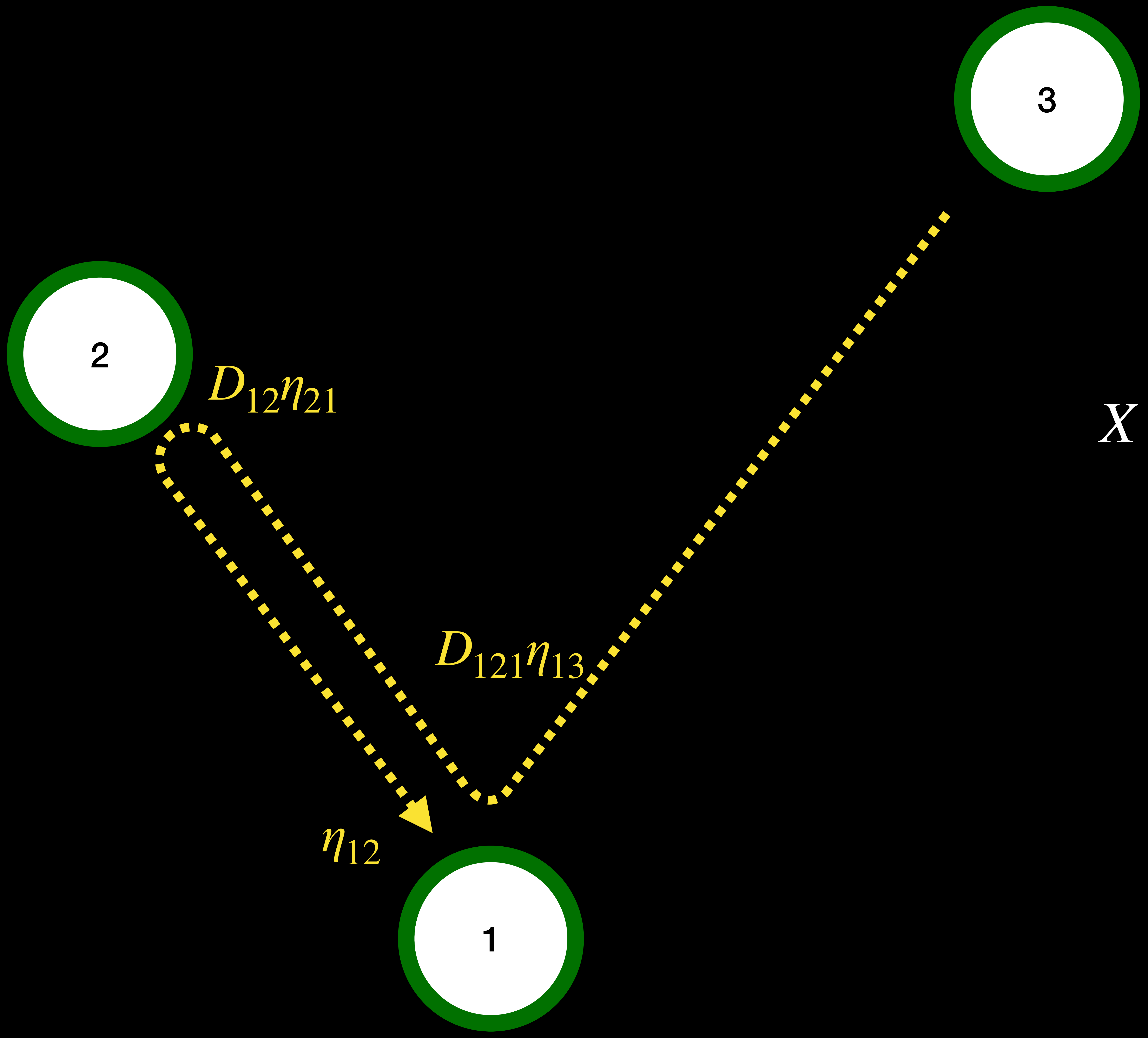


$$X = \eta_{12} + D_{12}\eta_{21} + D_{121}\eta_{13}$$

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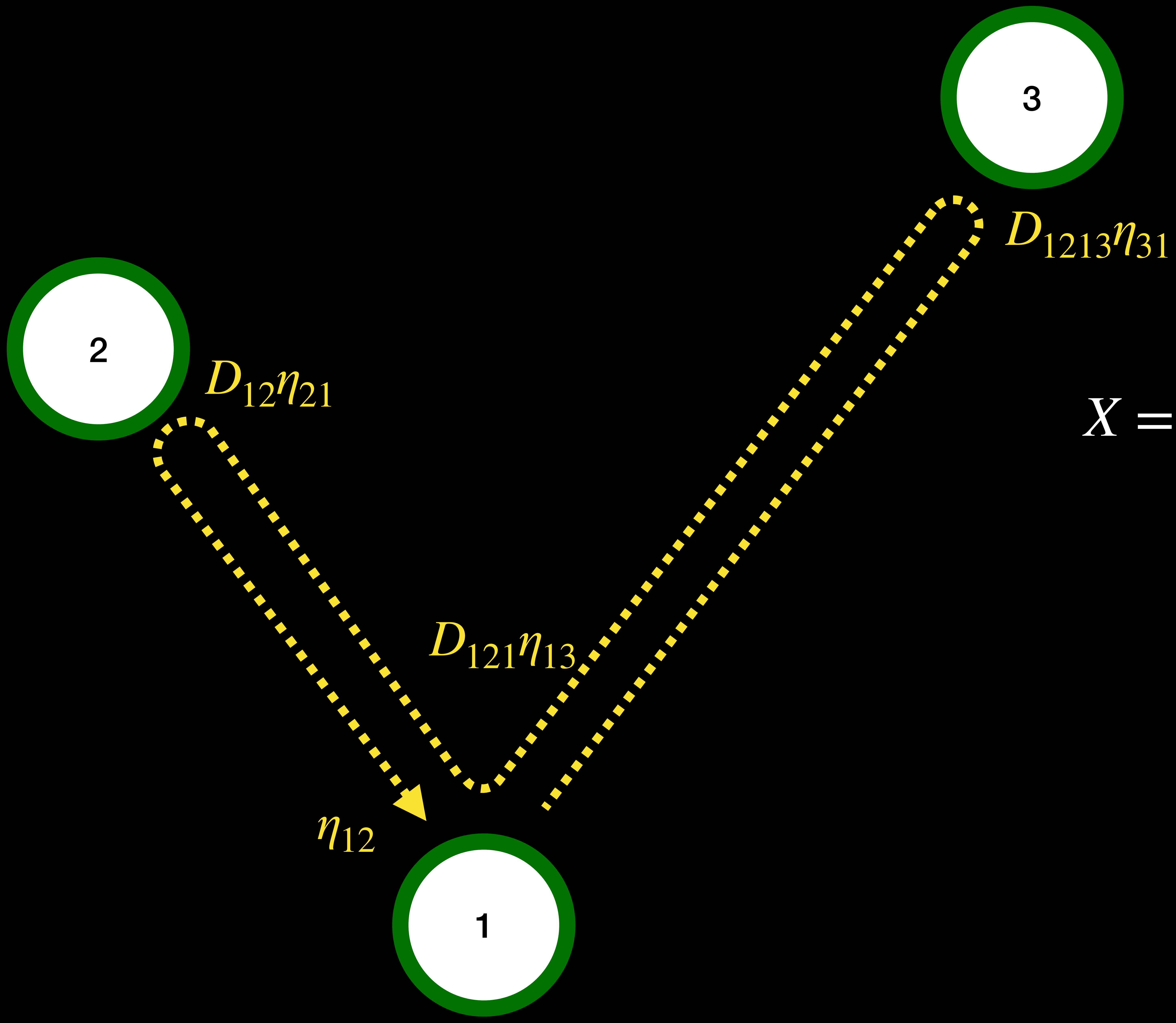


$$X = \eta_{12} + D_{12}\eta_{21} + D_{121}\eta_{13} + D_{1213}\eta_{31}$$

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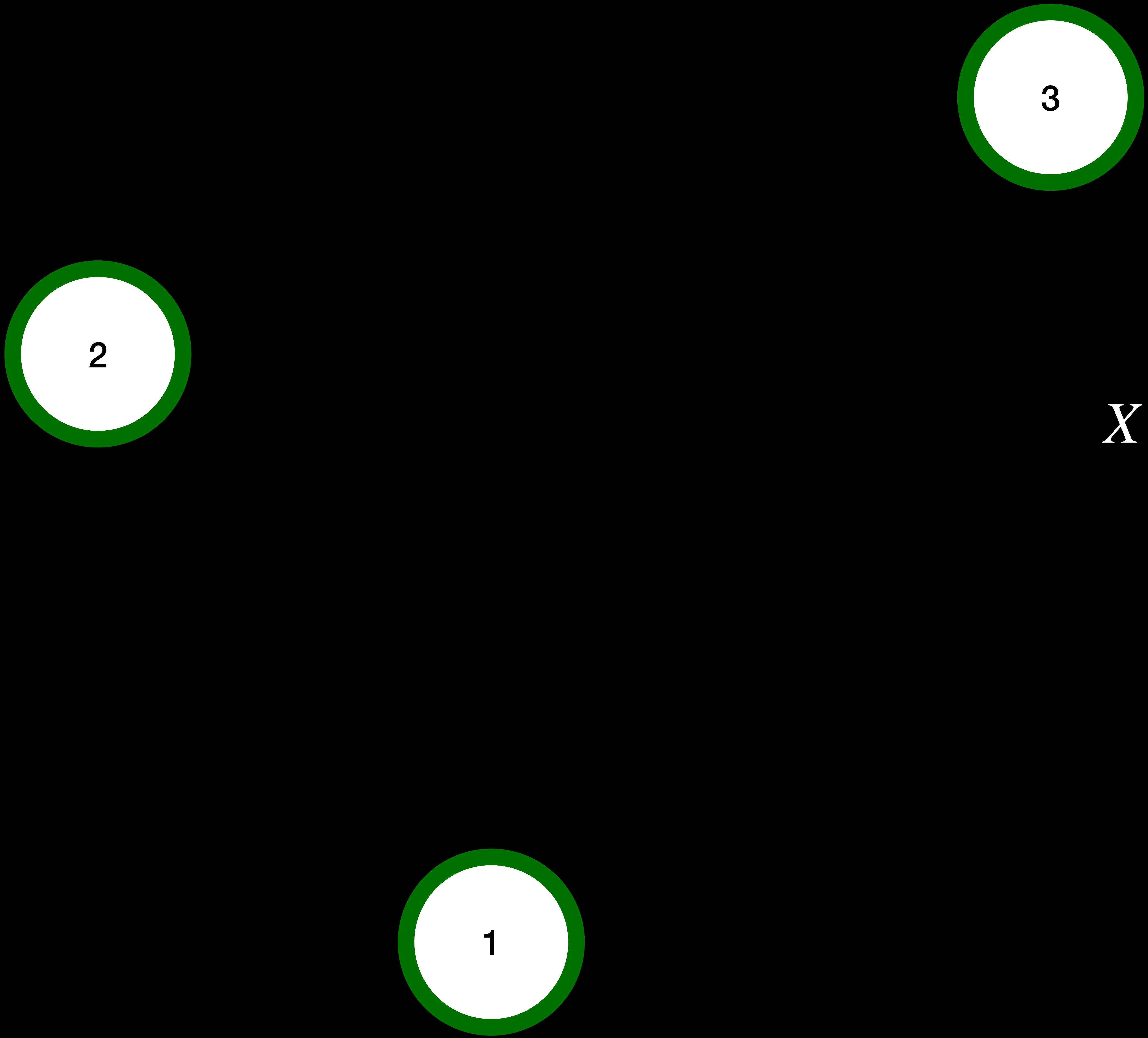


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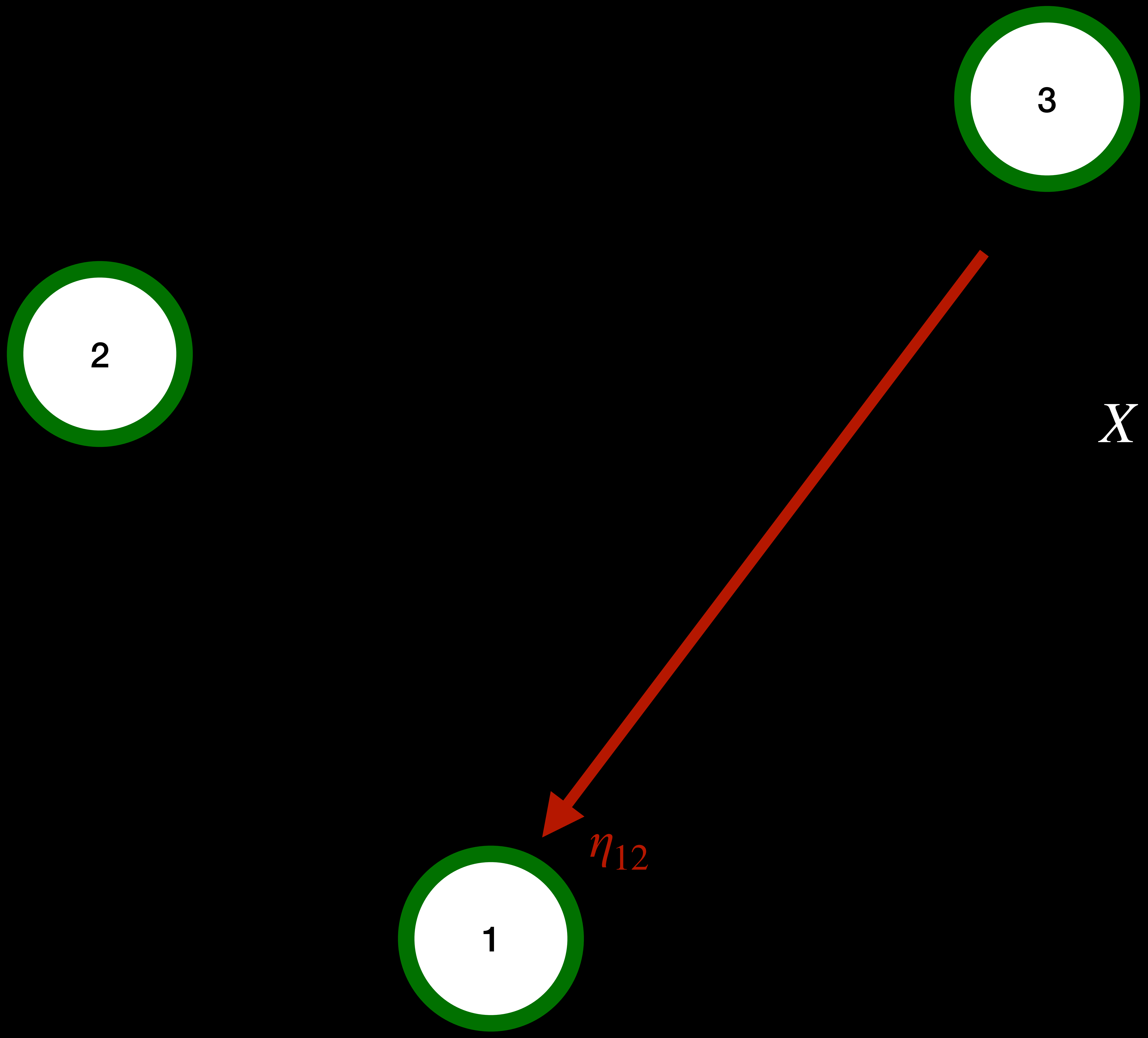


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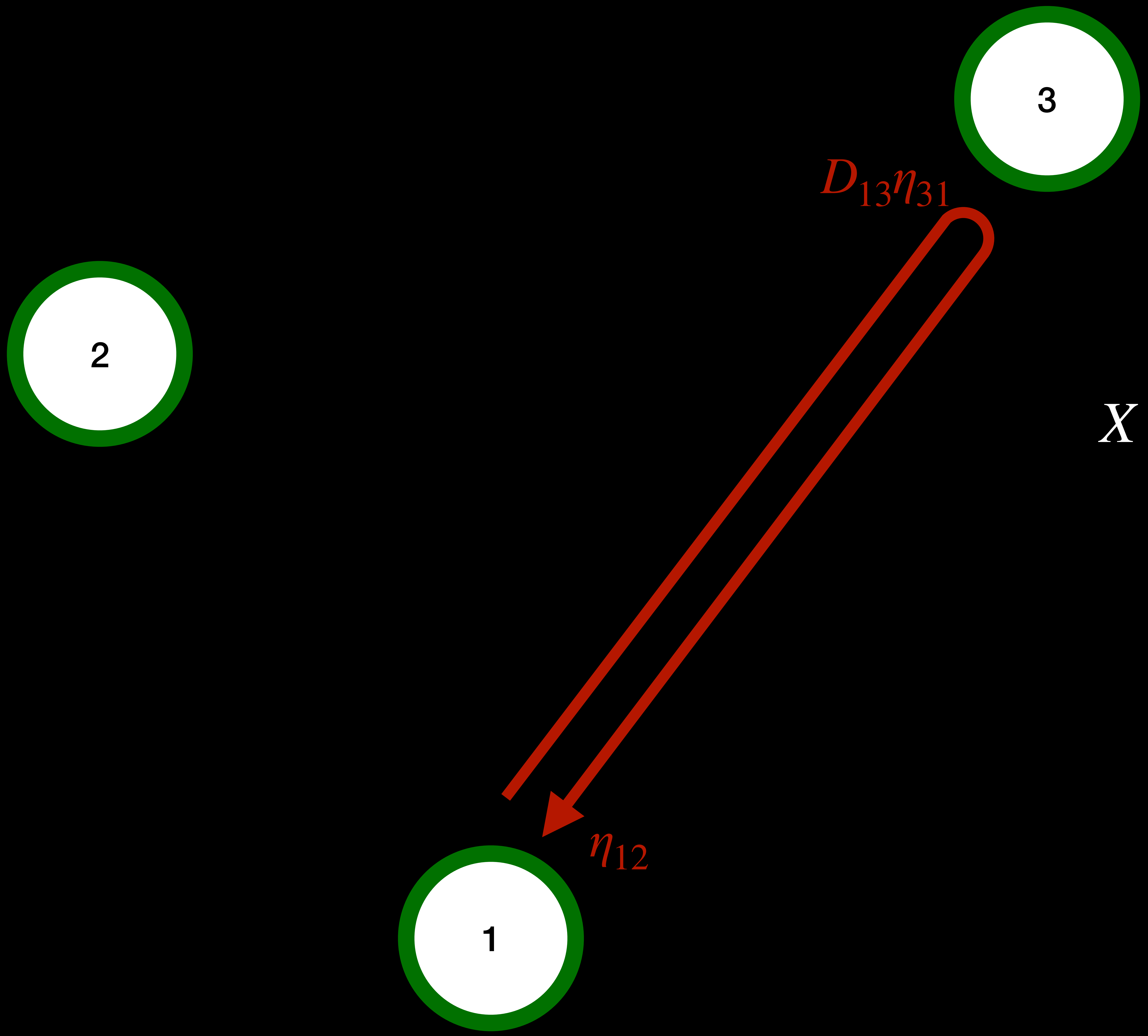


