

Figures of lecture 6

Stationary black holes and the no-hair theorem

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<https://relativite.obspm.fr/blackholes/paris25/>

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Home page for the lectures

<https://relativite.obspm.fr/blackholes/paris25/>

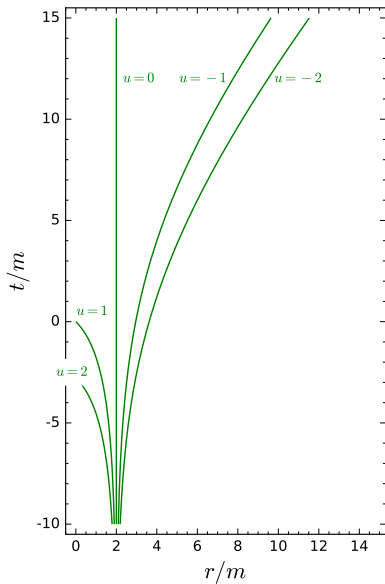
includes

- these slides
- the lecture notes (draft)
- some SageMath notebooks

Prerequisite

An introductory course on general relativity

Proof of the topology theorem



Null hypersurfaces $\mathcal{H}_u$ around the Schwarzschild horizon $\mathcal{H} = \mathcal{H}_{u=0}$

Topology of cross-sections of a stationary BH horizon for $n = 5$

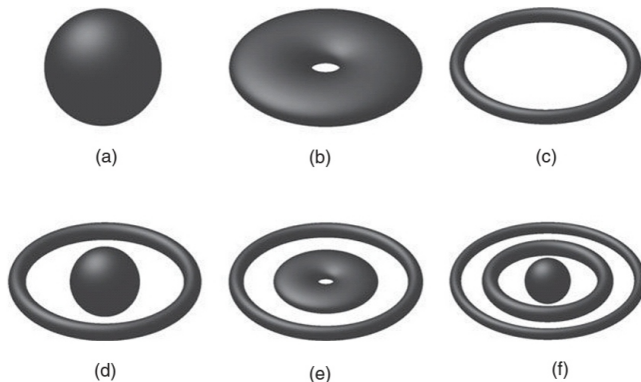
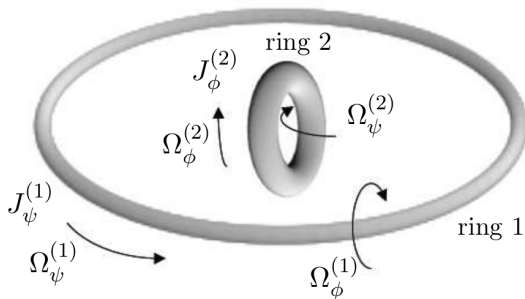


Fig. 10.11 Schematic illustration of 5D black objects: (a) MP black hole; (b) Fat black ring; (c) Thin black ring; (d) Black Saturn; (e) Di-ring; (f) Black Saturn with several rings.

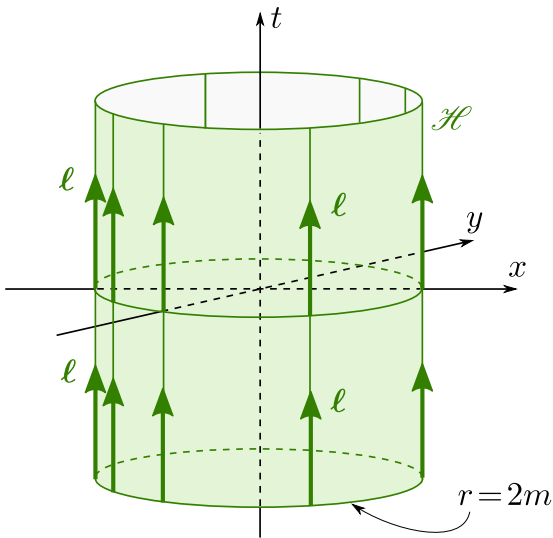
[Frolov & Zelnikov: *Introduction to Black Hole Physics*, Oxford Univ. Press (2011)]

Bicycling black rings ($n = 5$)



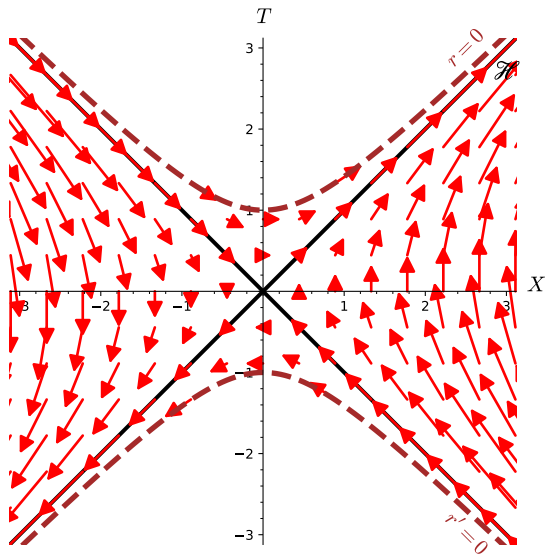
[Elvang & Rodriguez, JHEP04(2008)045]

