Figures of lecture 4

Geodesics and images in the Kerr metric

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https://relativite.obspm.fr/blackholes/paris23/

PSL graduate programs in Physics and in Astrophysics
ENS, Paris, France
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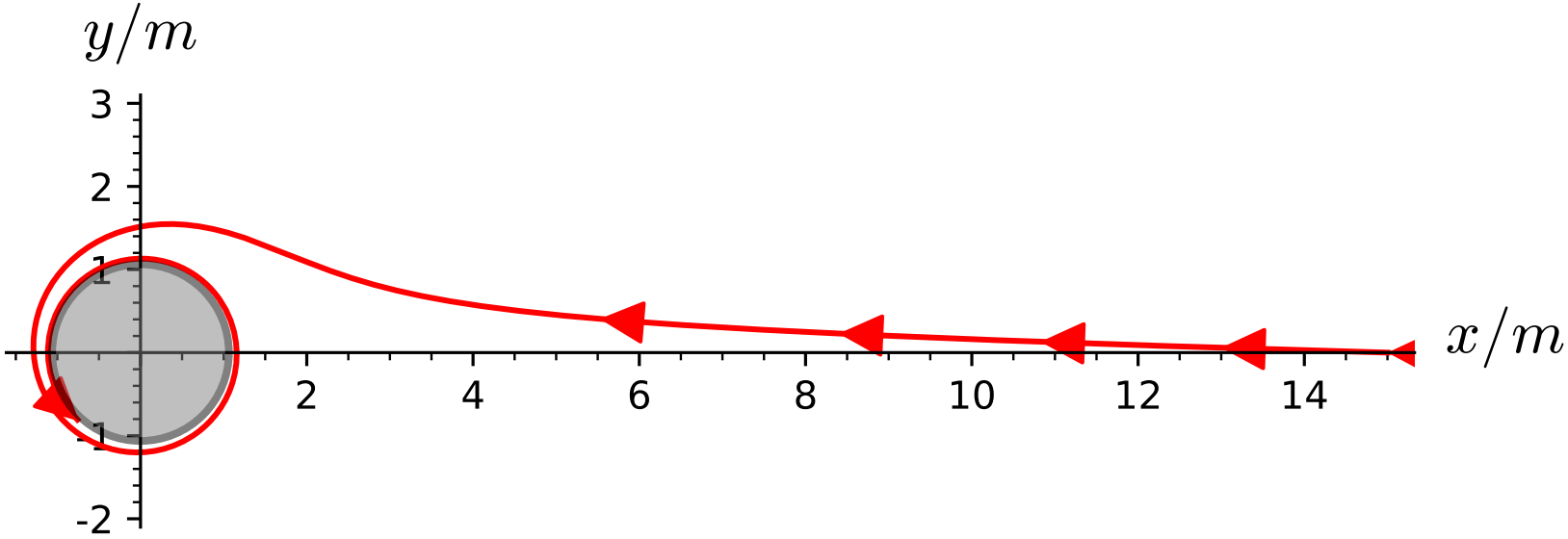
Home page for the lectures

https://relativite.obspm.fr/blackholes/paris23

includes

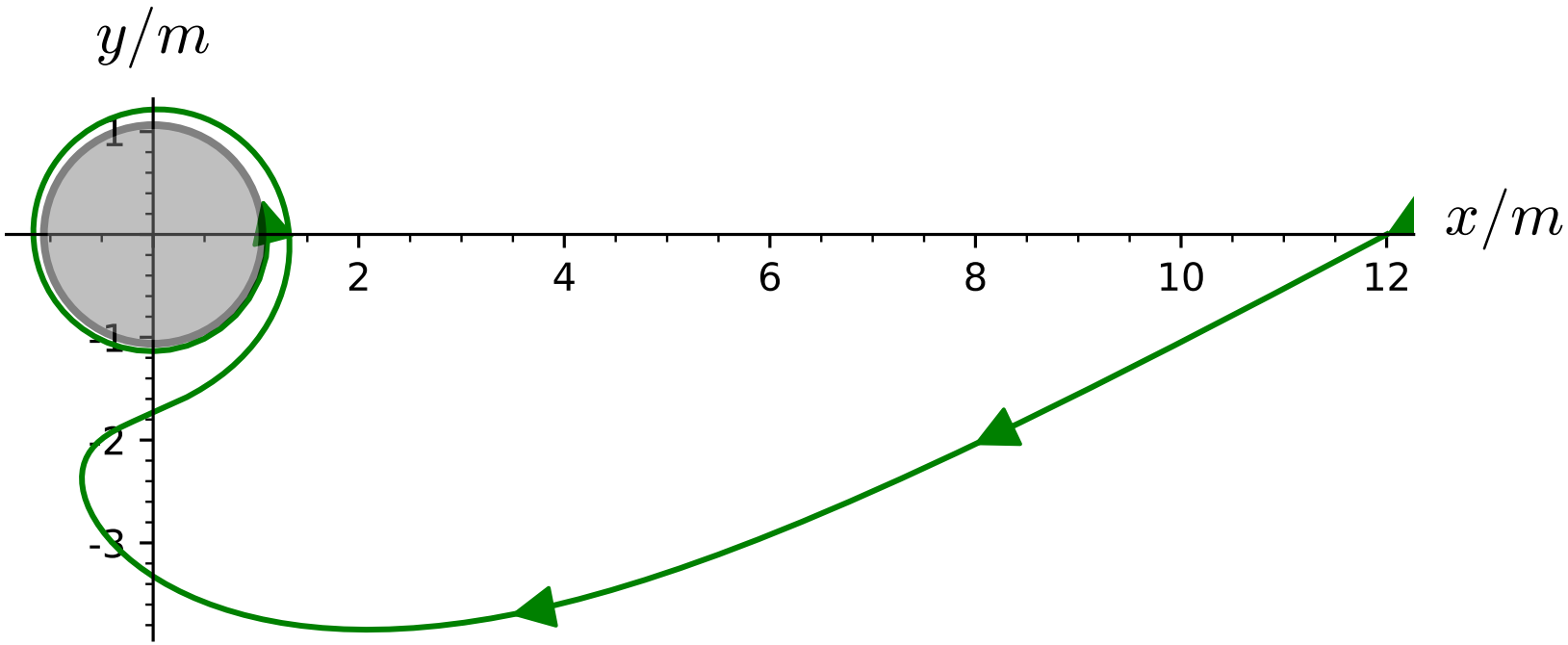
- the lecture notes (draft)
- some SageMath notebooks
- these slides

Lense-Thirring effect



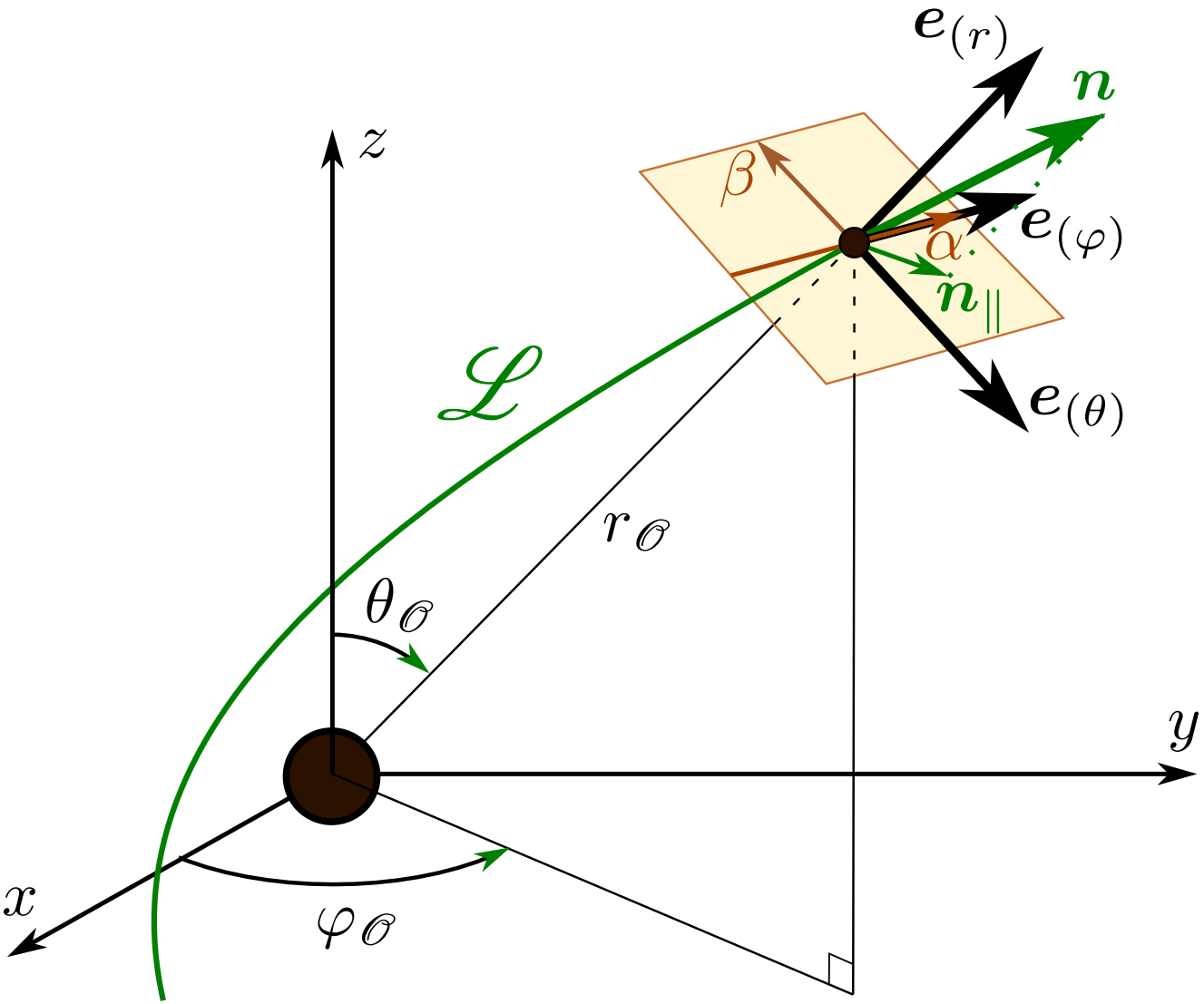
Trajectory in the equatorial plane of an incoming timelike geodesic with $E = \mu$, $L = 0$ and $Q = 0$, plunging into a Kerr black hole with $a = 0.998\, m$.

Lense-Thirring effect



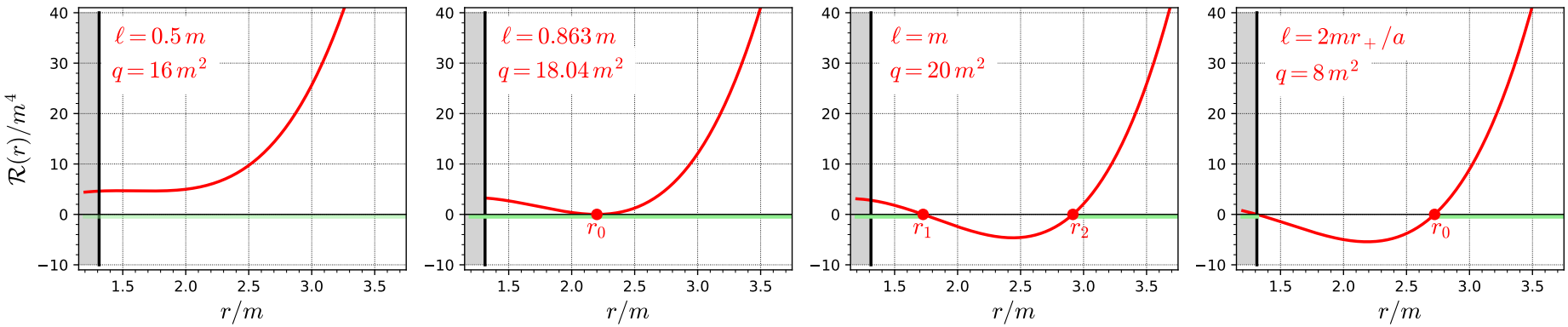
Trajectory in the equatorial plane of an incoming null geodesic with $L = -6E < 0$ and $Q = 0$, plunging into a Kerr black hole with $a = 0.998\,m$.

