Modelling proto-neutron star evolution

LUTh's student day

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LUTh (Laboratoire Univers et Théories), Observatoire de Paris, PSL

The formation of neutron stars

The core-collapse mechanism : infall

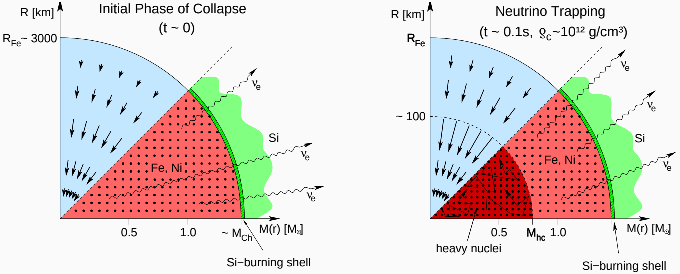


Figure 1: Core-collapse mechanism, figure extracted from Janka et al. (2007)

Iron core beyond the Chandrasekhar mass $M_{\mathrm{Ch}} \approx 1.2\,\mathrm{M}_{\odot} \Rightarrow$ collapse

Electron captures during the infall :

$$ {}^{A}_{Z}\mathrm{X} + e^{-} \rightarrow {}^{\quad A}_{Z-1}\mathrm{Y} + \nu_e $$

The limit of the zone at high densities and temperatures in which neutrinos are *trapped* because of their low mean free path is called the *neutrinosphere*

The core-collapse mechanism : bounce and shock

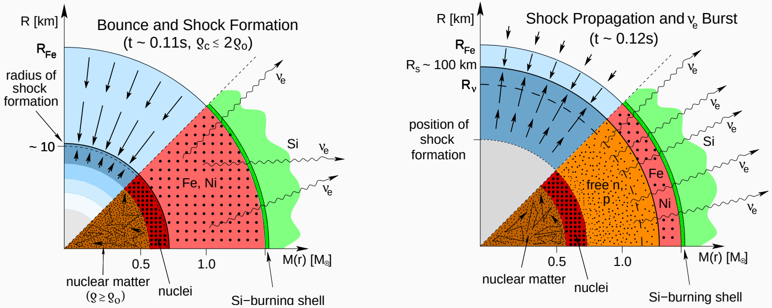


Figure 2: Core-collapse mechanism, figure extracted from Janka et al. (2007)

density of roughly nuclear saturation : $n_0 = 0.16\,\text{fm}^{-3}$

$\Rightarrow$ nuclei dissociation, core bounce and shock generation

shock propagation ν-burst when the shock reaches the neutrinosphere

exhaustion of the shock by dissociation of infalling material

The core-collapse mechanism : shock stalling and revival

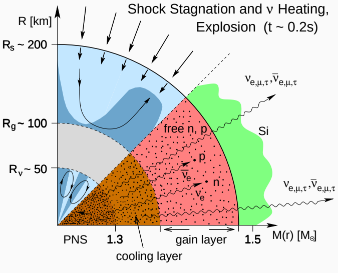


Figure 3: Core-collapse mechanism, figure extracted from Janka et al. (2007)

shock stalling and accretion

ν-heating (coupled with SASI and strong asymmetries) $\Rightarrow$ possible revival of the shock and final explosion

Proto Neutron Star and ν-emission

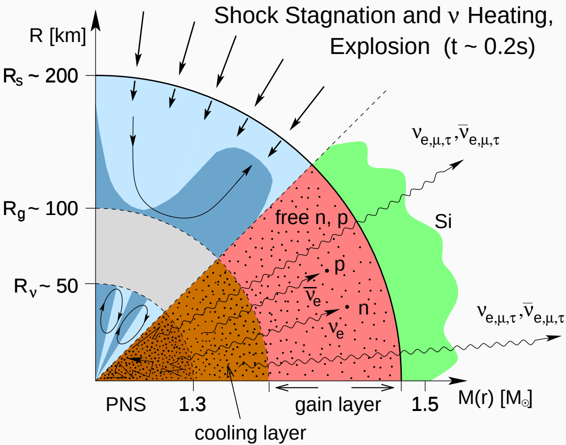


Figure 4: Core-collapse mechanism, figure extracted from Janka et al. (2007)

Relevant weak processes occuring during core-collapse

Neutrinos absorption/emission via charge exchange

$$p + e^- \leftrightarrows n + \nu_e \qquad p \leftrightarrows n + e^+ + \nu_e$$

$$n + e^+ \leftrightarrows p + \bar{\nu}_e \qquad n \leftrightarrows p + e^- + \bar{\nu}_e$$

$${}^{A}_{Z}\mathrm{X} + e^- \leftrightarrows {}_{Z-1}^{\;\;A}\mathrm{Y} + \nu_e$$

Thermal pair production of neutrinos

$$e^- + e^+ \leftrightarrows \nu + \bar{\nu}$$

$$N + N \leftrightarrows N + N + \nu + \bar{\nu} \quad \text{(nucleon bremsstrahlung)}$$