Example of non-rotating horizon: the Schwarzschild horizon



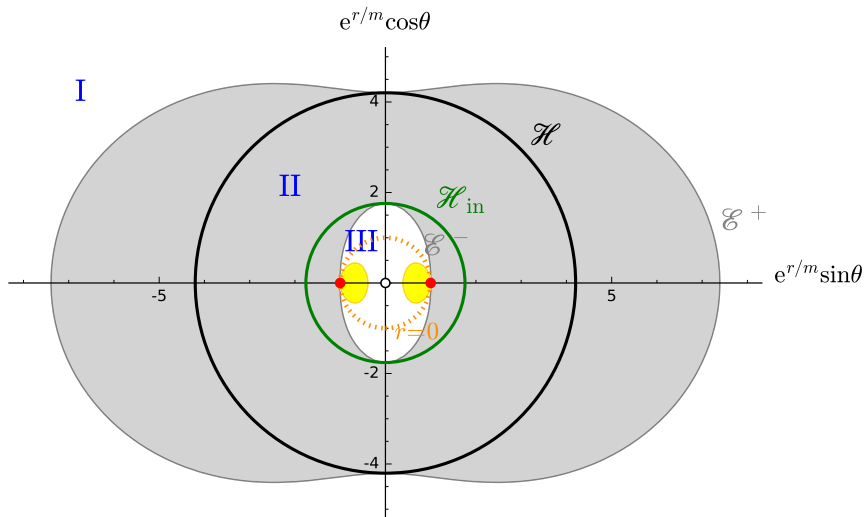
Stationary Killing vector ξ
on $\mathcal{H}$: $\xi = \ell$

Example of non-rotating horizon: the Schwarzschild horizon



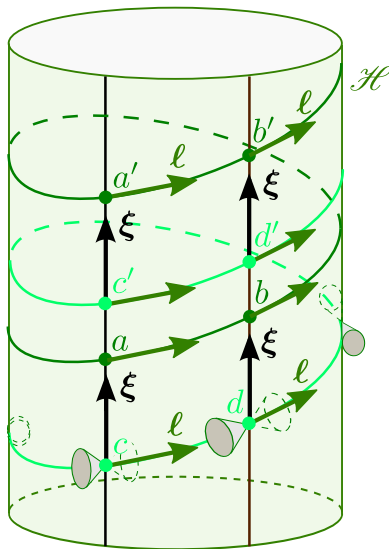
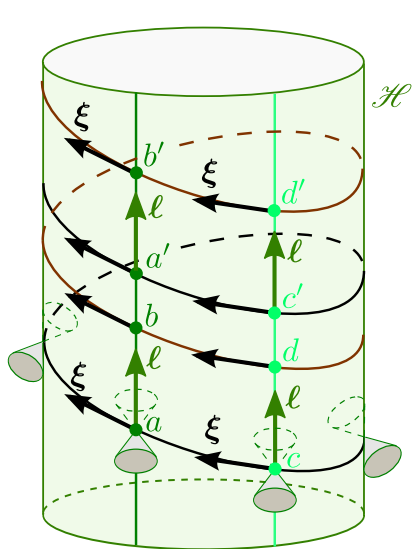
Stationary Killing vector ξ
in the maximal extension of
Schwarzschild spacetime
(Kruskal diagram)

Example of rotating horizon: the Kerr horizon

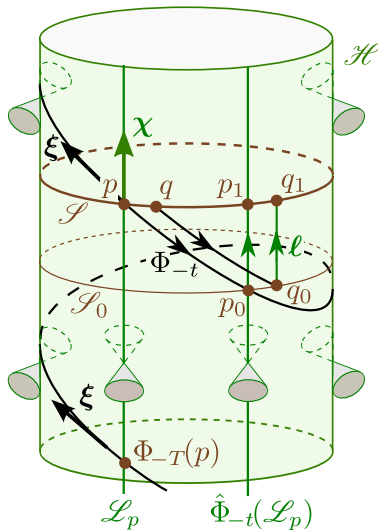


ξ spacelike in the ergoregion (grey)