Screen coordinates (α, β) of remote observer



Impact of a null geodesic $\mathscr{L}$ onto the screen of a remote observer $\mathscr{O}$

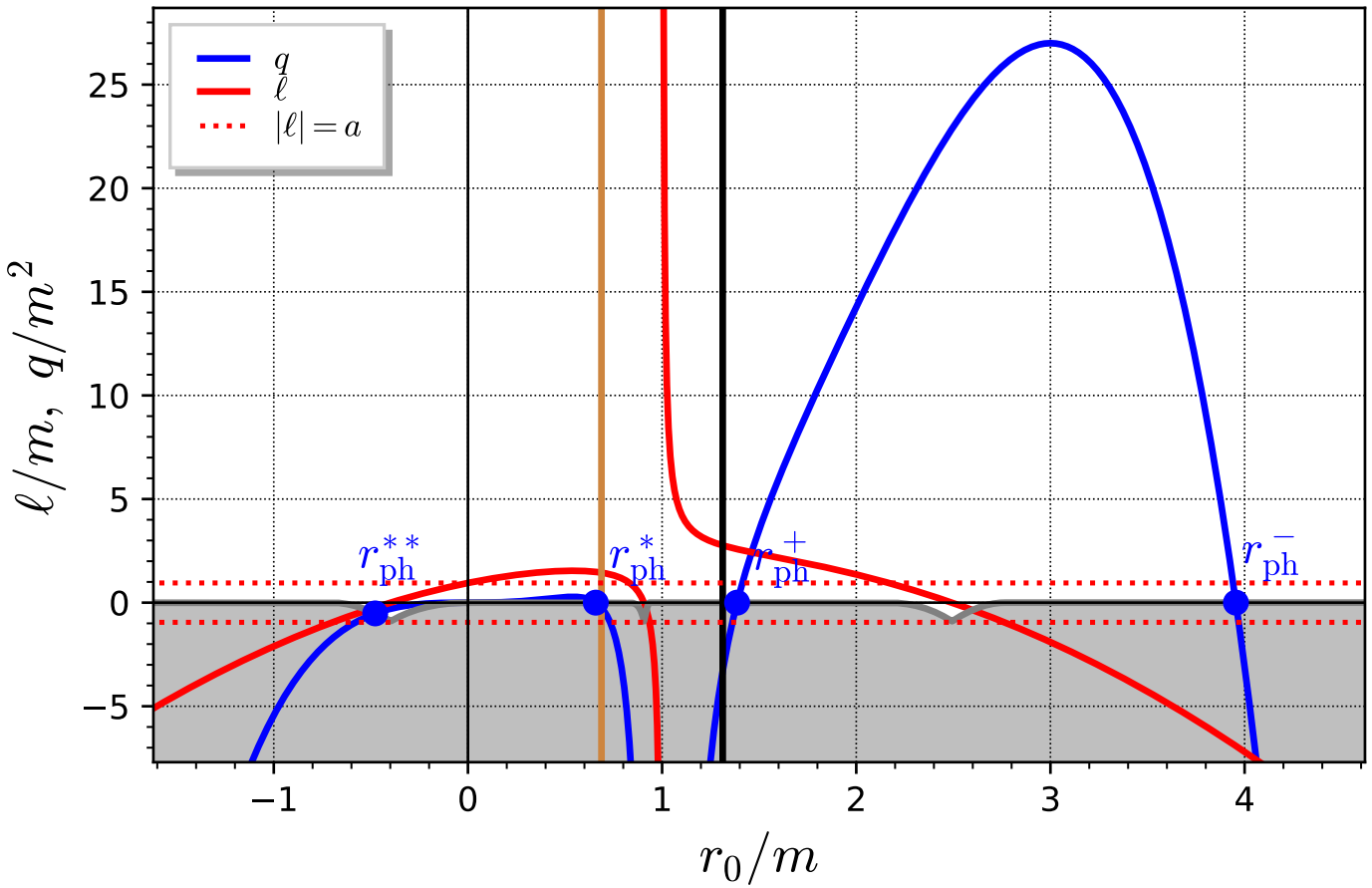
Quartic polynomial $\mathcal{R}(r)$ in $\mathscr{M}_{\mathrm{I}}$



(a) (b) (c) (d)

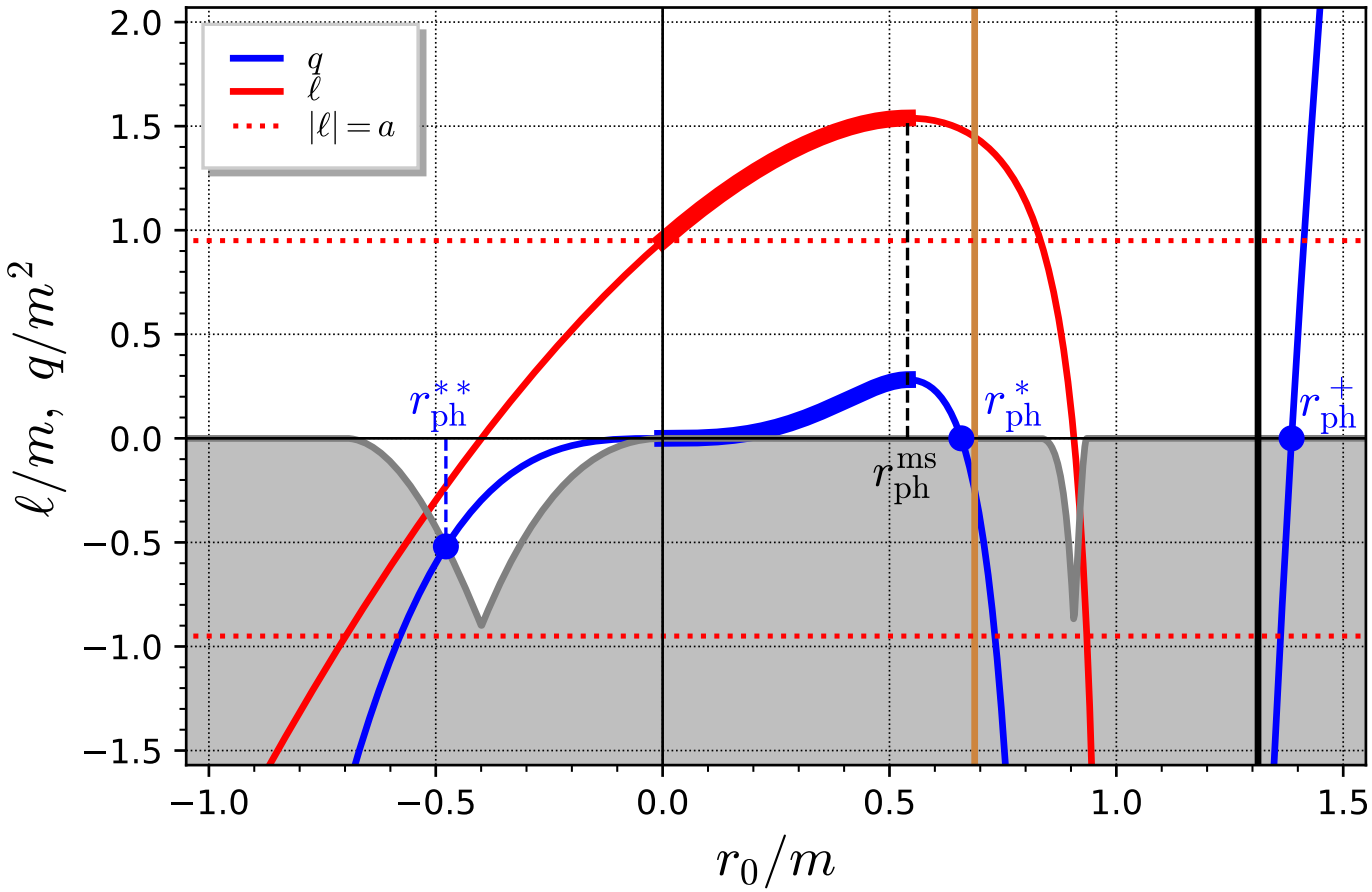
Quartic polynomial $\mathcal{R}(r)$ in the region $\mathscr{M}_{\mathrm{I}}$ for four values of (ℓ, q) and for $a = 0.95\,m$

Existence of spherical photon orbits



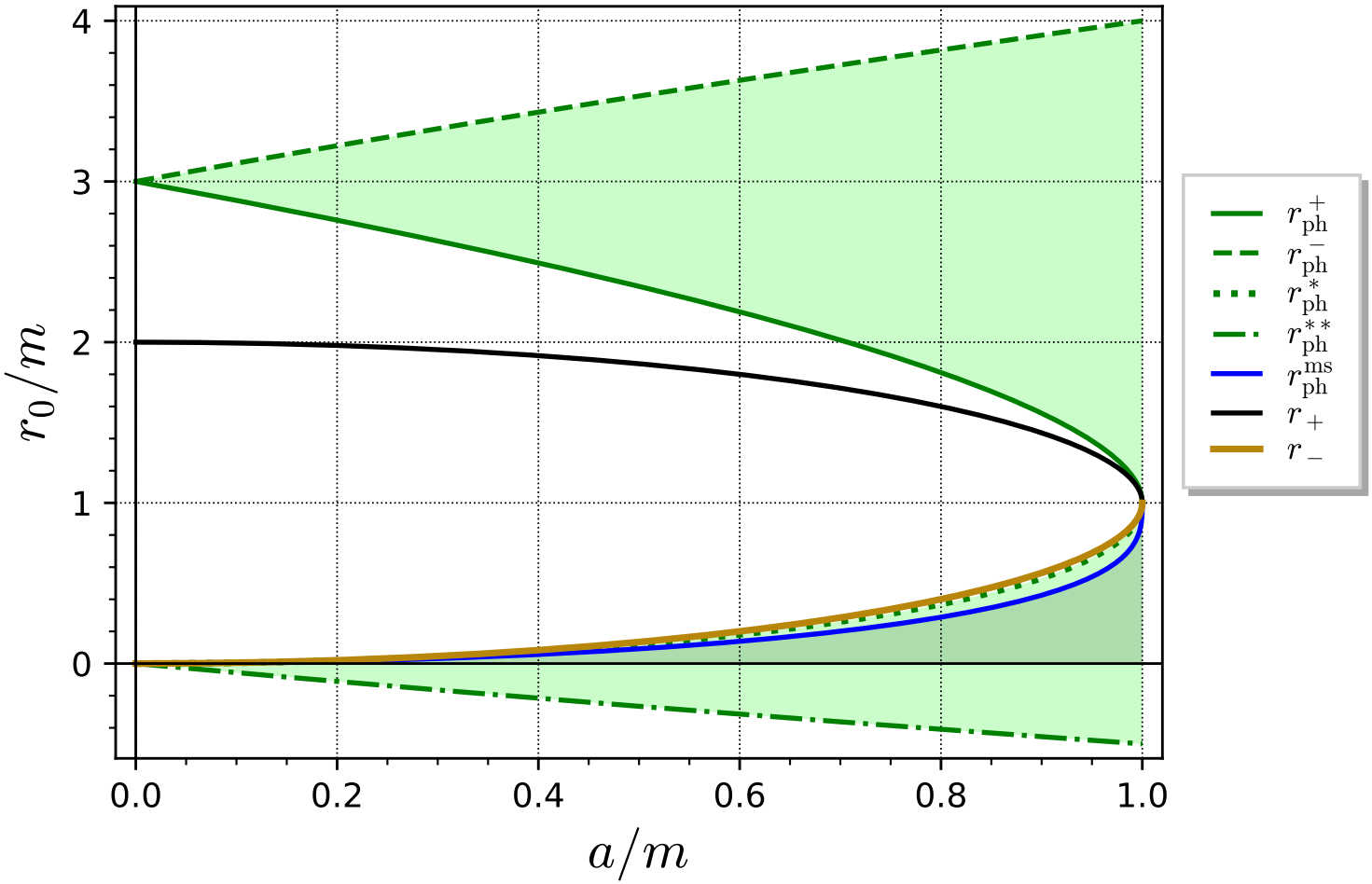
Functions $\ell_{\rm c}(r_0)$ (in red) and $q_{\rm c}(r_0)$ (in blue) giving the reduced angular momentum and reduced Carter constant of a spherical photon orbit of radius r_0 for $a = 0.95\,m$. Values of q in the grey zone are unphysical.

