The black hole no-hair theorem

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based on a collaboration with Philippe Grandclément, Carlos Herdeiro, Zakaria Meliani, Jérôme Novak, Thibaut Paumard, Guy Perrin, Eugen Radu, Claire Somé, Odele Straub and Frédéric H. Vincent

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The no-hair theorem

2 Theoretical alternatives to the Kerr black hole

3 Testing the no-hair theorem : some examples

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Outline

The no-hair theorem

Theoretical alternatives to the Kerr black hole

Testing the no-hair theorem : some examples

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 numbers :

- the total mass M
- the total specific angular momentum a = J/(Mc)
- the total electric charge Q

 \implies "a black hole has no hair" (John A. Wheeler)

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- \implies "a black hole has no hair" (John A. Wheeler)

Astrophysical black holes have to be electrically neutral :

• Q = 0: Kerr solution (1963)

Other special cases :

- a = 0: Reissner-Nordström solution (1916, 1918)
- a = 0 and Q = 0: Schwarzschild solution (1916)
- a = 0, Q = 0 and M = 0: Minkowski metric (1907)

The no-hair theorem : precise mathematical statement

Any spacetime $(\mathscr{M}, \boldsymbol{g})$ that

- is 4-dimensional
- is asymptotically flat
- is stationary
- is a solution of the vacuum Einstein equation : $\operatorname{Ric}(g) = 0$
- contains a black hole with a connected regular horizon
- does not contain any closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

domain of outer communications : black hole exterior

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Possible improvements : remove the hypotheses of analyticity and non-existence of closed timelike curves (analyticity removed recently but only for slowly rotating black holes [Alexakis, Ionescu & Klainerman, Duke Math. J. **163**, 2603 (2014)])

The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta} \,\mathrm{d}x^{\alpha} \,\mathrm{d}x^{\beta} = -\left(1 - \frac{2GMr}{c^{2}\rho^{2}}\right) c^{2}\mathrm{d}t^{2} - \frac{4GMar\sin^{2}\theta}{c^{2}\rho^{2}} c\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^{2}}{\Delta}\,\mathrm{d}r^{2}$$
$$+\rho^{2}\mathrm{d}\theta^{2} + \left(r^{2} + a^{2} + \frac{2GMa^{2}r\sin^{2}\theta}{c^{2}\rho^{2}}\right)\sin^{2}\theta\,\mathrm{d}\varphi^{2}$$

where

$$ho^2:=r^2+a^2\cos^2 heta$$
, $\Delta:=r^2-rac{2GM}{c^2}r+a^2$ and $r\in(-\infty,\infty)$

 \rightarrow spacetime manifold : $\mathscr{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r = 0 \& \theta = \pi/2\}$

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 \rightarrow Schwarzschild solution as the subcase a = 0:

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Basic properties of Kerr metric

- Asymptotically flat $(r
 ightarrow \pm \infty)$
- Stationary : metric components independent from \boldsymbol{t}
- Axisymmetric : metric components independent from arphi
- Not static when $a \neq 0$
- Contains a black hole $\iff 0 \le a \le m$, where $m := GM/c^2$ event horizon : $r = r_+ := m + \sqrt{m^2 - a^2}$
- Contains a curvature singularity at $ho=0\iff r=0$ and $heta=\pi/2$

 mass M : not a measure of the "amount of matter" inside the black hole, but rather a characteristic of the external gravitational field
 → measurable from the orbital period of a test particle in far circular orbit around the black hole (Kepler's third law)

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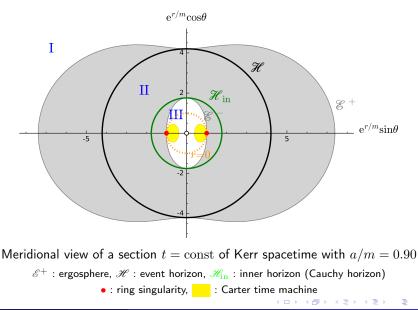
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Remark : the radius of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole "centre" and the event horizon. A well defined quantity is the area of the event horizon, A. The radius can be then defined from it : for a Schwarzschild black hole :

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3\left(\frac{M}{M_{\odot}}\right) \, \mathrm{km}$$

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Kerr spacetime



The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) case :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

 \implies No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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Rotating axisymmetric case :

