The Galactic central black hole and the no-hair theorem

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
Paris Sciences et Lettres Research University
92190 Meudon, France

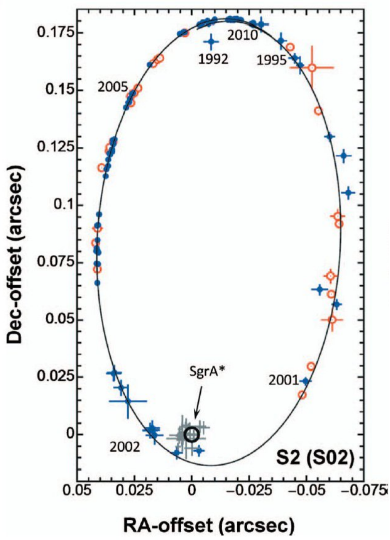
http://luth.obspm.fr/~luthier/gourgoulhon/

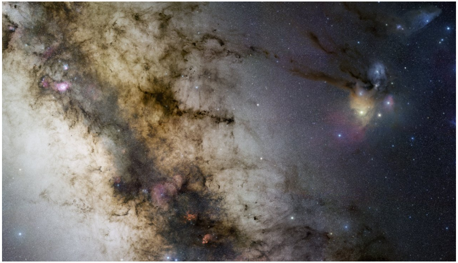
IPTA 2017 Student Week
CIEP, Sèvres, France
29 June 2017

Outline

Outline

The black hole at the centre of our galaxy : Sgr A*





[ESO (2009)]

Mass of Sgr A* black hole deduced from stellar dynamics :

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*
$P = 16\,{\rm yr}$, $r_{\rm per} = 120\,{\rm UA} = 1400\,R_{\rm S}$,
$V_{\rm per} = 0.02\,c$

[Genzel, Eisenhauer & Gillessen, RMP **82**, 3121 (2010)]

Next periastron passage : mid 2018

Can we see it from the Earth ?

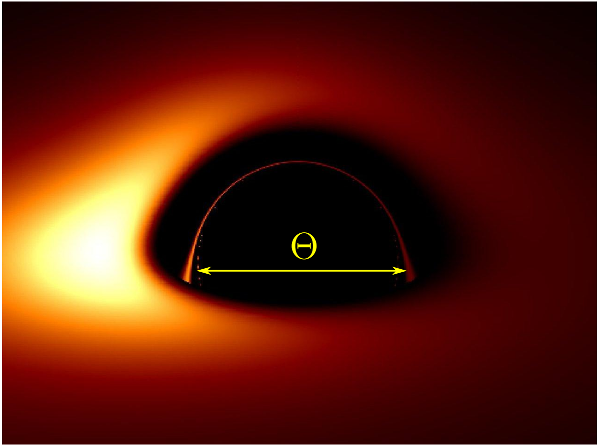


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_{\rm S}}{d}$$

Can we see it from the Earth ?

