Figures of lecture 7

Evolution and thermodynamics of black holes

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https://relativite.obspm.fr/blackholes/paris25/

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Home page for the lectures

https://relativite.obspm.fr/blackholes/paris25/

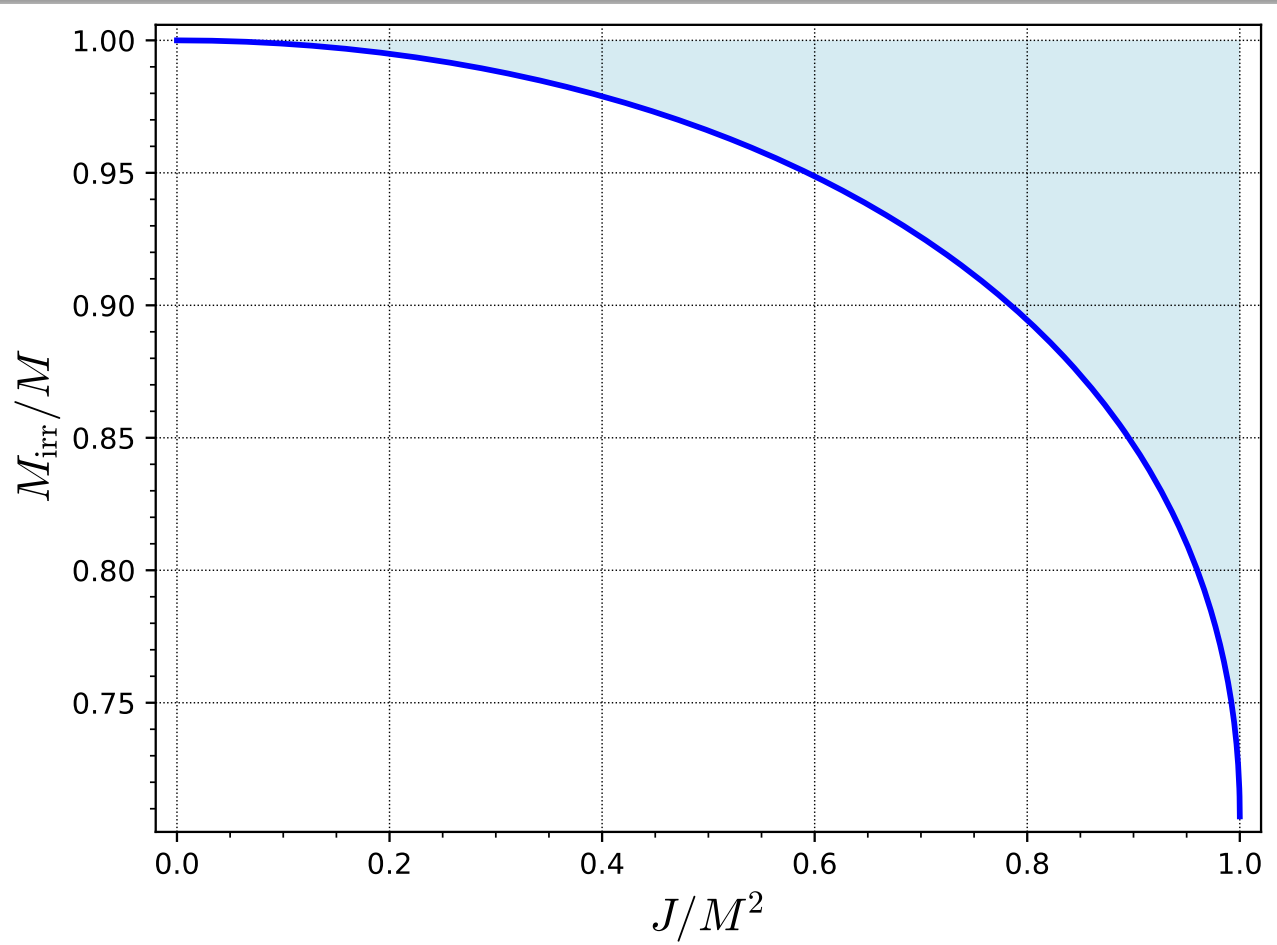
includes

- these slides
