General relativity computations with SageManifolds

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on behalf on SageManifolds team:

http://sagemanifolds.obspm.fr/authors.html

Journée GPhys 2018

Observatoire de Paris 8 June 2018

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The mission

Create a viable free open source alternative to Magma, Maple, Mathematica and Matlab.

Some advantages of SageMath

SageMath is free (GPL v2)

Freedom means

- everybody can use it, by downloading the software from http://sagemath.org
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SageMath is developing and spreading fast

...sustained by an enthusiastic community of developers

The SageManifolds project

SageManifolds: extending SageMath towards differential geometry and tensor calculus



Stereographic-coordinates frame on \mathbb{S}^2

- http://sagemanifolds.obspm.fr/
- \sim 75,000 lines of Python code (including comments and doctests)
- submitted to SageMath community as a sequence of \sim 50 tickets cf. list at https:

//trac.sagemath.org/ticket/18528

• a dozen of contributors (developers and reviewers)

cf. http://sagemanifolds.obspm.fr/ authors.html

SageManifolds 1.2 released on 5 May 2018 and fully included in SageMath 8.2

Click on the links below:

• Schwarzschild spacetime:

http://nbviewer.jupyter.org/urls/gitlab.obspm.fr/gourgoul/ SageMathTour/raw/master/Notebooks/demo_Schwarzschild.ipynb

 Surface gravity of a Schwarzschild black hole: http://nbviewer.jupyter.org/urls/gitlab.obspm.fr/gourgoul/ SageMathTour/raw/master/Notebooks/surface_gravity_Schwarz. ipynb