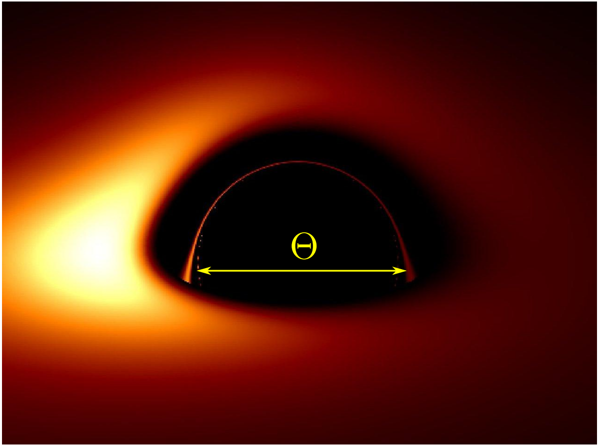


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

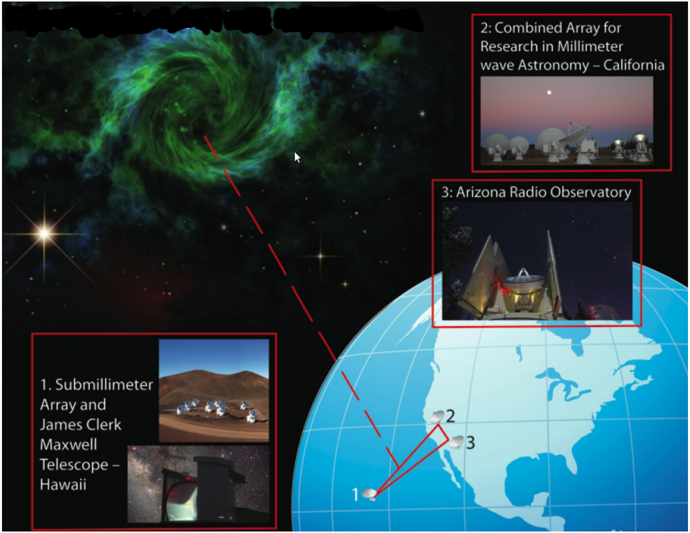
$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}$$

Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu\mathrm{as}$
M87 : $\Theta = 21\ \mu\mathrm{as}$
M31 : $\Theta = 20\ \mu\mathrm{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

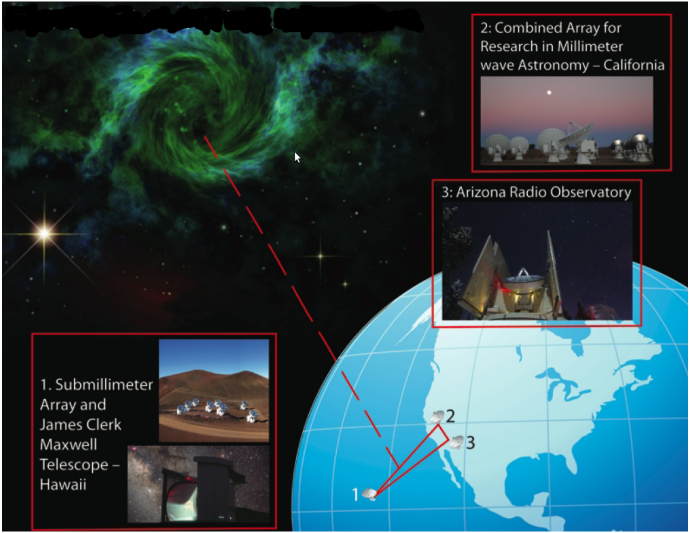
Reaching the μas resolution with VLBI



Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

Reaching the μas resolution with VLBI



Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu$as [Doeleman et al., Nature **455**, 78 (2008)]

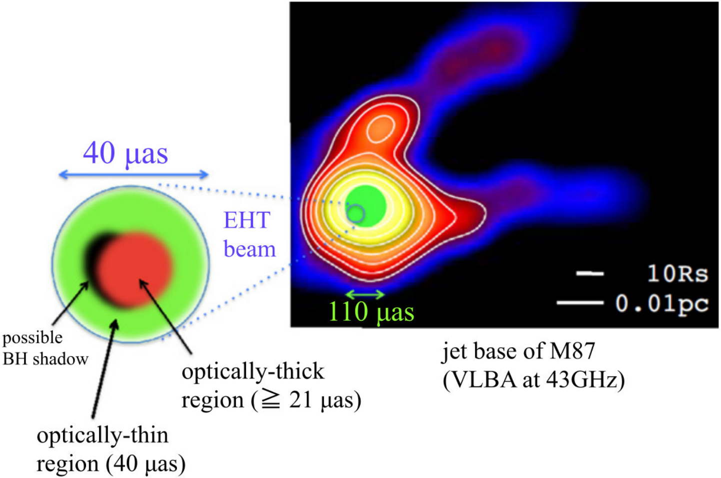
The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : $1.3 \text{ mm} \to 0.8 \text{ mm}$
- increase the number of stations ; in particular add ALMA