Existence of spherical photon orbits



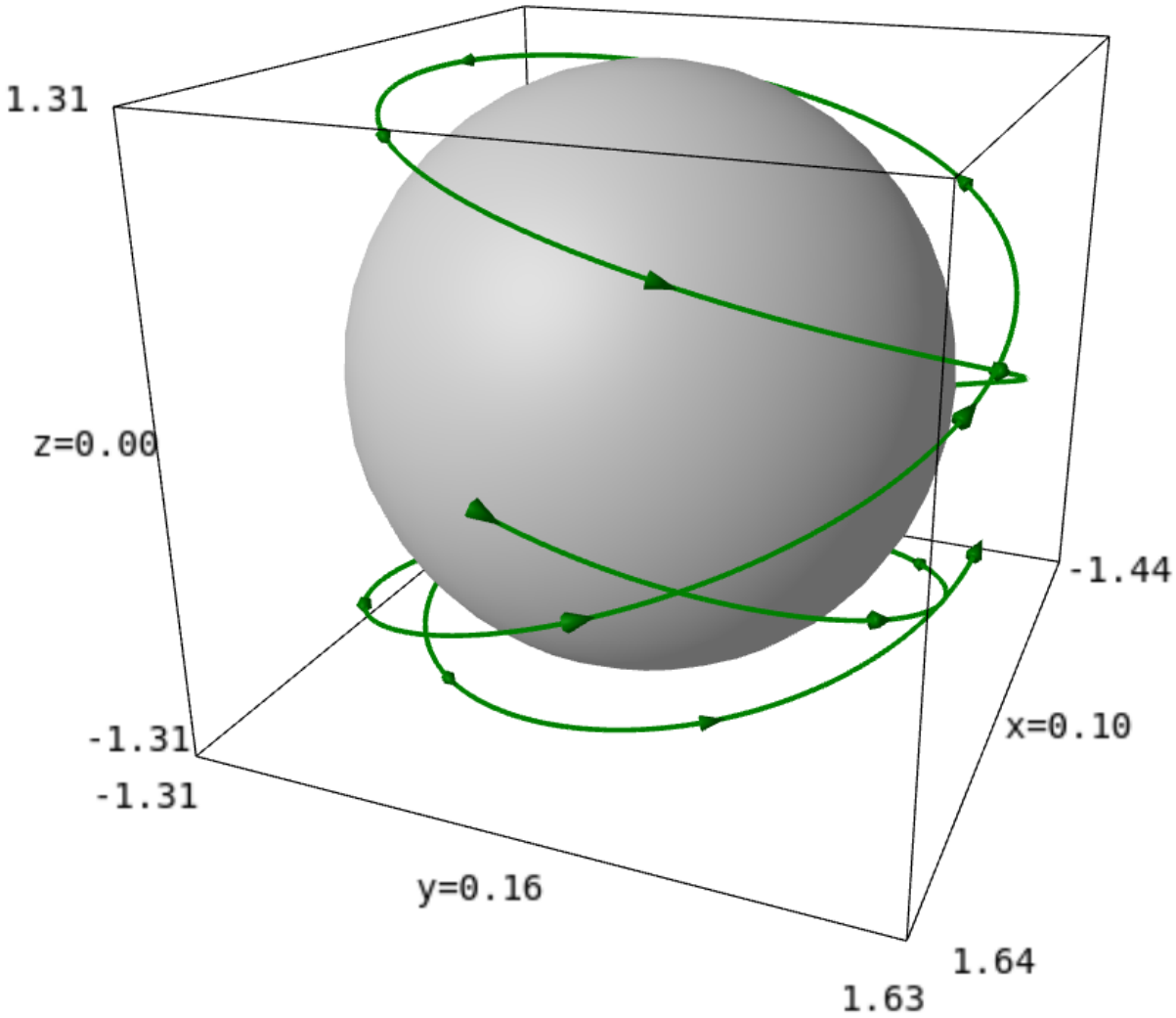
Functions $\ell_{\rm c}(r_0)$ (in red) and $q_{\rm c}(r_0)$ (in blue) giving the reduced angular momentum and reduced Carter constant of a spherical photon orbit of radius r_0 for $a = 0.95\,m$. Values of q in the grey zone are unphysical.

Range of spherical photon orbits



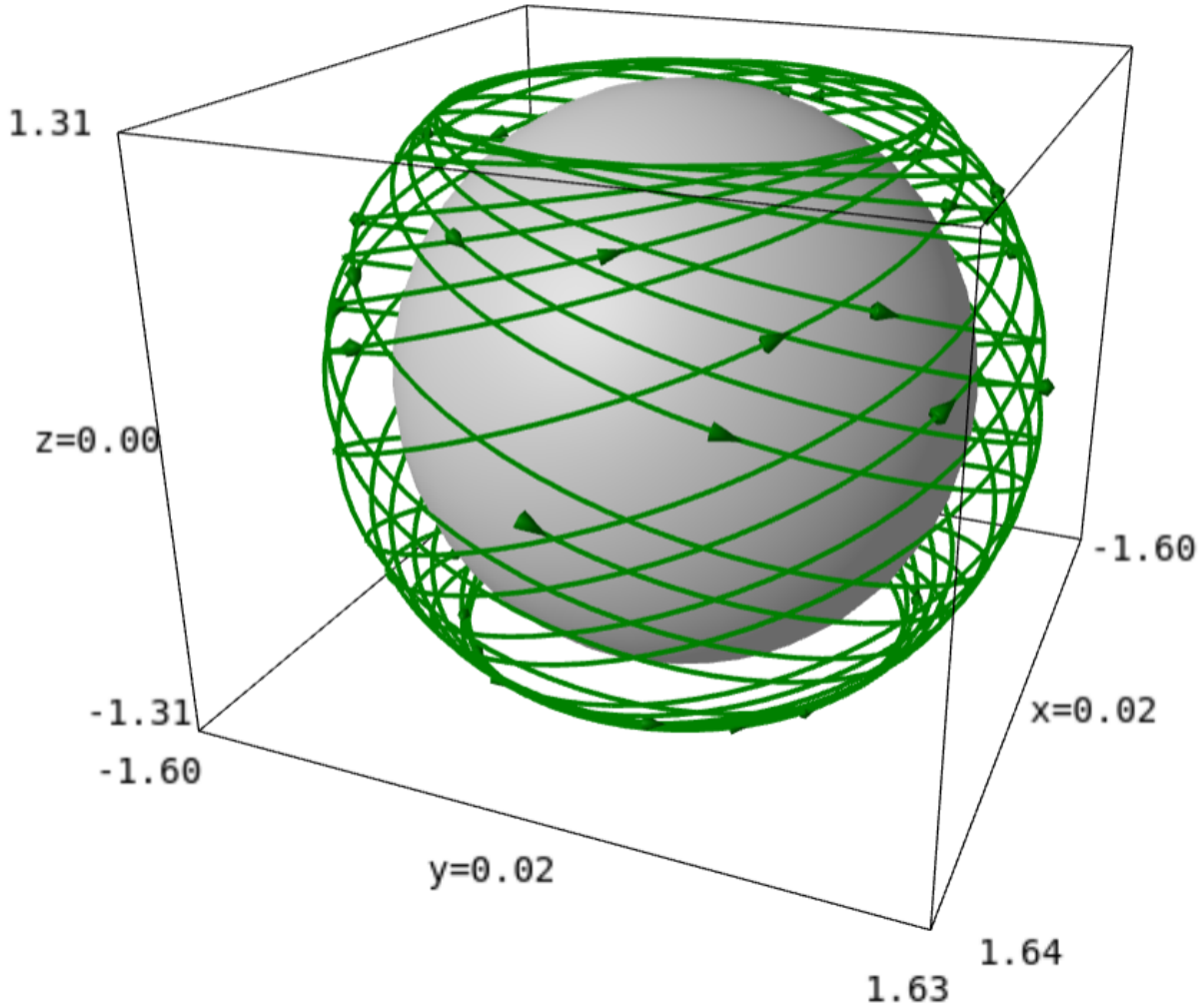
Domain of existence of spherical photon orbits in the (a, r_0) plane (in green). The shaded area correspond to stable spherical orbits.

Spherical photon orbit at $r_0 = 1.6\,m$



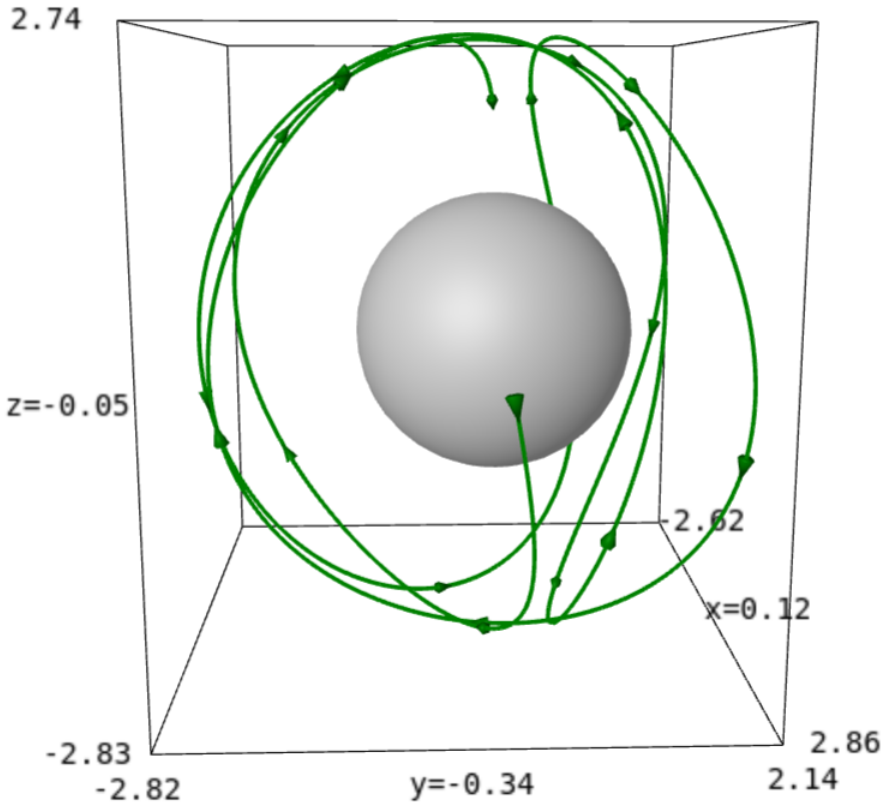
Spherical photon orbit at $r_0 = 1.6\,m$ around a Kerr black hole with $a = 0.95\,m$. Plot for $0 < \lambda < 7.7\,m/E$.

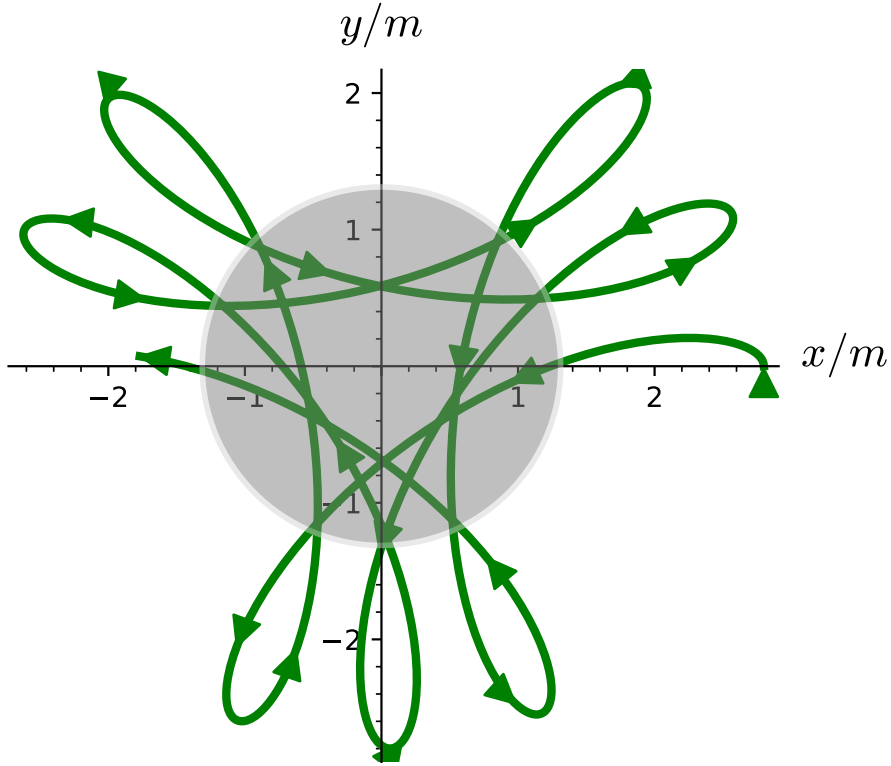
Spherical photon orbit at $r_0 = 1.6\,m$



Spherical photon orbit at $r_0 = 1.6\,m$ around a Kerr black hole with $a = 0.95\,m$. Plot for $0 \leq \lambda \leq 70\,m/E$.

