Last orbits of binary neutron stars and binary black holes

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Plan

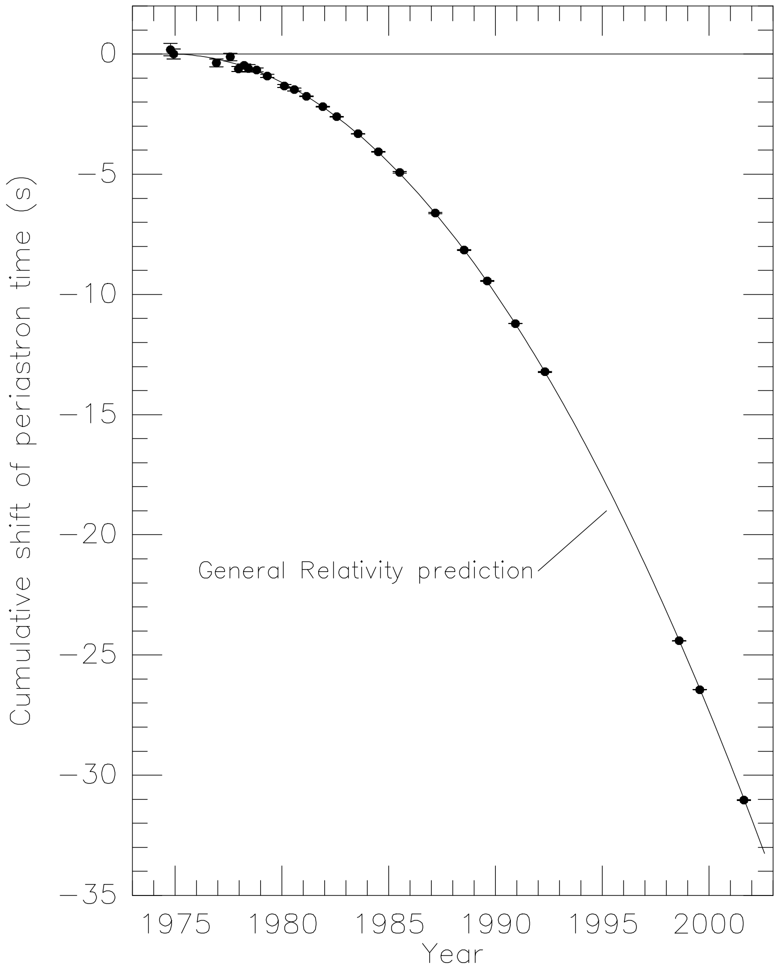
1. Evolution of binary compact objects
2. The helical Killing vector approach
3. Numerical technique
4. Results for binary neutron stars
5. Results for binary black holes

1

Evolution of binary compact objects

Observational evidences for binary neutron stars

Binary pulsars with $M_1 > 1.3\, M_\odot$ and $M_2 > 1.3\, M_\odot$:



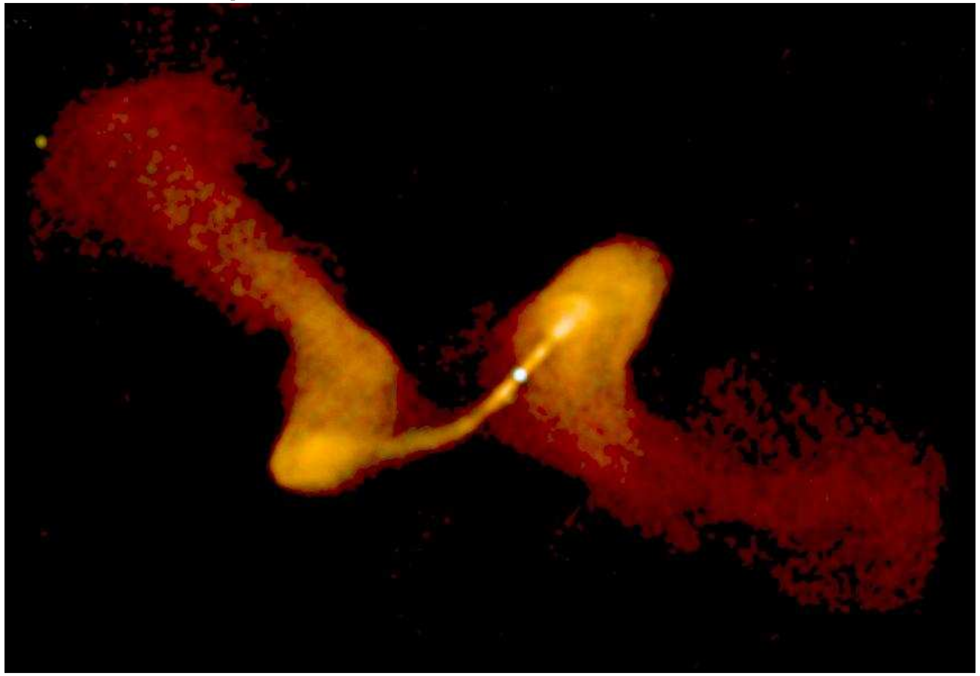
[from Weisber & Taylor (2002)]

PSR B1913+16	$M_1 = 1.44\, M_\odot$	$M_2 = 1.39\, M_\odot$
PSR B1534+12	$M_1 = 1.34\, M_\odot$	$M_2 = 1.34\, M_\odot$
PSR B2127+11C	$M_1 = 1.34\, M_\odot$	$M_2 = 1.37\, M_\odot$

$\leftarrow$ Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 due to gravitational radiation reaction $\Longrightarrow$ merger in 140 Myr.

Observational evidences for binary black holes

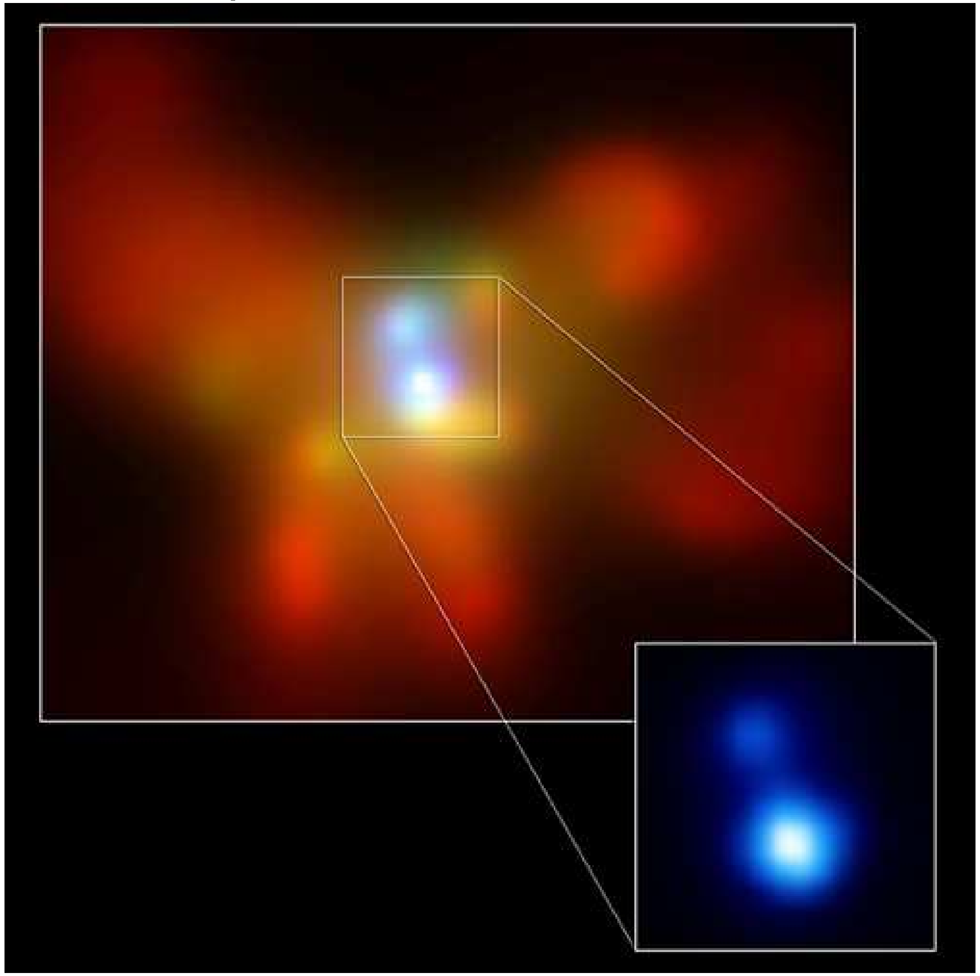
... in the past



Change of direction of NGC 326 jet

[Merrit & Eckers, Science **297**, 1310 (2002)]

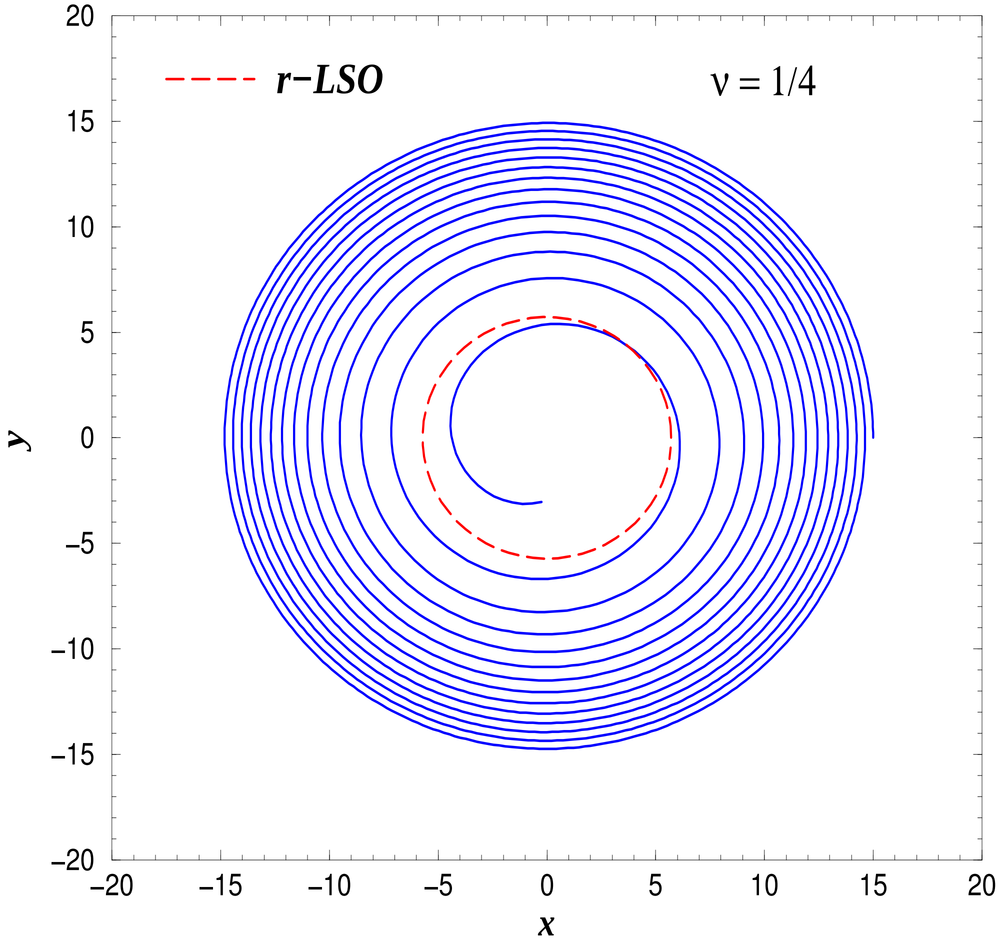
... in the present



X-ray view of double nucleus
of galaxy NGC 6240 (Chandra satellite)

[Komossa et al., ApJ **582**, L15 (2003)]