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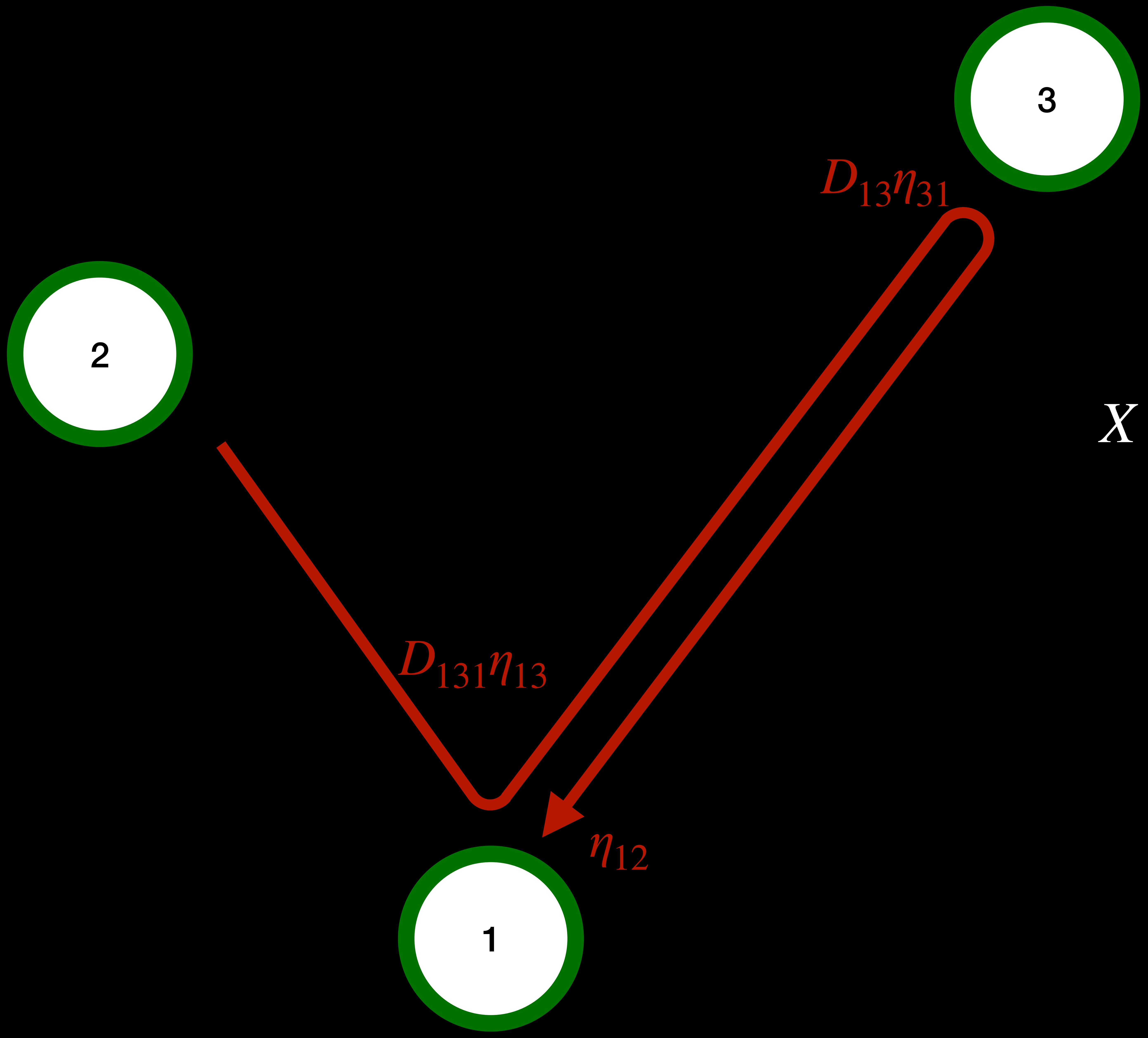


$$X = \eta_{12} + D_{12}\eta_{21} + D_{121}\eta_{13} + D_{1213}\eta_{31}$$
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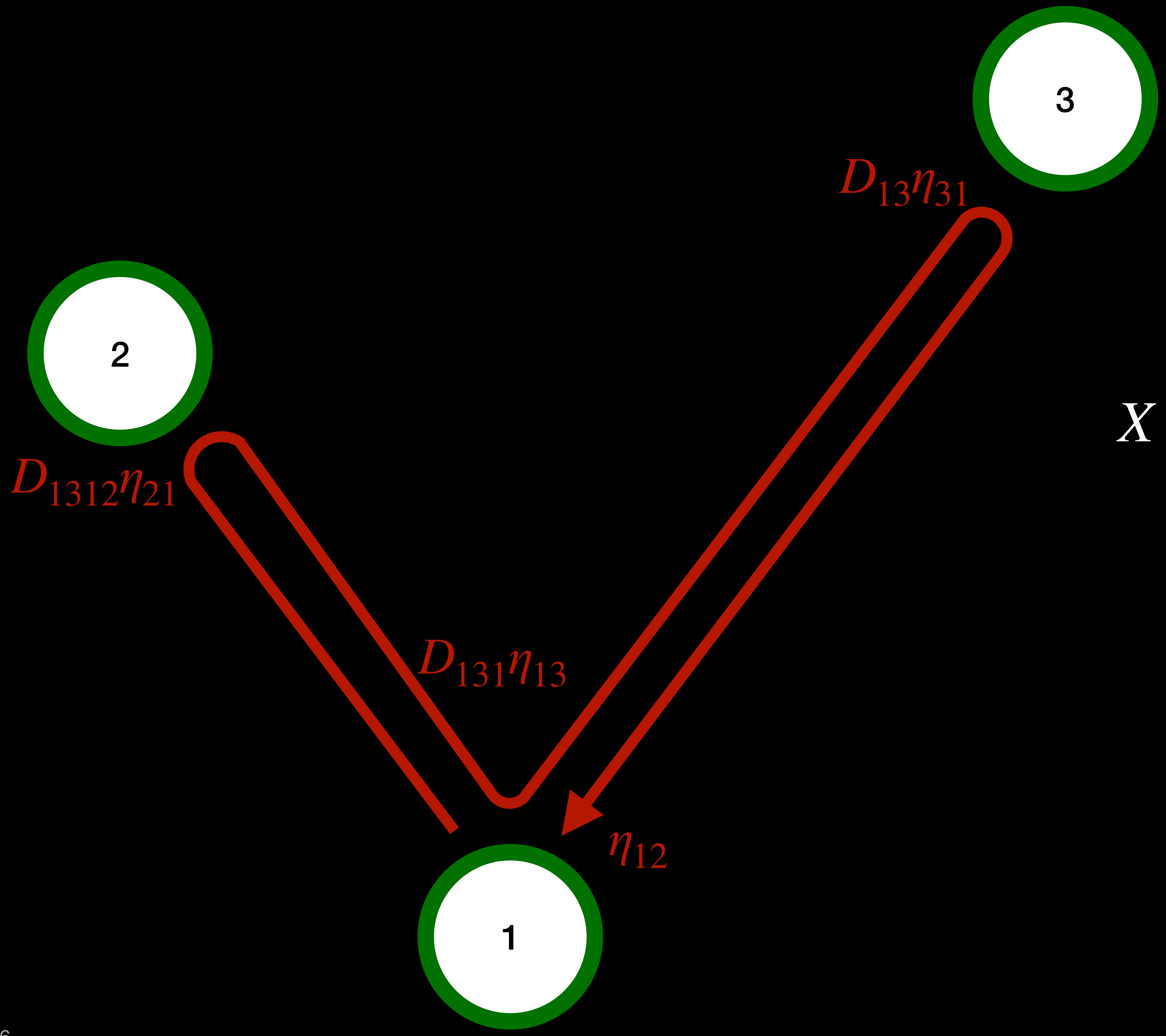


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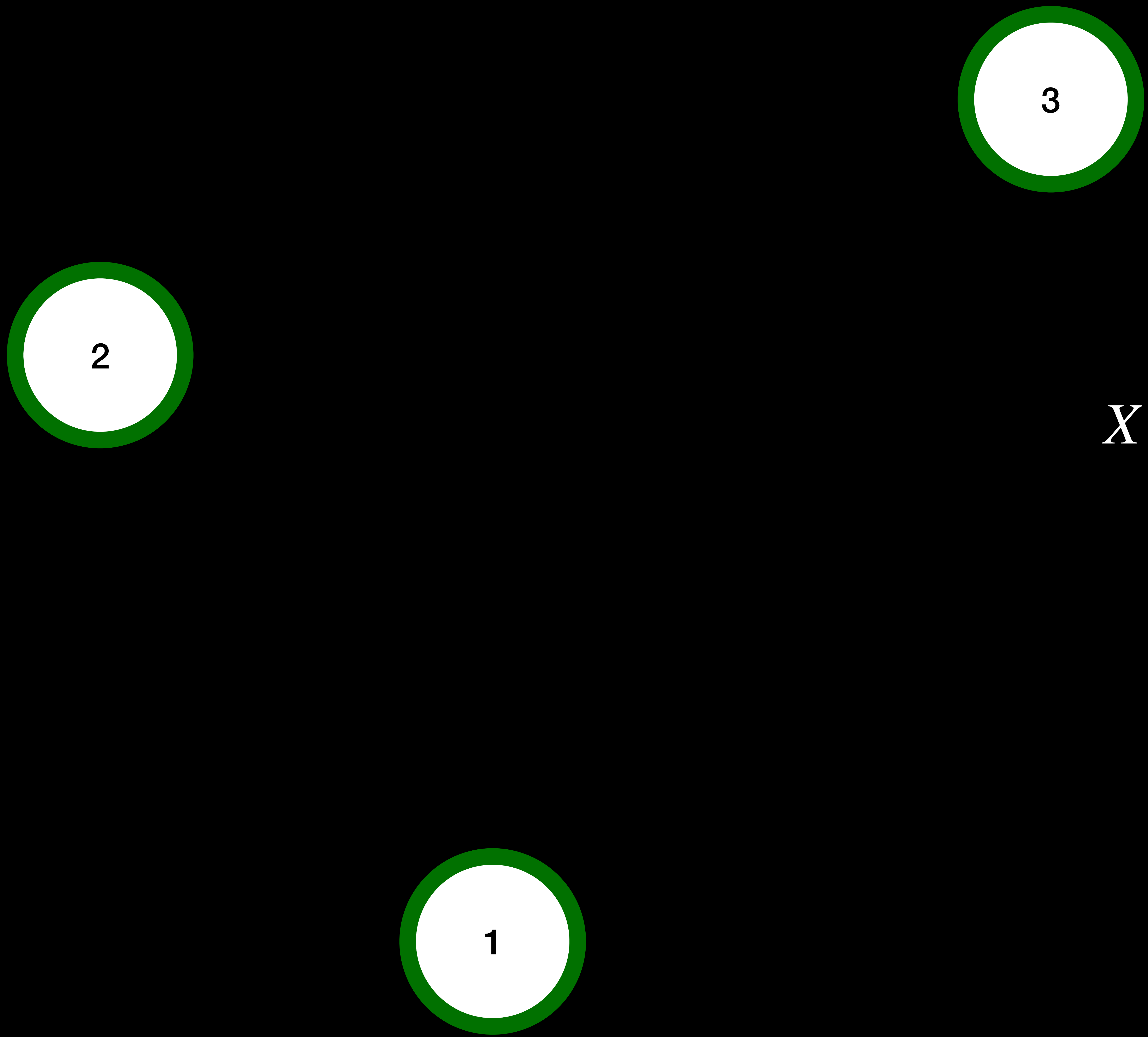


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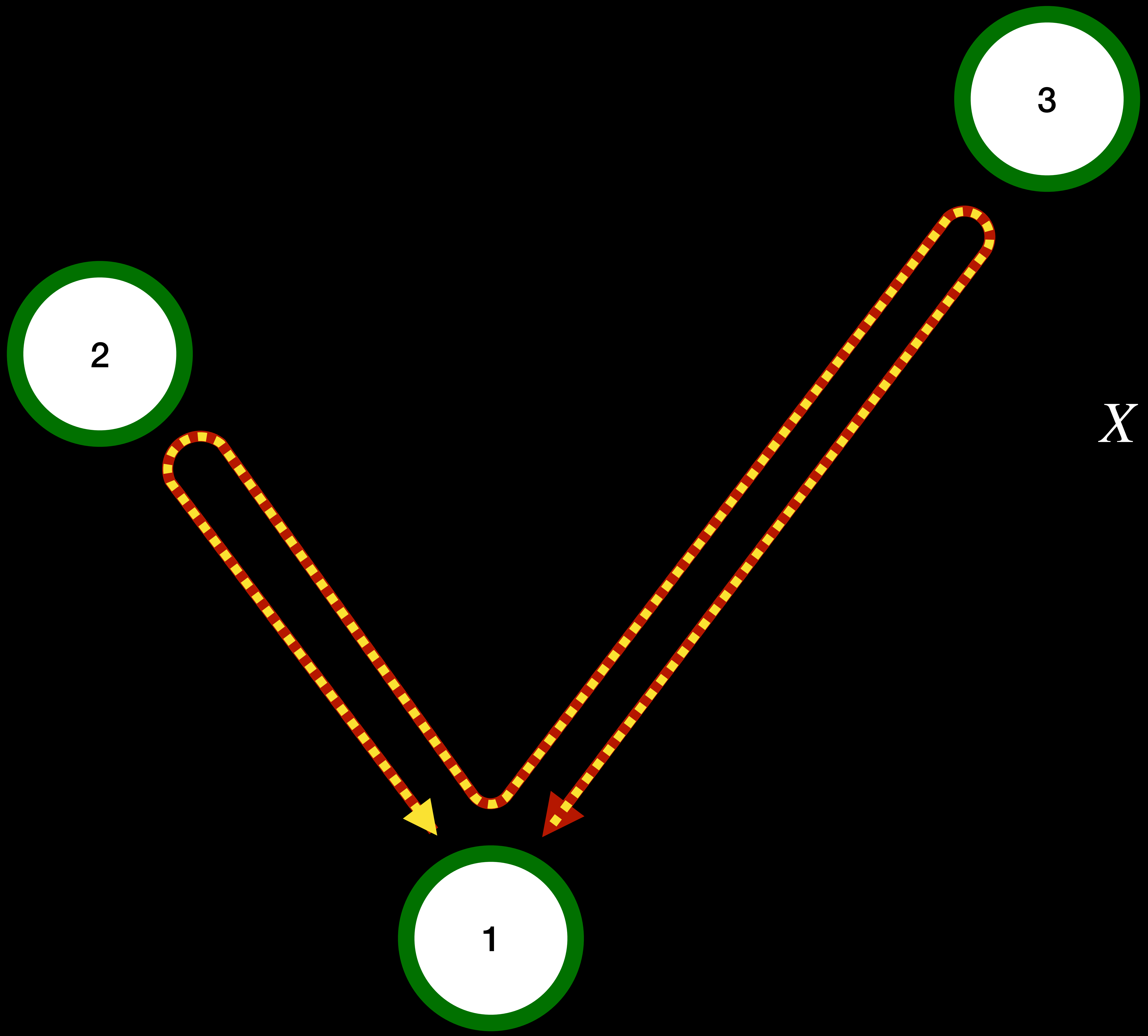


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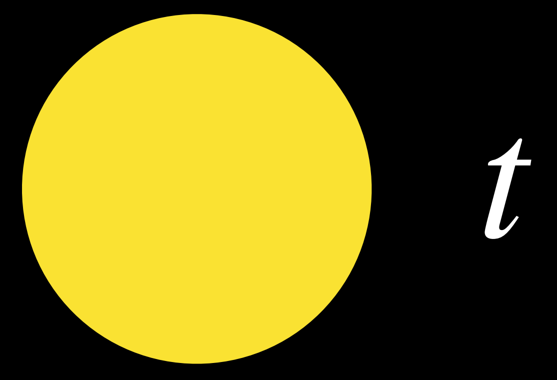
$$-\eta_{12} - D_{13}\eta_{31} - D_{131}\eta_{13} - D_{1312}\eta_{21}$$

$$= (D_{12131} - D_{13121})\Phi_1$$

# TDI with desynchronized clocks

# Timescales in LISA

- TCB time $t$
  - Defined as the time shown of a perfect clock sitting at the solar system baricenter
  - Global timescale, used for data analysis + ‘standard’ TDI