Neutrino scattering

$$N + \nu \leftrightarrows N + \nu$$

$${}^{A}_{Z}\mathrm{X} + \nu \leftrightarrows {}^{A}_{Z}\mathrm{X} + \nu$$

$$e^{\pm} + \nu \leftrightarrows e^{\pm} + \nu$$

Proto-neutron star structure

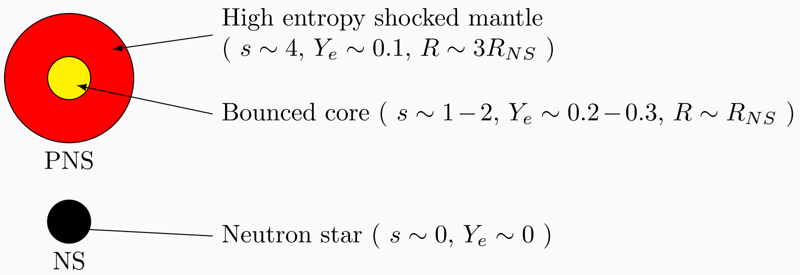


Figure 5: Schematic representation of a proto neutron star structure (PNS), compared to the corresponding cold catalysed neutron star (NS)

PNS cooling : $T_{PNS} \sim 10\,\text{MeV}$ $(10^{11}\,\text{K}) \Rightarrow T_{NS} \sim 10\,\text{keV}$ $(10^{8}\,\text{K})$

main mechanism : energy loss and deleptonization via emission of ν_e,ν_μ,ν_τ
$\Rightarrow$ mantle contraction with Kelvin-Helmoltz mechanism :
cooling via radiation $\rightarrow$ heating via contraction $\rightarrow$ cooling...

Relevant timescales

Acoustic timescale :

$$t_{\text{ac}} = \frac{R}{c_{\text{sound}}} = \left(\frac{R}{10\,\text{km}}\right)\left(\frac{c_{\text{sound}}}{10^{8}\,\text{m}\,\text{s}^{-1}}\right)^{-1} \times 10^{-1}\,\text{ms}$$

Deleptonization timescale :

$$t_{\text{delep}} = \frac{Y_e N_B}{L_{\nu,n}} \approx \left(\frac{Y_e}{0.2}\right)\left(\frac{M}{1.6\,\text{M}_{\odot}}\right)\left(\frac{L_{\nu,n}}{10^{55}\,\text{s}^{-1}}\right)^{-1} \times 30\,\text{s}$$

Where N_B is the total baryon number, M is the total mass and $L_{\nu,n}$ the total neutrino number-luminosity.

Kelvin-Helmholtz (star contraction) timescale :

$$t_{\text{KH}} = \frac{GM^2}{RL_{\nu,e}} \approx \left(\frac{M}{1.6\,\text{M}_{\odot}}\right)^{2}\left(\frac{R}{10\,\text{km}}\right)^{-1}\left(\frac{L_{\nu,e}}{10^{52}\,\text{erg}\,\text{s}^{-1}}\right)^{-1} \times 30\,\text{s}$$

Where $L_{\nu,e}$ is the total luminosity.

Timescales

$$t_{\text{ac}} = 10^{-1}\,\text{ms}$$
$$t_{\text{delep}} = 30\,\text{s}$$
$$t_{\text{Kelvin-Helmoltz}} = 30\,\text{s}$$

We want to simulate $\sim 60\,\text{s}$ but the acoustic timescale limits timesteps to $\delta t \sim 10\,\mu\text{s}$

$\Rightarrow$ we use a quasi-stationary approximation to average acoustic effects and evolve the PNS over KH-time

Open questions on PNS evolution

- how do uncertainties on microphysics (EoS and weak cross sections) influence the cooling ?
- how and when the NS does the crust form ? and what influence does it have on cooling ?
- what is the influence of the neutrino transport scheme
- to which extent convection effects contributes to the cooling ?
- what are the effects of rotation (meridional circulation, horizontal turbulence, magneto-dynamo...)
- what is the GW emission of a PNS ?

PNS modelling within the quasi-stationnary approximation

Hydrostatic approximation

We assume the star contracts slowly :
$\frac{\partial n_B}{\partial t} \sim 0$, $\frac{\partial p}{\partial t} \sim 0$, $\frac{\partial g_{\mu\nu}}{\partial t} \sim 0$

(but we still have $\frac{\partial s}{\partial t} \neq 0$ and $\frac{\partial Y_e}{\partial t} \neq 0$!)

$\Rightarrow$ p is computed via the TOV equations

Closure is obtained with a hot equation of state for dense matter[1] :
$(p, s, Y_e) \mapsto$ density, temperature, composition, chemical potentials, ...