No Birkhoff theorem

Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL 71, 1669 (1993)])

 \implies The Kerr metric is specific to rotating black holes (in 4-dimensional general relativity)

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Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k \in \mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- \mathcal{M}_k : mass 2^k -pole moment
- \mathcal{J}_k : angular momentum 2^k -pole moment
- \implies For the Kerr metric, all the multipole moments are determined by (M,a) :
 - $\mathcal{M}_0 = M$
 - $\mathcal{J}_1 = aM = J/c$

•
$$\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}$$
 (*)

 $\leftarrow \mathsf{mass} \mathsf{ quadrupole} \mathsf{ moment}$

- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- • •

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 - $\mathcal{M}_0 = M$
 - $\mathcal{J}_1 = aM = J/c$ • $\mathcal{M}_2 = -a^2M = -\frac{J^2}{c^2M}$ (*) \leftarrow mass quadrupole moment
 - $\mathcal{J}_3 = -a^3 M$
 - $\mathcal{M}_4 = a^4 M$
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Measuring the three quantities M, J, M_2 provides a compatibility test w.r.t. the Kerr metric, by checking (*)

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Galactic central BH and no-hair theorem

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Theoretical alternatives to the Kerr black hole

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Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

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Beyond general relativity

The compact object is a black hole but in a theory that differs from GR :

- Einstein-Gauss-Bonnet with dilaton
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Einstein-Yang-Mills
- ...

Theoretical alternatives to the Kerr black hole Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- ullet gravitation only described by a metric tensor g
- ullet field equation involving only derivatives of g up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ (\Longrightarrow weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

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However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

 ${\sf GR}$ is generally considered as a low-energy limit of a more fundamental theory :

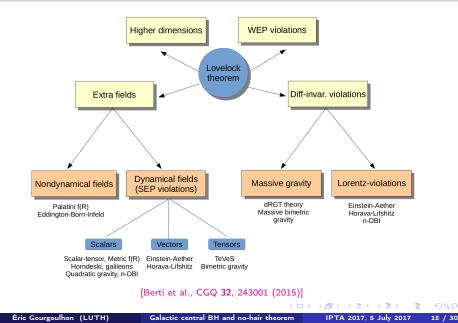
- string theory
- loop quantum gravity

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Theoretical alternatives to the Kerr black hole

Extensions of general relativity



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Observational tests

Search for

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 - ring-down phase of binary black hole mergers (LIGO, Virgo, LISA)
 - EMRI : extreme-mass-ratio binary inspirals (LISA)

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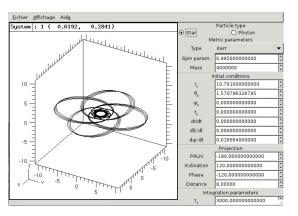
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- pulsar orbiting Sgr A* : the Holly Grail !

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Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]

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Galactic central BH and no-hair theorem

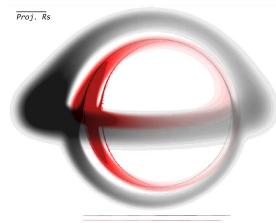
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Testing the no-hair theorem : some examples

Measuring the spin from the black hole silhouette

Ray-tracing in the Kerr metric (spin parameter a)

Accretion structure around Sgr A* modelled as a ion torus, derived from the *polish doughnut* class [Abramowicz, Jaroszynski & Sikora (1978)]



Radiative processes included : thermal synchrotron, bremsstrahlung, inverse Compton

- $\leftarrow \text{ Image of an ion torus} \\ \text{computed with Gyoto for the} \\ \text{inclination angle } i = 80^\circ:$
 - black : a = 0.5M
 - red : a = 0.9M

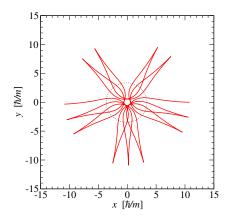
[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A 543, A83 (2012)]

An example : rotating boson stars

Boson star = localized configurations of a self-gravitating massive complex scalar field $\Phi \equiv$ "Klein-Gordon geons" [Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of Kadath [Grandclément, JCP 229, 3334 (2010)]
- Timelike geodesics computed by means of Gyoto

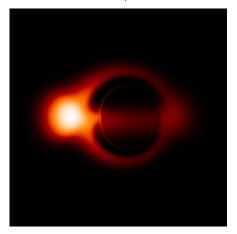


Intially-at-rest orbit around a rotating boson star based on a free scalar field $\Phi = \phi(r, \theta)e^{i(\omega t + 2\varphi)}$ with $\omega = 0.75 m/\hbar$.