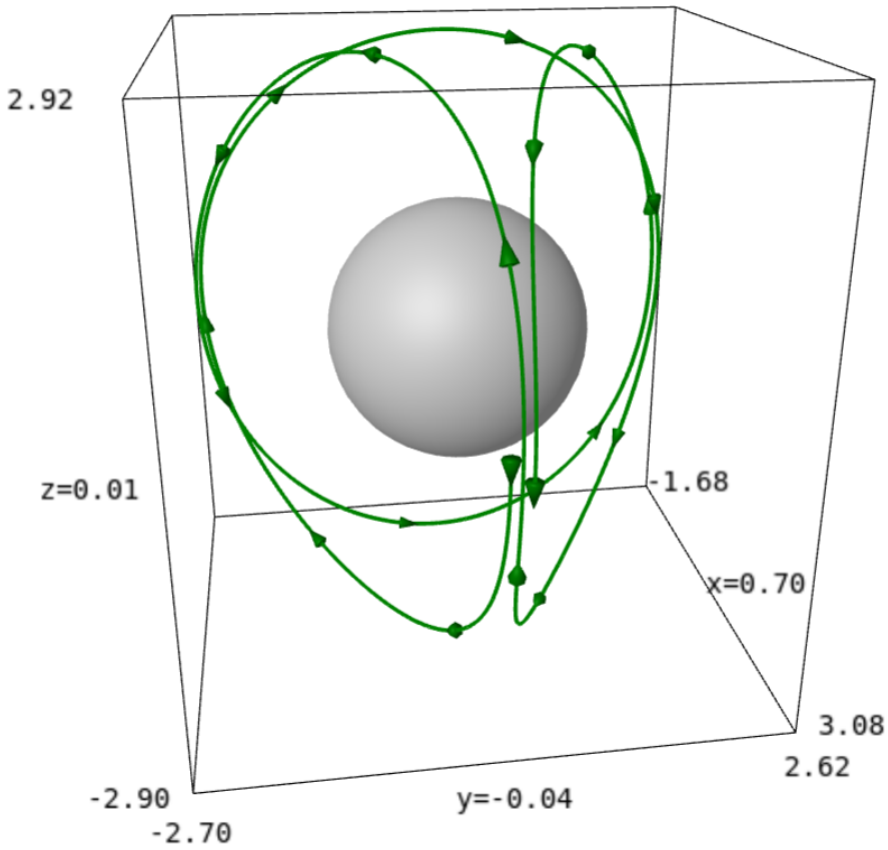
Spherical photon orbit at $r_0 = 2.8\,m$

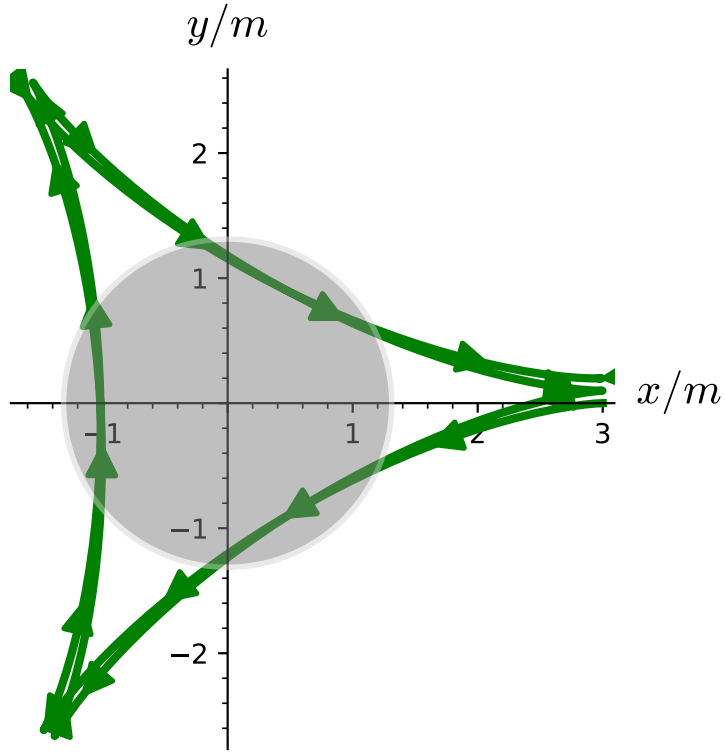




Spherical photon orbit at $r_0 = 2.8\,m$ around a Kerr black hole with $a = 0.95\,m$

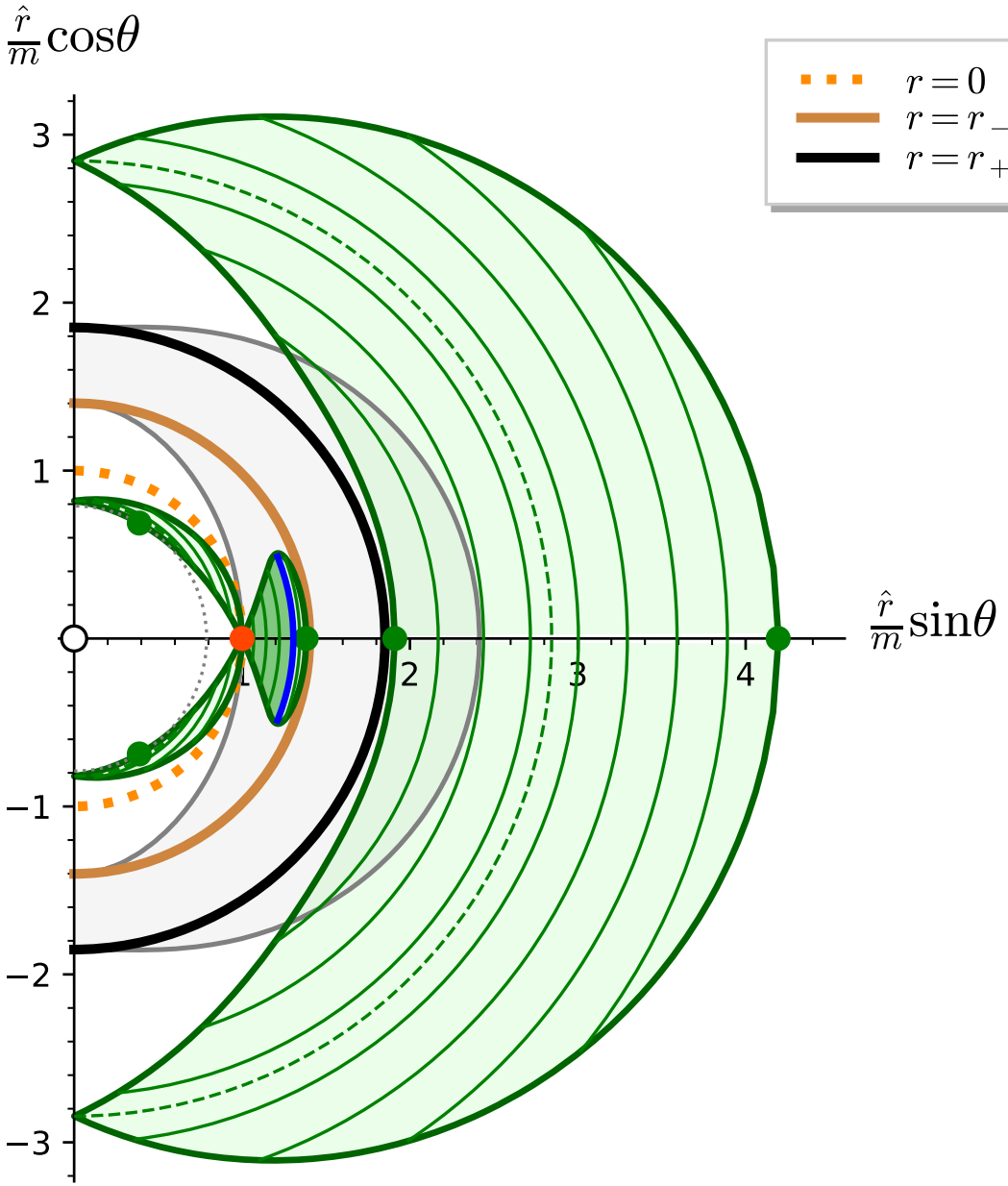
Spherical photon orbit at $r_0 = 3\,m$





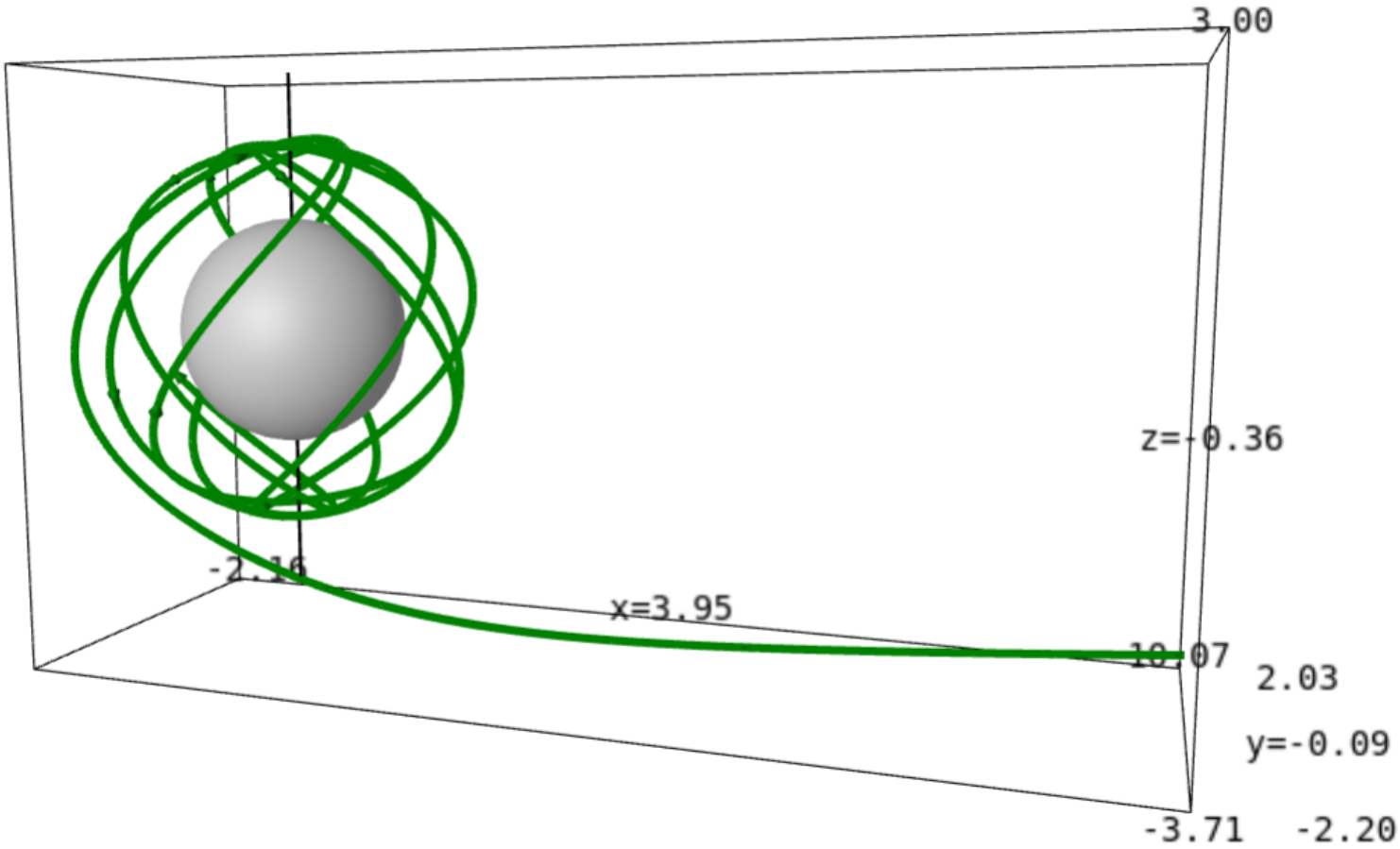
Spherical photon orbit at $r_0 = 3\,m$ around a Kerr black hole with $a = 0.95\,m$

Photon region



Trace of the photon region (pale and dark green areas) in a meridional plane $(t,\varphi)=\text{const}$ of a Kerr spacetime with $a=0.95\,m$

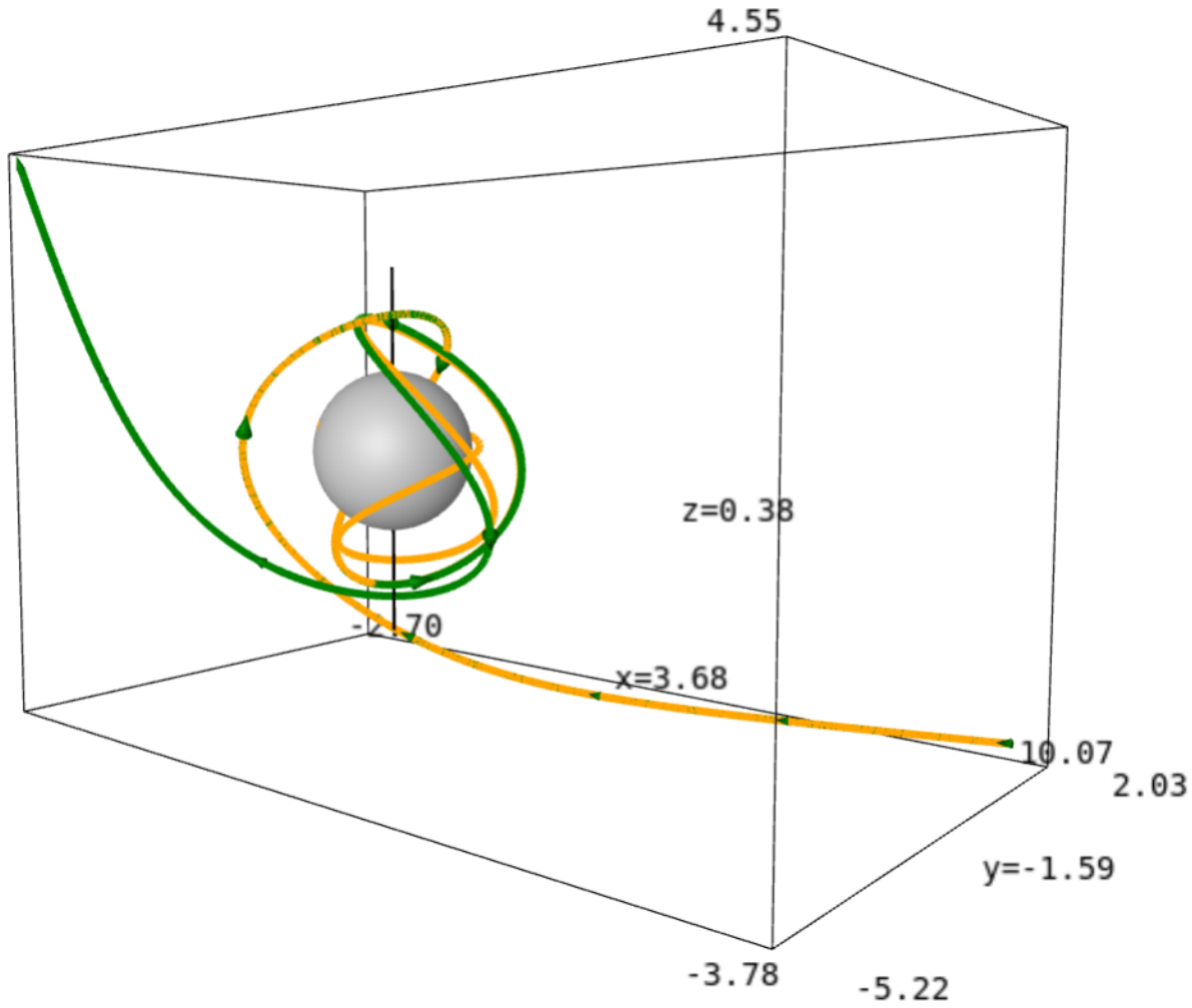
Critical null geodesic



Critical null geodesic of parameter $r_0 = 2.2\,m$ in a Kerr spacetime with $a = 0.95\,m$. For an interactive 3D view, see the SageMath notebook

https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Kerr_null_geod_plots.ipynb