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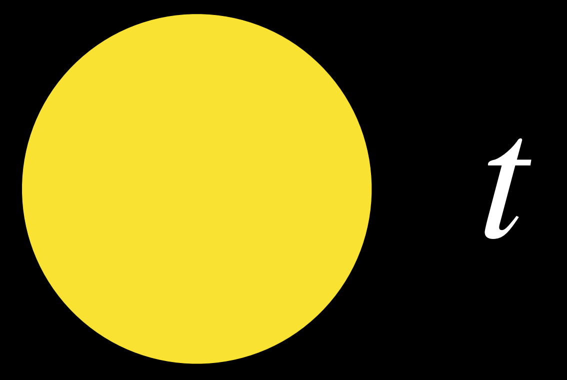


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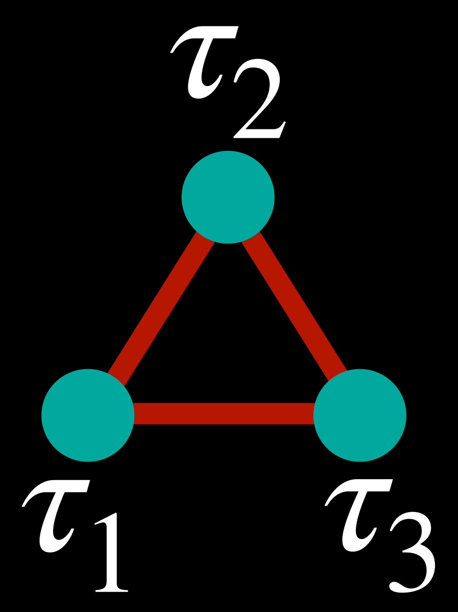
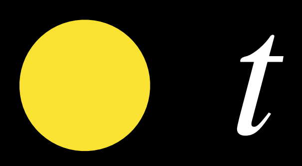
- One proper time $\tau_i$ for each spacecraft $i$ ($i = 1,2,3$)




  - Defined as the time shown of a perfect clock sitting in spacecraft $i$
  - Related to $t$ (and each other) by General Relativity
  - Used for describing physics inside one spacecraft



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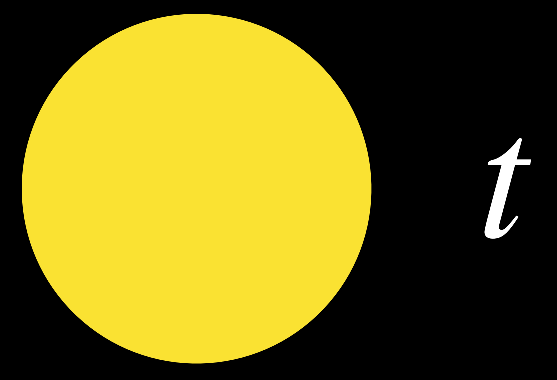


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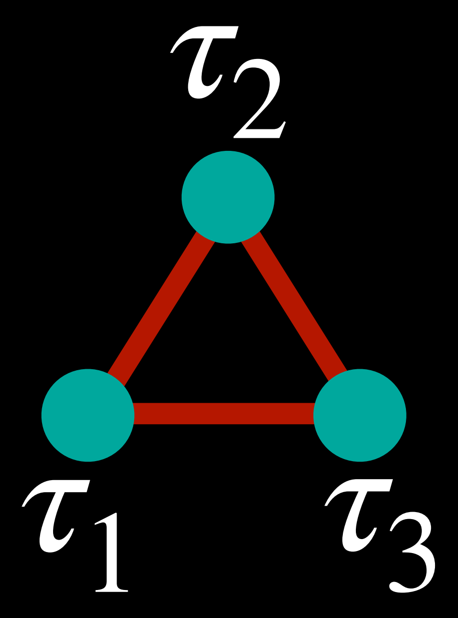
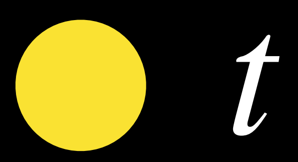
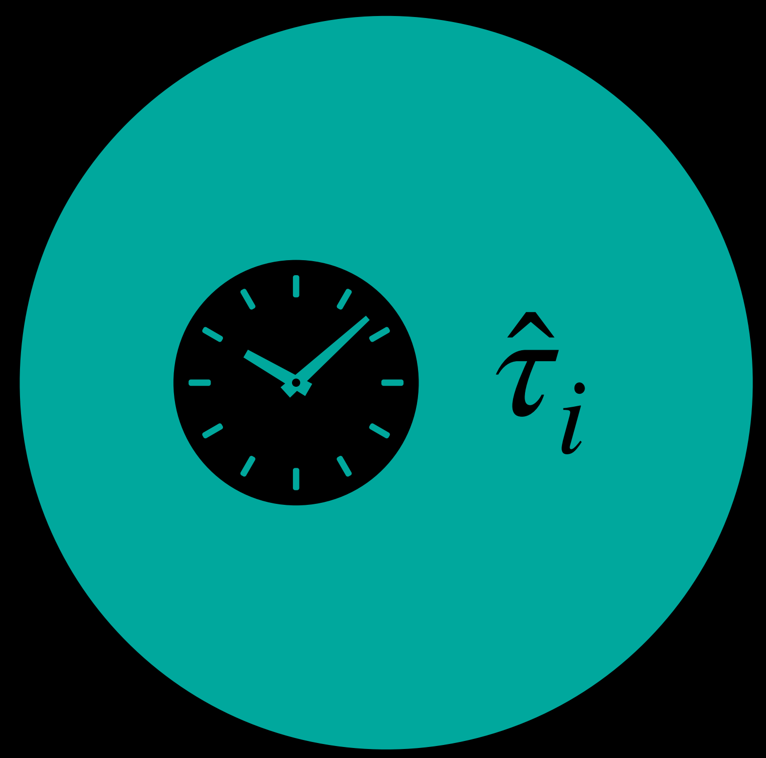



- One onboard clock time $\hat{\tau}_i$ for each spacecraft $i$ $(i = 1,2,3)$
  - Defined as the time shown of the actual clock sitting in spacecraft $i$
  - Differs from $\tau_i$ by instrumental imperfections
  - Only timescale directly accessible by the satellites



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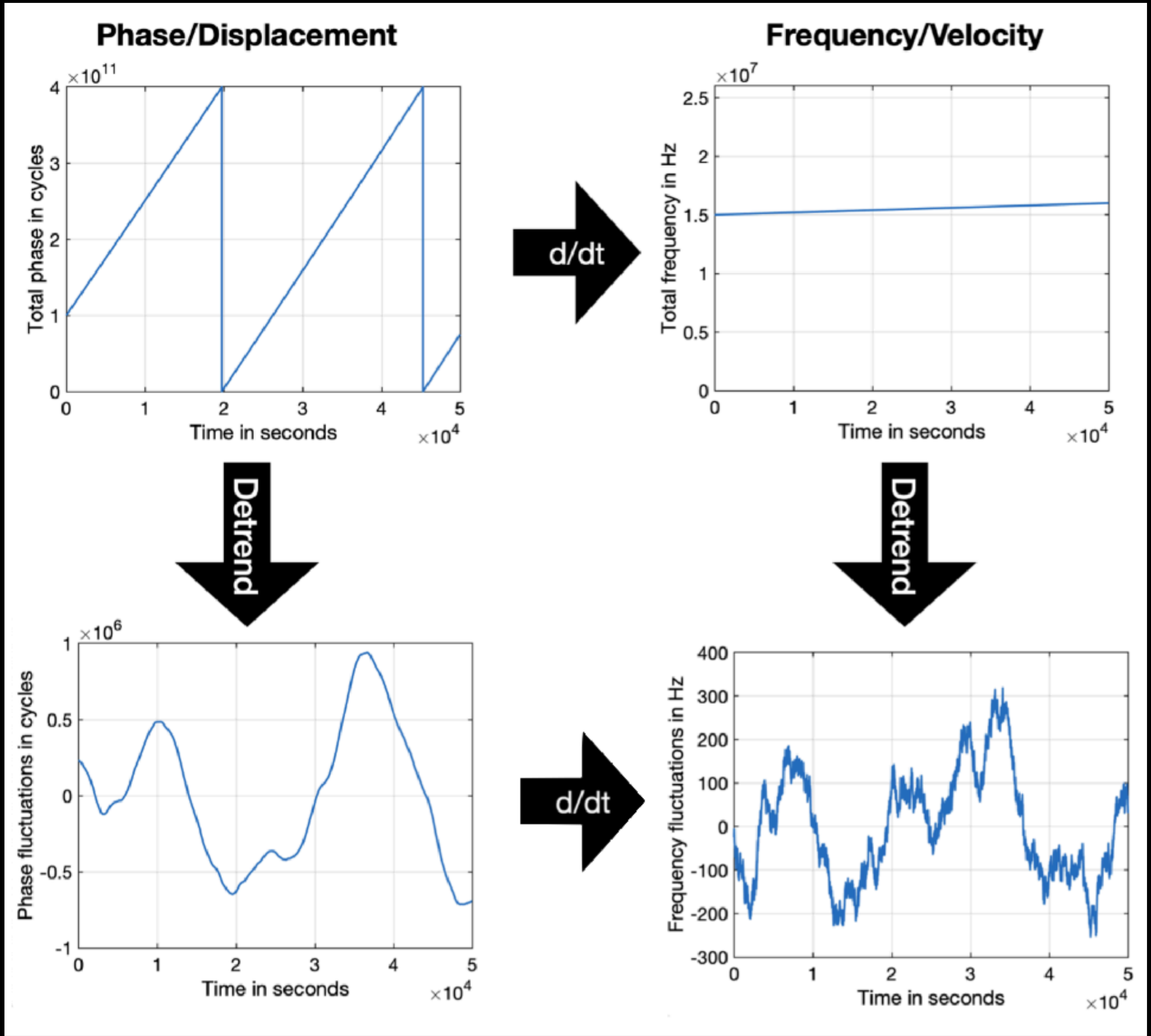
- LISA measures total phase/ frequency of MHz interferometric beat notes



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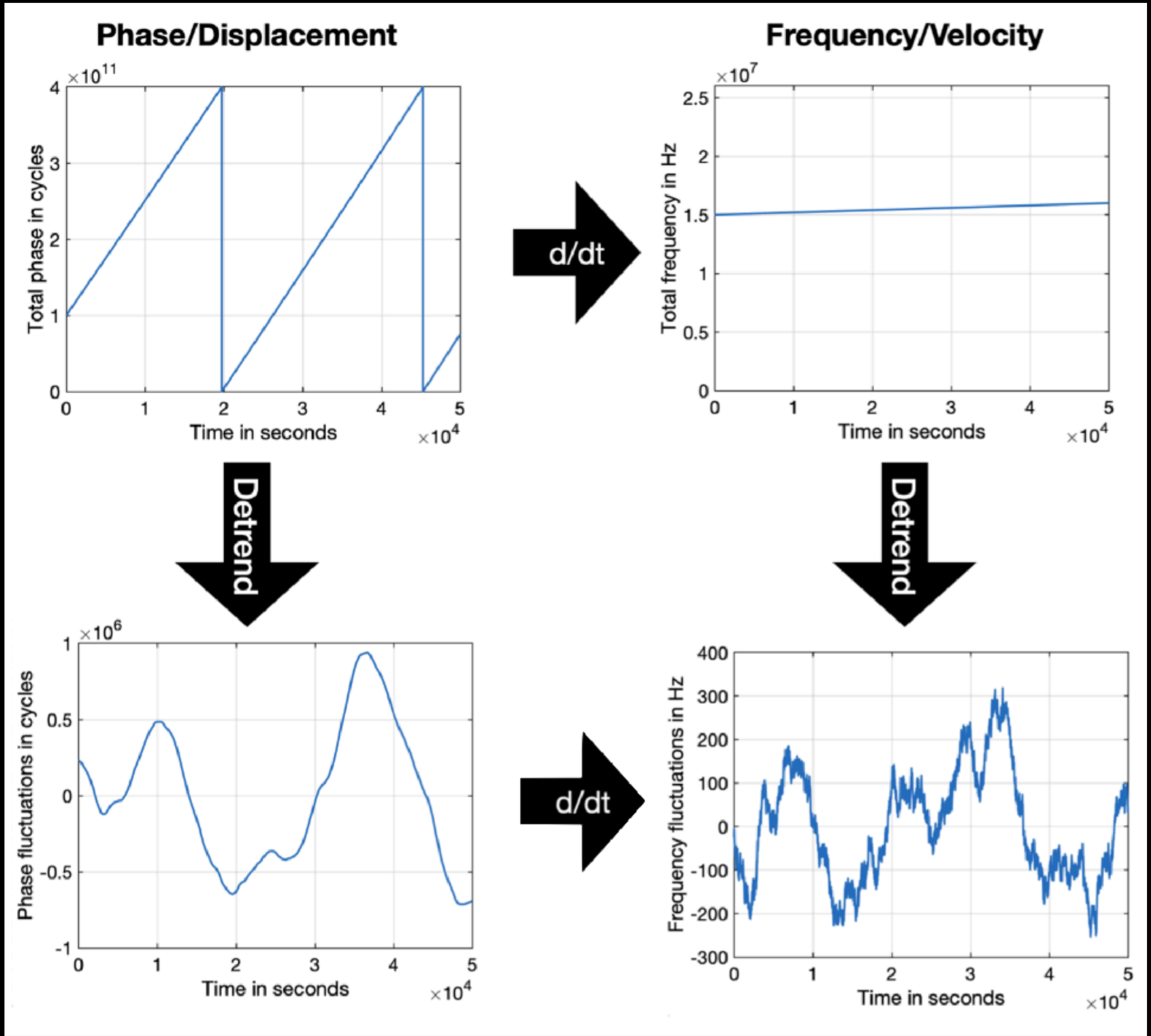

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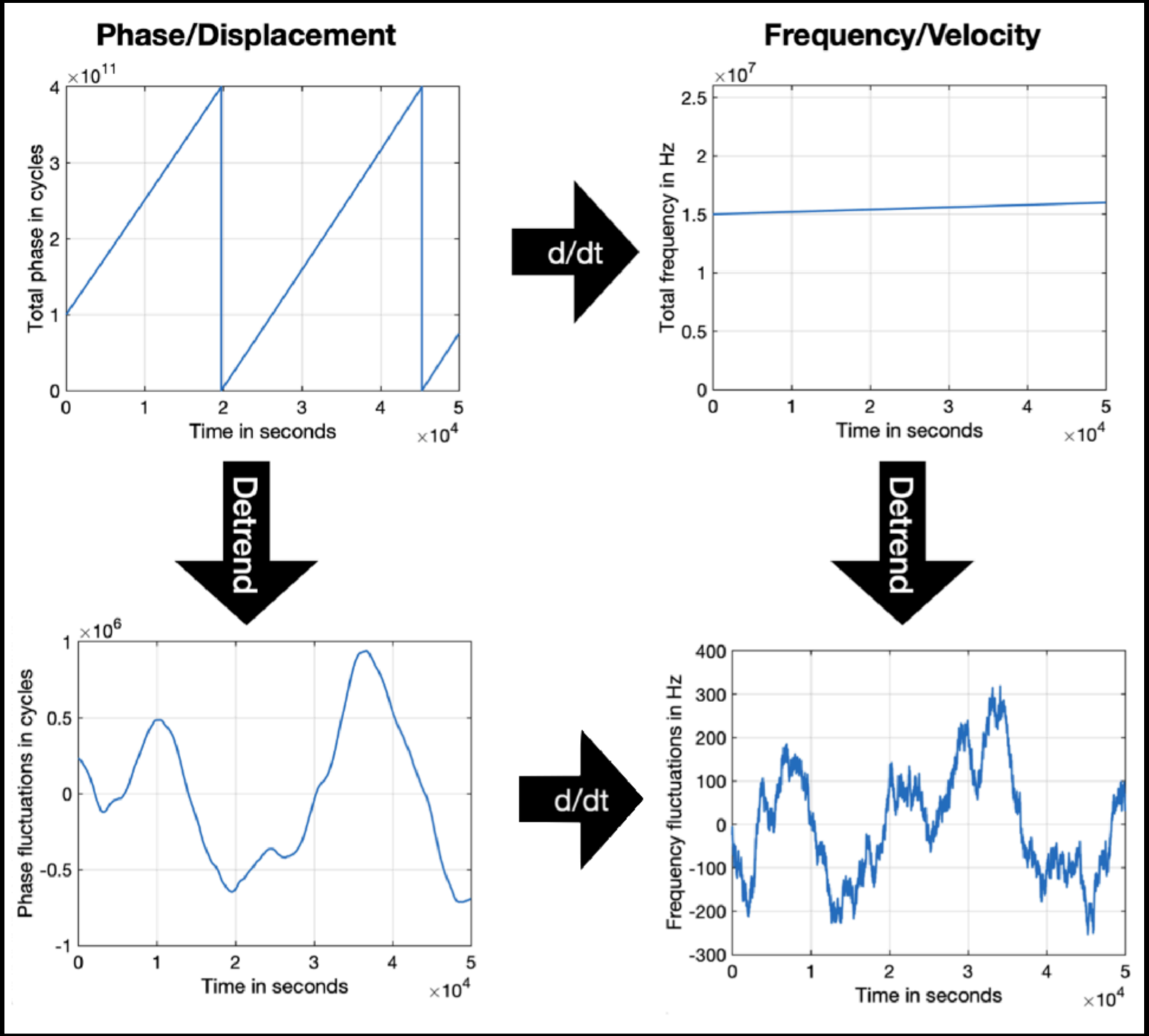