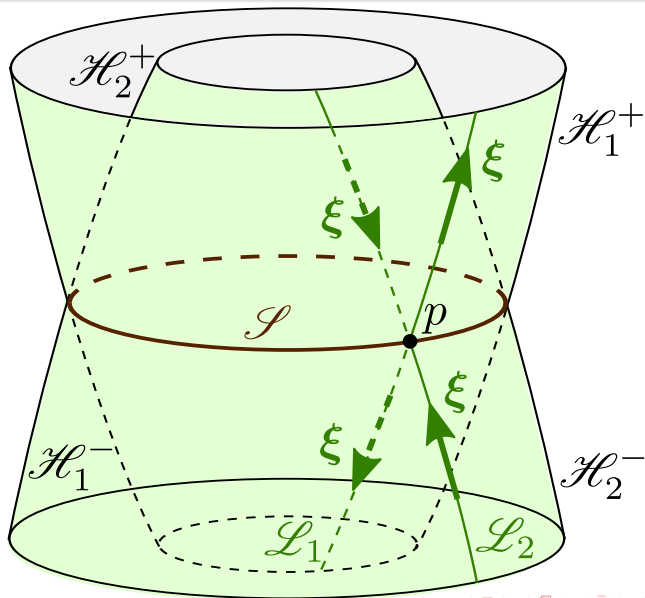
Example of rotating horizon: the Kerr horizon



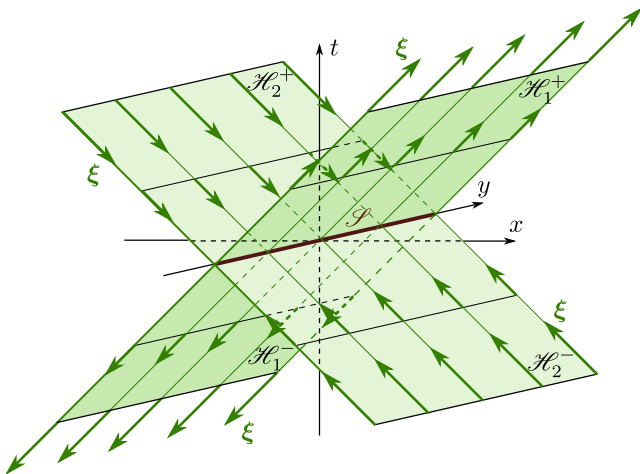
Proving that cross-sections of a rotating horizon are axisymmetric



Bifurcate Killing horizon



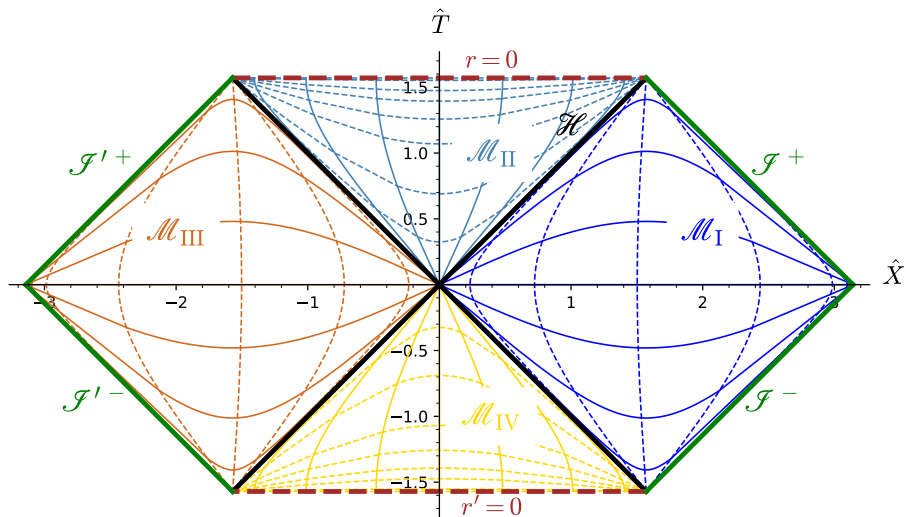
Example of bifurcate Killing horizon in Minkowski spacetime



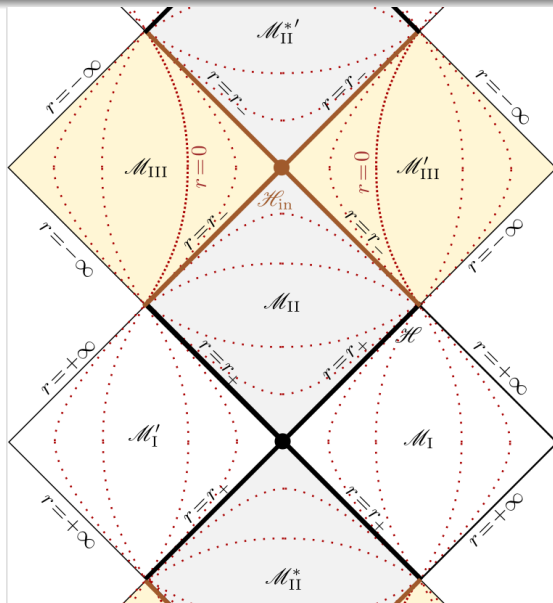
Killing vector =
generator of Lorentz
boosts in the (t, x)
plane:

$$\xi = x\partial_t + t\partial_x$$

Bifurcate Killing horizon in Schwarzschild spacetime



Bifurcate Killing horizons in Kerr spacetime with $0 < a < m$



No bifurcate Killing horizon for extremal Kerr ($a = m$)