- the lecture notes (draft)
- some SageMath notebooks

Prerequisite

An introductory course on general relativity

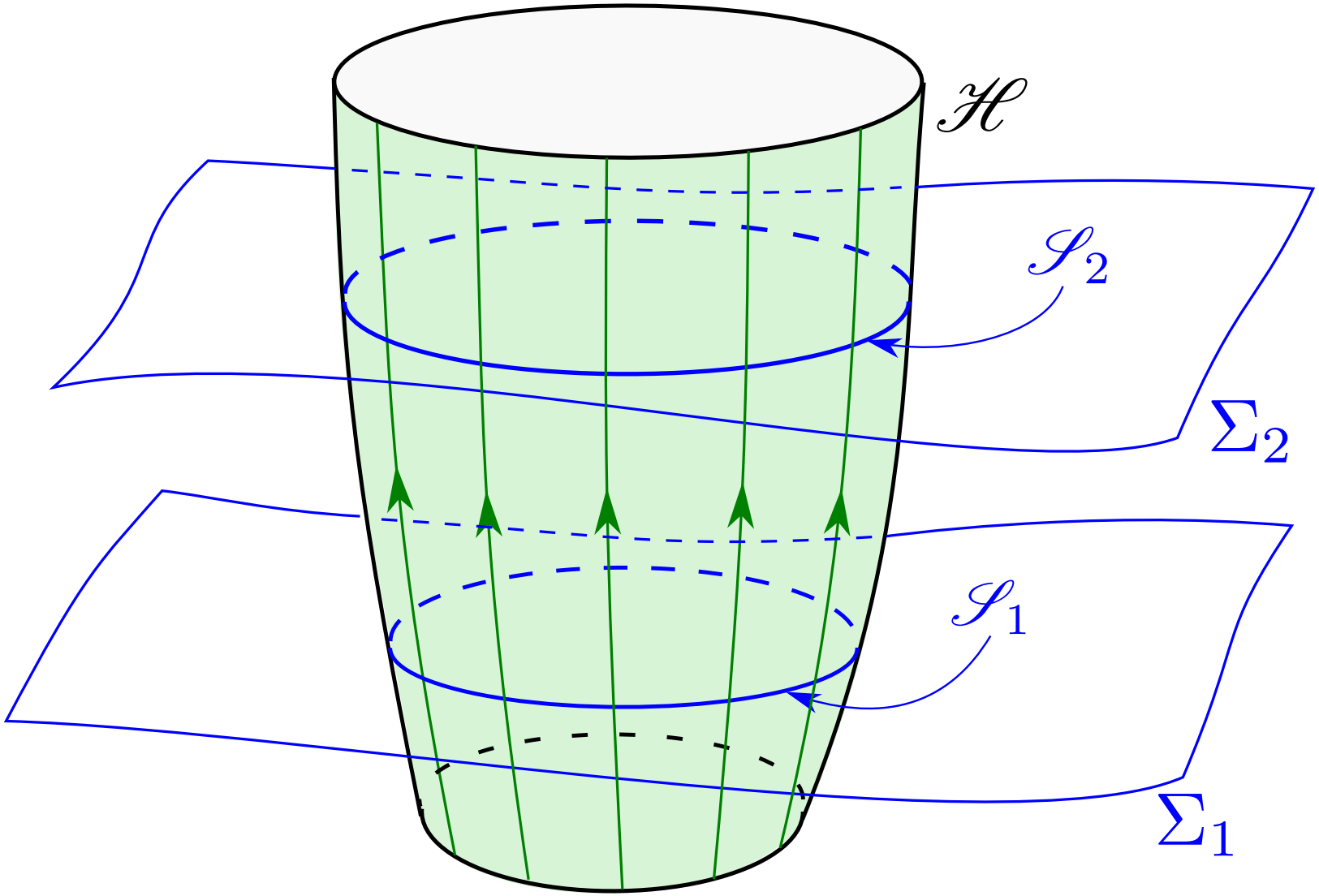
Irreducible mass



Irreducible mass $M_{\rm irr}$ of a Kerr black hole of mass M, as a function of the black hole angular momentum J.
Shadded area = maximal energy that can be extracted by a Penrose process bringing J down to 0