Inspiraling motion



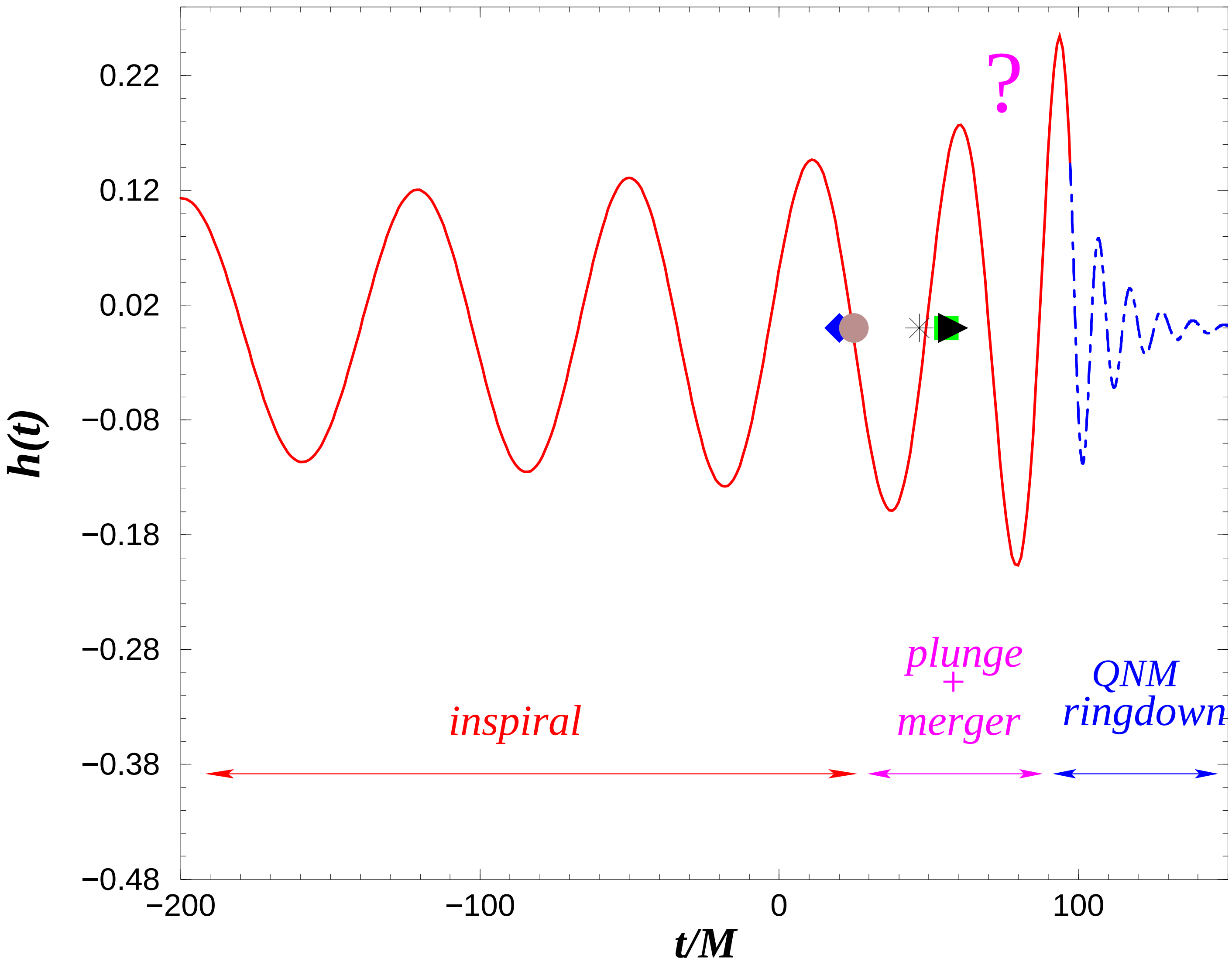
2.5-PN Effective One Body computation
[Buonanno & Damour, PRD **62**, 064015 (2000)]]

Evolution of binary black holes or neutron stars entirely driven by

gravitational radiation reaction

Another effect of gravitational wave emission:
circularisation of the orbits: $e \longrightarrow 0$

Gravitational waveform



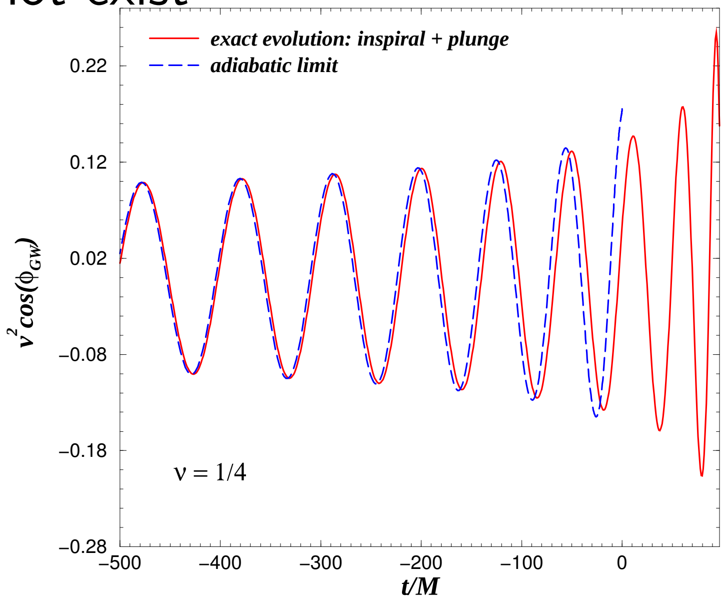
[from Buonanno & Damour, PRD **62**, 064015 (2000)]

End of inspiral: the last stable orbit

Very small mass ratio (Schwarzschild spacetime) : there exists an *innermost stable circular orbit (ISCO)* :

$$R_{\rm ISCO}^{\rm Schw} = 6M \qquad \qquad \Omega_{\rm ISCO}^{\rm Schw} = 6^{-3/2} M^{-1} \simeq 0.068\, M^{-1}$$

Equal mass ratio : gravitational radiation dissipation $\Longrightarrow$ strictly circular orbits do not exist



The ISCO is then defined in terms of the conservative part in the equation of motions, which give rise to circular orbits (adiabatic approximation). Consider a sequence of circular orbits of smaller and smaller radius, mimicking the inspiral. The ISCO is defined as the *turning point* in the *binding energy* of this sequence.

$\longleftarrow$ [Buonanno & Damour, PRD **62**, 064015 (2000)]

Computing quasiequilibrium configurations of close binary compact objects

Last orbits of the inspiral

- Initial motivation: provide initial data for numerical computation of the plunge and merger
- But these configurations have interest from their own: they can lead to the (adiabatic) ISCO, which may be observed in gravitational waveforms

Note: gravitational radiation reaction makes the orbital eccentricity to vanish $\Rightarrow$ one must deal only with circular orbits

2

The Helical Killing Vector (HKV) approach

Basics

Problem treated:

Binary black holes or neutron stars in the pre-coalescence stage
$\Rightarrow$ the notion of **orbit** has still some meaning

Basic idea:

Construct an approximate, but full spacetime (i.e. **4-dimensional**) representing 2 orbiting compact objects
Previous numerical treatments (IVP) : 3-dimensional (initial value problem on a spacelike 3-surface)
4-dimensional approach $\Rightarrow$ rigorous definition of orbital angular velocity

Formulation of the problem :

Binary NS : [Gourgoulhon, Grandclément, Taniguchi, Marck & Bonazzola, PRD **63**, 064029 (2001)]

Binary BH : [Gourgoulhon, Grandclément & Bonazzola, PRD **65**, 044020 (2002)]

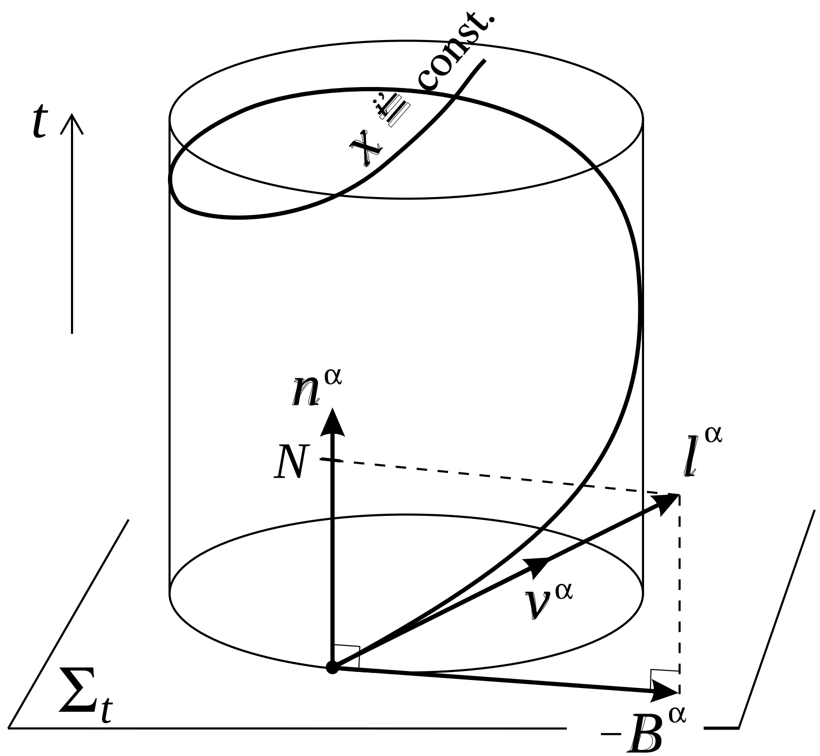
Helical symmetry

Physical assumption: when the two compact objects are sufficiently far apart, the radiation reaction can be neglected $\Rightarrow$ closed orbits
Gravitational radiation reaction circularizes the orbits $\Rightarrow$ circular orbits

Geometrical translation: there exists a Killing vector field $\boldsymbol{\ell}$ such that:

far from the system (asymptotically inertial coordinates $(t_0, r_0, \theta_0, \varphi_0)$),

$$\boldsymbol{\ell} \rightarrow \frac{\partial}{\partial t_0} + \Omega \frac{\partial}{\partial \varphi_0}$$



Helical symmetry: discussion

Helical symmetry is exact

- in **Newtonian gravity** and in **2nd order Post-Newtonian gravity**
- in general relativity for a non-axisymmetric system (binary) only with **standing gravitational waves**

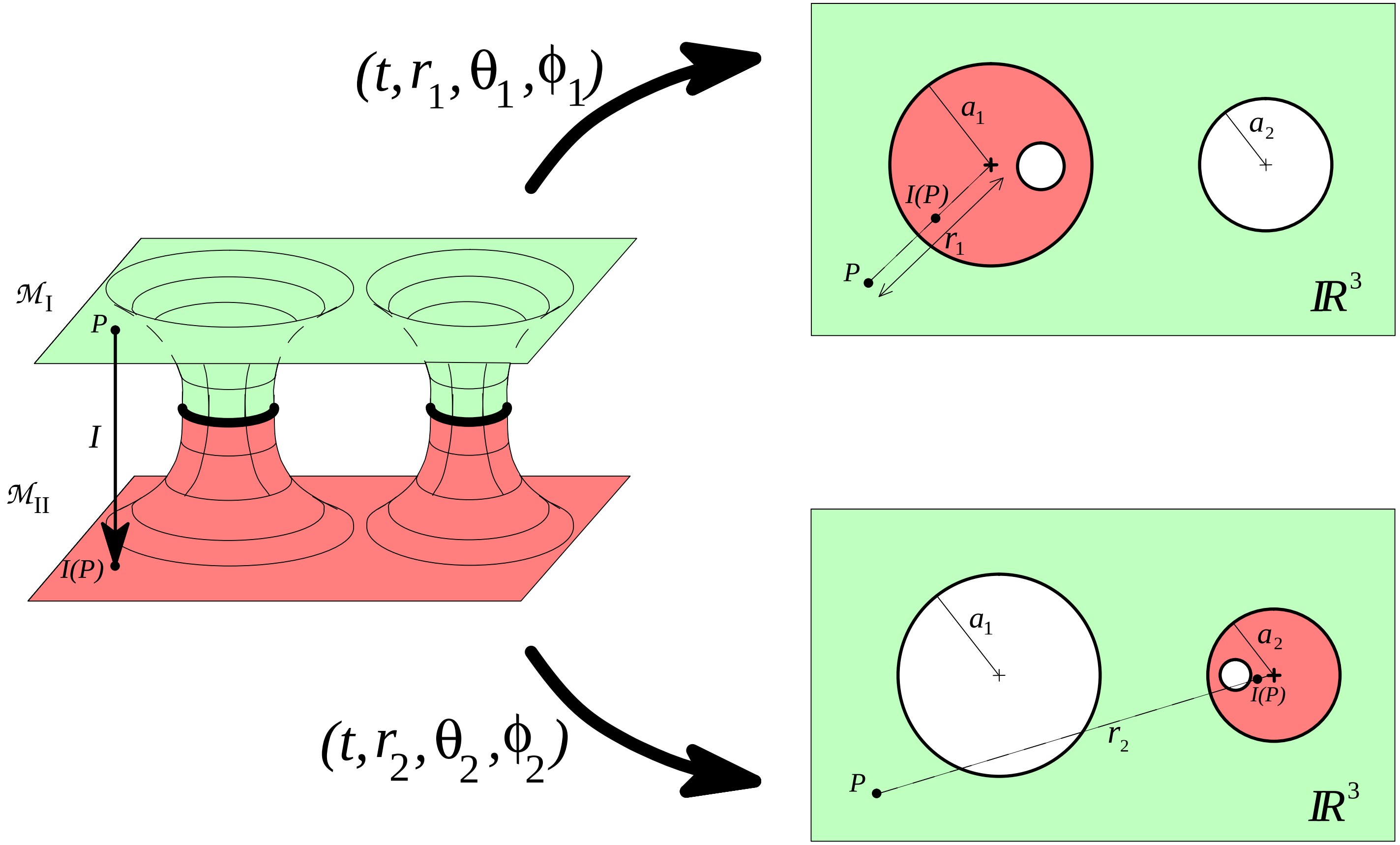
But a spacetime with a helical Killing vector and standing gravitational waves cannot be asymptotically flat in full GR [Gibbons & Stewart 1983].