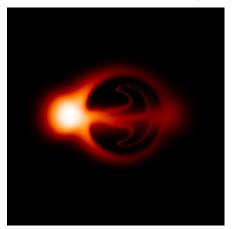
[Granclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]

Image of an accretion torus : comparing with a Kerr BH

Kerr BH a/M = 0.9



Boson star k = 1, $\omega = 0.70 \, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG 33, 105015 (2016)]

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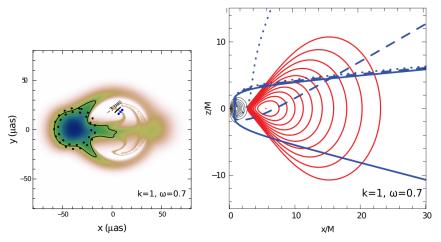
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Testing the no-hair theorem : some examples

Strong light bending in rotating boson star spacetimes



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG 33, 105015 (2016)]

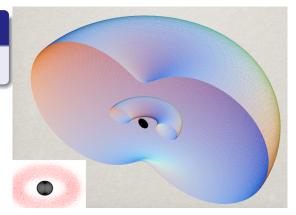
Hairy black holes

Herdeiro & Radu discovery (2014)

A black hole can have a complex scalar hair

Stationary axisymmetric configuration with a self-gravitating massive complex scalar field Φ and an event horizon

$$\begin{split} \Phi(t,r,\theta,\varphi) &= \Phi_0(r,\theta) e^{i(\omega t + k\varphi)} \\ \omega &= k \Omega_{\rm H} \end{split}$$

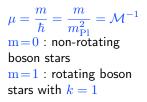


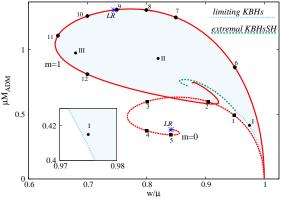
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[Herdeiro & Radu, PRL 112, 221101 (2014)]

Herdeiro-Radu hairy black holes

- Configuration I : rather Kerr-like
- Configuration II : not so Kerr-like
- Configuration III : very non-Kerr-like





[Cunha, Herdeiro, Radu Rúnarsson, PRL 115, 211102 (2015)]

TABLE I. KBHsSH configurations considered in the present study. M is the ADM mass, $M_{\rm H}$ is the horizon's Komar mass, J is the total Komar angular momentum and $J_{\rm H}$ is the horizon's Komar angular momentum.

	М	$M_{ m H}$	J	$J_{ m H}$	$\frac{M_{\rm H}}{M}$	$\frac{J_{\rm H}}{J}$	$\frac{J}{M^2}$	$rac{J_{\mathrm{H}}}{M_{\mathrm{H}}^2}$
Configuration I Configuration II	0.415 <i>M</i> 0.933 <i>M</i>	0.393 <i>M</i> 0.234 <i>M</i>	$0.172 \mathcal{M}^2$ $0.740 \mathcal{M}^2$	$0.150\mathcal{M}^2$ $0.115\mathcal{M}^2$	95% 25%	87% 15%	0.999 0.850	0.971 2.10
Configuration III	$0.955\mathcal{M}$	0.018M	$0.85M^{2}$	$0.002M^2$	1.8%	2.4%	0.894	6.20 (~
Éric Gourgoulhon (LUTH) Galactic central BH and no-hair theorem IPTA 2017, 5 July 2017						5 July 2017	24 / 30	

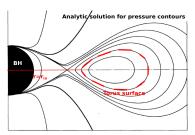
Images of a magnetized accretion torus

Accretion torus model of [Vincent, Yan, Straub, Zdziarski & Abramowicz, A&A 574, A48 (2015)]