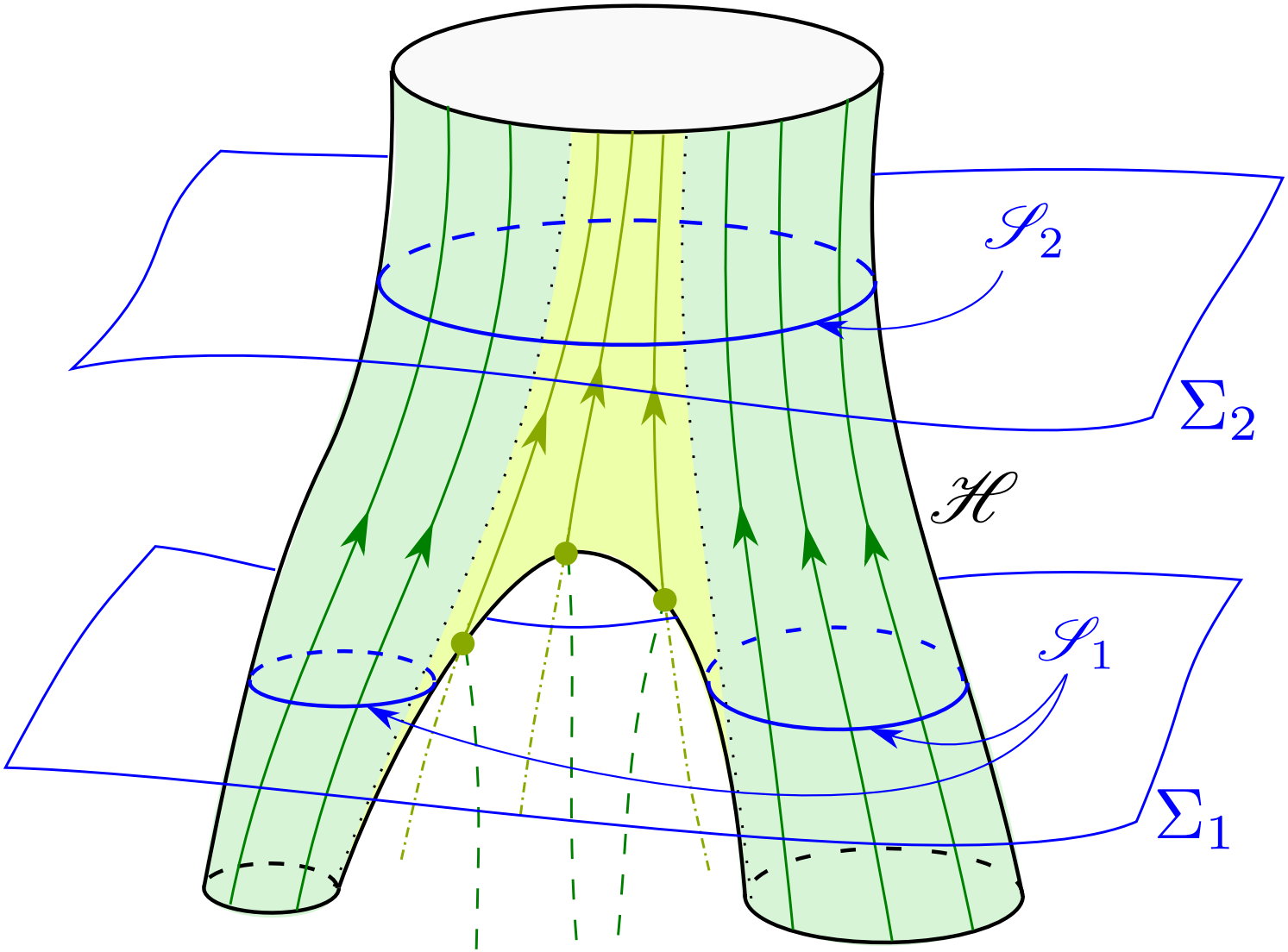
Area theorem

Smooth part of the event horizon

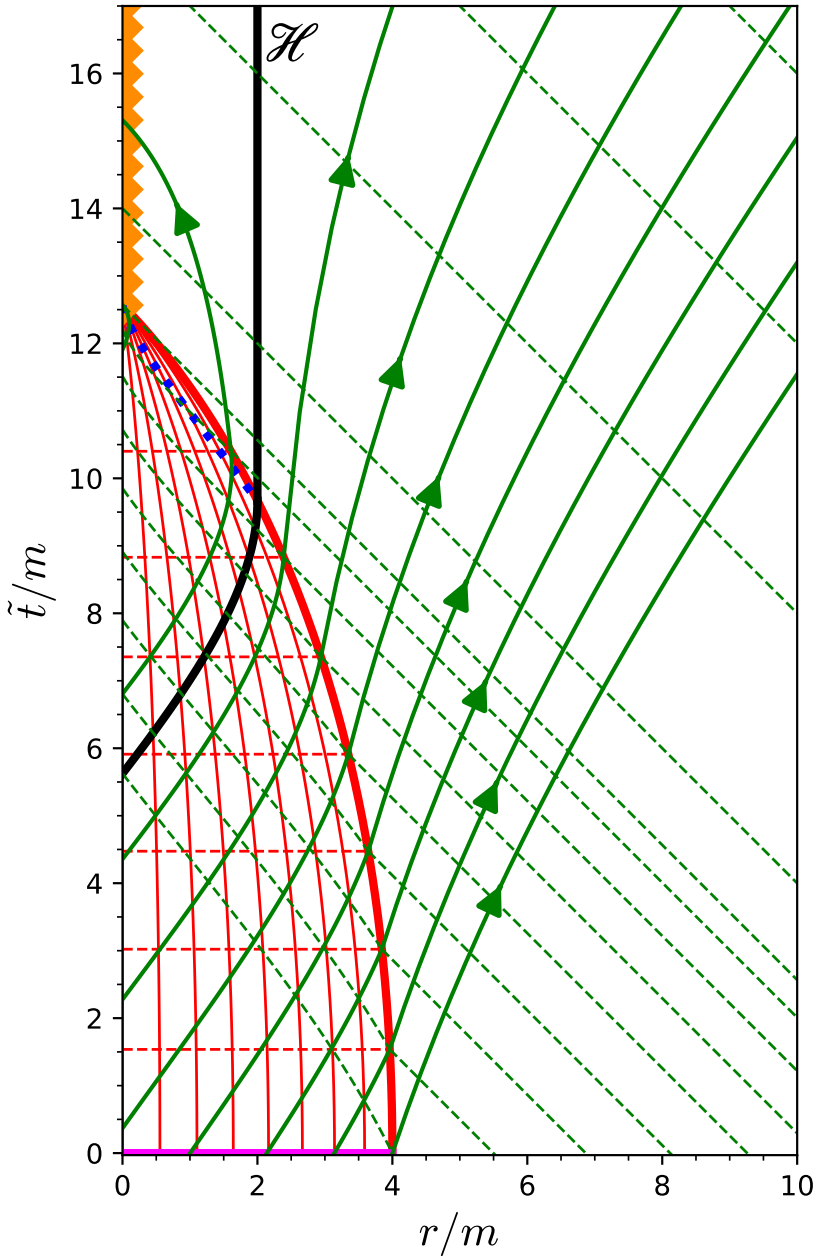


Area theorem