We have used a truncated version of GR (the **Isenberg-Wilson-Mathews** approximation, which will be described below) which (i) admits the helical Killing vector and (ii) is asymptotically flat.

Spacetime manifold

Topology : for binary NS : $\mathbb{R}^4$

for binary BH : $\mathbb{R} \times$ Misner-Lindquist



Canonical mapping: $I: \quad (t, r_1, \theta_1, \varphi_1) \mapsto \left(t, \frac{a_1^2}{r_1}, \theta_1, \varphi_1\right)$

Fluid equation of motion

Neutron star fluid = perfect fluid : $\mathbf{T} = (e+p)\mathbf{u}\otimes\mathbf{u} + p\mathbf{g}$.

Carter-Lichnerowicz equation of motion for zero-temperature fluids:

$$\nabla\cdot\mathbf{T} = 0 \iff \begin{cases} \mathbf{u}\cdot\mathbf{dw} = 0 & (1) \\ \nabla\cdot(n\mathbf{u}) = 0 & (2) \end{cases}$$

$\mathbf{w} := h\mathbf{u}$: co-momentum 1-form
$\mathbf{dw}$: vorticity 2-form

with n = baryon number density and $h = (e+p)/(m_{\rm B}n)$ specific enthalpy.

Cartan identity : Killing vector $\boldsymbol{\ell} \Longrightarrow \pounds_{\boldsymbol{\ell}}\mathbf{w} = 0 = \boldsymbol{\ell}\cdot\mathbf{dw} + \mathbf{d}(\boldsymbol{\ell}\cdot\mathbf{w})$ (3)

Two cases with a first integral : $\boxed{\boldsymbol{\ell}\cdot\mathbf{w} = \mathrm{const}}$ (4)

- **Rigid motion:** $\mathbf{u} = \lambda\boldsymbol{\ell}$: (3) + (1) ⇔ (4) ; (2) automatically satisfied
- **Irrotational motion:** $\mathbf{dw} = 0 \Leftrightarrow \mathbf{w} = \boldsymbol{\nabla}\Psi$: (3) ⇔ (4) ; (1) automatically satisfied

(2) ⇔ $\frac{n}{h}\nabla\cdot\nabla\Psi + \nabla\left(\frac{n}{h}\right)\cdot\nabla\Psi = 0$

Astrophysical relevance of the two rotation states

- **Rigid motion (synchronized binaries)** (also called **corotating binaries**) : the viscosity of neutron star matter is far too low to ensure synchronization of the stellar spins with the orbital motion [Kochanek, ApJ **398**, 234 (1992)], [Bildsten & Cutler, ApJ **400**, 175 (1992)] $\Longrightarrow$ not realistic state of rotation

- **Irrotational motion:** good approximation for neutron stars which are not initially millisecond rotators, because then $\Omega_{\rm spin} \ll \Omega_{\rm orb}$ at the late stages.

Rotation state in the binary BH case

Choice: rotation synchronized with the orbital motion (corotating system)

Justifications:

- the only rotation state fully compatible with the helical symmetry
  [Friedman, Uryu & Shibata, PRD **65**, 064035 (2002)]
- for close systems, black hole "effective viscosity" might be very efficient in synchronizing the spins with the orbital motion
  [e.g. Price & Whelan, PRL **87**, 231101 (2001)]

Geometrical translation: the two horizons are Killing horizons associated with $\boldsymbol{\ell}$:

$$\boldsymbol{\ell} \cdot \boldsymbol{\ell}|_{\mathcal{H}_1} = 0 \qquad \text{and} \qquad \boldsymbol{\ell} \cdot \boldsymbol{\ell}|_{\mathcal{H}_2} = 0 \ .$$

[cf. the rigidity theorem for a Kerr black hole]

Einstein equations

Framework: 3+1 formalism with maximal slicing: $K = 0$

Isenberg-Wilson-Mathews approximation: conformally flat spatial metric: $\boldsymbol{\gamma} = \Psi^4 \boldsymbol{f}$

$\Rightarrow$ spacetime metric : $ds^2 = -N^2 dt^2 + \Psi^4 f_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$

Amounts to solve 5 of the 10 Einstein equations (one more than IVP !) :

$$\underline{\Delta}\Psi = -\Psi^5 \left(2\pi E + \frac{1}{8} \hat{A}_{ij} \hat{A}^{ij} \right) \qquad \textit{(Hamiltonian constraint)}$$

$$\underline{\Delta}\beta^i + \frac{1}{3} \bar{\nabla}^i \bar{\nabla}_j \beta^j = 16\pi N \Psi^4 J^i + 2\hat{A}^{ij} \left(\bar{\nabla}_j N - 6N \bar{\nabla}_j \ln \Psi \right) \qquad \textit{(momentum constraint)}$$

$$\underline{\Delta} N = N\Psi^4 \left[4\pi(E + S) + \hat{A}_{ij} \hat{A}^{ij} \right] - 2\bar{\nabla}_j \ln \Psi \, \bar{\nabla}^j N \qquad \textit{(trace of } \frac{\partial K_{ij}}{\partial t} = \cdots \textit{)}$$

with $\hat{A}_{ij} := \Psi^{-4} K_{ij}$ and $\hat{A}^{ij} := \Psi^4 K^{ij}$

Extrinsic curvature : helical symmetry $\Rightarrow$ $2NK_{ij} = D_i \beta_j + D_j \beta_i$

$$\hat{A}^{ij} = \frac{1}{2N} (\bar{L}\beta)^{ij} \text{ with } (\bar{L}\beta)^{ij} := \bar{\nabla}^i \beta^j + \bar{\nabla}^j \beta^i - \frac{2}{3} \bar{\nabla}_k \beta^k f^{ij} \qquad \textit{(traceless part)}$$

$$\bar{\nabla}_i \beta^i = -6\beta^i \bar{\nabla}_i \ln \Psi \qquad \textit{(trace part)}$$

Boundary conditions

Inner boundary (binary BH only):		Spatial infinity:
isometry condition on γ_{rr}:		asymptotic flatness:
$\left(\frac{\partial\Psi}{\partial r_1} + \frac{\Psi}{2r_1}\right)\Big\vert_{\mathcal{S}_1} = 0$	$\left(\frac{\partial\Psi}{\partial r_2} + \frac{\Psi}{2r_2}\right)\Big\vert_{\mathcal{S}_2} = 0$	$\Psi \to 1$ when $r \to \infty$
corotating black holes:		definition of $\boldsymbol{\ell}$:
$\boldsymbol{\beta}\vert_{\mathcal{S}_1} = 0$	$\boldsymbol{\beta}\vert_{\mathcal{S}_2} = 0$	$\boldsymbol{\beta} \to \Omega\frac{\partial}{\partial\varphi_0}$ when $r \to \infty$
isometry condition on N:		asymptotic flatness:
$N\vert_{\mathcal{S}_1} = 0$	$N\vert_{\mathcal{S}_2} = 0$	$N \to 1$ when $r \to \infty$

Additional equations in the fluid case (binary NS)

Baryon number conservation for irrotational flows:

$$n\underline{\Delta}\Psi + \bar{\nabla}_i n \, \bar{\nabla}^i \Psi = \cdots$$

$\longrightarrow$ singular ($n = 0$ at the stellar surface) elliptic equation to be solved for Ψ.

First integral of fluid motion $\boldsymbol{\ell} \cdot \mathbf{w} = \mathrm{const}$ writes $\quad hN\frac{\Gamma}{\Gamma_0} = \mathrm{const} \quad (5)$

with Γ : Lorentz factor between fluid co-moving observer and co-orbiting observer (= 1 for synchronized binaries)

Γ_0 : Lorentz factor between co-orbiting observer and asymptotically inertial observer

$\longrightarrow$ solve (5) for the specific enthalpy h.

From h compute the fluid proper energy density e, pressure p and baryon number n via an equation of state:

$$e = e(h), \qquad p = p(h), \qquad n = n(h)$$

Determination of Ω : NS case

First integral of fluid motion:

$$hN\frac{\Gamma}{\Gamma_0} = \mathrm{const}$$

The Lorentz factor Γ_0 contains Ω: at the Newtonian limit, $\ln\Gamma_0$ is nothing but the centrifugal potential: $\ln\Gamma_0 \sim \frac{1}{2}(\mathbf{\Omega}\times\mathbf{r})^2$.

At each step of the iterative procedure, Ω and the location of the rotation axis are then determined so that the stellar centers (density maxima) remain at fixed coordinate distance from each other.

Determination of Ω : BH case

Virial assumption: $O(r^{-1})$ part of the metric $(r \to \infty)$ same as Schwarzschild

[The only quantity "felt" at the $O(r^{-1})$ level by a distant observer is the total mass of the system.]

A priori

$$\Psi \sim 1 + \frac{M_{\rm ADM}}{2r} \qquad \text{and} \qquad N \sim 1 - \frac{M_{\rm K}}{r}$$

Hence

$$\text{(virial assumption)} \iff M_{\rm ADM} = M_{\rm K}$$

Note

$$\text{(virial assumption)} \iff \Psi^2 N \sim 1 + \frac{\alpha}{r^2}$$

Link with the classical virial theorem

Einstein equations $\Rightarrow$

$$\underline{\Delta} \ln(\Psi^2 N) \; = \; \Psi^4 \left[4\pi S_i^{\ i} + \frac{3}{4} \hat{A}_{ij} \hat{A}^{ij} \right] - \frac{1}{2} \left[\bar{\nabla}_i \ln N \bar{\nabla}^i \ln N + \bar{\nabla}_i \ln(\Psi^2 N) \bar{\nabla}^i \ln(\Psi^2 N) \right]$$

No monopolar $1/r$ term in $\Psi^2 N \iff$

$$\int_{\Sigma_t} \left\{ 4\pi S_i^{\ i} + \frac{3}{4} \hat{A}_{ij} \hat{A}^{ij} - \frac{\Psi^{-4}}{2} \left[\bar{\nabla}_i \ln N \bar{\nabla}^i \ln N + \bar{\nabla}_i \ln(\Psi^2 N) \bar{\nabla}^i \ln(\Psi^2 N) \right] \right\} \Psi^4 \sqrt{f} \, d^3x \; = \; 0$$