Near-critical null geodesics

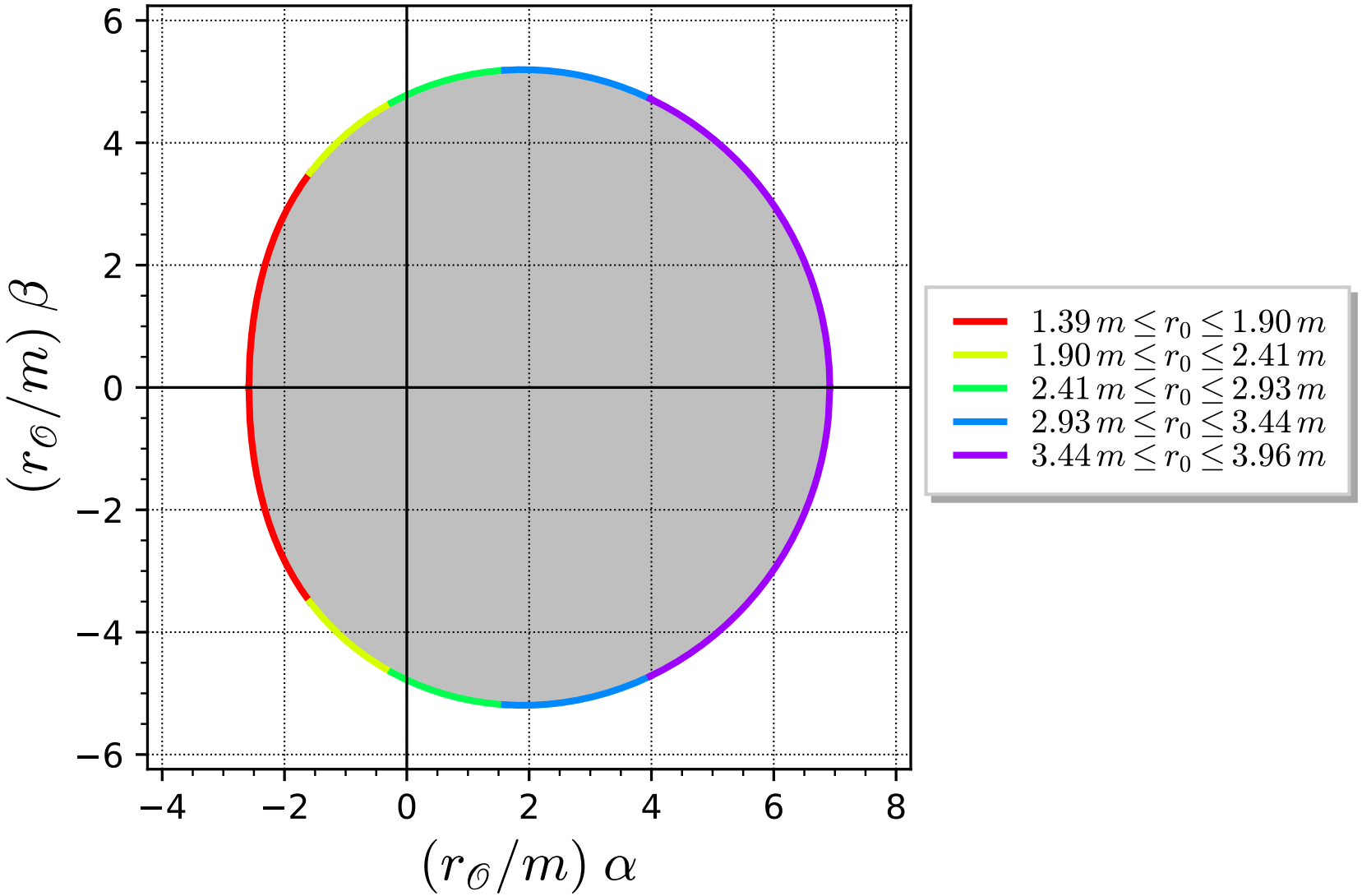


Both geodesics have $q = q_c(r_0)$ with $r_0 = 2.2\,m$ $(a = 0.95\,m)$
green: $\ell = 1.0001\,\ell_c(r_0)$, orange: $\ell = 0.9999\,\ell_c(r_0)$