Generic case



Horizon growth in the Oppenheimer-Snyder collapse

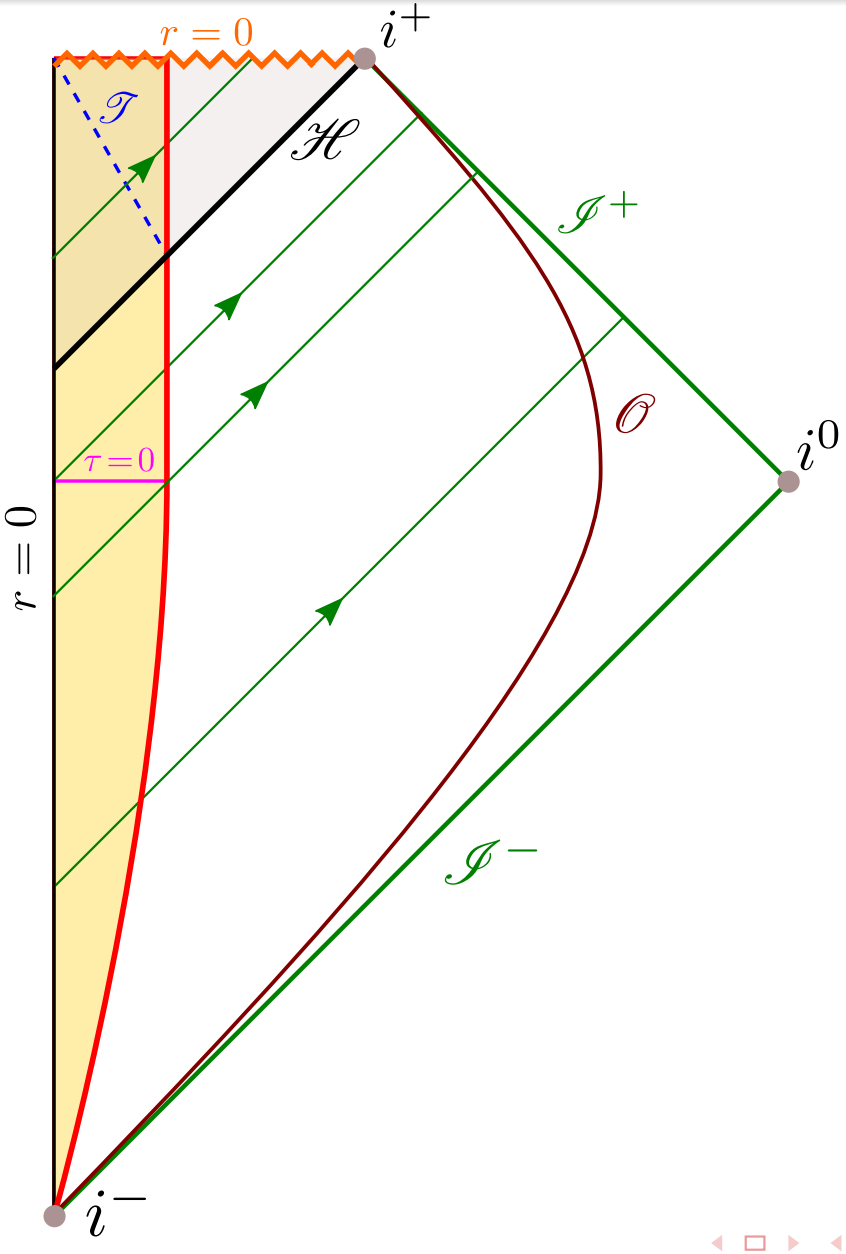


Collapse of a ball of pressureless