Newtonian limit is the classical virial theorem:

$$2E_{\rm kin} + 3P + E_{\rm grav} = 0$$

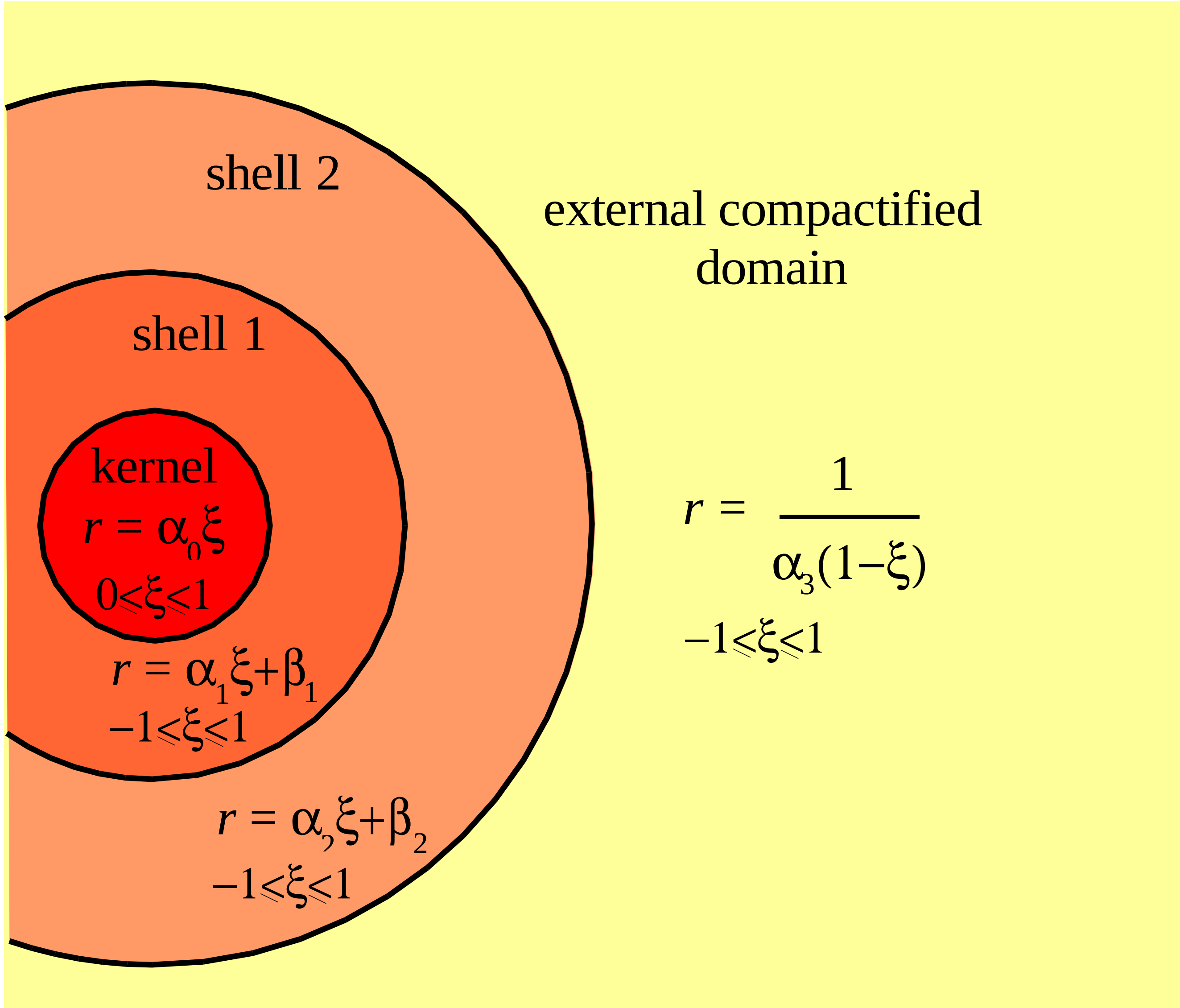
3

Numerical technique

Spectral methods developed in Meudon

- Multidomain three-dimensional spectral method
- Spherical-type coordinates (r, θ, φ)
- Expansion functions: r : Chebyshev; θ : cosine/sine or associated Legendre functions; φ : Fourier
- Domains = spherical shells + 1 nucleus (contains $r = 0$)
- Entire space ($\mathbb{R}^3$) covered: compactification of the outermost shell
- Adaptative coordinates : domain decomposition with spherical topology
- Multidomain PDEs: patching method (strong formulation)
- Numerical implementation: C++ codes based on LORENE

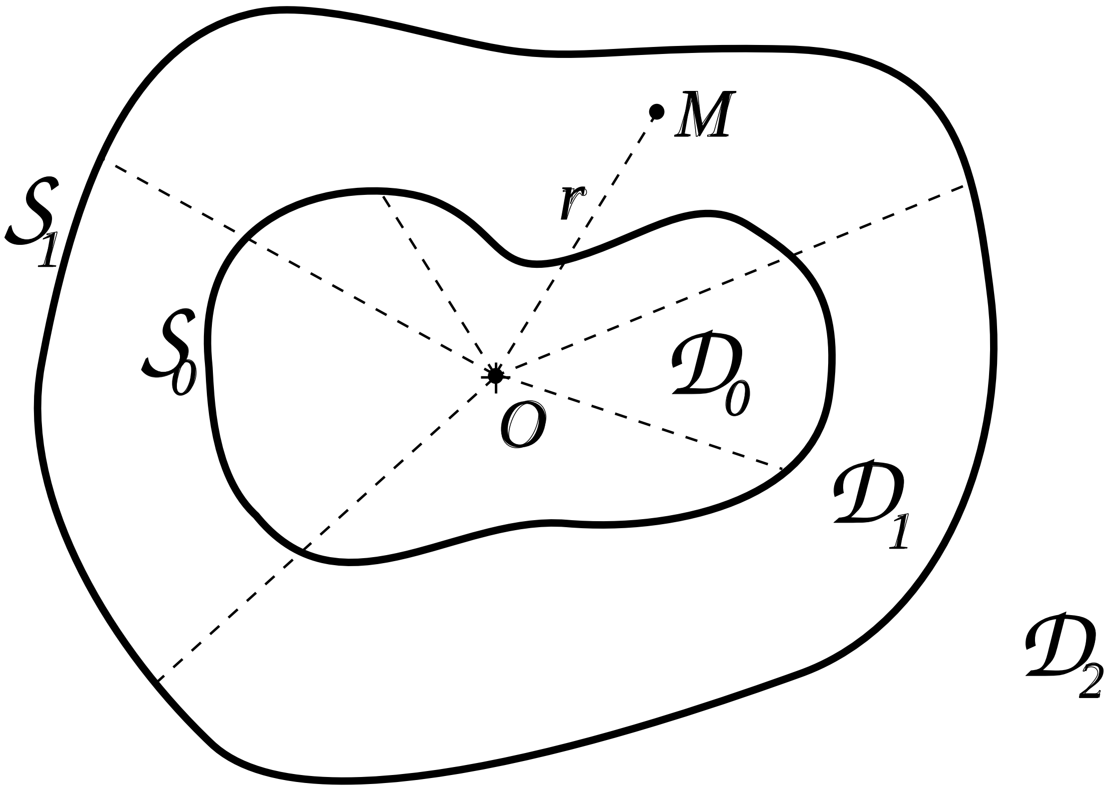
Domain decomposition



physical coordinates
(r, θ, φ)

comput. coordinates
(ξ, θ', φ')

Starlike domain decomposition



$\mathcal{N}$ nonoverlapping starlike domains:

- $\mathcal{D}_0$: *nucleus*
- $\mathcal{D}_q \quad (1 \leq q \leq \mathcal{N} - 2)$: *shell*
- $\mathcal{D}_{\mathcal{N}-1}$: *external domain*

$$\mathcal{D}_0 \cup \mathcal{D}_1 \cup \cdots \cup \mathcal{D}_{\mathcal{N}-1} = \mathbb{R}^3$$

Mapping computational space → physical space

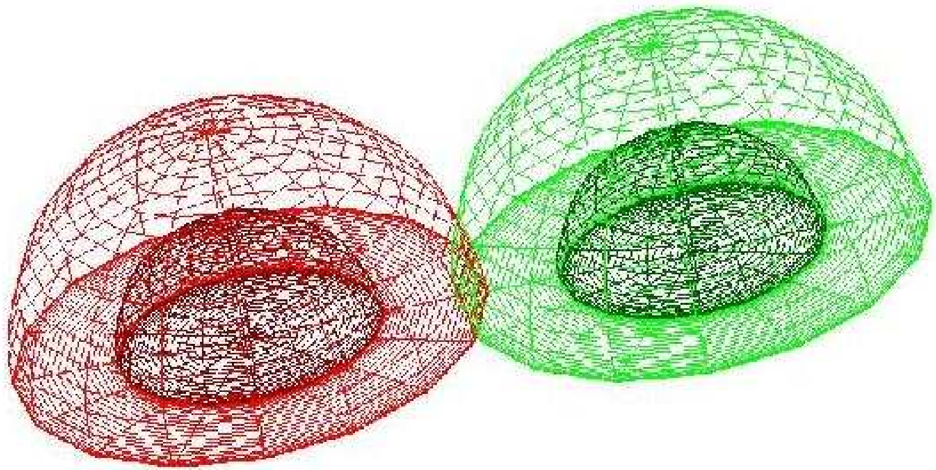
Mapping for domain $\mathcal{D}_q$: $$\begin{array}{rcl} [-1+\delta_{0q}, 1] \times [0,\pi] \times [0, 2\pi[& \longrightarrow & \mathcal{D}_q \\ (\xi, \theta', \varphi') & \longmapsto & (r, \theta, \varphi) \end{array}$$

Radial mapping : $\theta = \theta'$ and $\varphi = \varphi'$

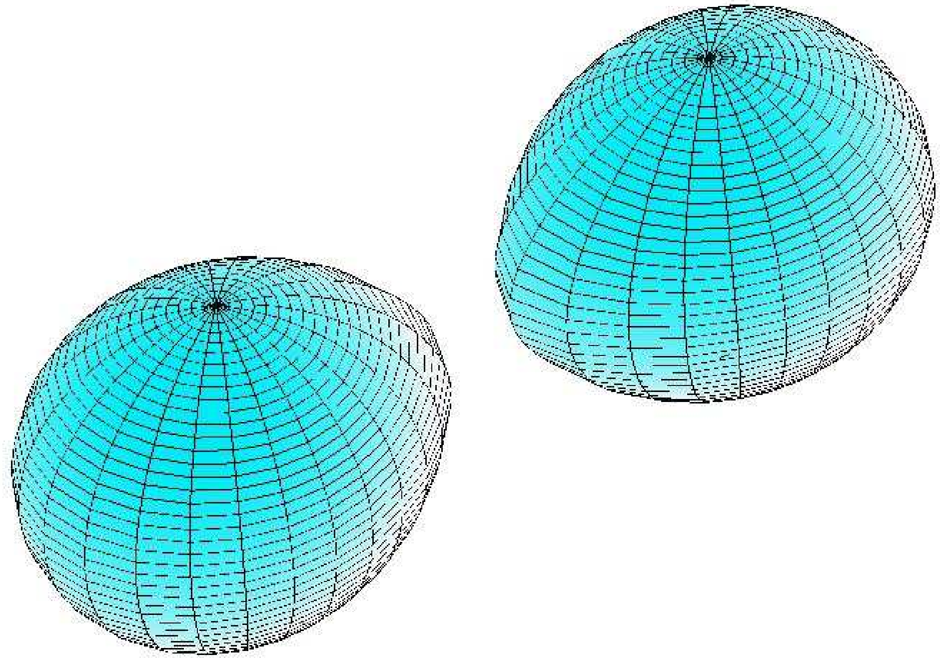
- in the nucleus: $\xi \in [0,1]$ $$r = \alpha_0 \left[\xi + \left(3\xi^4 - 2\xi^6\right) F_0(\theta,\varphi) + \frac{1}{2}\left(5\xi^3 - 3\xi^5\right) G_0(\theta,\varphi) \right]$$
- in the shells: $\xi \in [-1,1]$ $$r = \alpha_q \left[\xi + \frac{1}{4}\left(\xi^3 - 3\xi + 2\right) F_q(\theta,\varphi) + \frac{1}{4}\left(-\xi^3 + 3\xi + 2\right) G_q(\theta,\varphi) \right] + \beta_q$$
- in the external domain: $\xi \in [-1,1]$ $$\frac{1}{r} = \alpha_{\rm ext} \left[\xi + \frac{1}{4}\left(\xi^3 - 3\xi + 2\right) F_{\rm ext}(\theta,\varphi) - 1 \right]$$

[Bonazzola, Gourgoulhon & Marck, Phys. Rev. D **58**, 104020 (1998)]