https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/

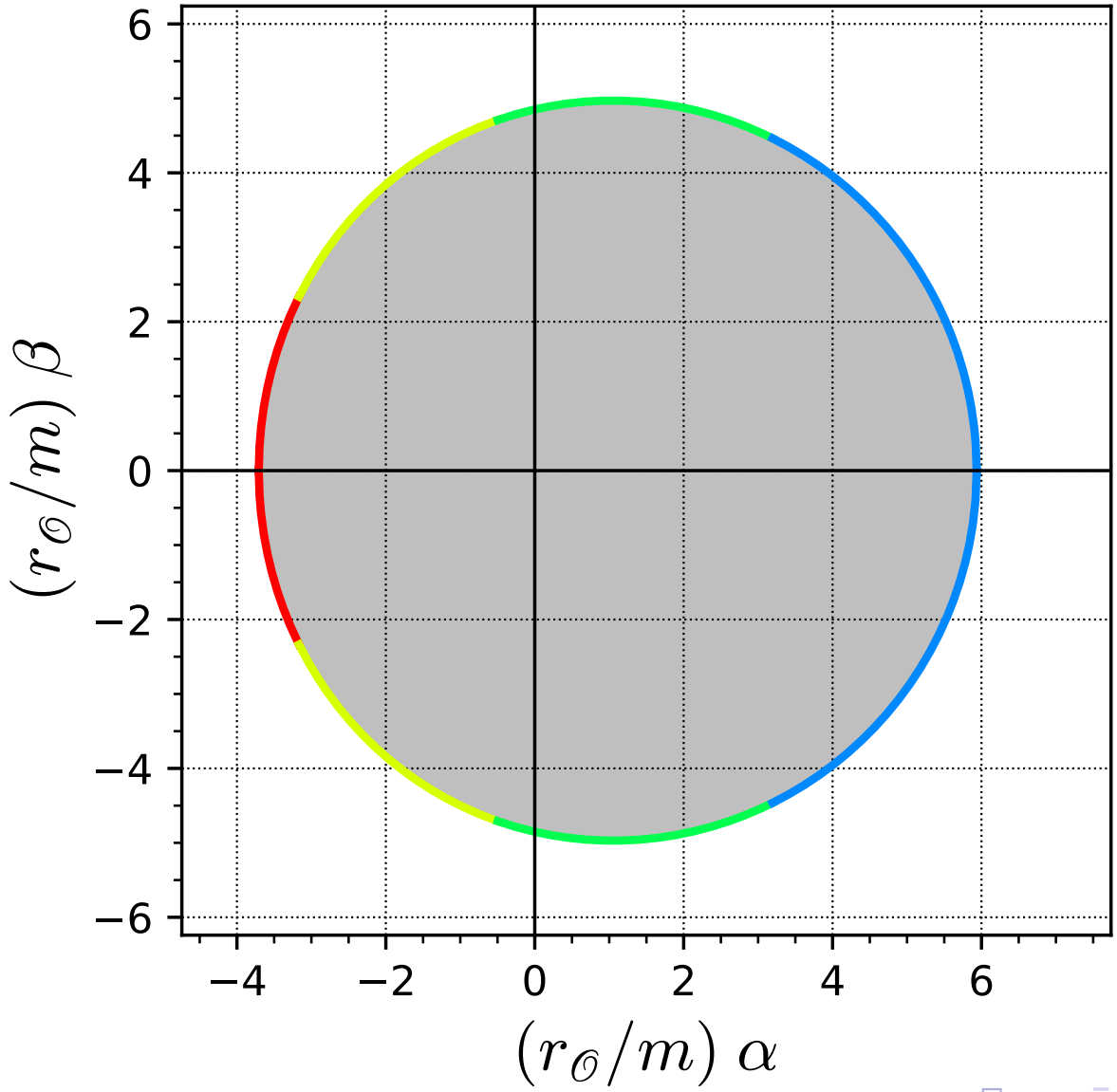
Critical curve on the observer's screen

$a = 0.95\,m$, $\theta_{\mathscr{O}} = \pi/2$



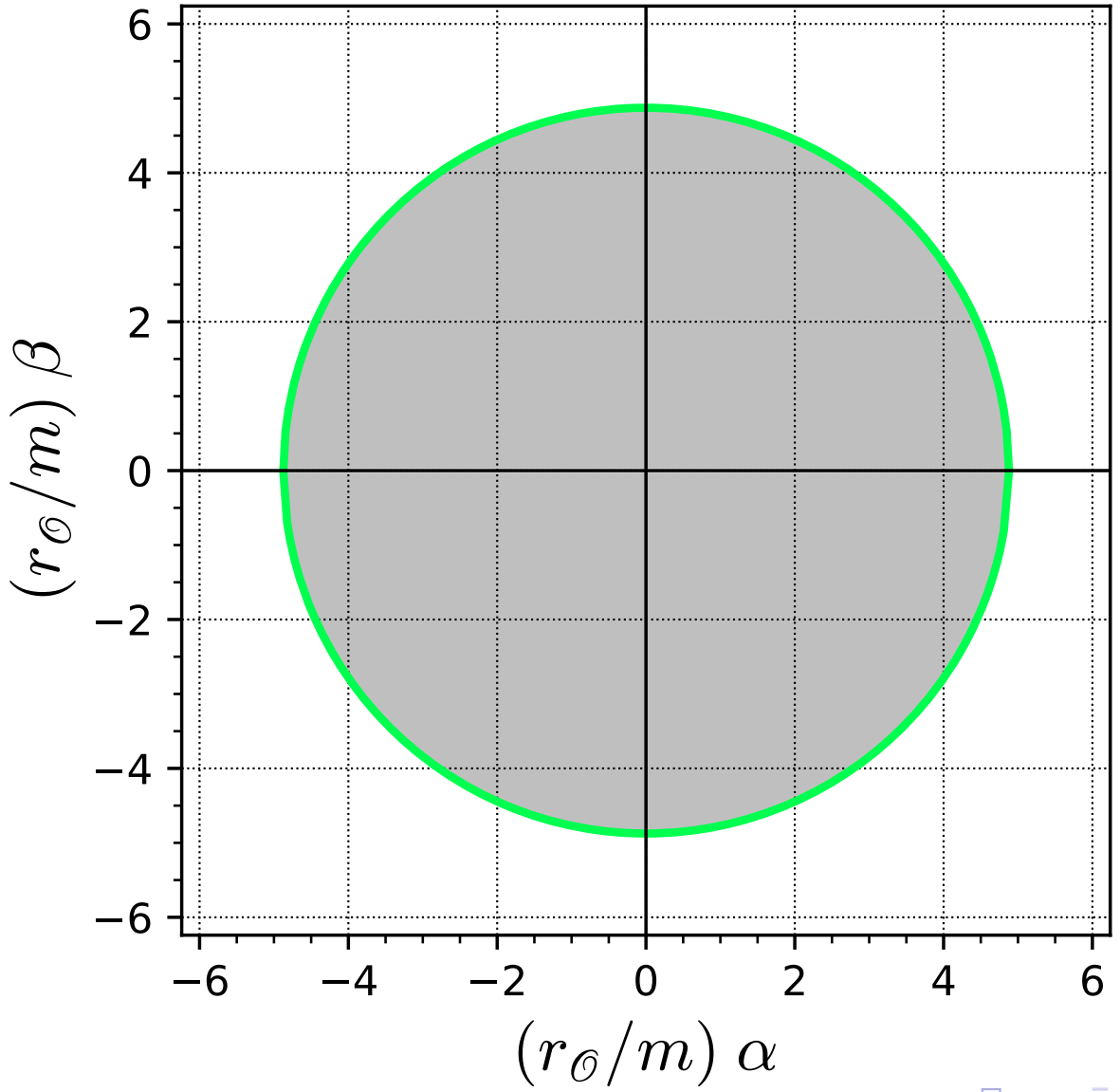
Critical curve on the observer's screen

$a = 0.95\,m$, $\theta_{\mathscr{O}} = \pi/6$



Critical curve on the observer's screen

$a = 0.95\,m$, $\theta_{\mathscr{O}} = 0$

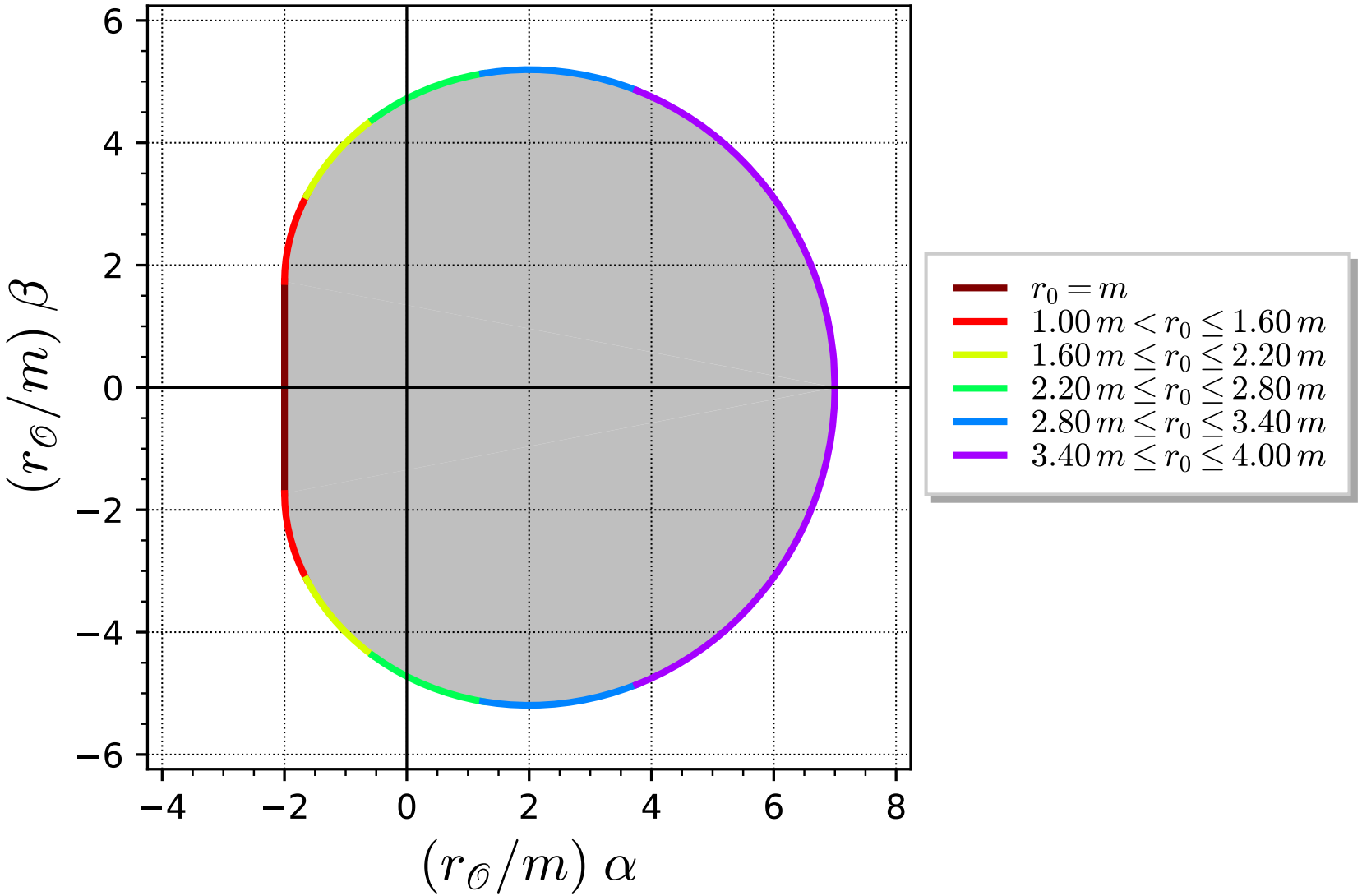


Scale factor $m/r_{\mathscr{O}}$

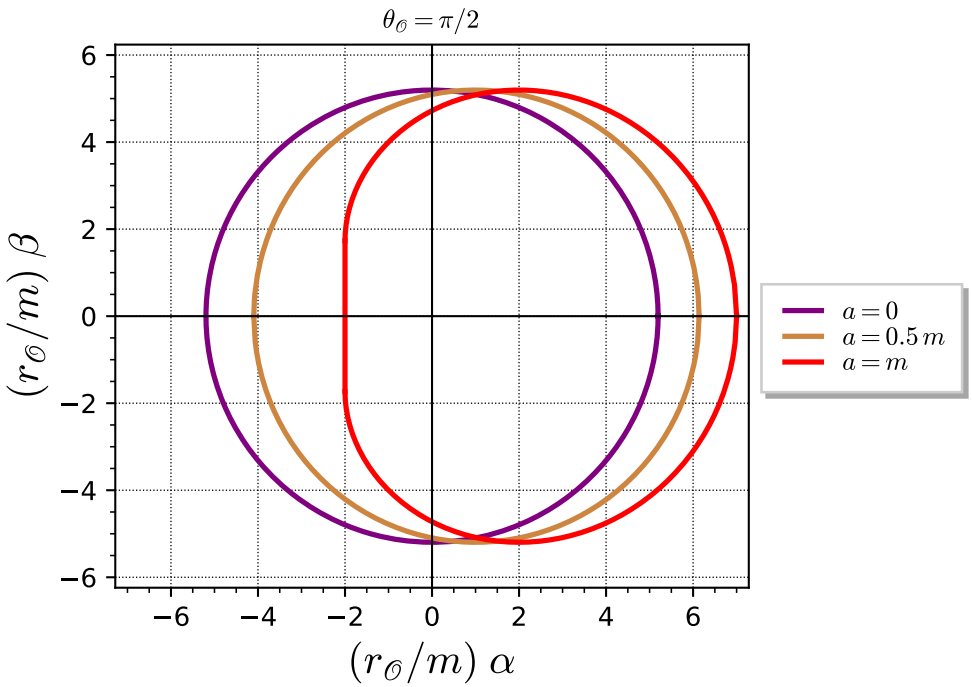
	Sgr A*	M87*	M31*	Cyg X-1
m $[M_\odot]$	$4.1\ 10^6$	$6.2\ 10^9$	$1.5\ 10^8$	15
$r_{\mathscr{O}}$ [kpc]	8.12	$1.67\ 10^4$	$7.6\ 10^2$	1.86
$m/r_{\mathscr{O}}$	$2.4\ 10^{-11}$	$1.8\ 10^{-11}$	$9.4\ 10^{-12}$	$3.9\ 10^{-16}$
$m/r_{\mathscr{O}}$ [μas]	5.0	3.7	1.9	$8.0\ 10^{-5}$

Critical curve for an extremal Kerr black hole

$a = m$, $\theta_{\mathscr{O}} = \pi/2$



Critical curve and spin parameter a



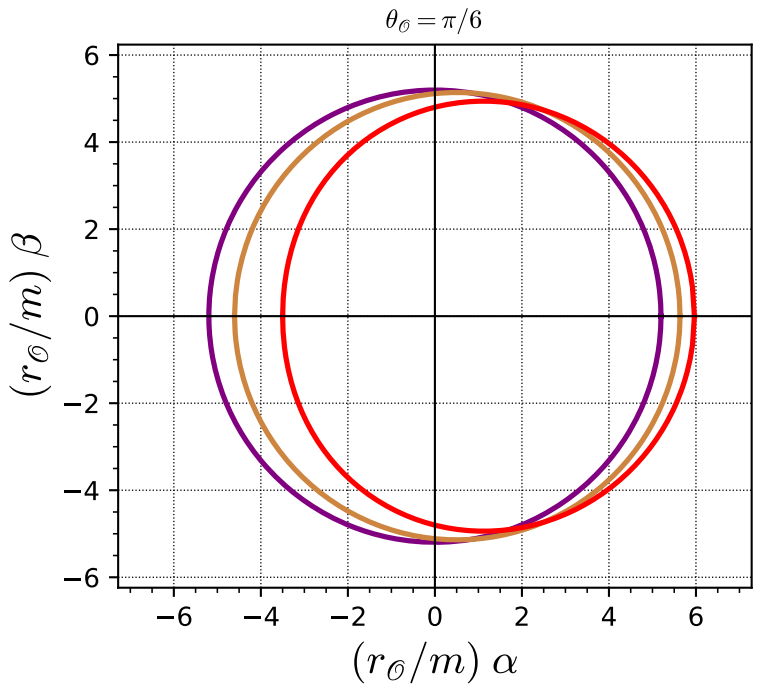
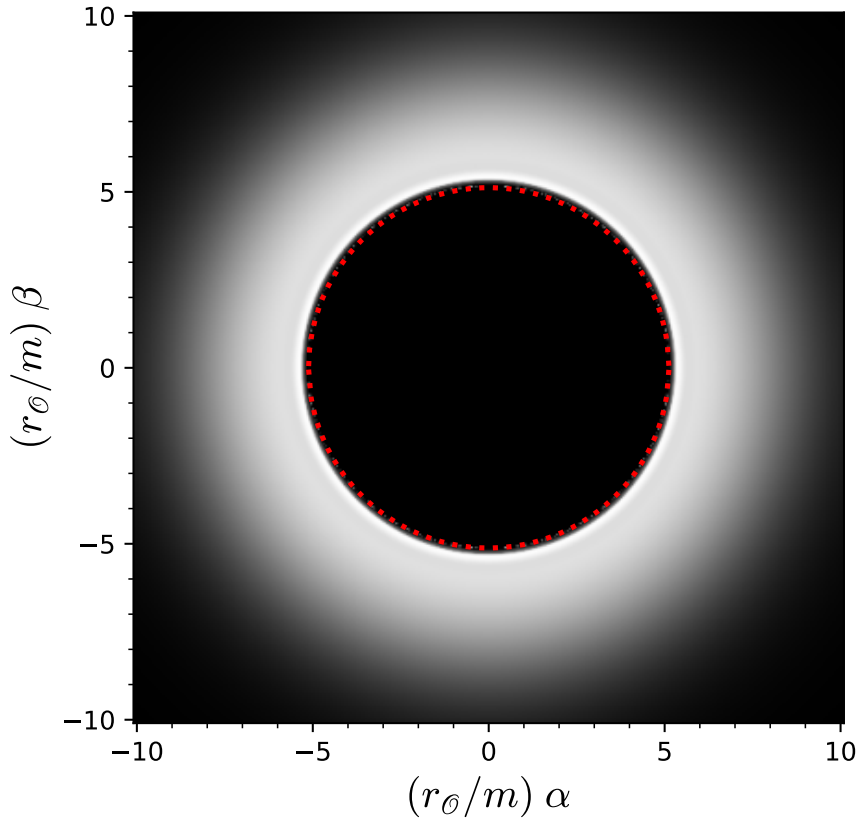


Image of an accretion disk

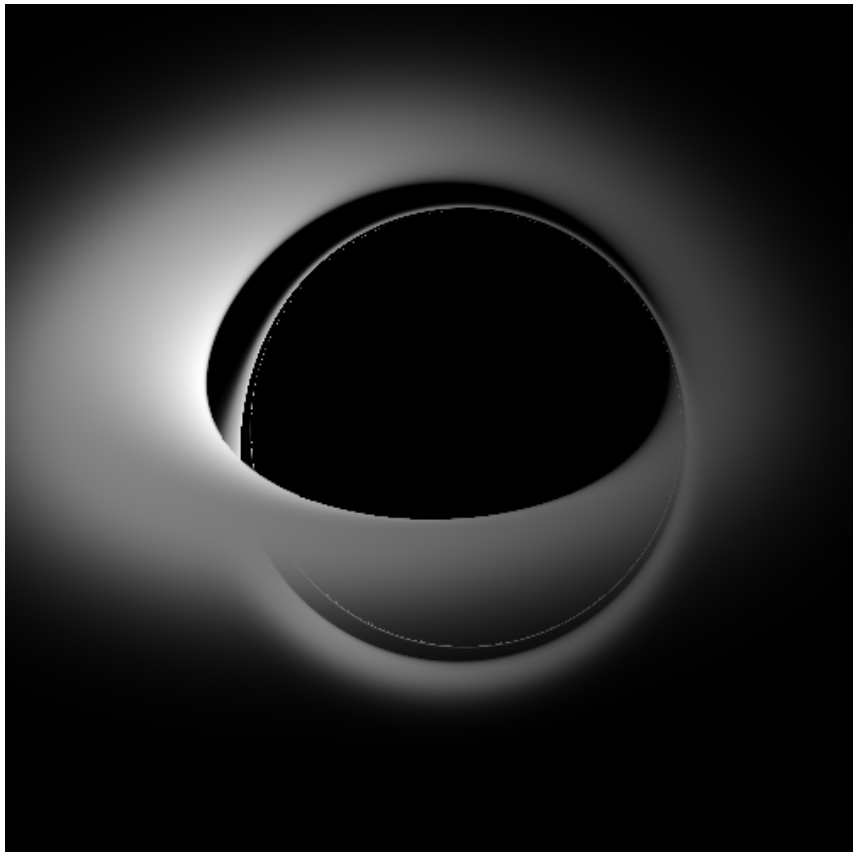
$a = 0.5\,m$, $\theta_{\mathscr{O}} = 0$