matter (dust) initially at rest

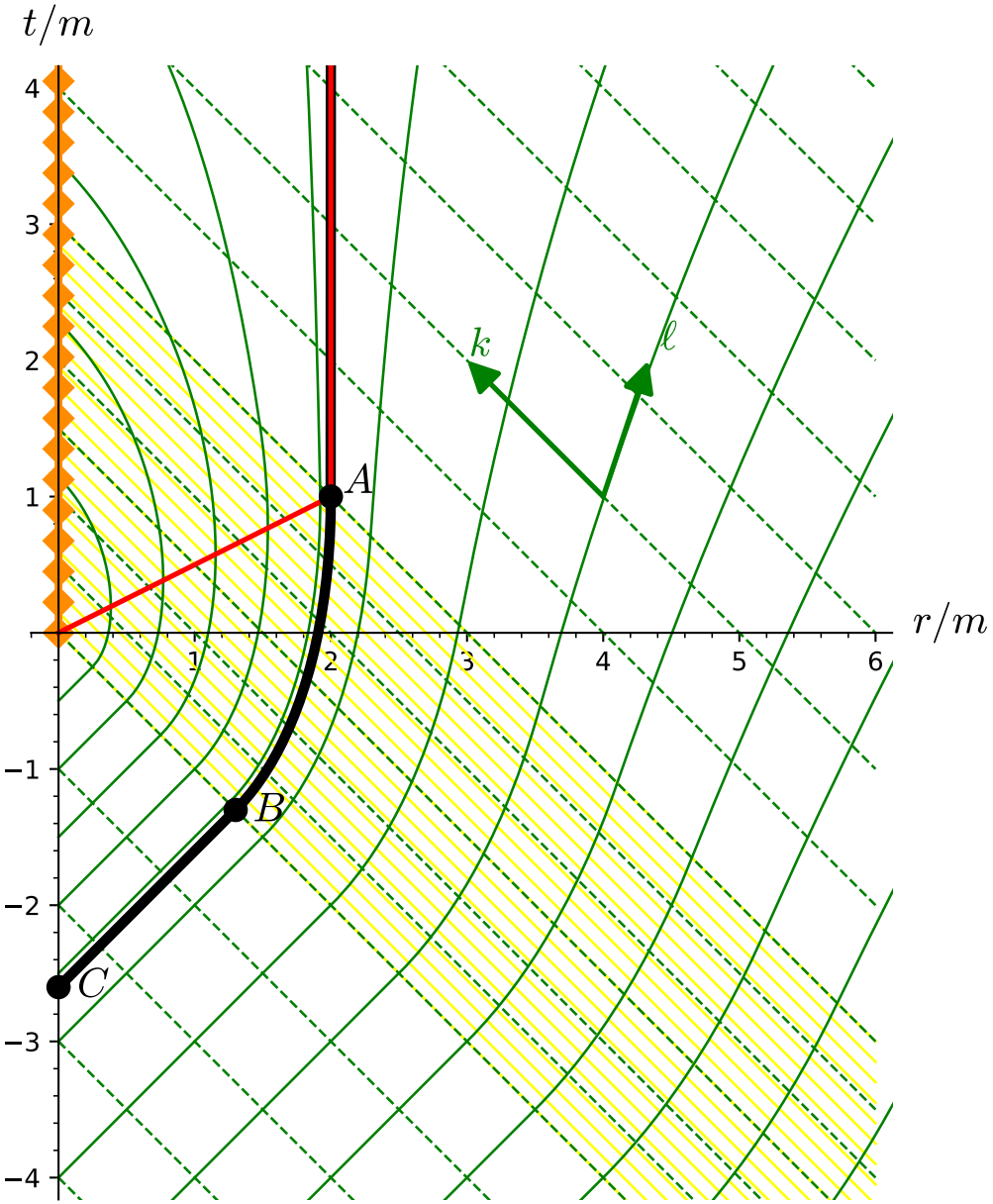
r = areal radius
$\Longrightarrow$ area of a $\tilde{t} = \text{const}$ section of $\mathscr{H}$:
$A = 4\pi r^2$

SageMath notebook: https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Oppenheimer_Snyder.ipynb