Binary star with surface fitted coordinates



Double domain decomposition



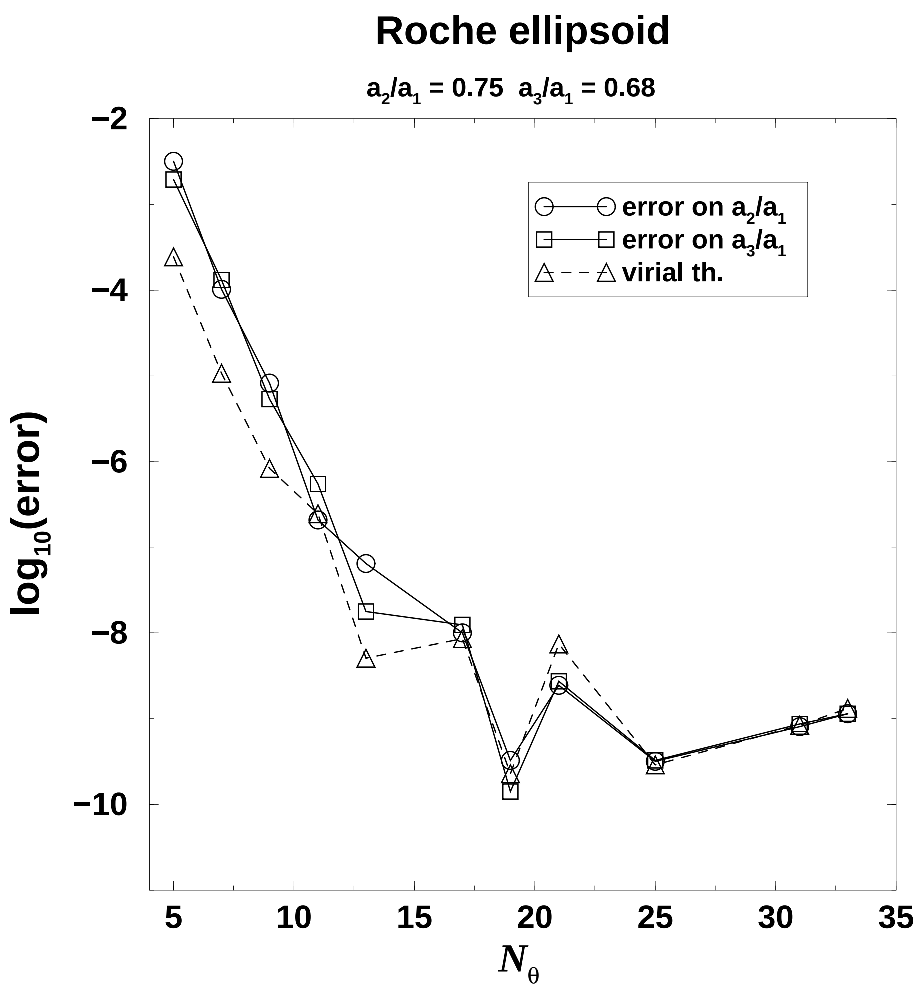
[Taniguchi, Gourgoulhon & Bonazzola, Phys. Rev. D **64**, 064012 (2001)]

Surface fitted coordinates:

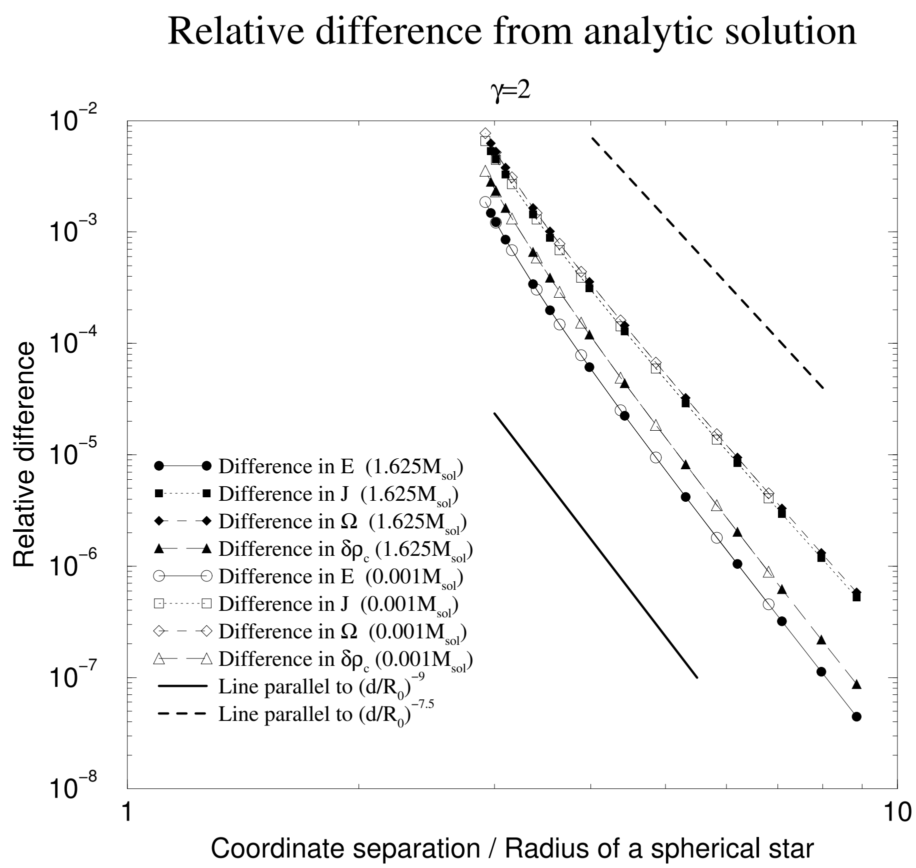
$F_0(\theta, \varphi)$ and $G_0(\theta, \varphi)$ chosen so that

$\xi = 1 \Leftrightarrow$ surface of the star

A test for binary NS: comparison with analytical solutions

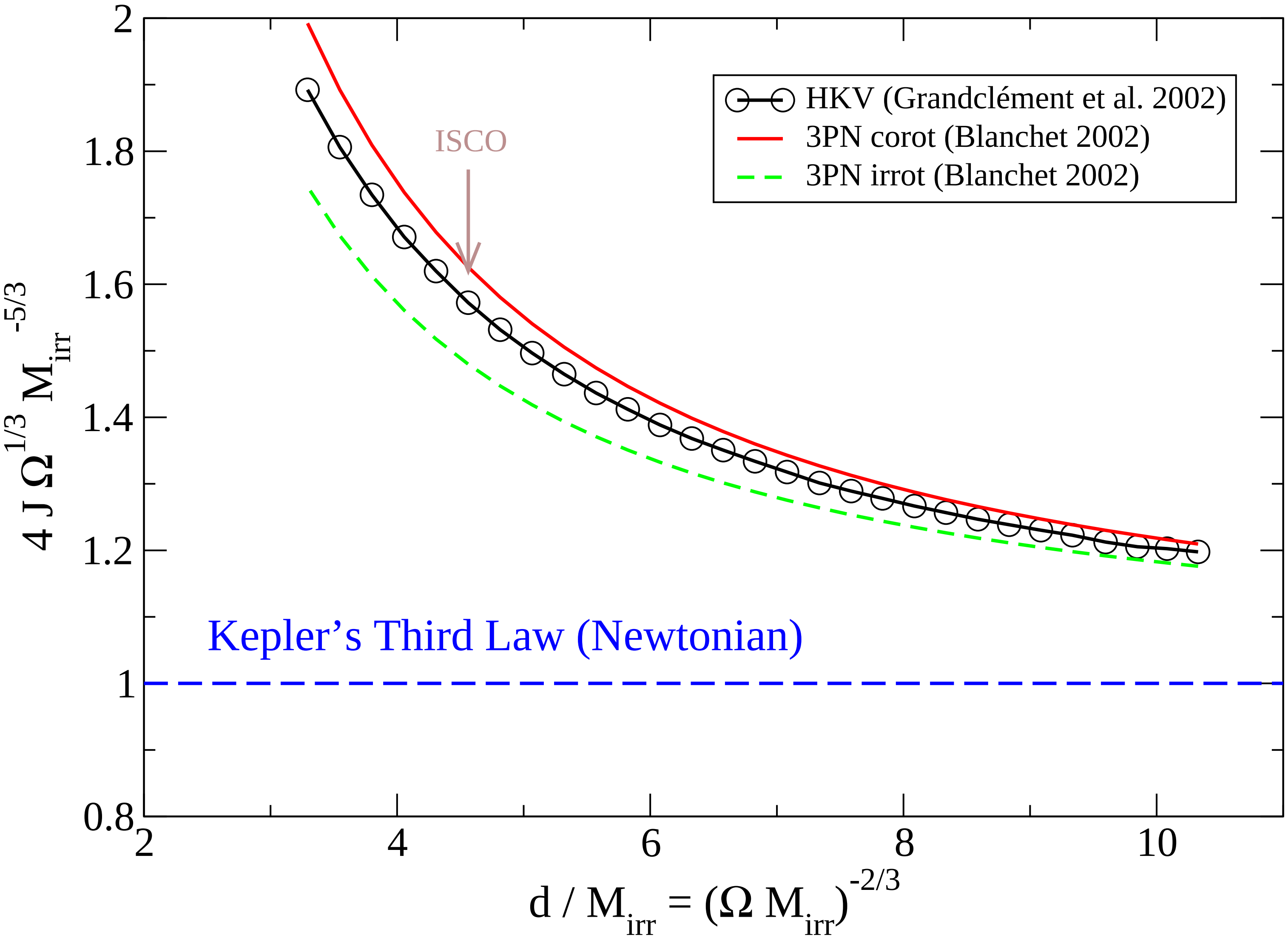


Difference w.r.t. the Roche solution



Difference w.r.t. Taniguchi & Nakamura approxim. solution

A test for binary BH: recovering Kepler's third law



Check of the determination of Ω at large separation.

4

Results for binary neutron stars

Defining an evolutionary sequence of binary NS

The gravitational radiation driven evolution of binary neutron stars preserves the baryon number.

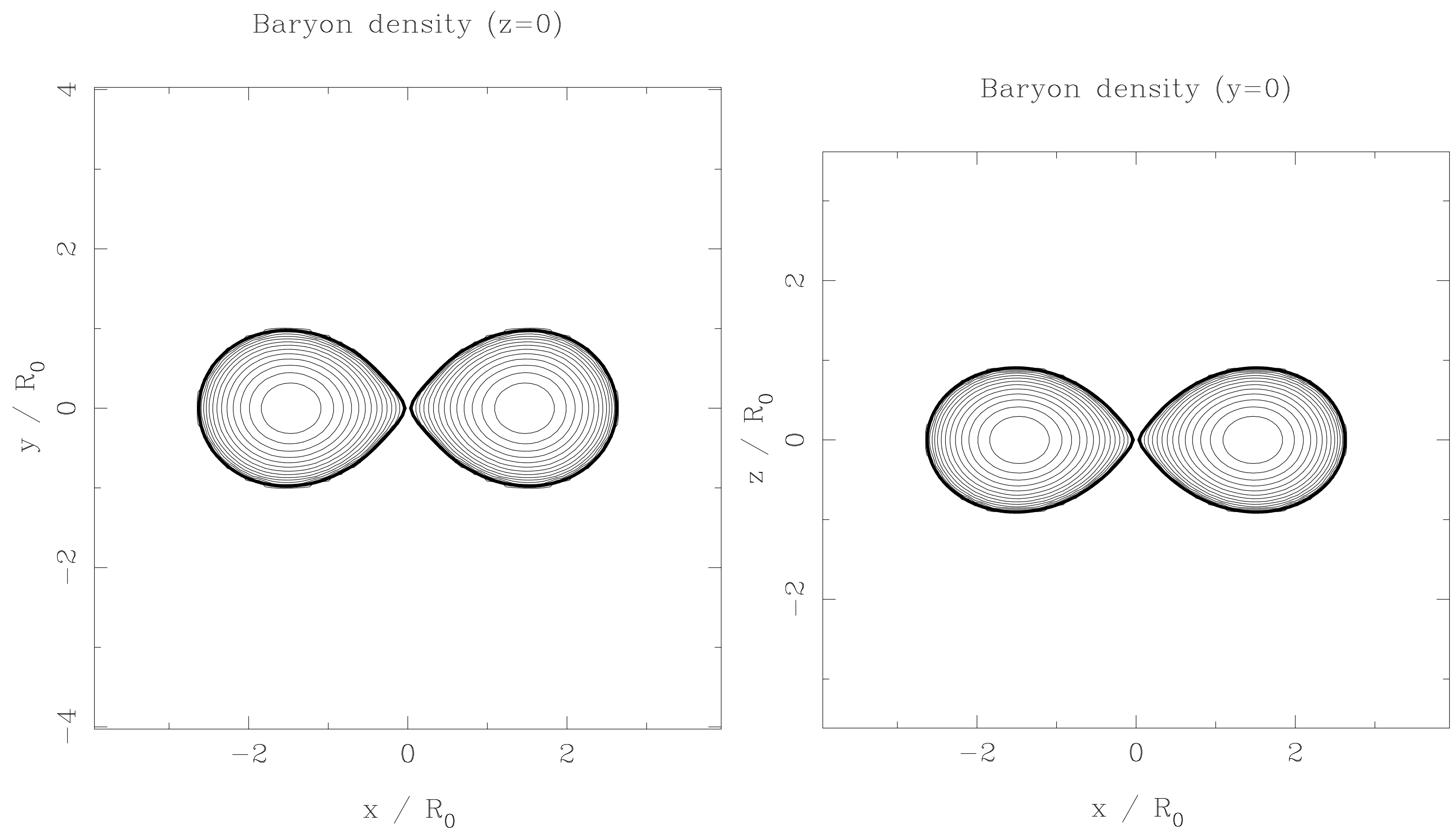
The quasi-stationary evolution of binary NS is modelled by computing a sequence of HKV configurations with