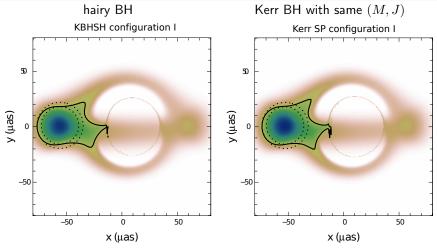
- non-self-gravitating perfect fluid
- polytropic EOS $\gamma=5/3$
- constant specific angular momentum $\ell = u_\varphi/(-u_t) = 3.6\,M$

[Abramowicz, Jaroszynski & Sikora, A&A 63, 221 (1978)]

- torus inner radius $r_{\rm in} \simeq 5.5 \, M$
- max electron density : $n_{\rm e} = 6.3 \ 10^{12} \ {\rm m}^{-3}$
- max electron temperature : $T_{\rm e} = 5.3 \ 10^{10} \ {\rm K}$
- isotropized magnetic field \implies synchrotron radiation
- \bullet gas-to-magnetic pressure ration $\beta=10$
- observer inclination angle : $\theta=85^\circ$



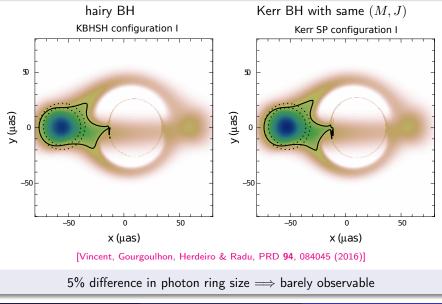
Configuration I Gyoto-simulated images of Sgr A* at f = 250 GHz



[Vincent, Gourgoulhon, Herdeiro & Radu, PRD 94, 084045 (2016)]

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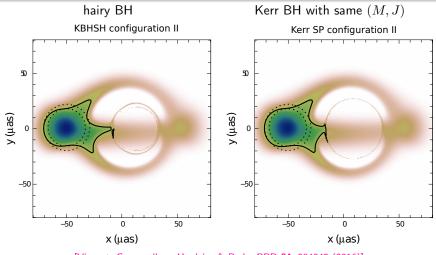


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Configuration II Gyoto-simulated images of Sgr A* at f = 250 GHz

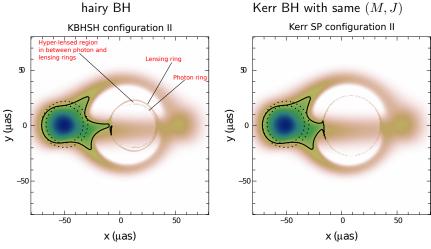


[Vincent, Gourgoulhon, Herdeiro & Radu, PRD 94, 084045 (2016)]

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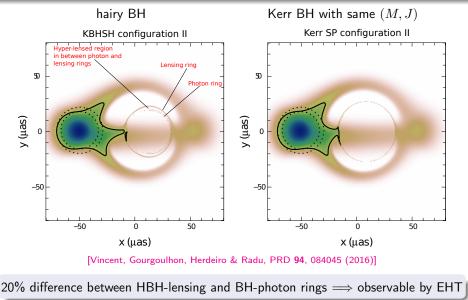


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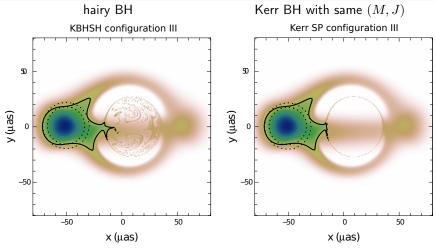
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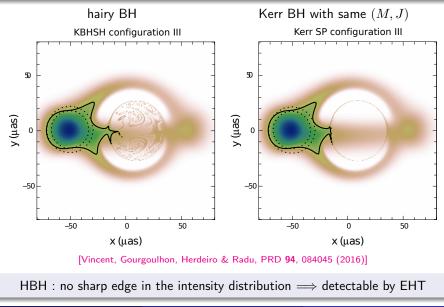
Configuration III Gyoto-simulated images of Sgr A* at f = 250 GHz



[Vincent, Gourgoulhon, Herdeiro & Radu, PRD 94, 084045 (2016)]

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Configuration III Gyoto-simulated images of Sgr A* at f = 250 GHz



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Galactic central BH and no-hair theorem

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Conclusions

After a century marked by the Golden Age (1965-1975), which culminated with the no-hair theorem, the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

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It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime, notably by finding some voliation of the no-hair theorem.

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The GW150914 event was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.