Carter-Penrose diagram of the Oppenheimer-Snyder collapse



Horizon growth in the Vaidya collapse

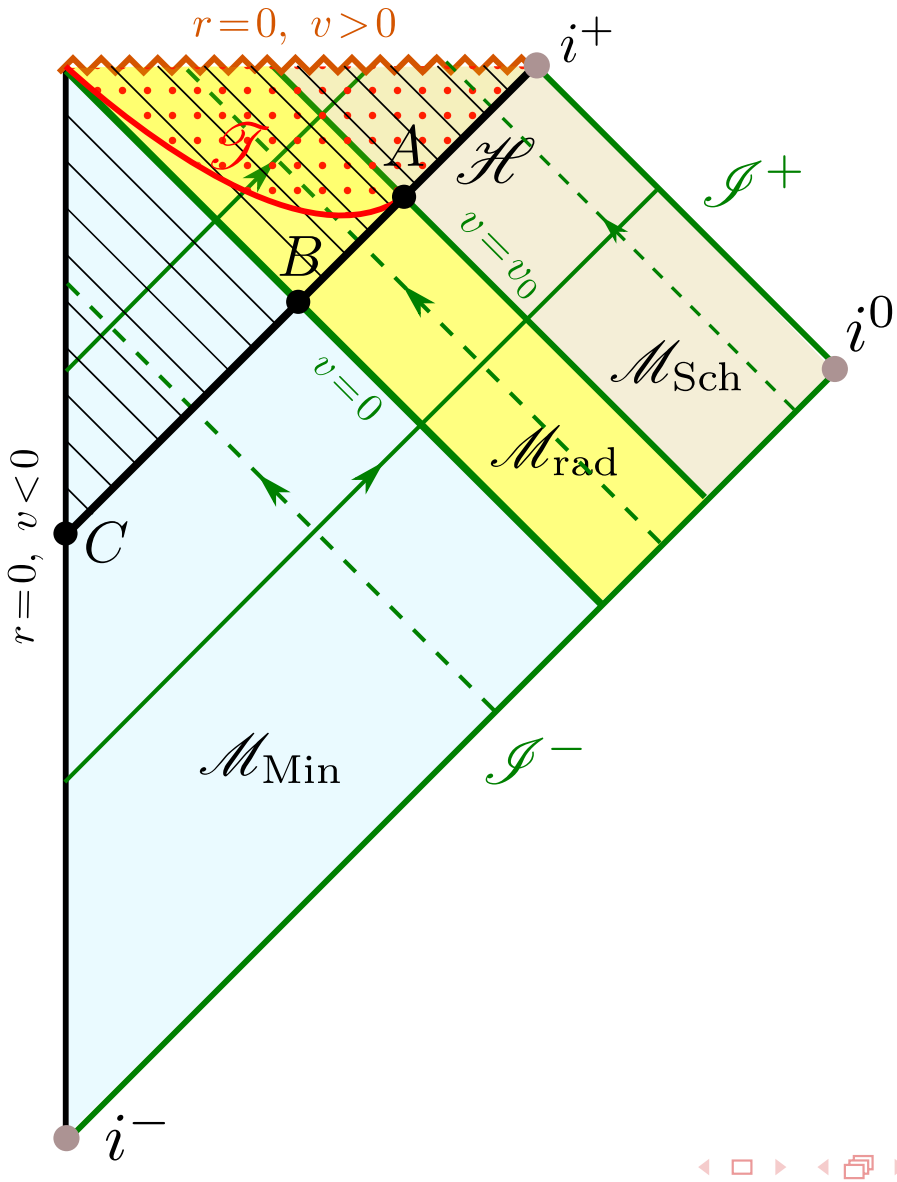


Collapse of shell of electromagnetic radiation

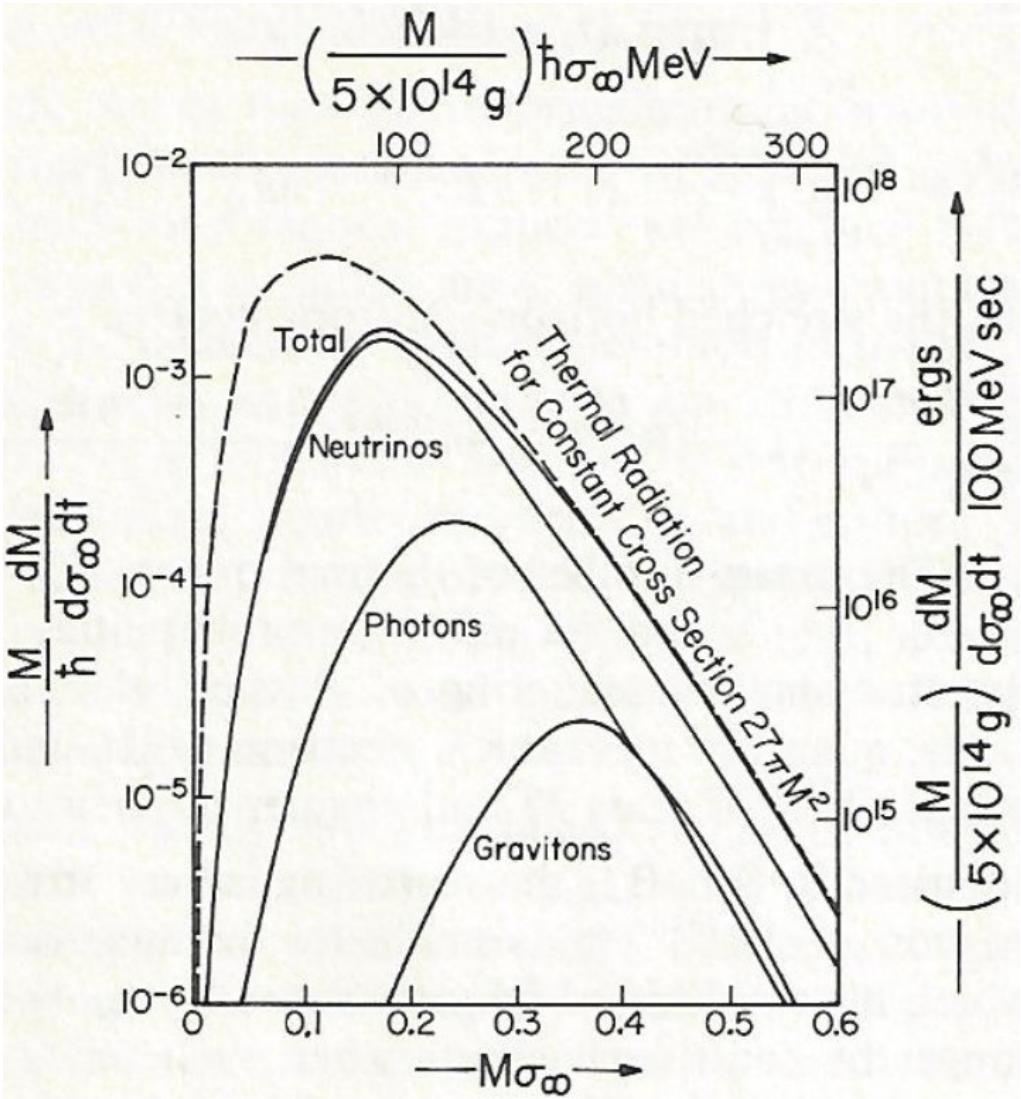
r = areal radius
$\Longrightarrow$ area of a $t = \text{const}$ section of $\mathscr{H}$: $A = 4\pi r^2$

SageMath notebook: https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Vaidya.ipynb

Carter-Penrose diagram of the Vaidya collapse



Hawking radiation



Spectrum of Hawking radiation for a Schwarzschild black hole ($\kappa = (4M)^{-1}$) of mass $M \gg 10^{14}$ kg

$\sigma_\infty/(2\pi)$ = frequency at infinity

[Thorne, Zurek & Price (1986)]