- decreasing separation
- fixed total baryon number of each star

Results for Newtonian polytropic stars

End of the sequence:

- synchronized binaries: contact between the two stars

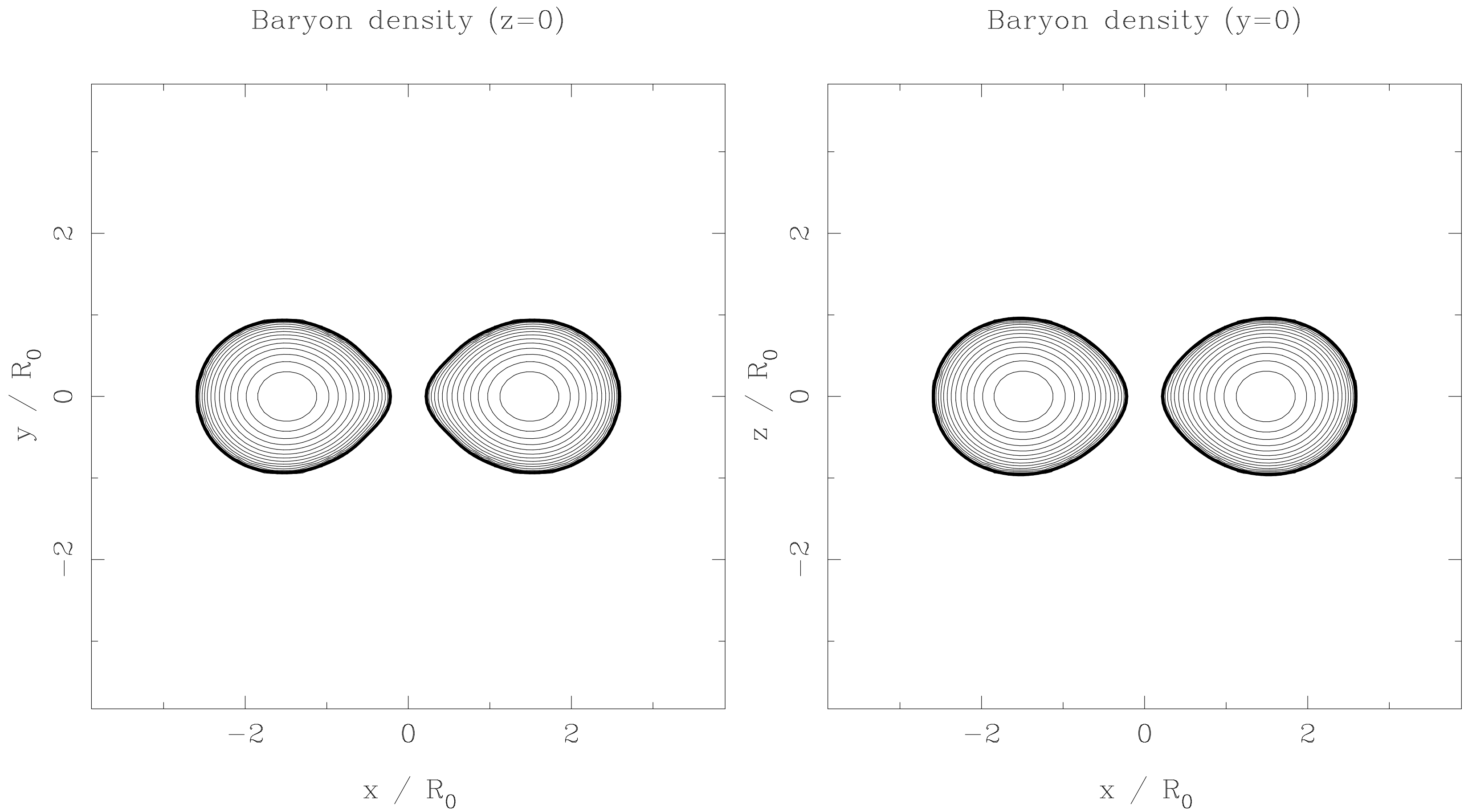


$\gamma = 3$ synchronized configuration [Taniguchi, Gourgoulhon & Bonazzola, PRD **64**, 064012 (2001)]

Results for Newtonian polytropic stars (con't)

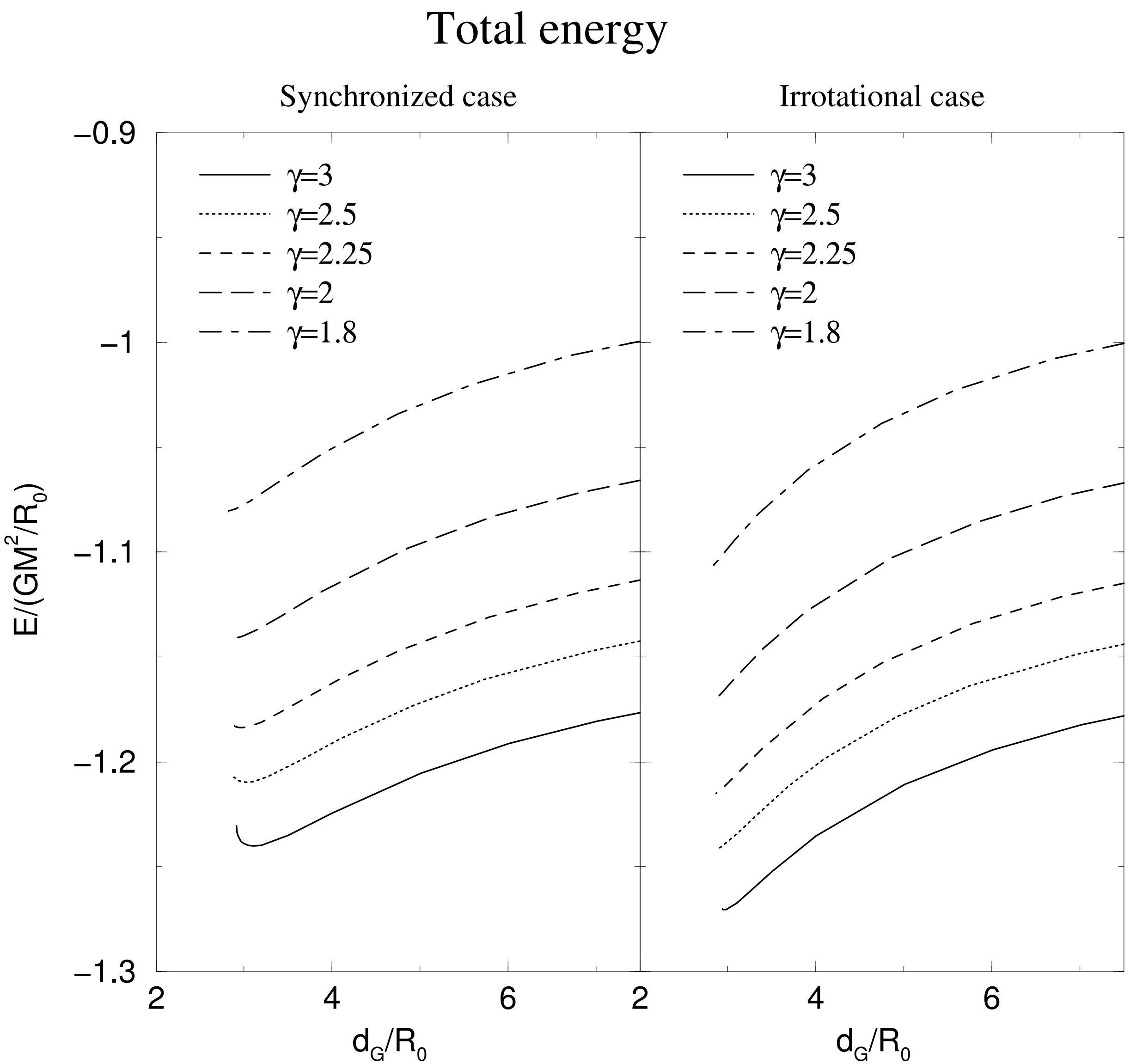
End of the sequence:

- irrotational binaries: mass-shedding detached configuration



$\gamma = 3$ irrotational configuration [Taniguchi, Gourgoulhon & Bonazzola, PRD **64**, 064012 (2001)]

Results for Newtonian polytropic stars (con't)



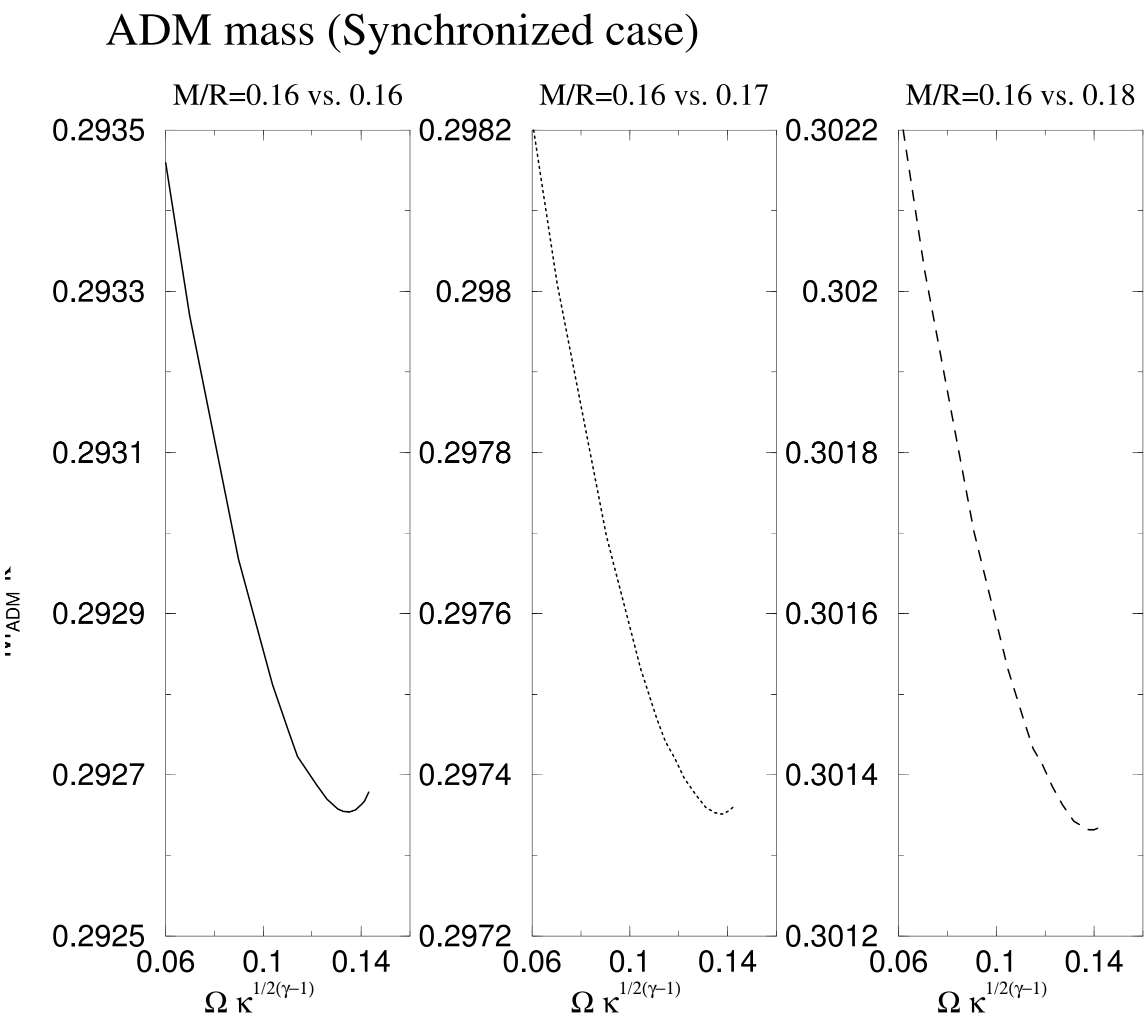
Last stable orbit: minimum of energy along the sequence

For irrotational binaries: exists before mass-shedding only for $\gamma \gtrsim 2.3$