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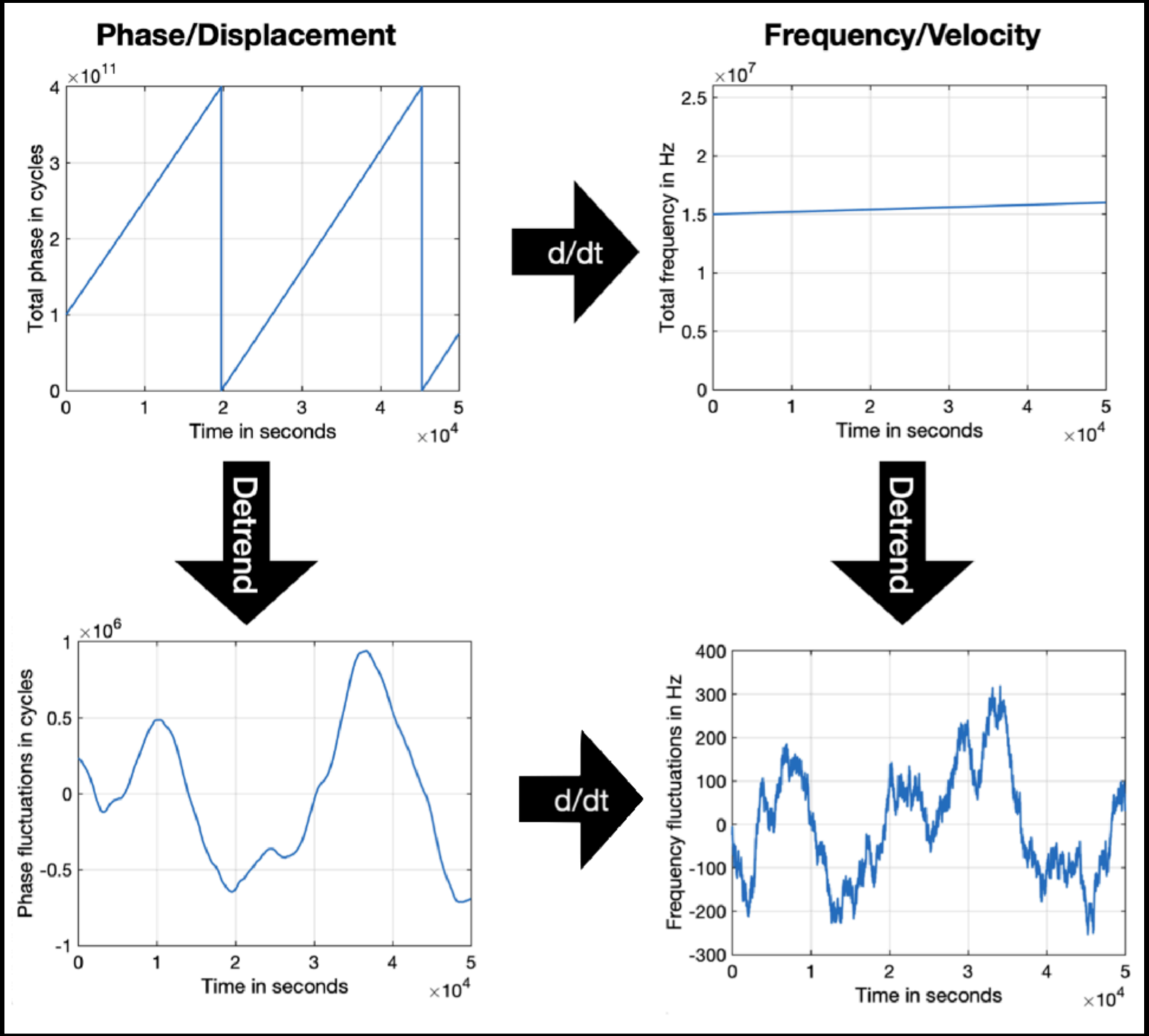

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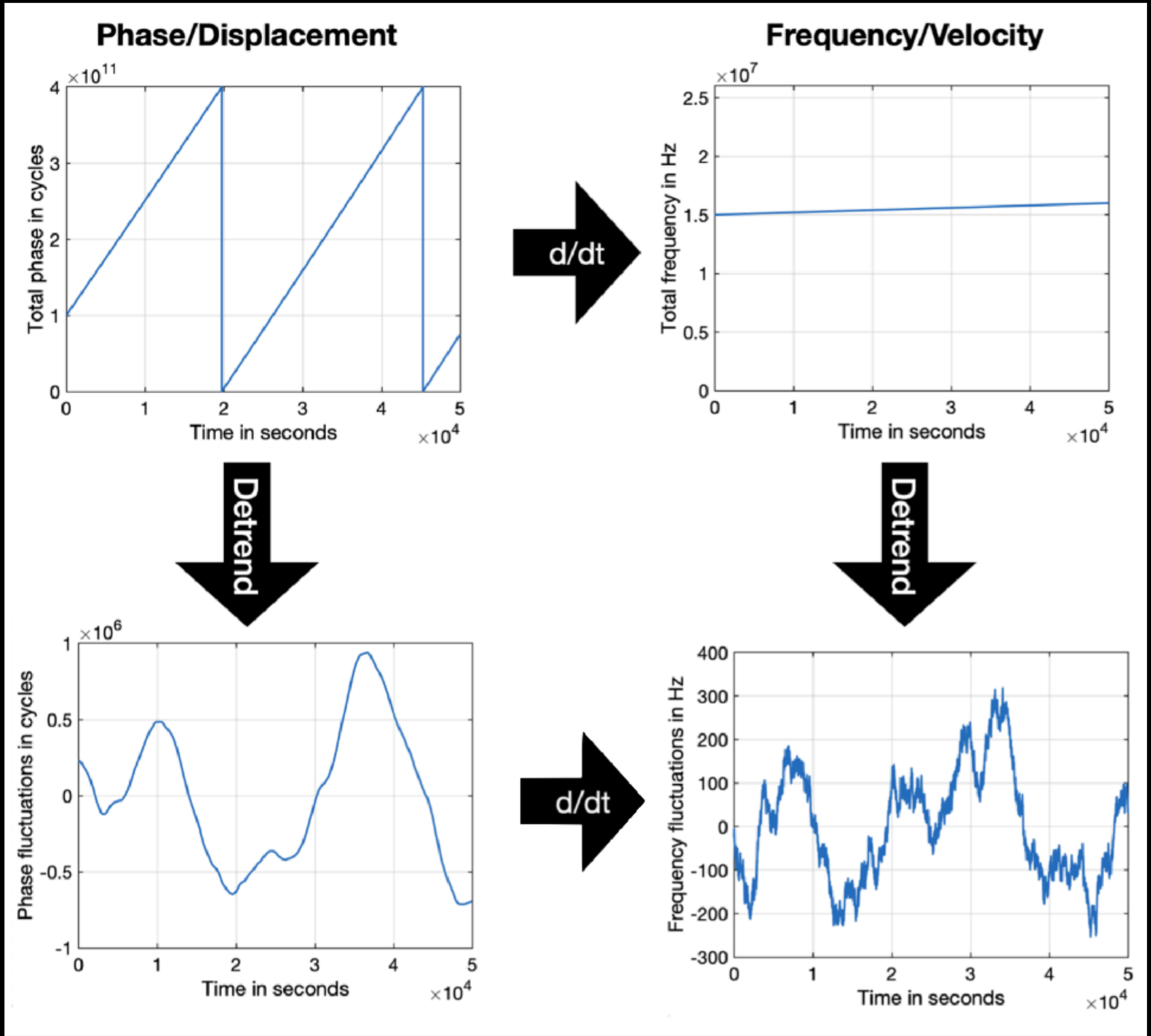
- Note: any time shift applied to total phase/frequency requires same 50 fs/$\sqrt{\mathrm{Hz}}$ precision



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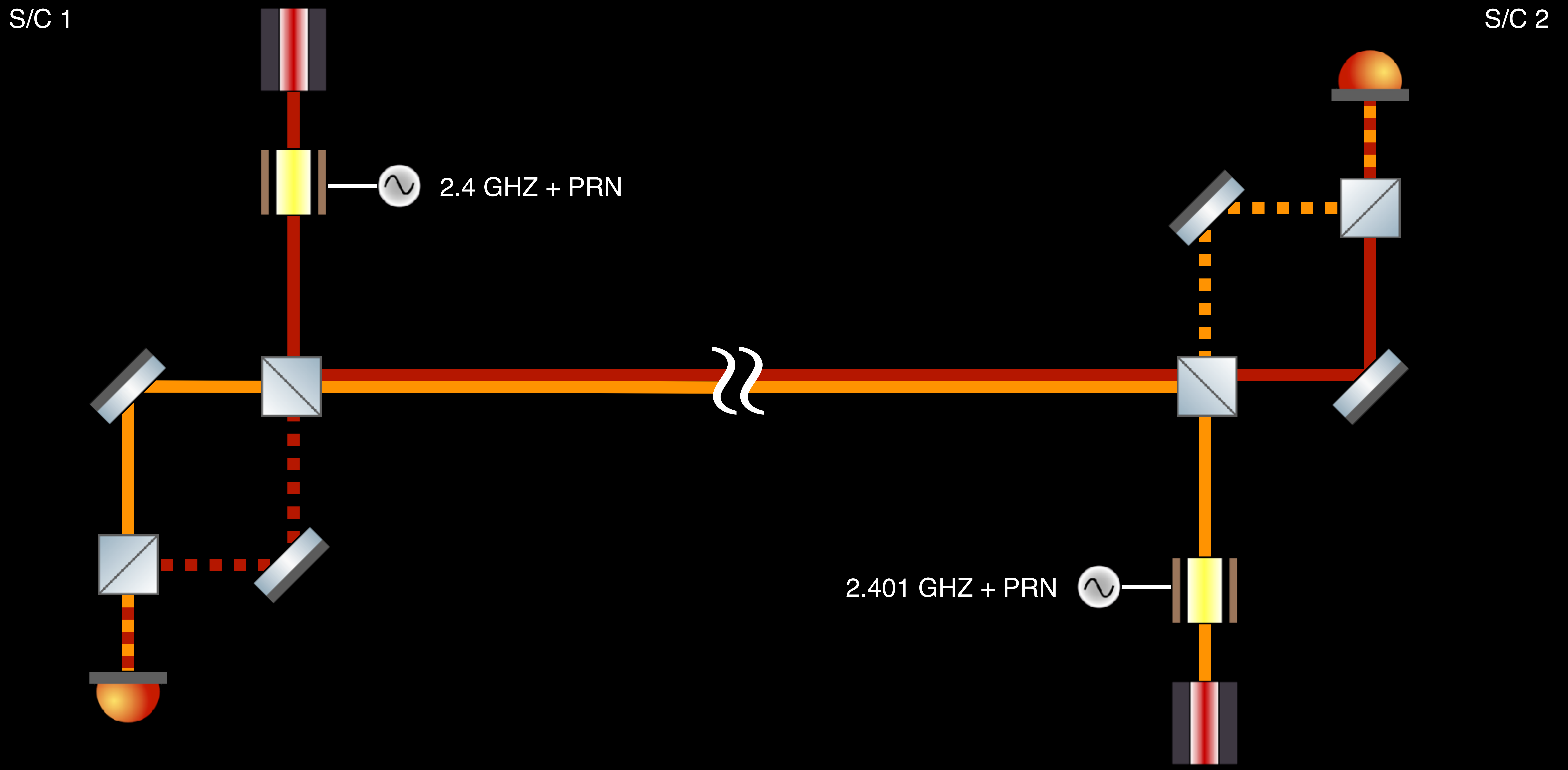

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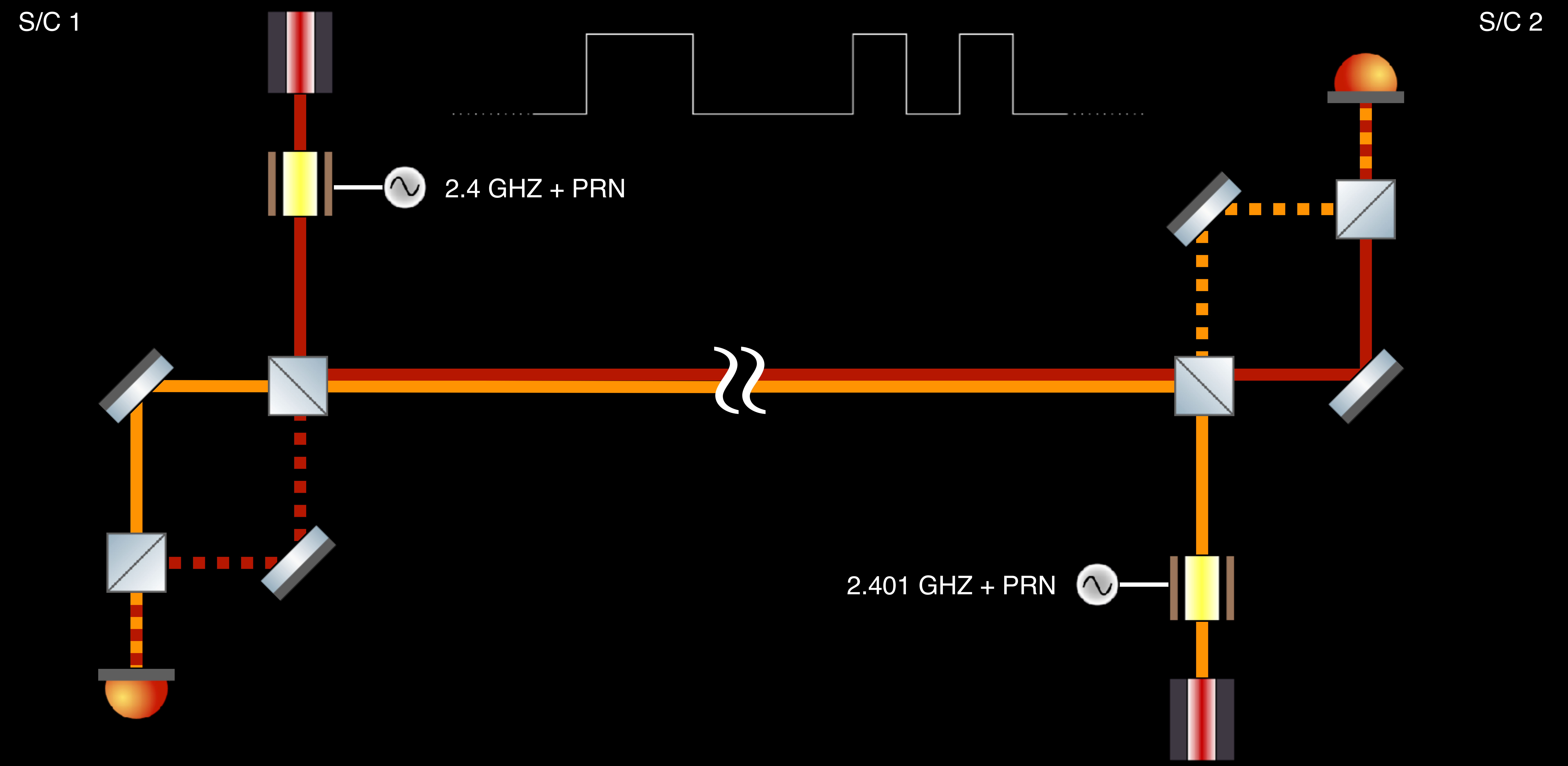