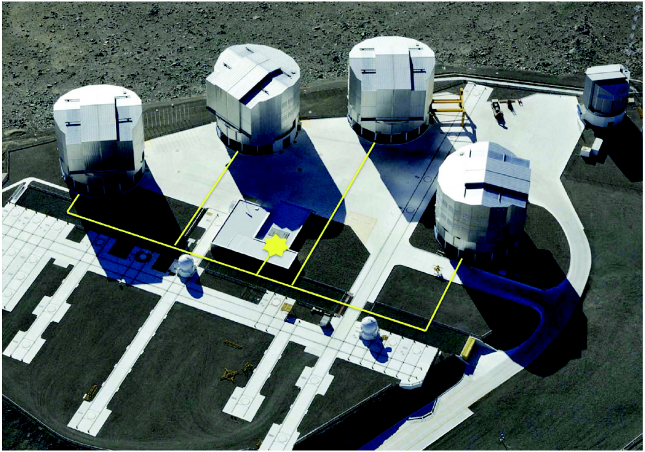
Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
August 2015 : VLBI observations involving ALMA and VLBA

VLBA and EHT observations of M87



[Kino et al., ApJ **803**, 30 (2015)]

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu$as

cf. Guy Perrin's talk next Wednesday

Near-infrared optical interferometry : GRAVITY

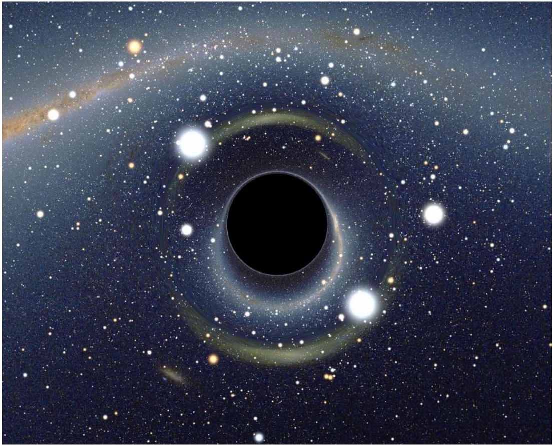
[MPE/GRAVITY team]

July 2015 : GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory

Fall 2016 : observations have started !

Outline

What is a black hole ?



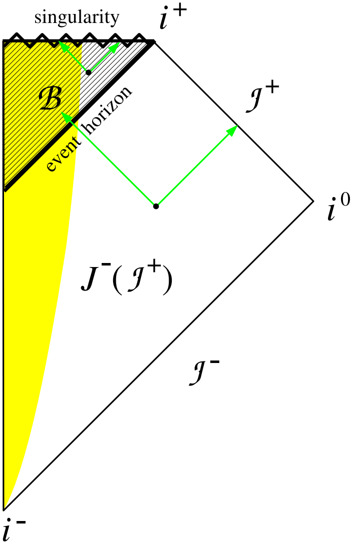
[Alain Riazuelo, 2007]

... for the layman :

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

What is a black hole ?



Textbook definition [Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

where

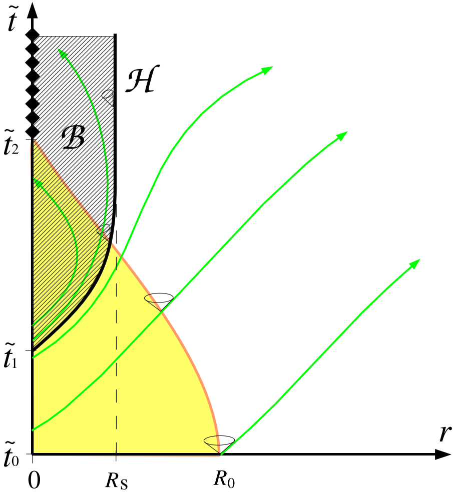
- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = (complete) future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathscr{I}^+)$
(boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

What is a black hole ?



Textbook definition [Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

where

- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = (complete) future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathscr{I}^+)$
(boundary of $J^-(\mathscr{I}^+)$)

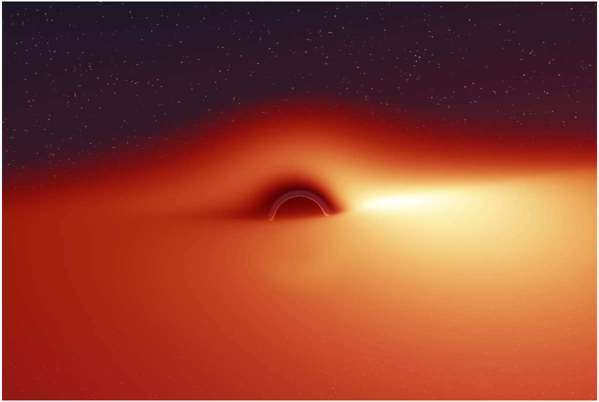
$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

What is a black hole ?