[Taniguchi, Gourgoulhon & Bonazzola, PRD **64**, 064012 (2001)]

Results for relativistic polytropic stars

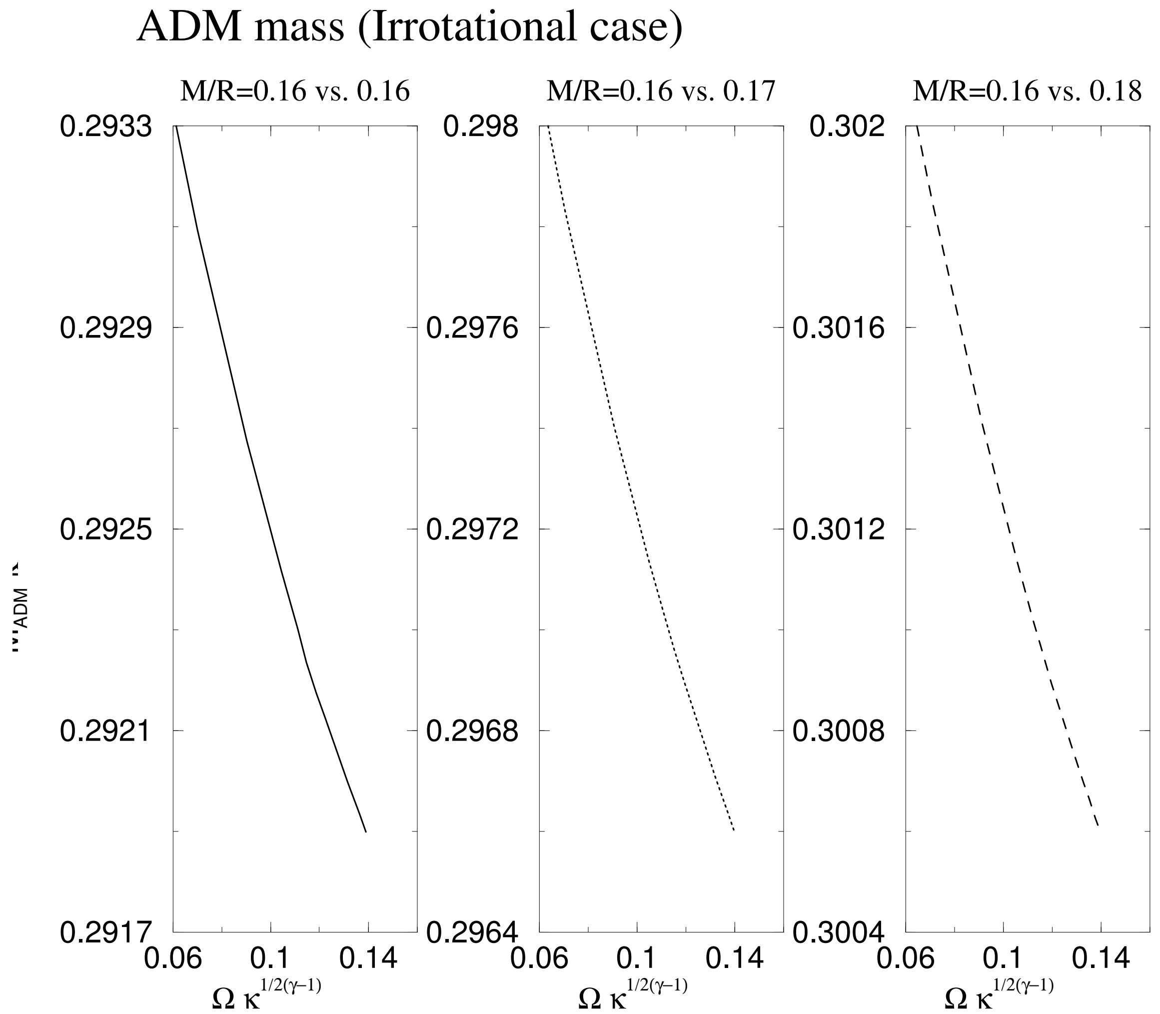
$\gamma = 2$ synchronized configurations with different mass ratios



[Taniguchi & Gourgoulhon, PRD **66**, 104019 (2002)]

Results for relativistic polytropic stars (con't)

$\gamma = 2$ irrotational configurations with different mass ratios

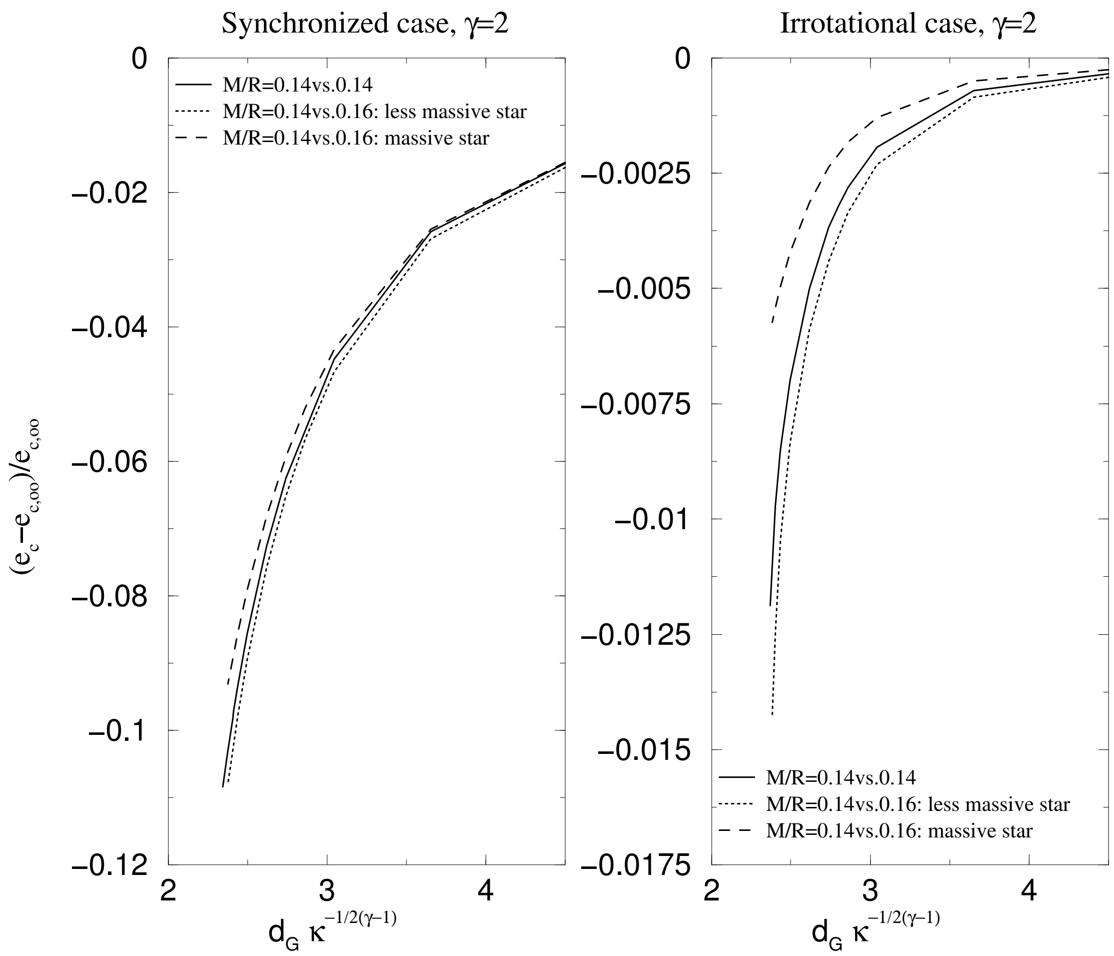


[Taniguchi & Gourgoulhon, PRD **66**, 104019 (2002)]

Results for relativistic polytropic stars (con't)

Stability against gravitational collapse of each star

Relative change in central energy density



[Taniguchi & Gourgoulhon, PRD **66**, 104019 (2002)]

5

Results for binary black holes

Defining an evolutionary sequence: BH case

An evolutionary sequence is defined by:

$$\left.\frac{dM_{\rm ADM}}{dJ}\right|_{\rm sequence} = \Omega$$

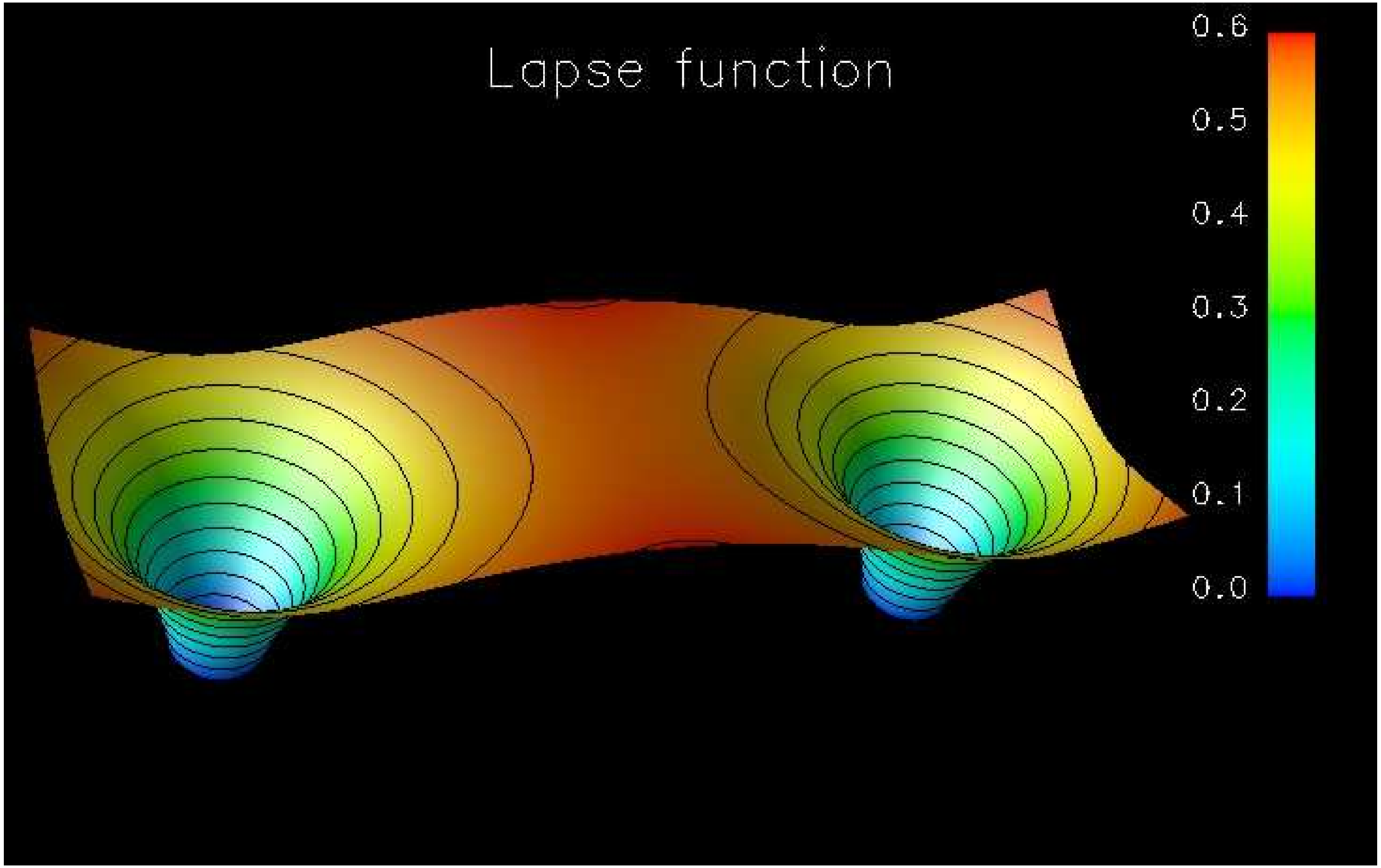
This is equivalent to requiring the constancy of the horizon area of each black hole, by virtue of the First law of thermodynamics for binary black holes :

$$dM_{\rm ADM} = \Omega\, dJ + \frac{1}{8\pi}\left(\kappa_1\, dA_1 + \kappa_2\, dA_2\right)$$

recently established by Friedman, Uryu & Shibata [PRD **65**, 064035 (2002)].