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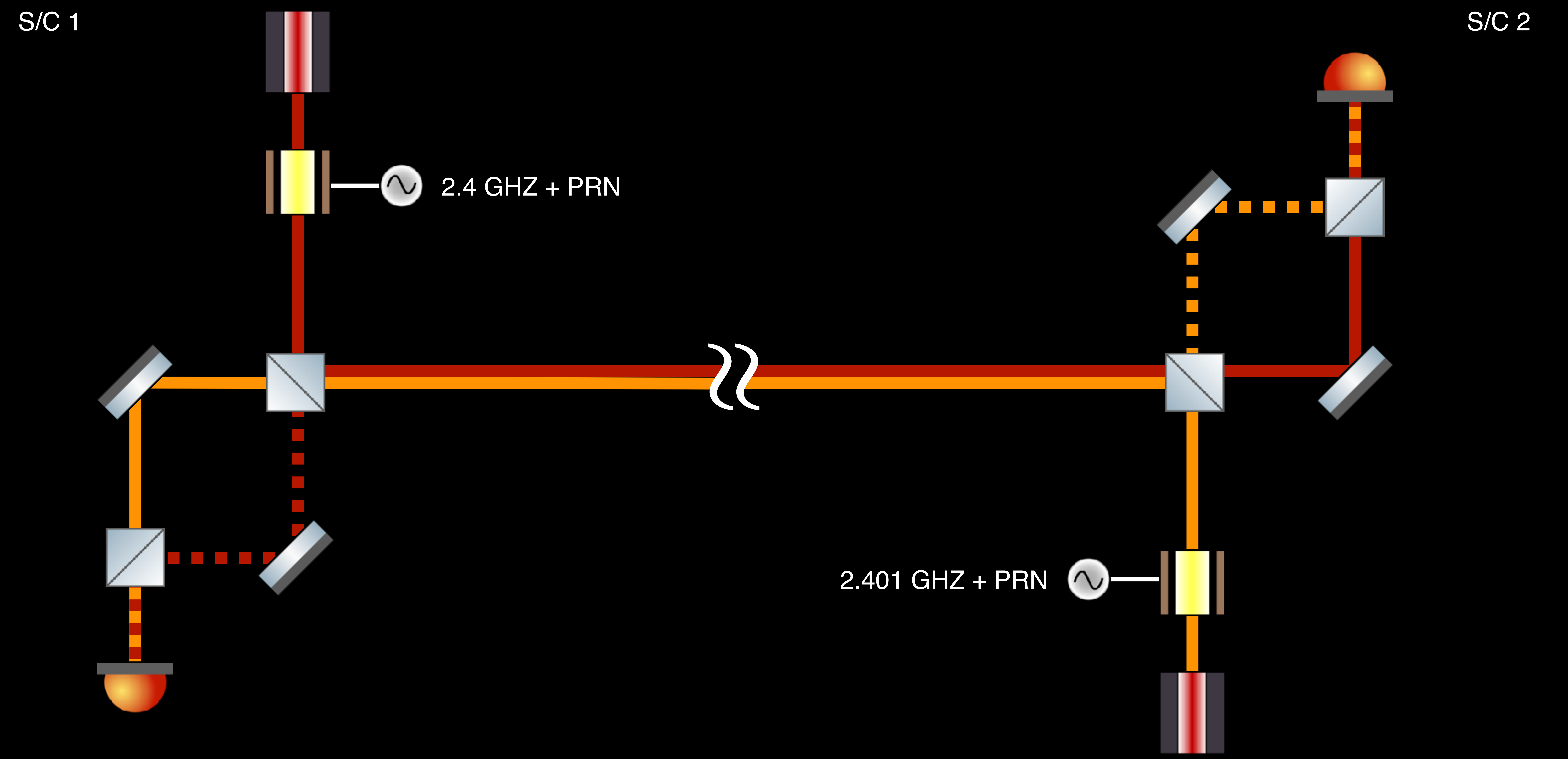

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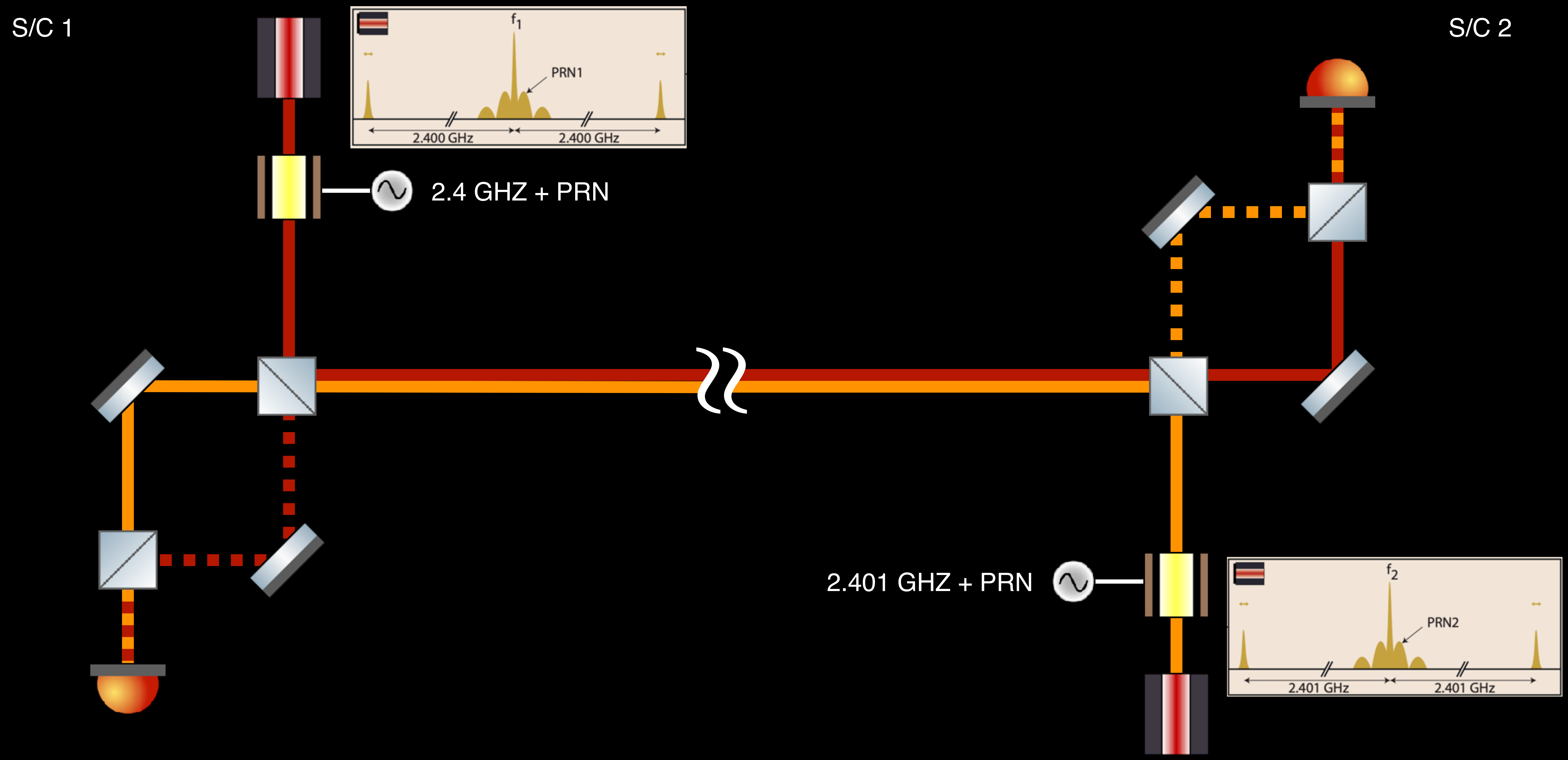
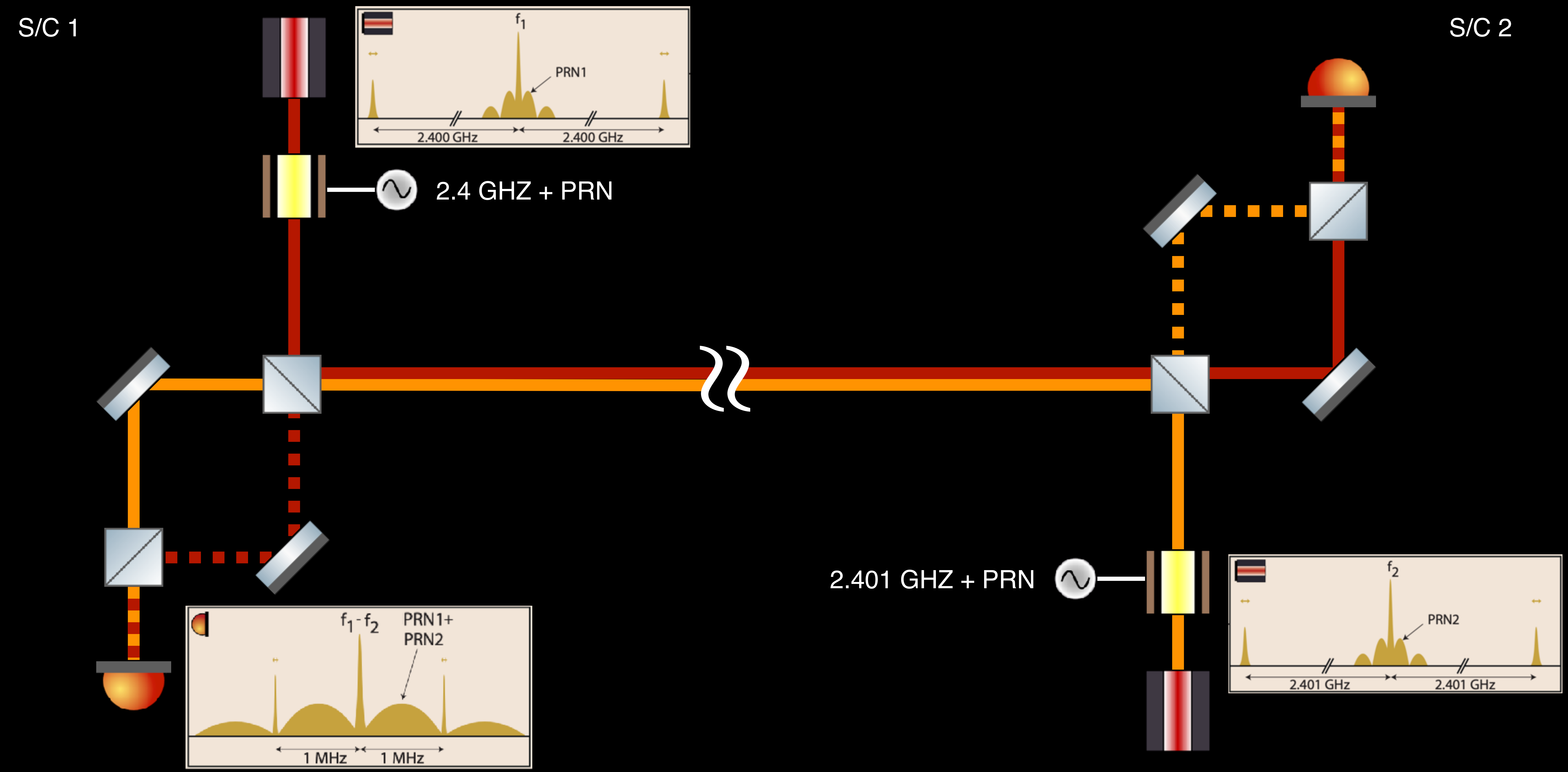
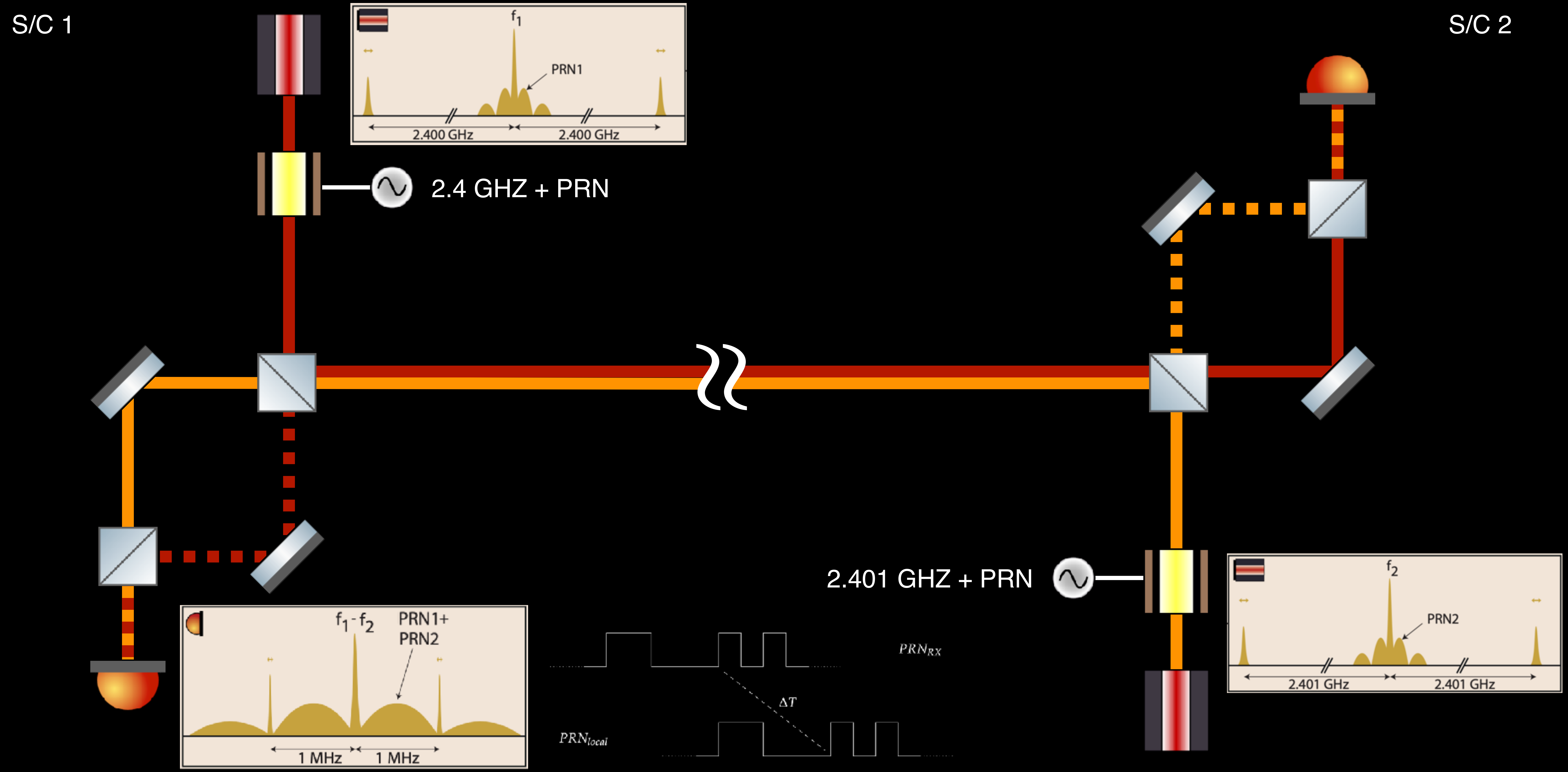


# Determining the delay: pseudo-ranging measurements



# Determining the delay: pseudo-ranging measurements



# TDI with desynchronized clocks

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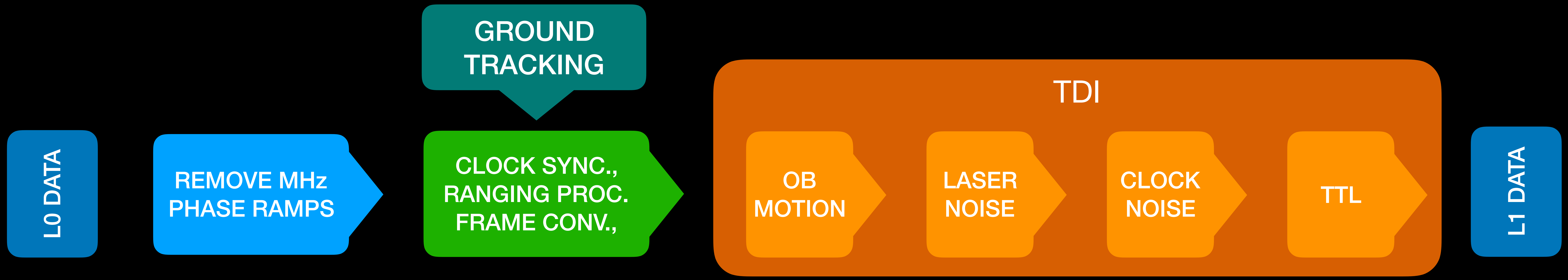
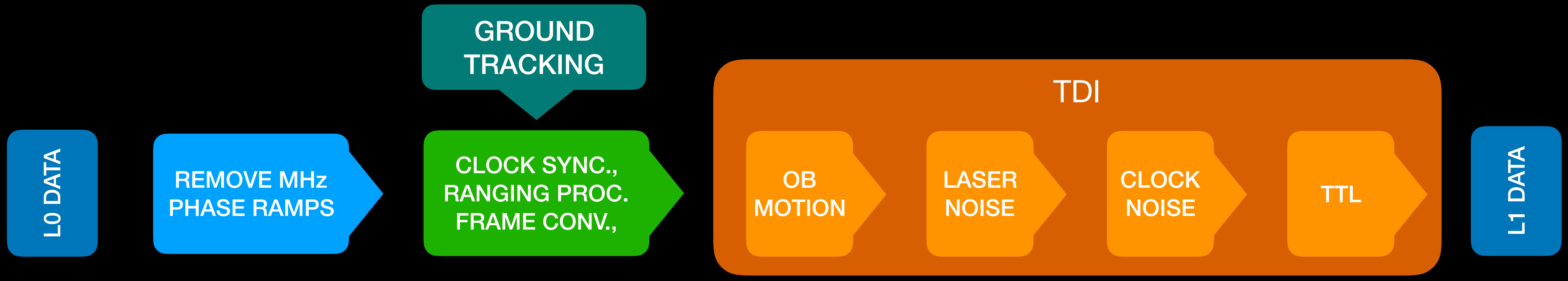
- ‘Baseline’ pipeline (simplified):