... for the astrophysicist : a very deep gravitational potential well

Release of potential gravitational energy by **accretion** on a black hole : up to 42% of the mass-energy mc^2 of accreted matter !

NB : thermonuclear reactions release less than 1% mc^2



Matter falling in a black hole forms an **accretion disk** [Lynden-Bell (1969), Shakura & Sunayev (1973)]

[J.-A. Marck (1996)]

Main properties of black holes (1/3)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.

Main properties of black holes (1/3)

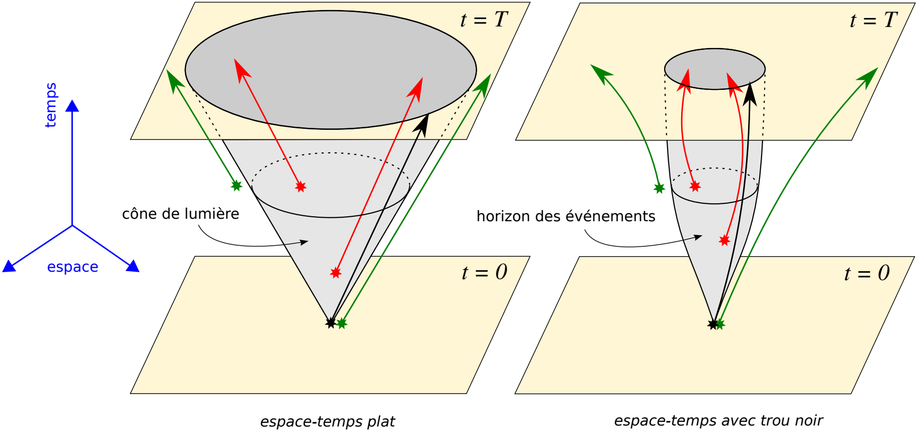
- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.
- The singularity marks the limit of validity of general relativity : to describe it, a quantum theory of gravitation would be required.

Main properties of black holes (1/3)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.
- The singularity marks the limit of validity of general relativity : to describe it, a quantum theory of gravitation would be required.
- The event horizon $\mathcal{H}$ is a global structure of spacetime : no physical experiment whatsoever can detect the crossing of $\mathcal{H}$.

Main properties of black holes (2/3)

The event horizon as a null cone



Main properties of black holes (3/3)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation

Main properties of black holes (3/3)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its “mean density” by $\bar{\rho} = M/(4/3\pi R^3)$, then

$$\bar{\rho}_{BH} \propto \frac{1}{M^2}$$

$\Longrightarrow$ a black hole is a compact object : $\dfrac{M}{R}$ large, not $\dfrac{M}{R^3}$!

Main properties of black holes (3/3)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its "mean density" by $\bar{\rho} = M/(4/3\pi R^3)$, then
  - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6 \ \mathrm{kg\,m^{-3}} \sim 2 \ 10^{-4} \ \rho_{\text{white dwarf}}$

$\Longrightarrow$ a black hole is a compact object : $\dfrac{M}{R}$ large, not $\dfrac{M}{R^3}$!

Main properties of black holes (3/3)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its "mean density" by $\bar{\rho} = M/(4/3\pi R^3)$, then
  - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6\ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-4}\ \rho_{\text{white dwarf}}$
  - for the BH at the centre of M87 : $\bar{\rho} \sim 2\ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-3}\ \rho_{\text{water}}$!

$\Longrightarrow$ a black hole is a compact object : $\dfrac{M}{R}$ large, not $\dfrac{M}{R^3}$!

Main properties of black holes (3/3)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its "mean density" by $\bar{\rho} = M/(4/3\pi R^3)$, then
  - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-4}\ \rho_{\text{white dwarf}}$
  - for the BH at the centre of M87 : $\bar{\rho} \sim 2 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-3}\ \rho_{\text{water}}$!