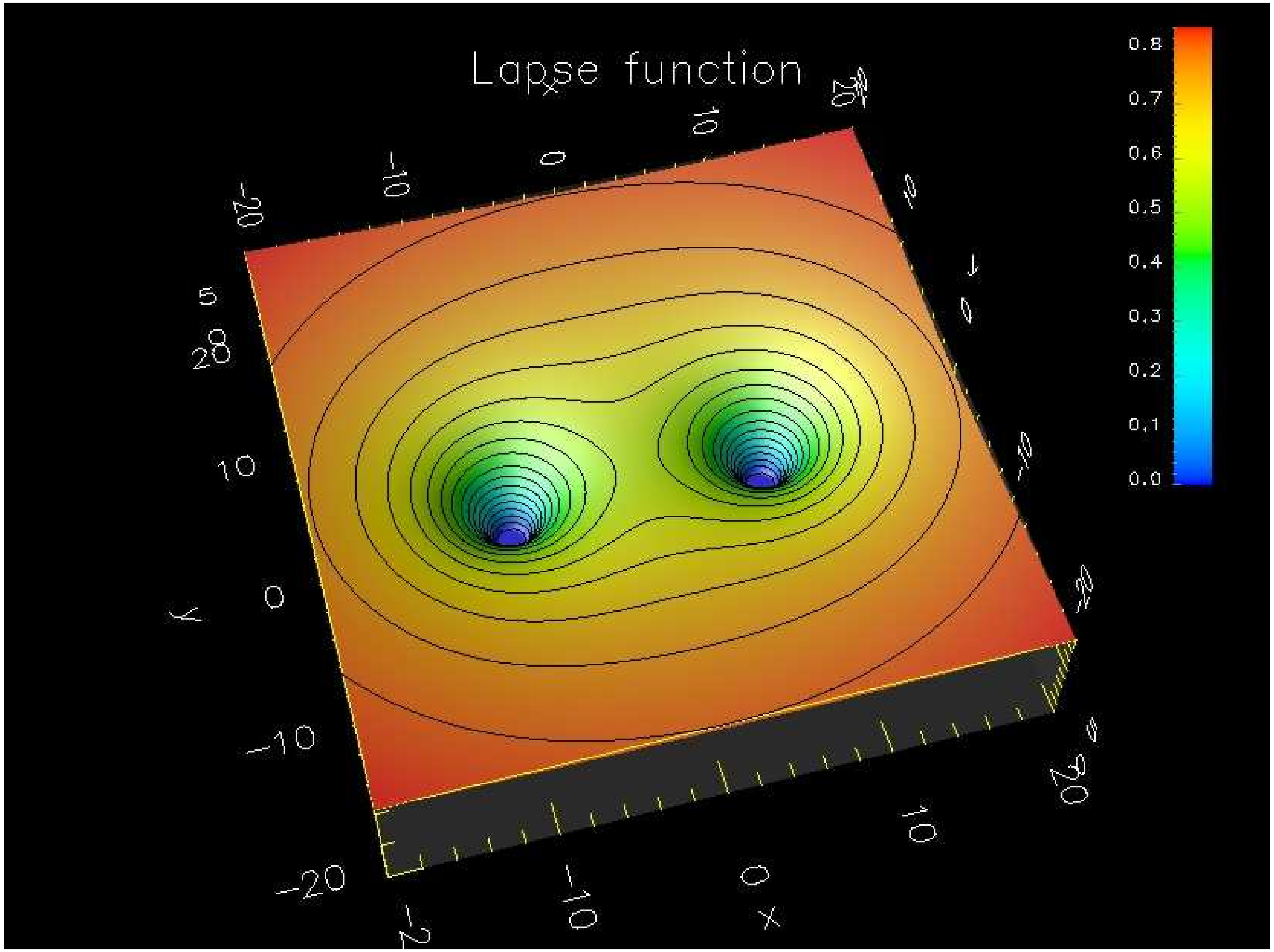
Note: Within the helical symmetry framework, a minimum in $M_{\rm ADM}$ along a sequence at fixed horizon area locates a change of orbital stability (ISCO) [Friedman, Uryu & Shibata, PRD **65**, 064035 (2002)].

ISCO configuration



[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

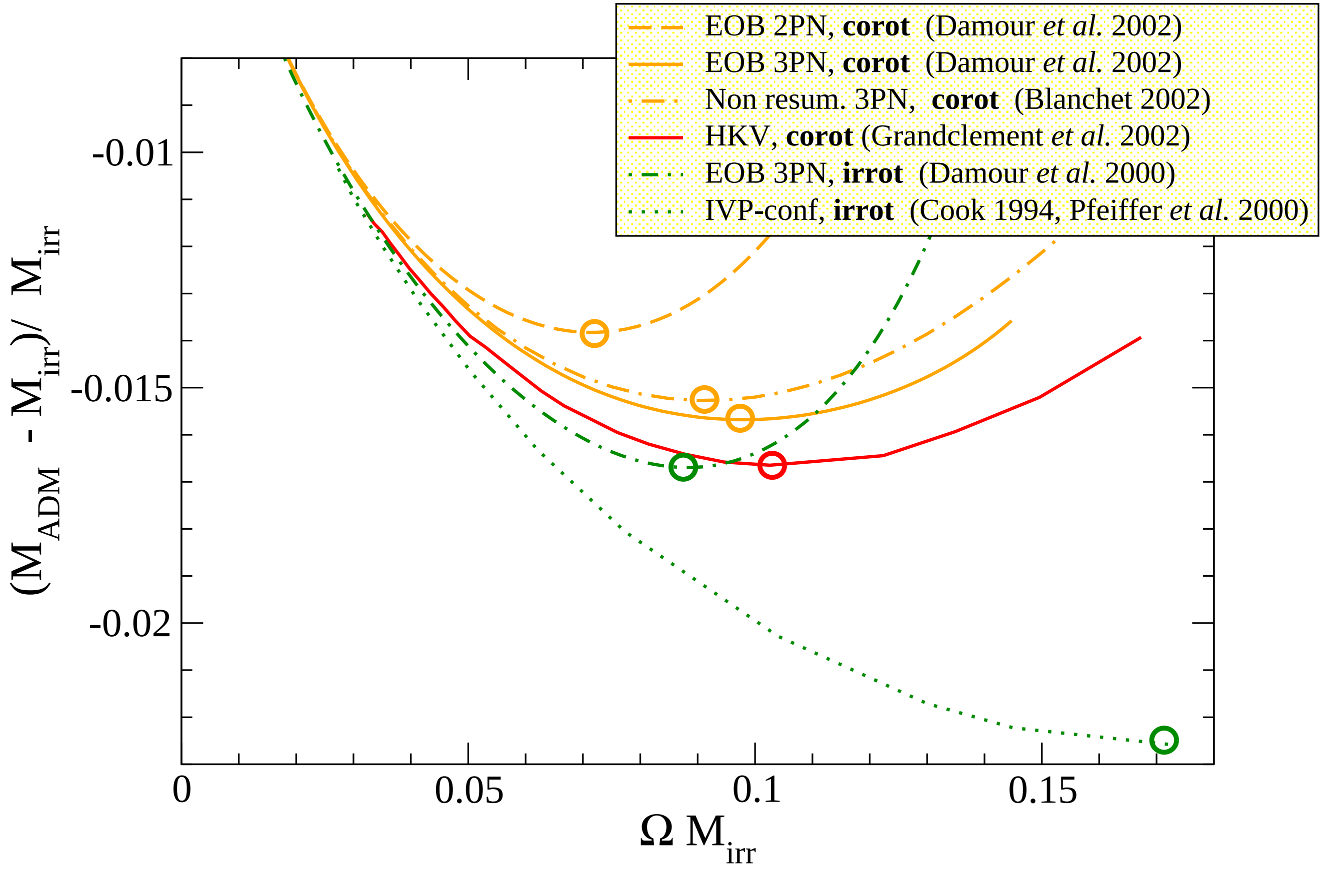
ISCO configuration



[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

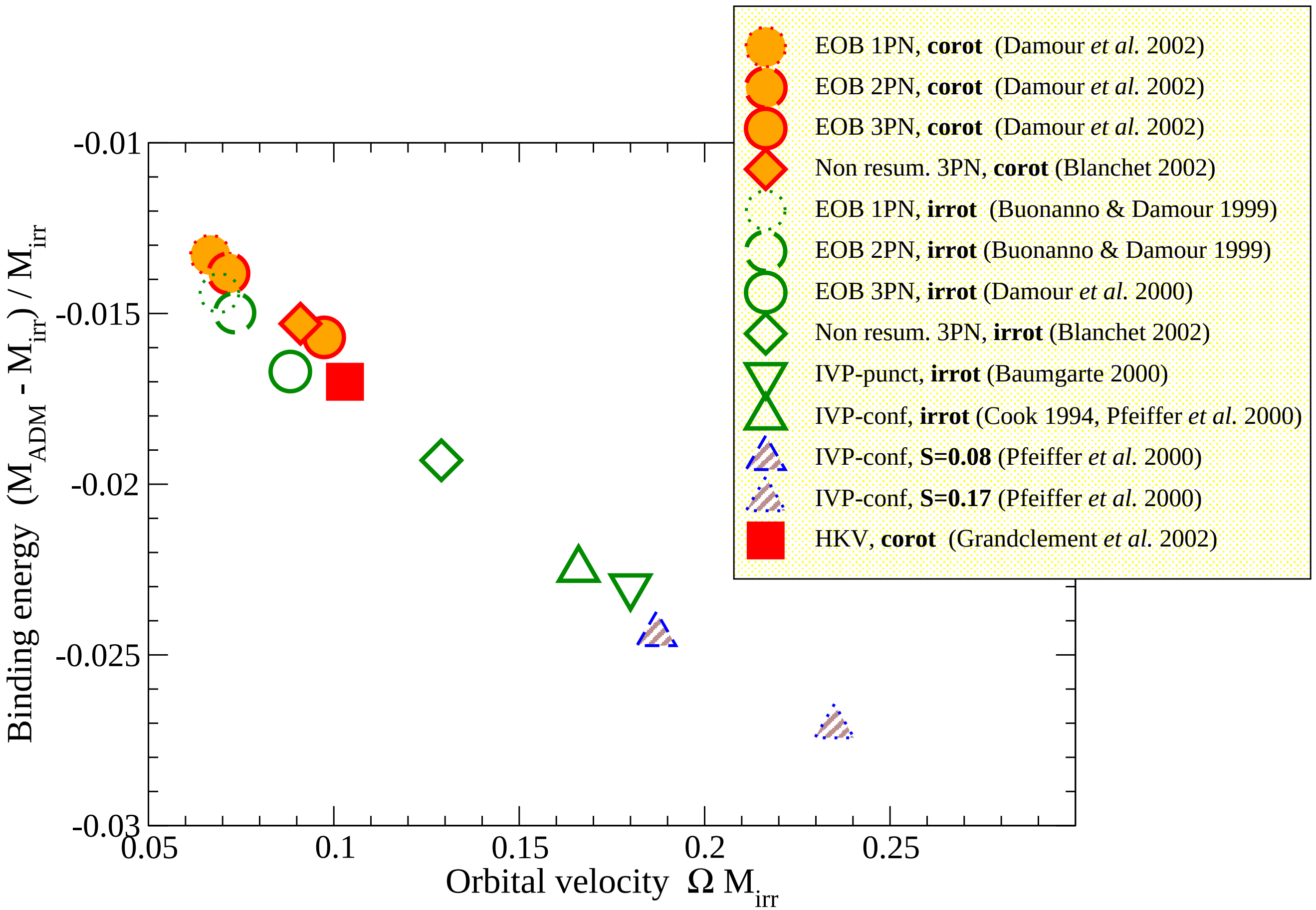
Comparison with Post-Newtonian computations

Binding energy along an evolutionary sequence of equal-mass binary black holes



[Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Location of the ISCO



Gravitational wave frequency:

$$f = 320 \frac{\Omega M_{\rm ir}}{0.1} \frac{20\, M_\odot}{M_{\rm ir}} \ {\rm Hz}$$

[Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Conclusions and future prospects

- Conclusions for binary BH:
  - ⋆ The classical Bowen-York extrinsic curvature does not represent well binary black holes in quasiequilibrium orbital motion
  - ⋆ The helical Killing vector approach results in very good agreement with post-Newtonian computations

- Future studies for binary NS:
  - ⋆ using EOS from nuclear physics (M. Bejger, P. Haensel, J.L. Zdunik)
  - ⋆ computing binary systems of strange quark stars (D. Gondek-Rosińska)

- Next computational step: relaxing the conformal flatness hypothesis, while keeping the helical symmetry :
  - ⋆ within the classical 3+1 formalism (F. Limousin)
  - ⋆ within the Ehlers-Geroch quotient formalism (C. Klein, J. Novak)