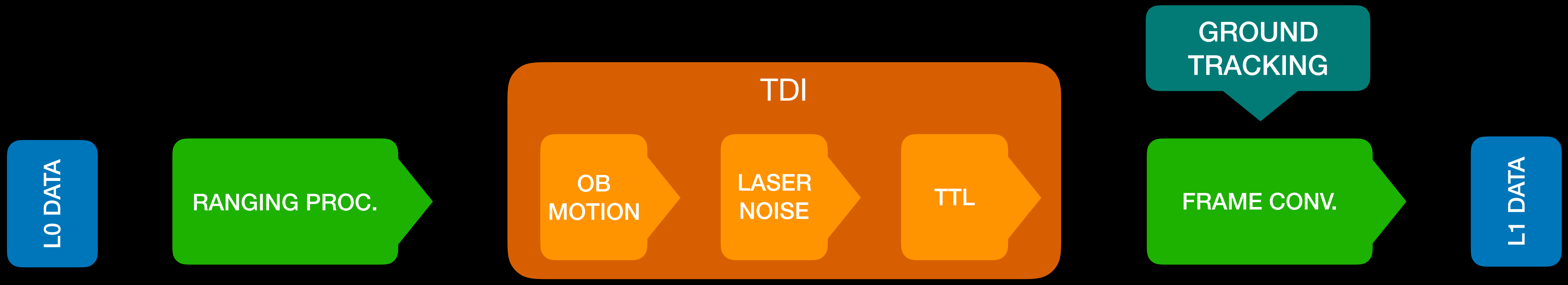


# TDI with desynchronized clocks

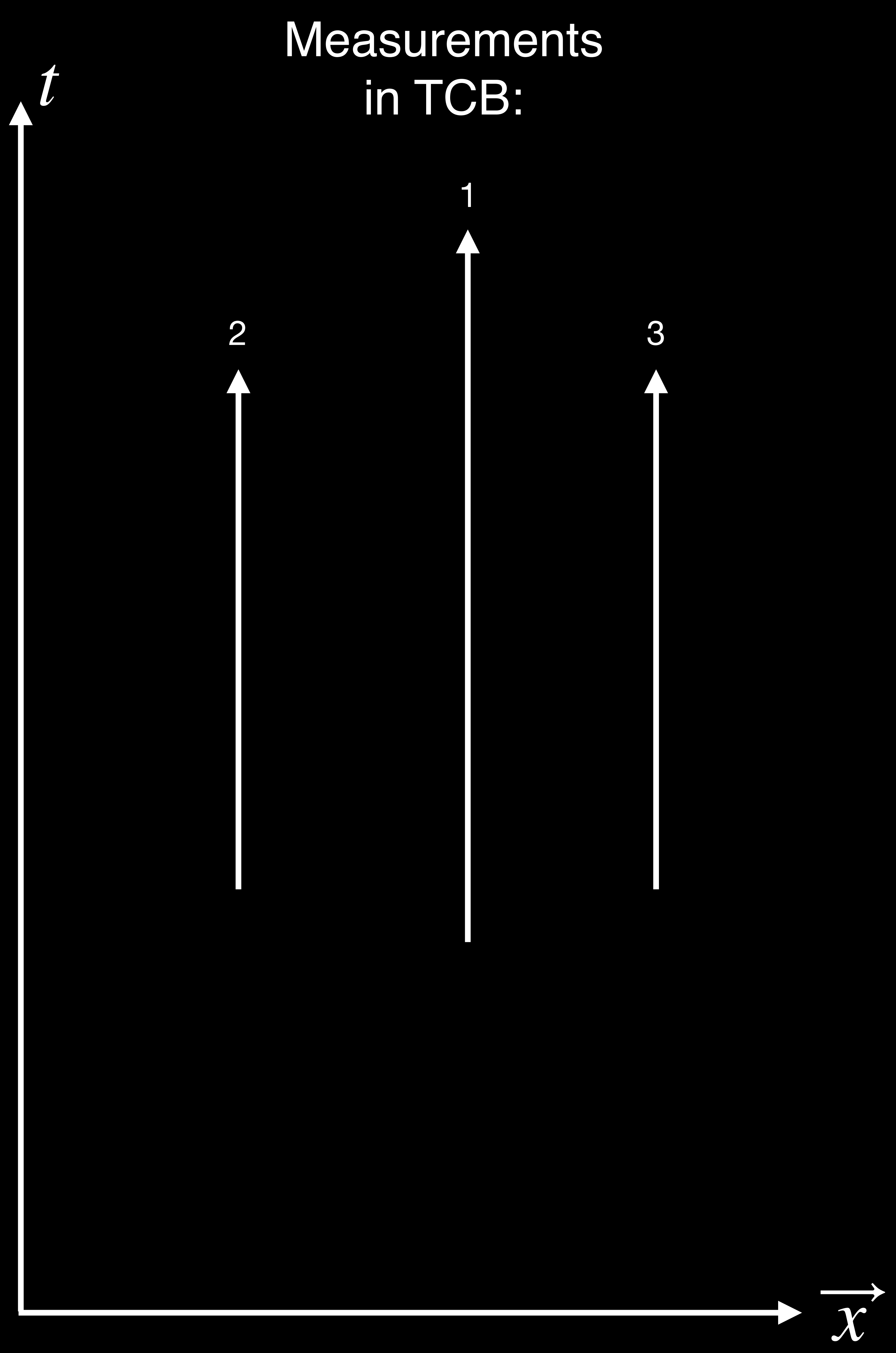
- 'Baseline' pipeline (simplified):




- Alternative pipeline:



# Geometric TDI with clock times



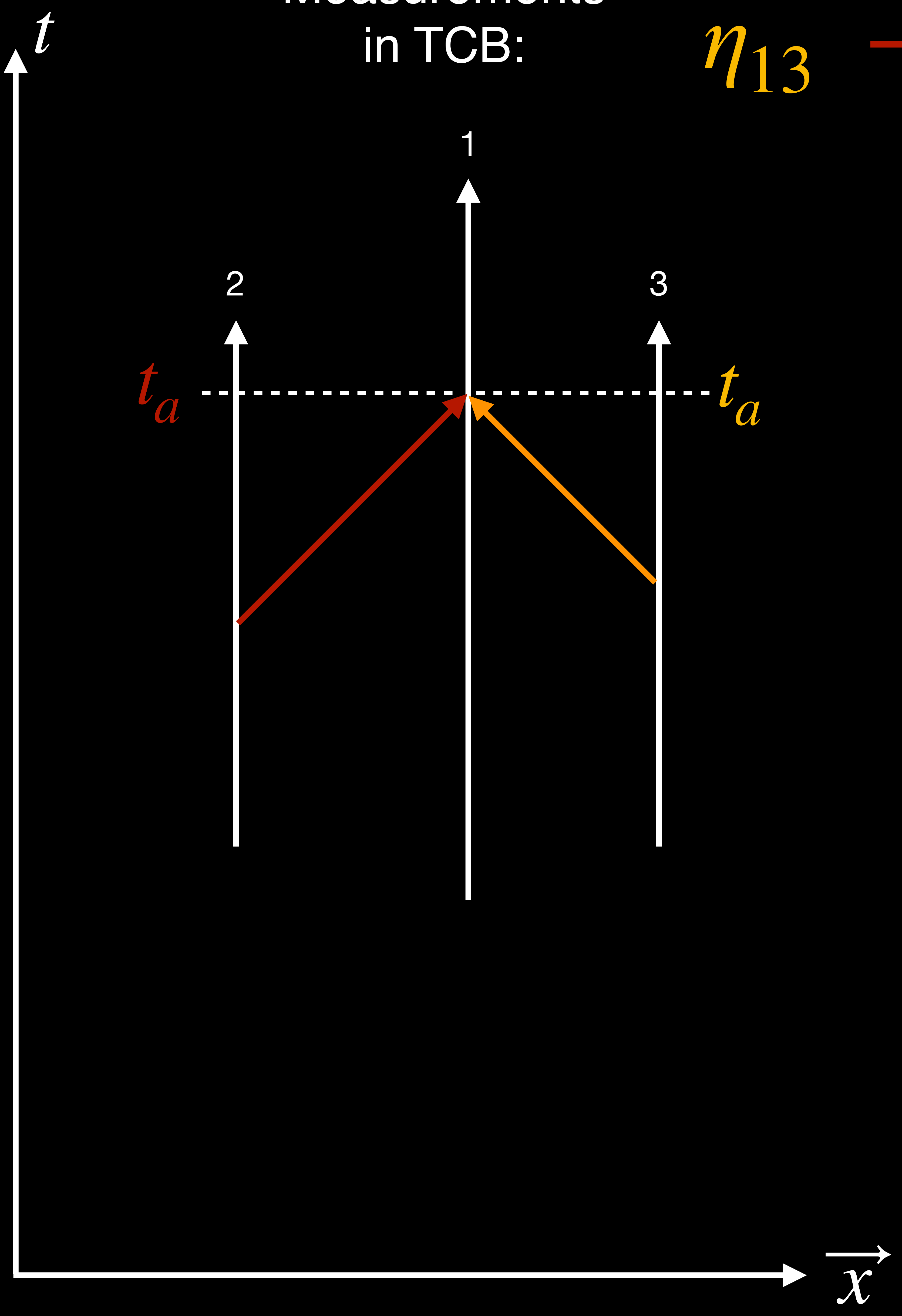


23

# Geometric TDI with clock times

Measurements
in TCB:

$\eta_{13} \; - \eta_{12}$



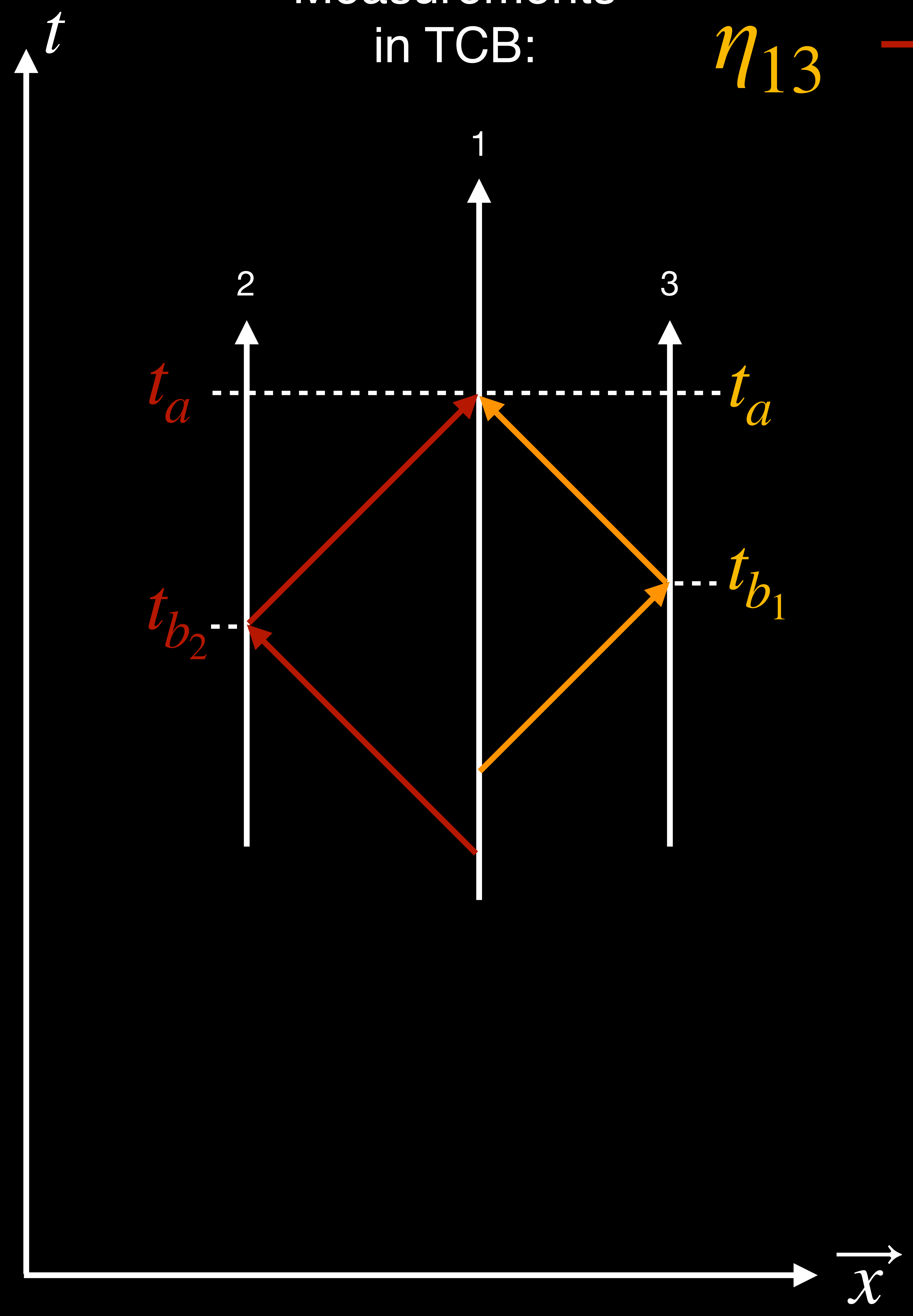



23

# Geometric TDI with clock times

Measurements in TCB:

$$\eta_{13} \quad - \eta_{12} \quad + D_{13}\eta_{31} - D_{12}\eta_{21}$$



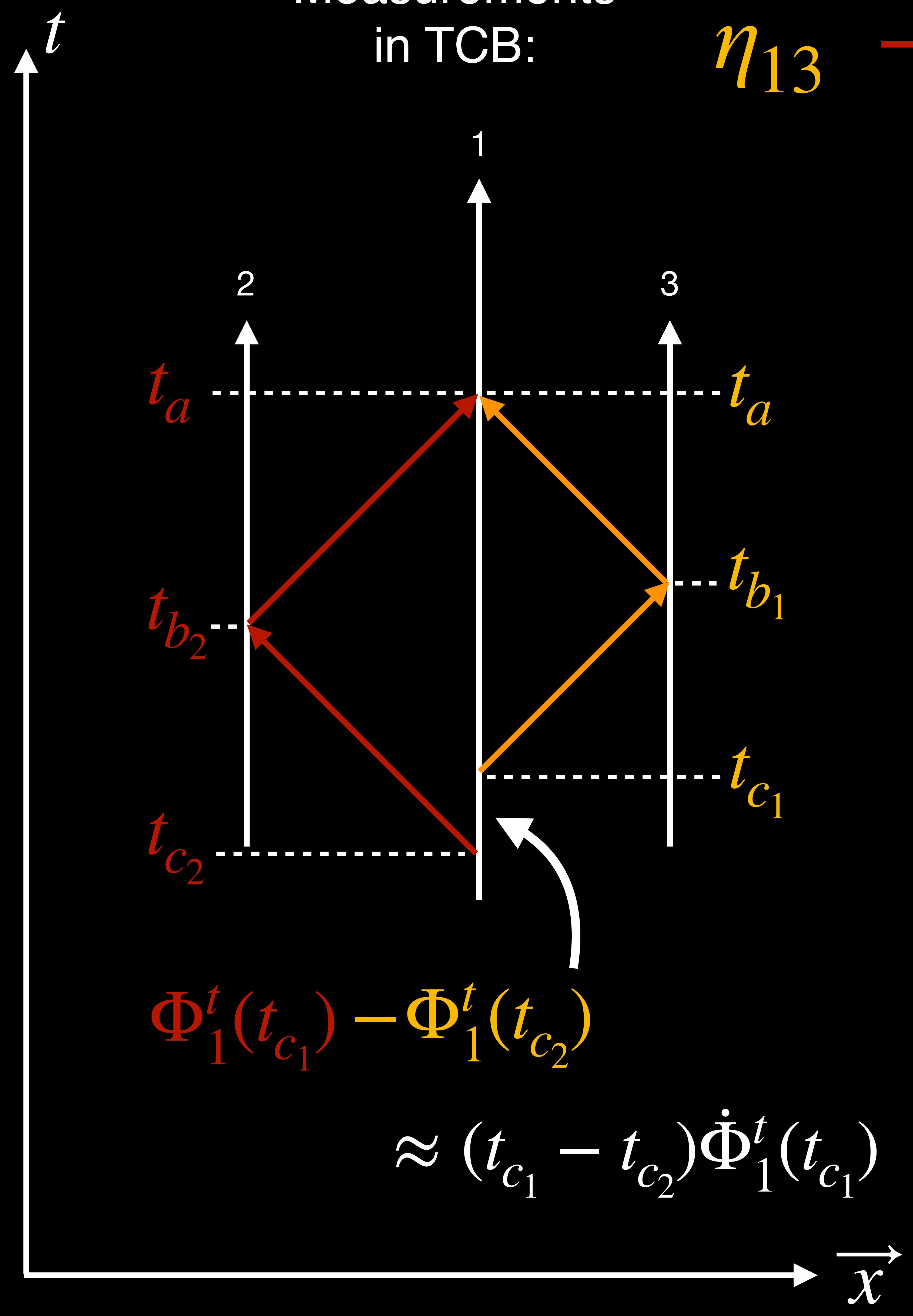



23

# Geometric TDI with clock times

Measurements in TCB:

$$\eta_{13} - \eta_{12} + D_{13}\eta_{31} - D_{12}\eta_{21}$$



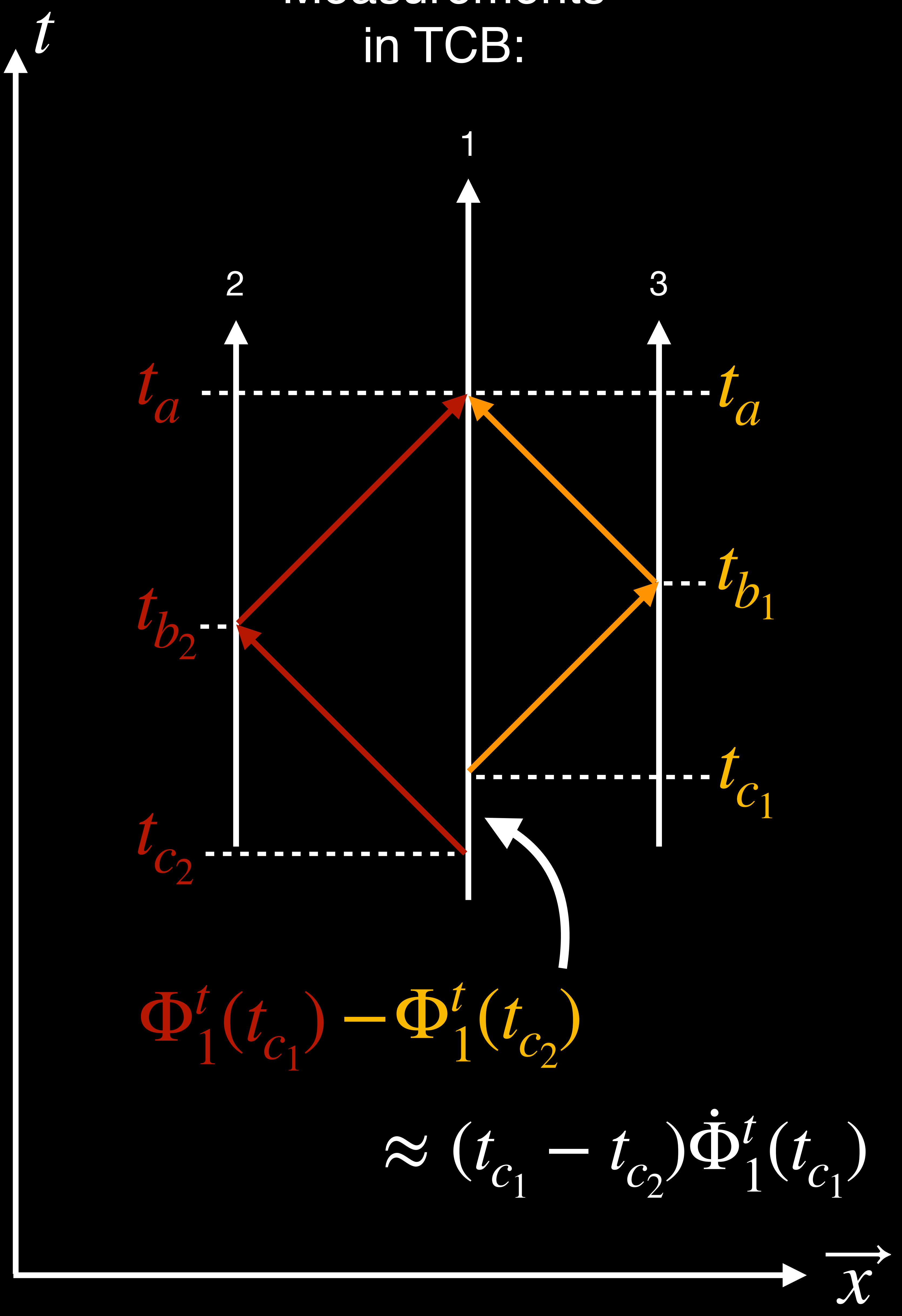

$$\Phi_1^t(t_{c_1}) - \Phi_1^t(t_{c_2})$$

$$\approx (t_{c_1} - t_{c_2})\dot{\Phi}_1^t(t_{c_1})$$

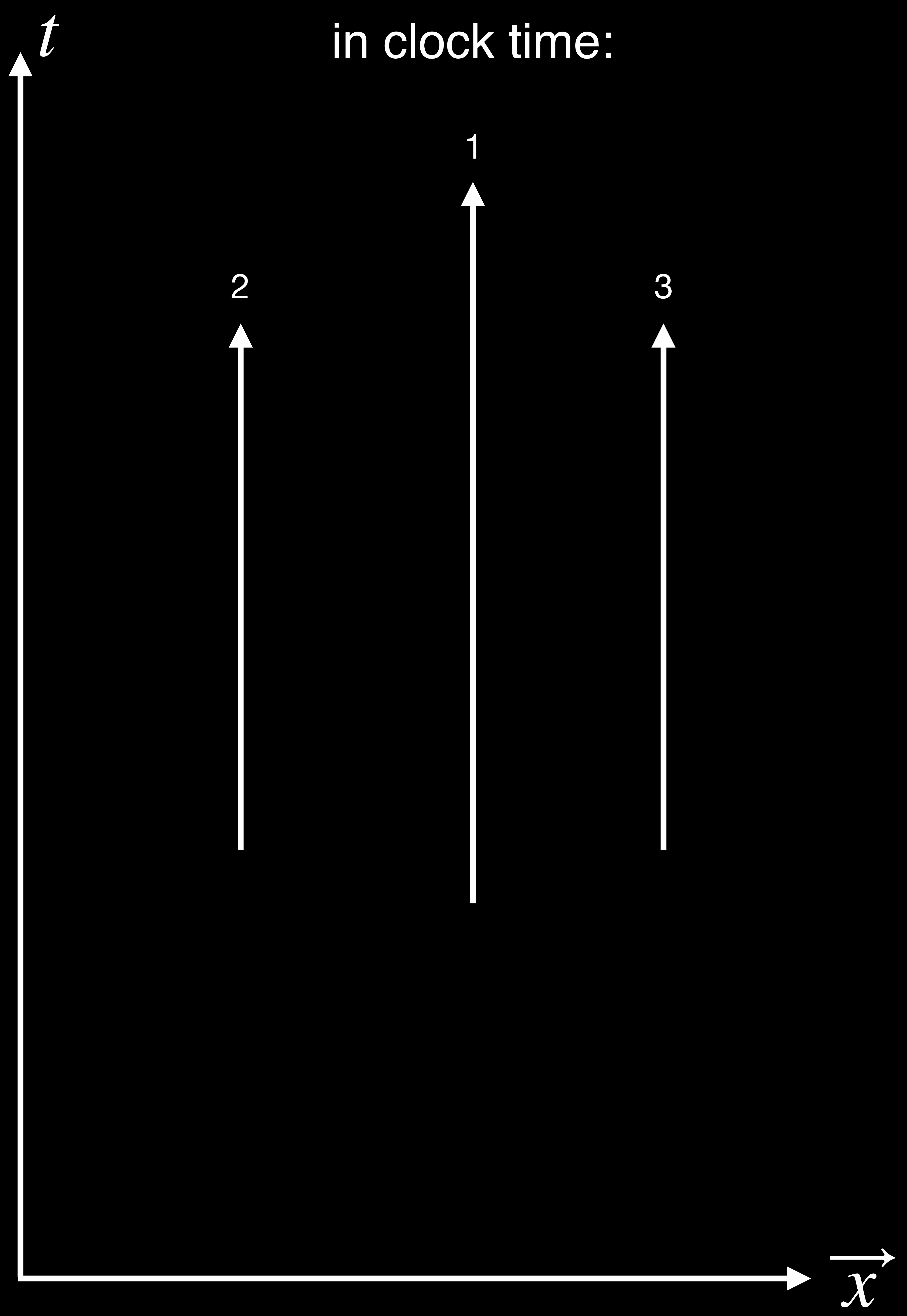

23

# Geometric TDI with clock times

Measurements
in TCB:



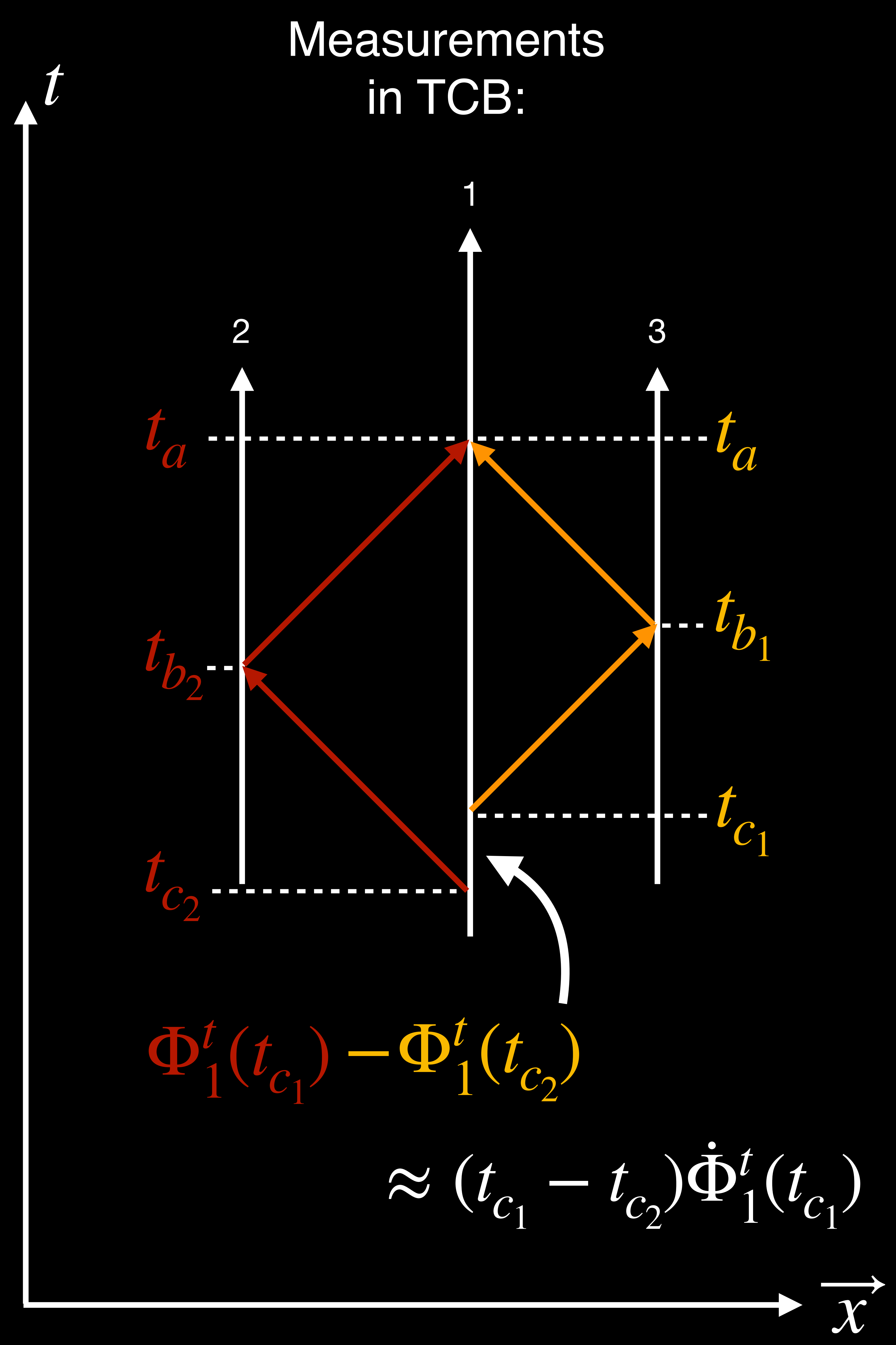

Measurements
in clock time:

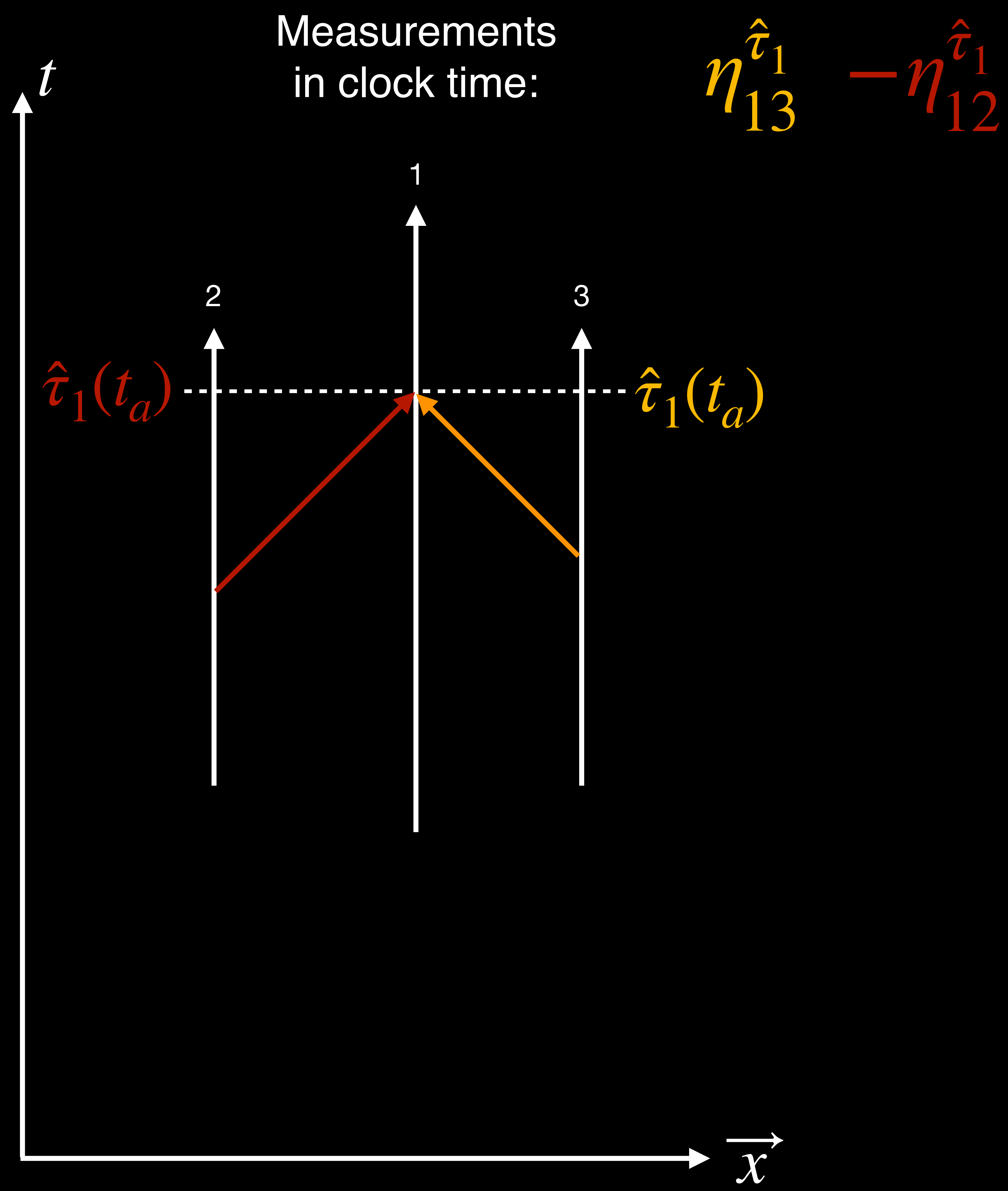




24

# Geometric TDI with clock times

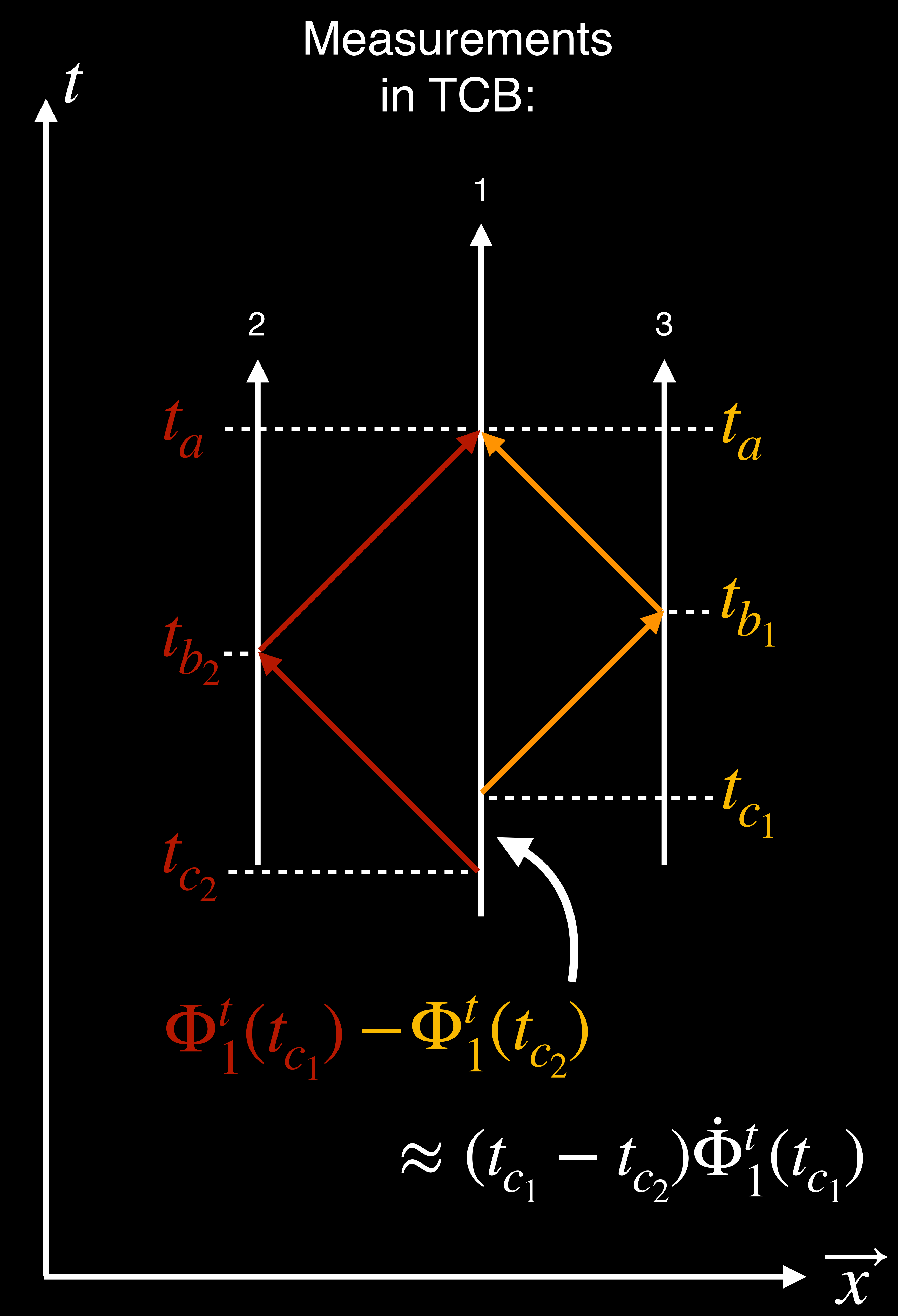


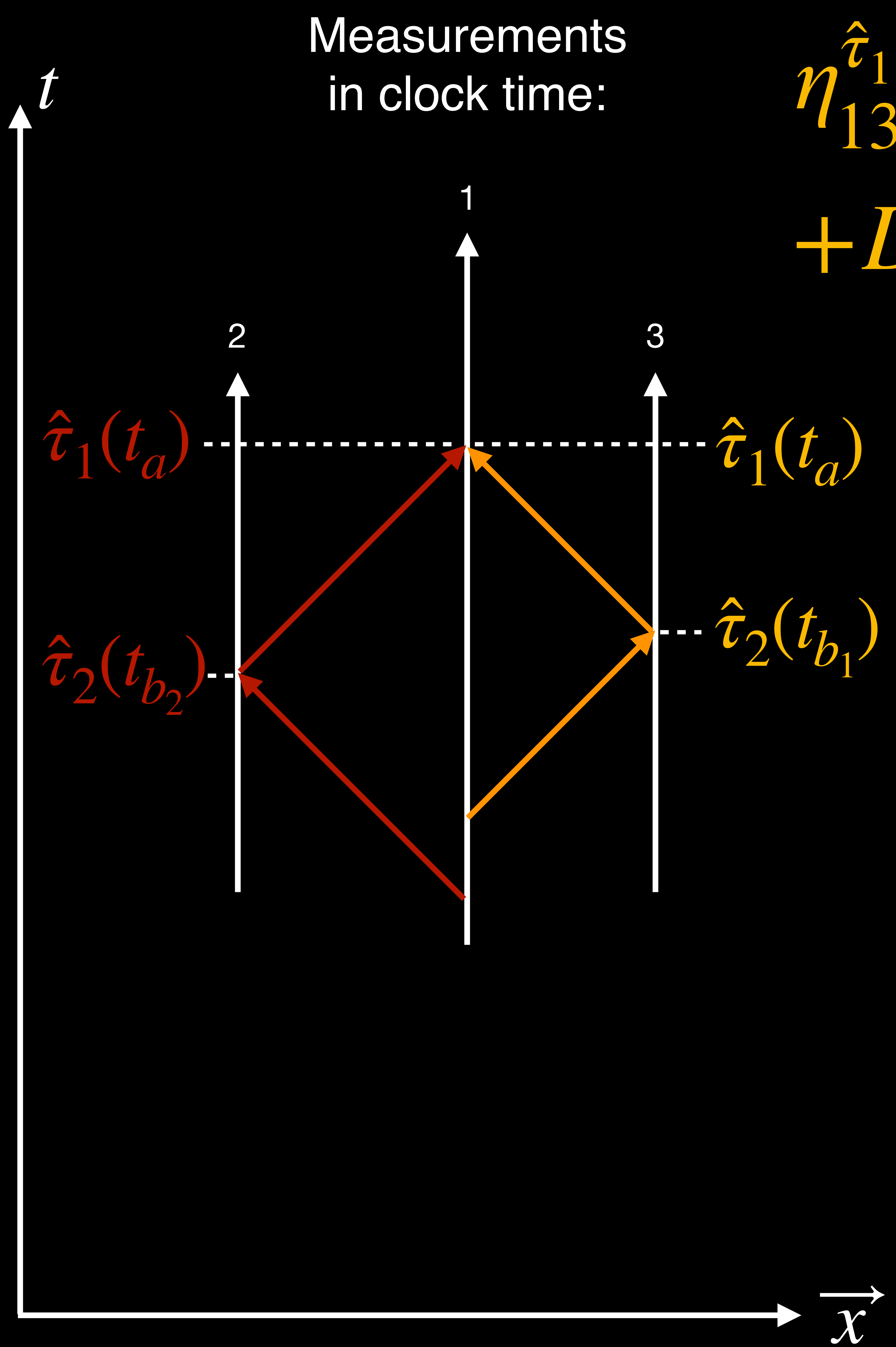





24

# Geometric TDI with clock times

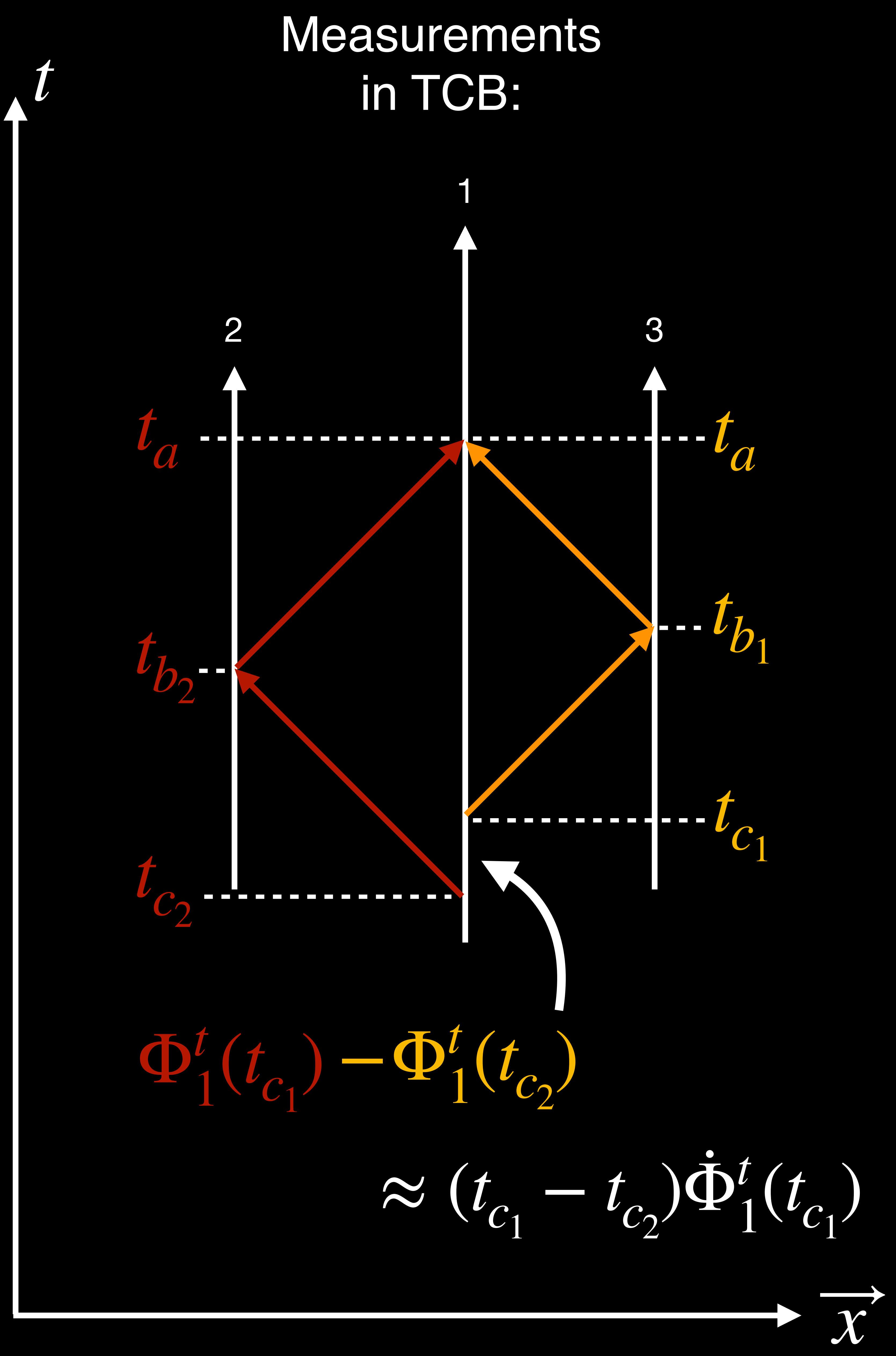


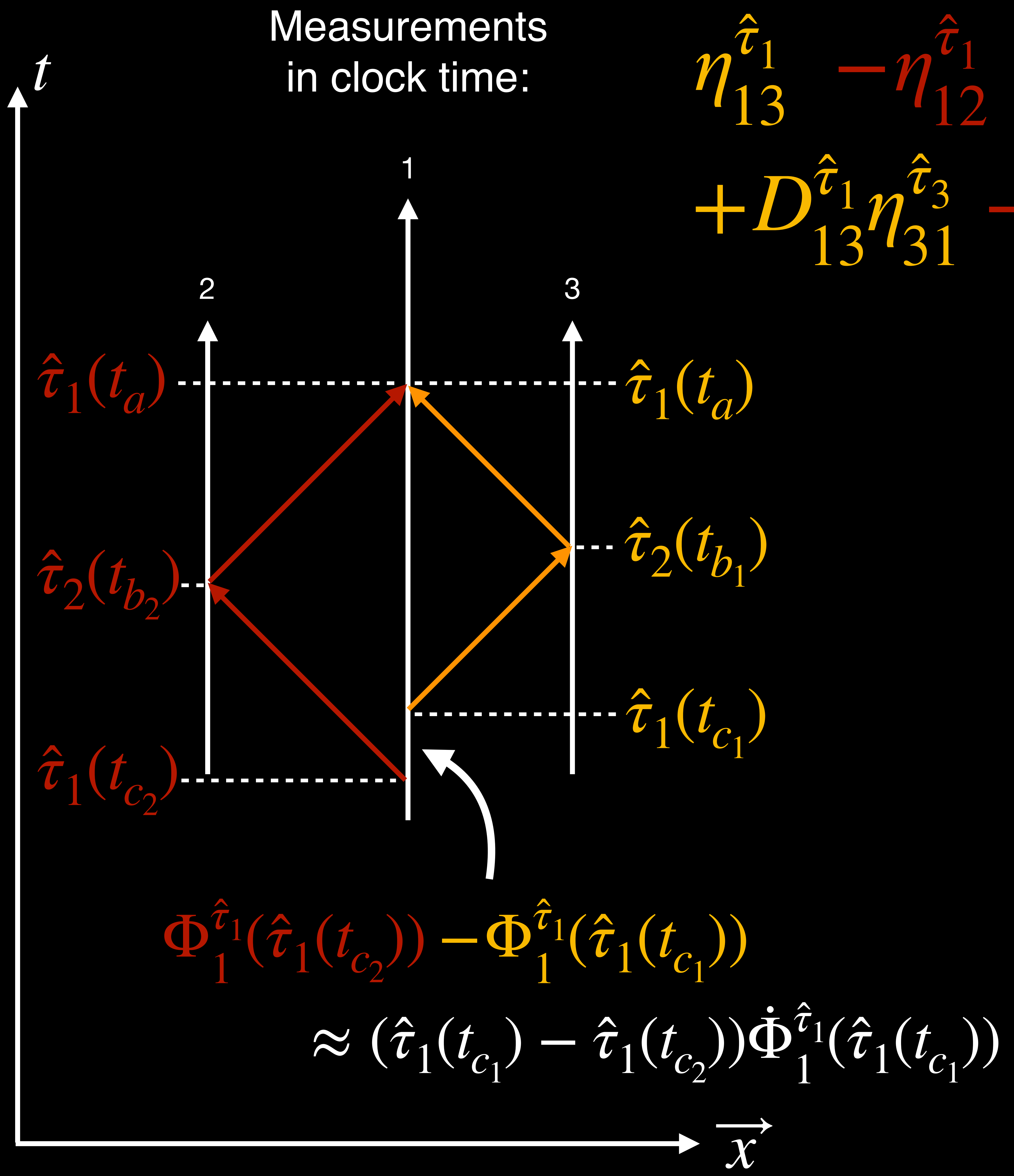




$$\eta_{13}^{\hat{\tau}_1} - \eta_{12}^{\hat{\tau}_1} + D_{13}^{\hat{\tau}_1}\eta_{31}^{\hat{\tau}_3} - D_{12}^{\hat{\tau}_1}\eta_{21}^{\hat{\tau}_2}$$


24

# Geometric TDI with clock times



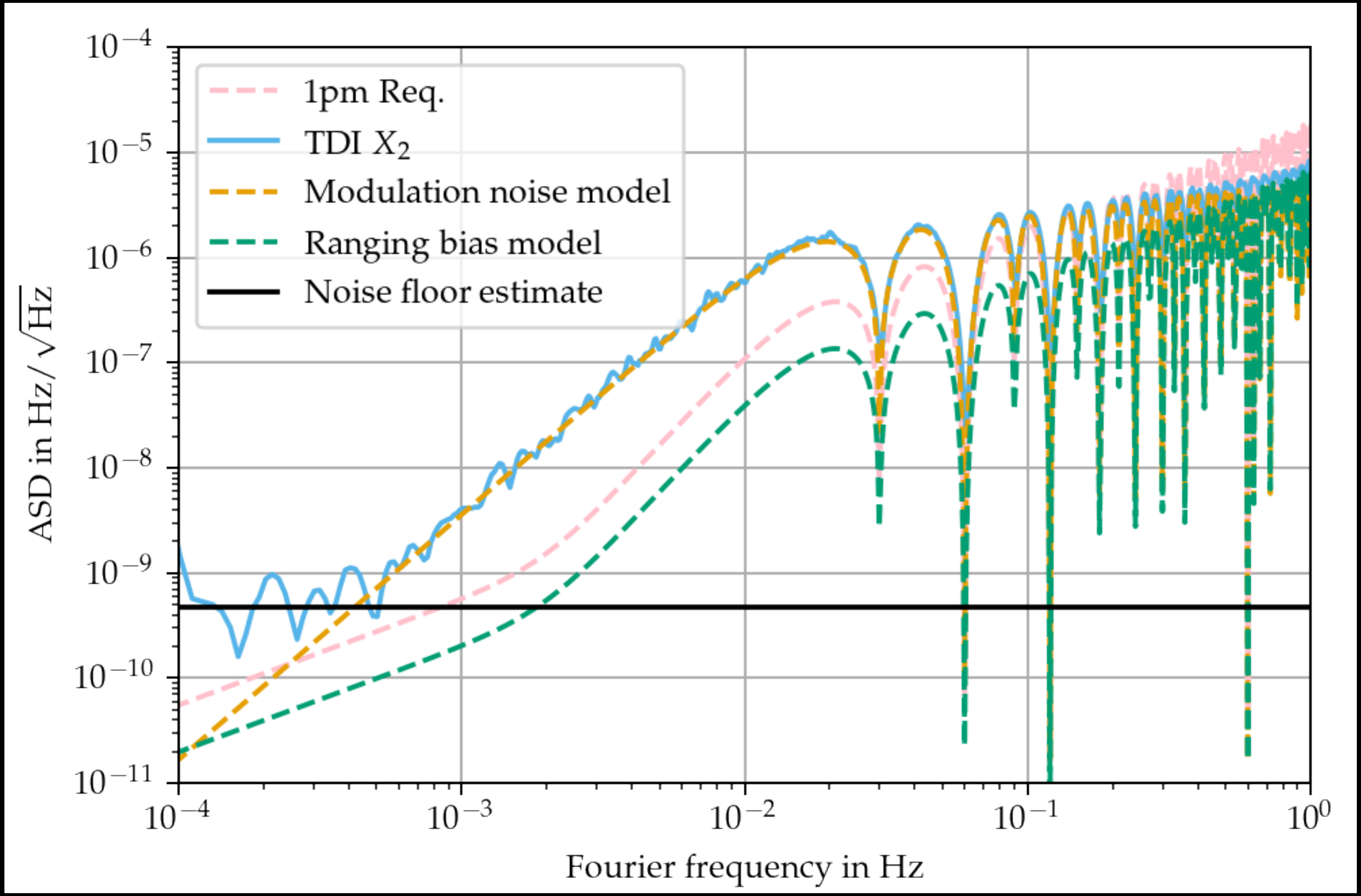




$$\eta^{\hat{\tau}_1}_{13} - \eta^{\hat{\tau}_1}_{12} + D^{\hat{\tau}_1}_{13}\eta^{\hat{\tau}_3}_{31} - D^{\hat{\tau}_1}_{12}\eta^{\hat{\tau}_2}_{21}$$


24

# TDI with desynchronized clocks + total frequency: performance

- Perform simulation with:
  - Realistic orbits
  - Realistic laser, clock, sideband and PRN noise
  - Neglect ultimately limiting secondary noises
- Performance is unaffected by large clock drifts + offsets
- Sideband noise enters identical to previous studies with frequency fluctuations + dedicated clock correction step
- Numerics are a problem: double precision not quite enough for 1 pm across whole band




Paper in preparation with J.B. Bayle, M. Staab and the SYRTE Theory and Metrology group

# Conclusion

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- TDI variables can be constructed directly from the unsynchronised data
- The necessary delays can be directly extracted from the onboard ranging measurements at 50 fs/$\sqrt{\mathrm{Hz}}$ precision
- This allows to simplify the L0-L1 pipeline
- Remark: Resulting TDI data still needs to be synchronised to TCB, but at much lower precision.