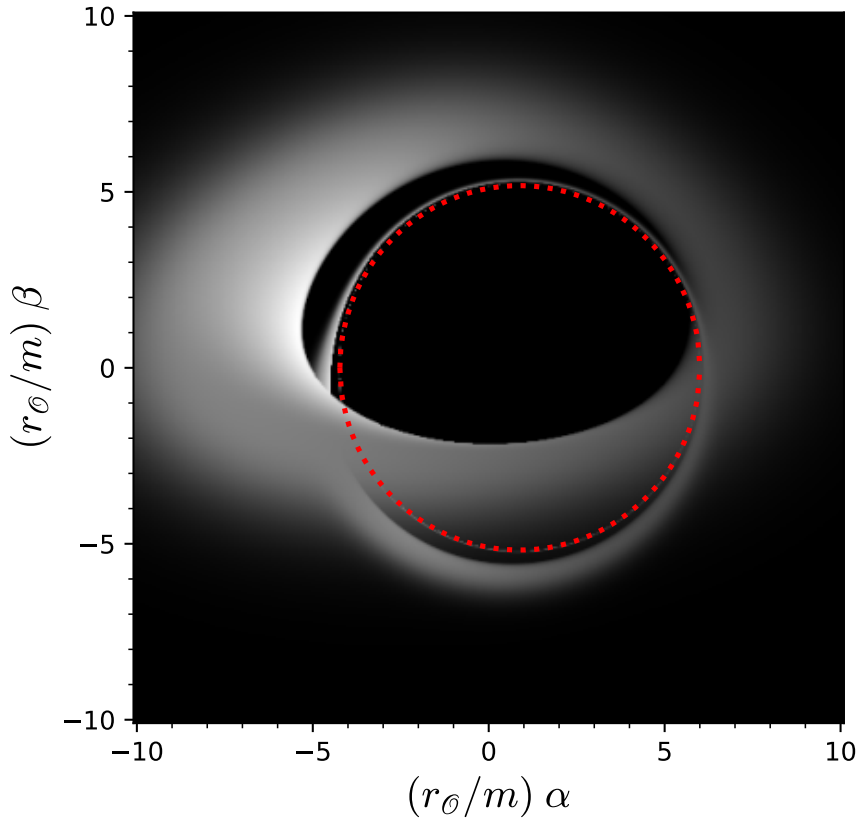


Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

Image of an accretion disk

$a = 0.5\,m$, $\theta_{\mathscr{O}} = \pi/3$

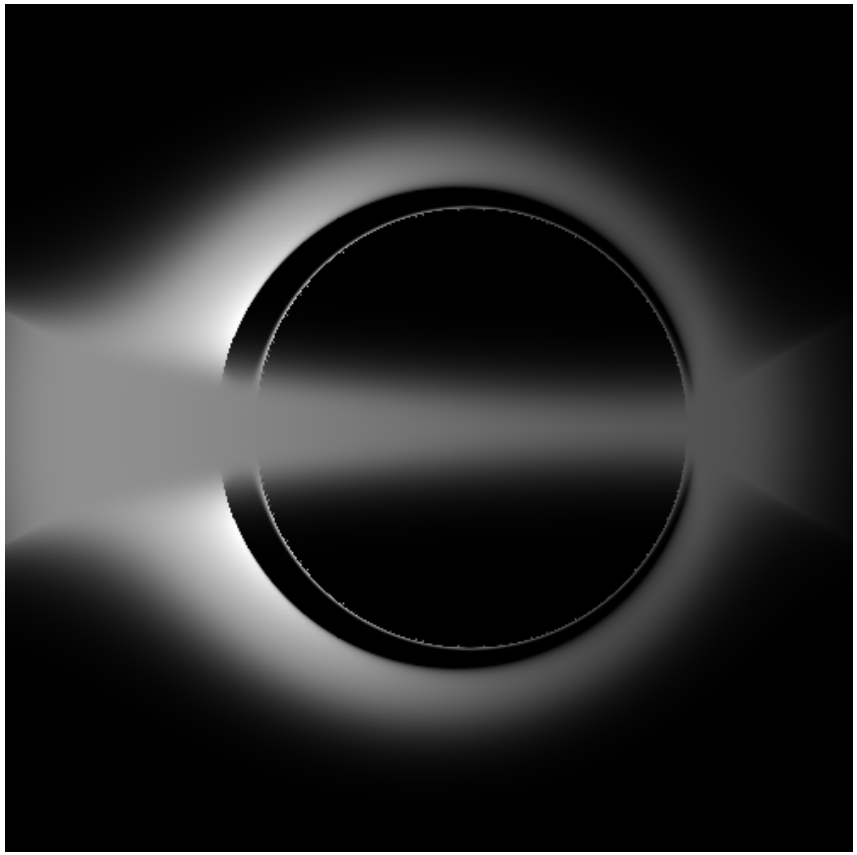


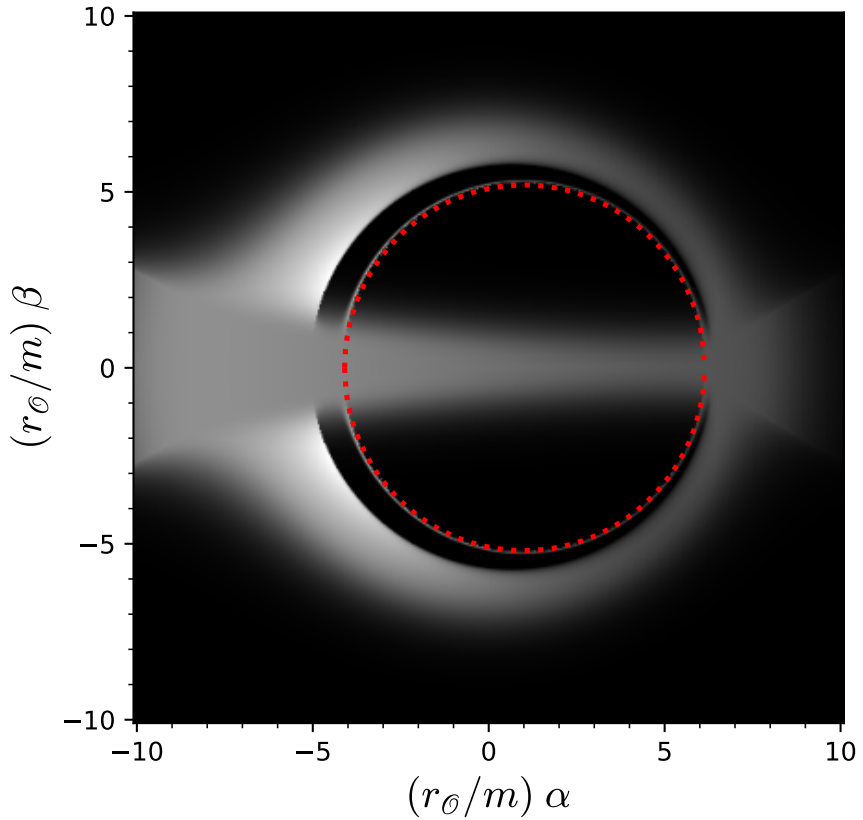


Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

Image of an accretion disk

$a = 0.5\,m$, $\theta_{\mathscr{O}} = \pi/2$

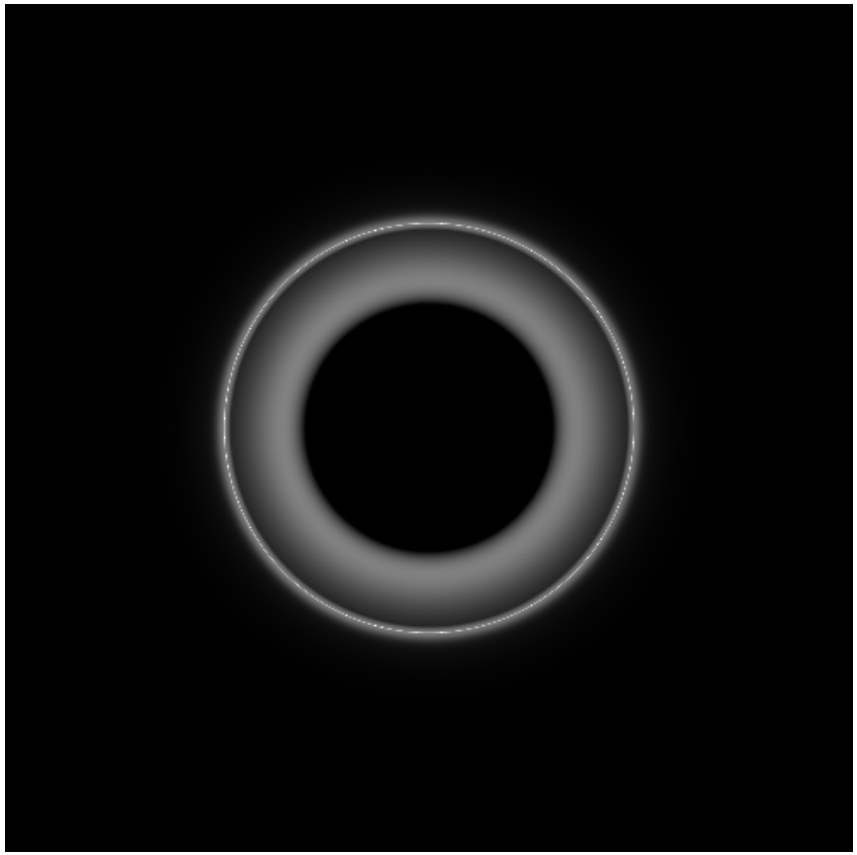


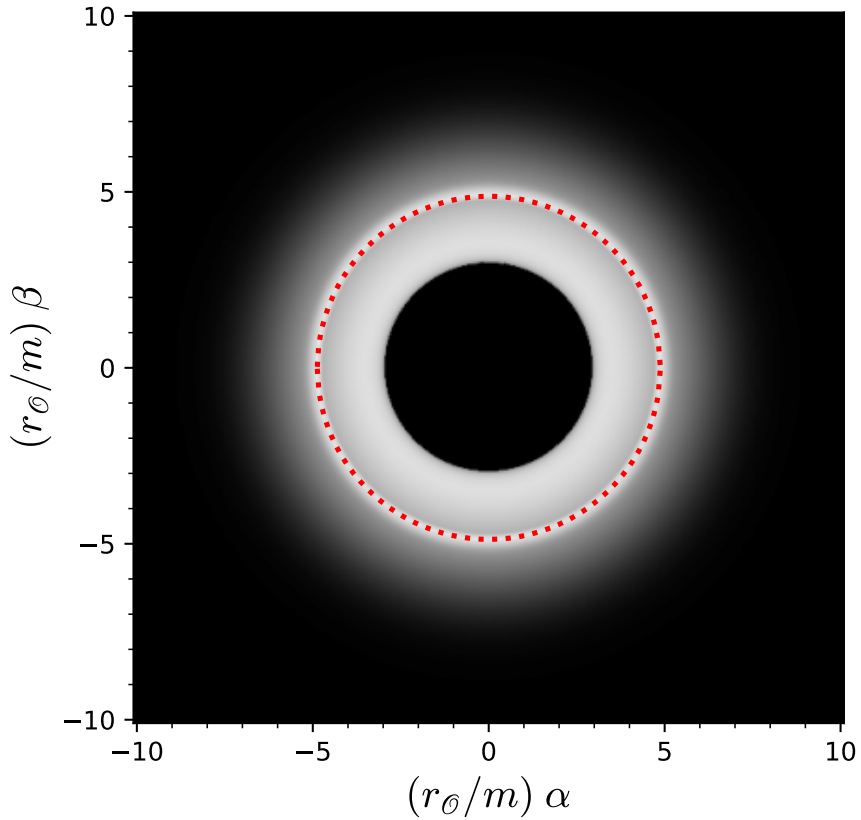


Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

Image of an accretion disk

$a = 0.95\,m$, $\theta_{\mathscr{O}} = 0$