[1] Oertel et al. 2017

Hydrostatic equilibrium - TOV equation

Metric in spherical symmetry :

$$ds^2 = -\alpha^2 c^2 dt^2 + \psi^2 dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Einstein equations :

$$\frac{1}{\psi} = \sqrt{1 - \frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \frac{E}{c^2}$$

$$\frac{d\ln\alpha}{dr} = \psi^2 \frac{G}{c^2}\left(\frac{m}{r^2} + 4\pi r \frac{p}{c^2}\right)$$

Hydrostatic equilibrium equation :

$$\frac{dp}{dr} = -(E + p)\frac{d\ln\alpha}{dr}$$

Evolution equations

Despite the quasi-stationary approximation, we still have $\frac{\partial Y_e}{\partial t} \neq 0$ and $\frac{\partial s}{\partial t} \neq 0$ and we use evolution equations for Y_e and s to compute the next quasi-stationary state

The time evolution of Y_e and s comes from the source of electrons s_n and the source of energy s_e :

$$\nabla_\mu (n_B Y_e u^\mu) = s_n$$
$$u_\nu \nabla_\mu (T^{\mu\nu}) = s_e$$

which can be recasted as

$$\frac{1}{\alpha c} \frac{DY_e}{Dt} = \frac{s_n}{n_B}$$
$$\frac{1}{\alpha c} \frac{Ds}{Dt} = \frac{\alpha s_e - \mu_e s_n}{n_B T}$$

s_n and s_e have to be computed with a neutrino radiation-transfert scheme

Neutrino radiation-transfert scheme

we need the source terms for evolution :

$$s_n = -\frac{1}{c}\left(\Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}\right)$$
$$s_e = -\frac{1}{c}\left(Q_{\nu_e} + Q_{\bar{\nu}_e} + 4Q_{\nu_x}\right)$$

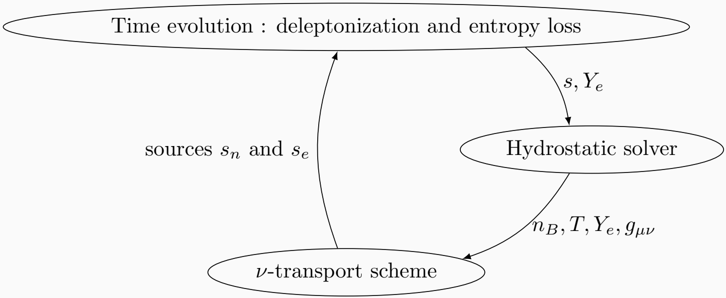
we use the Fast Multigroup Transport scheme[2] a *stationnary* approximation of the transport equation :

$$p^i \frac{\partial f}{\partial x^i} - \Gamma^i{}_{\mu\nu} p^\mu p^\nu \frac{\partial f}{\partial p^i} = u_\mu p^\mu \mathcal{B}[f]$$

at high optical depth we use the *two-stream approximation*
at low optical depth we use a *two-moment closure*

[2] Müller and Janka 2015.

The algorithm



Some exemples of results

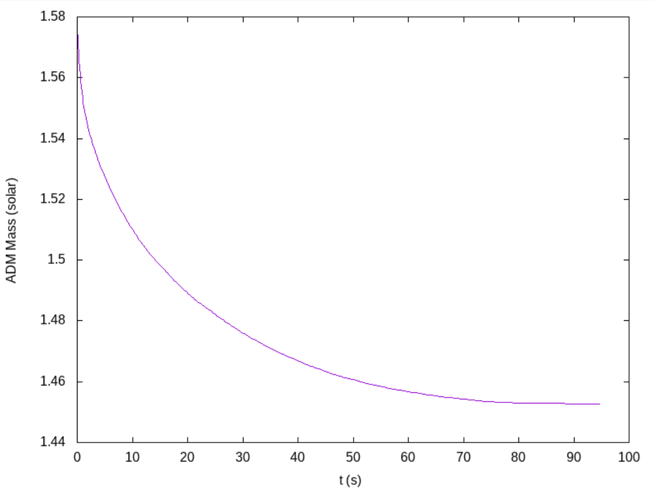


Figure 6: Evolution of the mass of the PNS

Some exemples of results

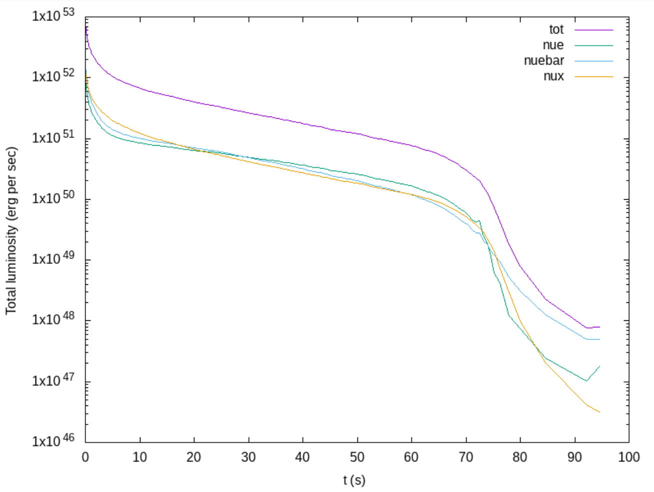


Figure 7: Evolution of the ν-luminosity of the PNS

Conclusion

- a code to model PNS cooling has been developed
- it is currently used to study influence of ν interaction rates and/or convection on the cooling
- currently writing a paper and the manuscript... you are invited to my PhD defense in June for more details on all this ;)