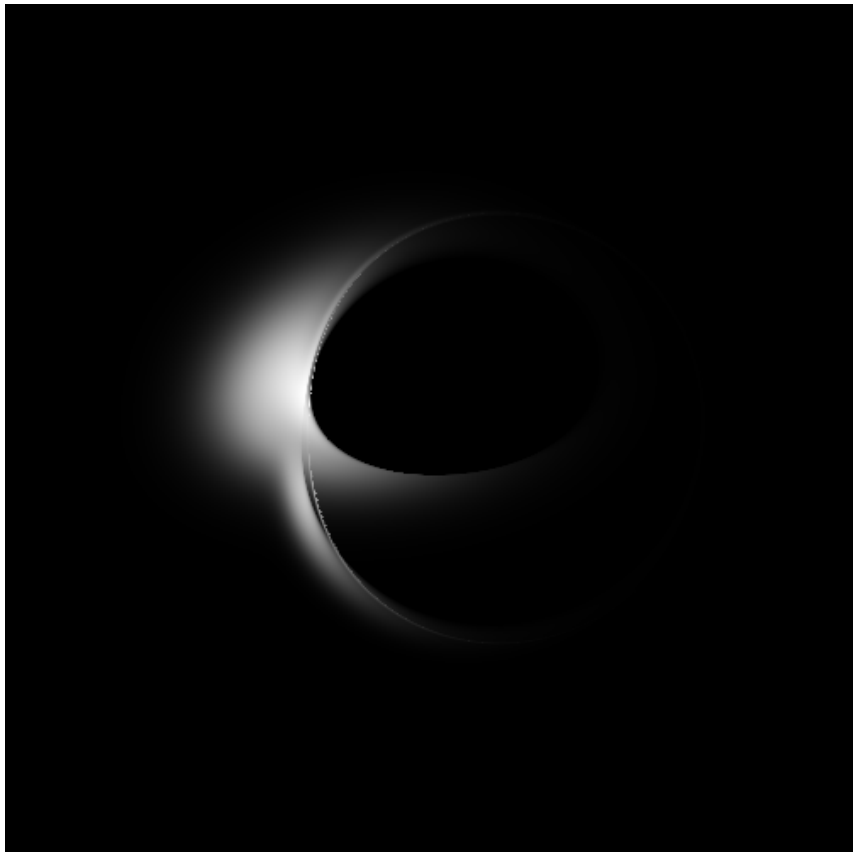


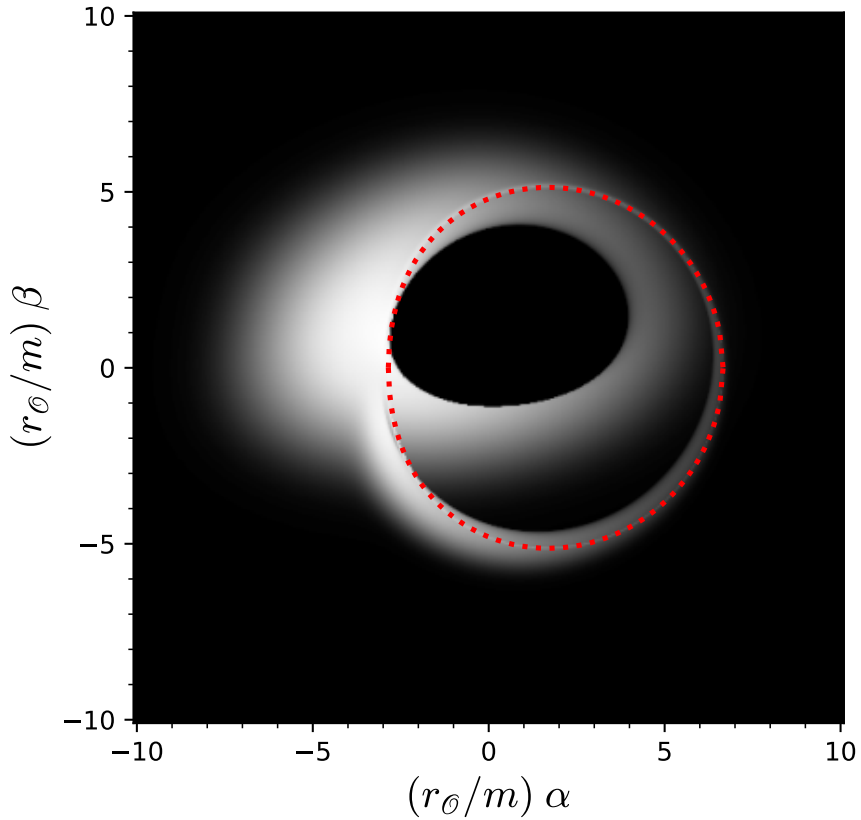


Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

Image of an accretion disk

$a = 0.95\,m$, $\theta_{\mathscr{O}} = \pi/3$

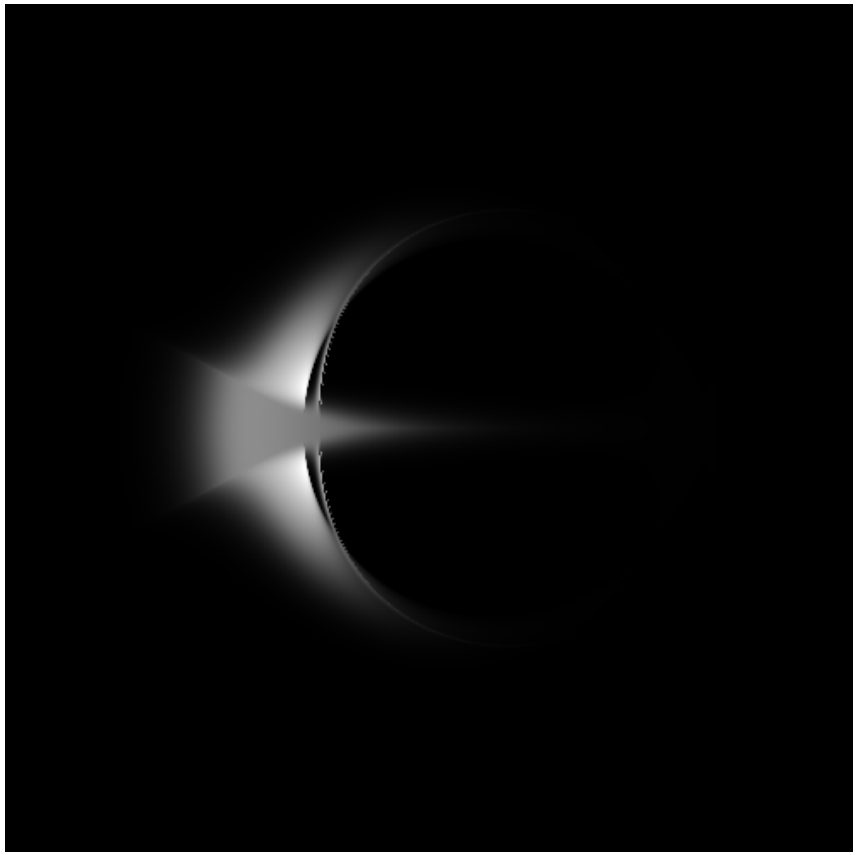


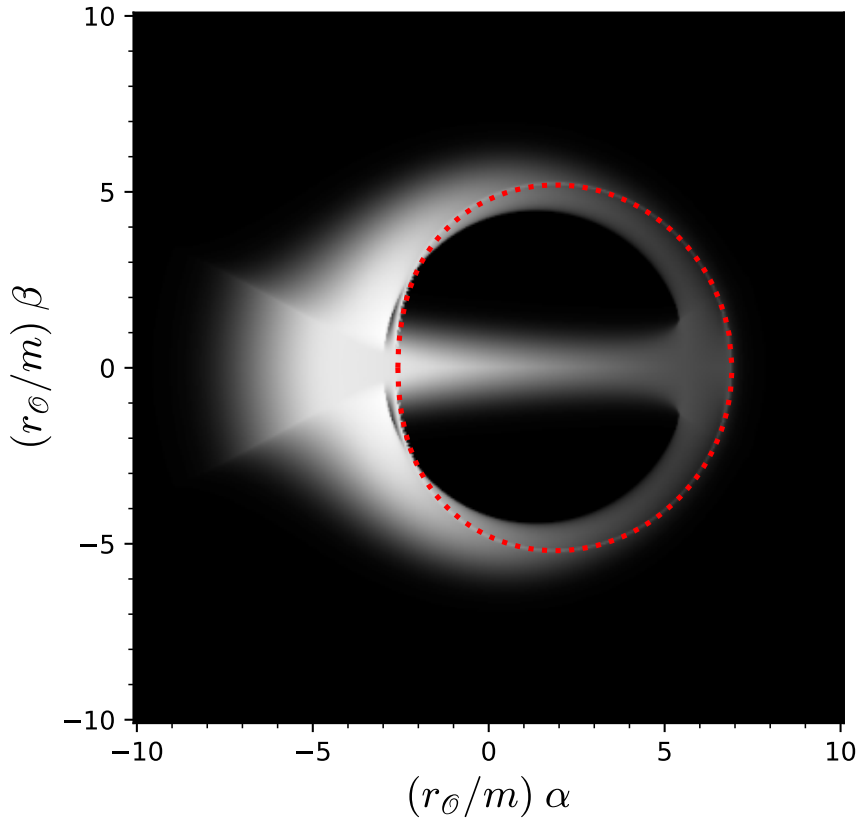


Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

Image of an accretion disk

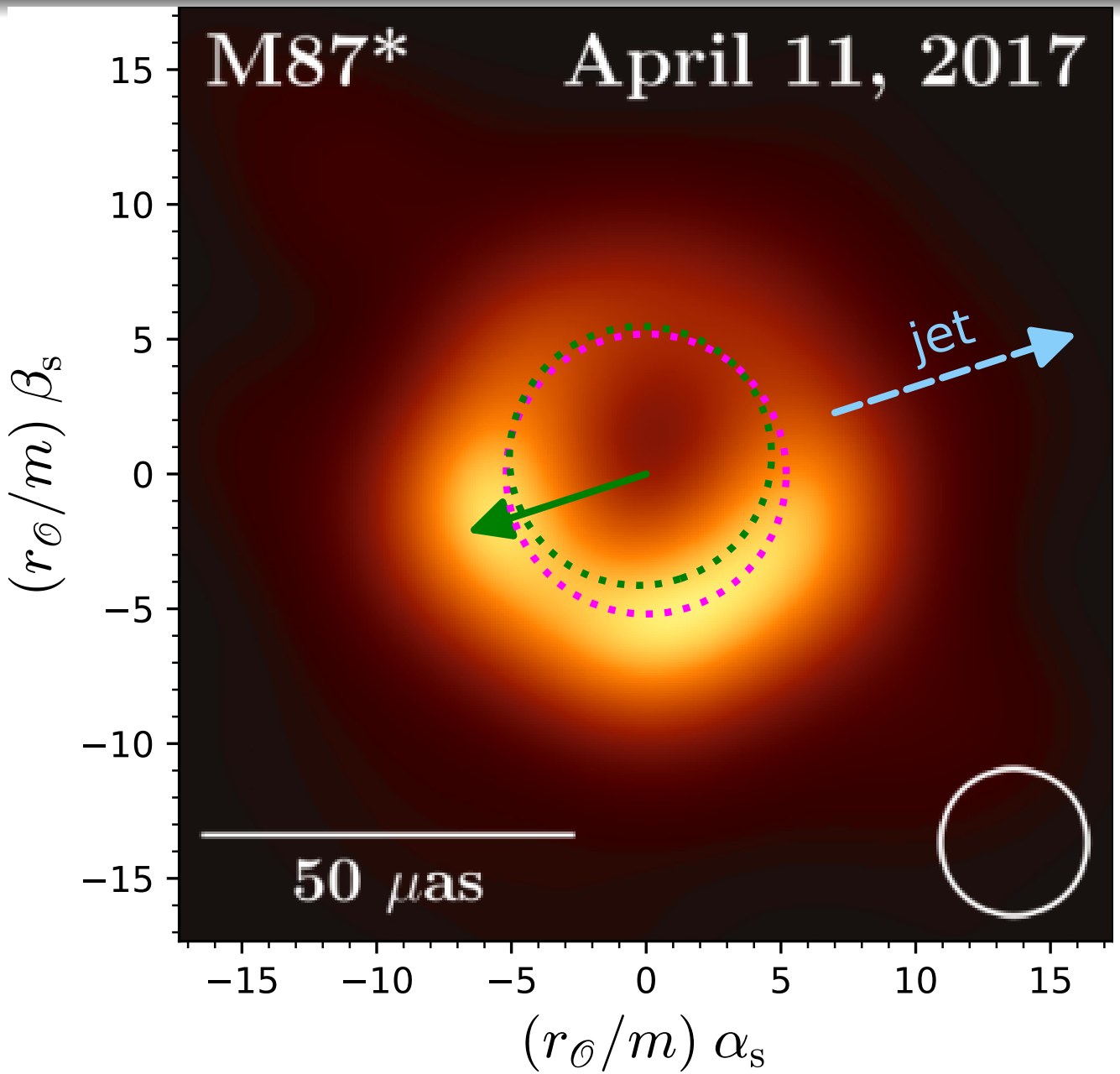
$a = 0.95\,m$, $\theta_{\mathscr{O}} = \pi/2$





Computation with Gyoto ray tracing code (https://gyoto.obspm.fr/)

EHT image of M87*



$\theta_{\mathscr{O}} = 163°$

- magenta: $a = 0$
- green: $a = m$