  $\Longrightarrow$ a black hole is a compact object : $\frac{M}{R}$ large, not $\frac{M}{R^3}$!
- Due to the non-linearity of general relativity, black holes can form in spacetimes without any matter, by collapse of gravitational wave packets.

Outline

The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\,dx^\alpha\,dx^\beta = -\left(1-\frac{2GMr}{c^2\rho^2}\right)c^2dt^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\,c\,dt\,d\varphi + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2GMa^2r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\,d\varphi^2$$

where

$$\rho^2 := r^2 + a^2\cos^2\theta,\ \Delta := r^2 - \frac{2GM}{c^2}r + a^2 \text{ and } \boxed{r\in(-\infty,\infty)}$$

→ spacetime manifold : $\mathscr{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r=0\ \&\ \theta=\pi/2\}$

→ 2 parameters : M : gravitational mass ; $a := \frac{J}{cM}$ reduced angular momentum

The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GMr}{c^2\rho^2}\right)c^2\mathrm{d}t^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\,c\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2$$
$$+\rho^2\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2GMa^2r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\,\mathrm{d}\varphi^2$$

where

$\rho^2 := r^2 + a^2\cos^2\theta$, $\Delta := r^2 - \frac{2GM}{c^2}r + a^2$ and $r\in(-\infty,\infty)$

→ spacetime manifold : $\mathscr{M} = \mathbb{R}^2\times\mathbb{S}^2\setminus\{r=0\ \&\ \theta=\pi/2\}$

→ 2 parameters : M : gravitational mass ; $a := \frac{J}{cM}$ reduced angular momentum

→ Schwarzschild solution as the subcase $a=0$:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GM}{c^2r}\right)c^2\mathrm{d}t^2+\left(1-\frac{2GM}{c^2r}\right)^{-1}\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2\right)$$

Basic properties of Kerr metric

- Asymptotically flat ($r \to \pm\infty$)
- Stationary : metric components independent from t
- Axisymmetric : metric components independent from φ
- Not static when $a \neq 0$
- Contains a black hole $\iff$ $\boxed{0 \leq a \leq m}$, where $m := GM/c^2$

  event horizon : $r = r_+ := m + \sqrt{m^2 - a^2}$
- Contains a curvature singularity at $\rho = 0 \iff r = 0$ and $\theta = \pi/2$

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

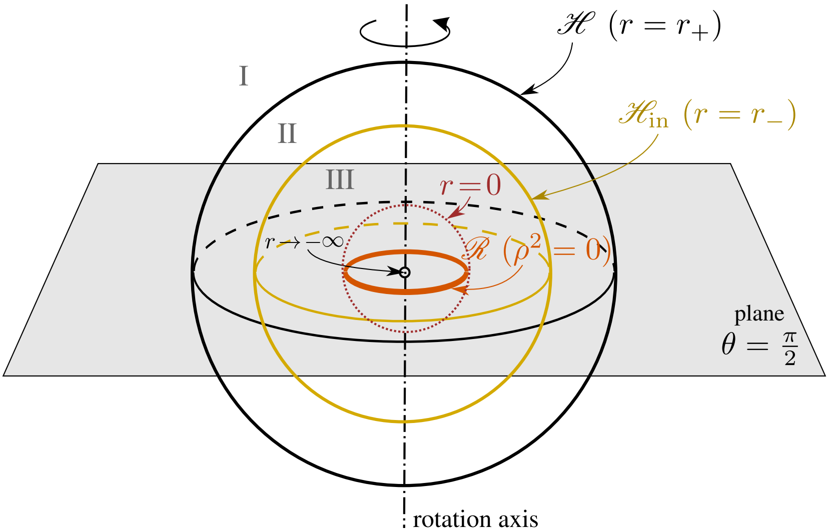
Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*
- angular momentum $J = aMc$ characterizes the *gravito-magnetic* part of the gravitational field
  $\rightarrow$ measurable from the precession of a gyroscope orbiting the black hole *(Lense-Thirring effect)*

Remark : the radius of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole "centre" and the event horizon. A well defined quantity is the area of the event horizon, A.
The radius can be then defined from it : for a Schwarzschild black hole :

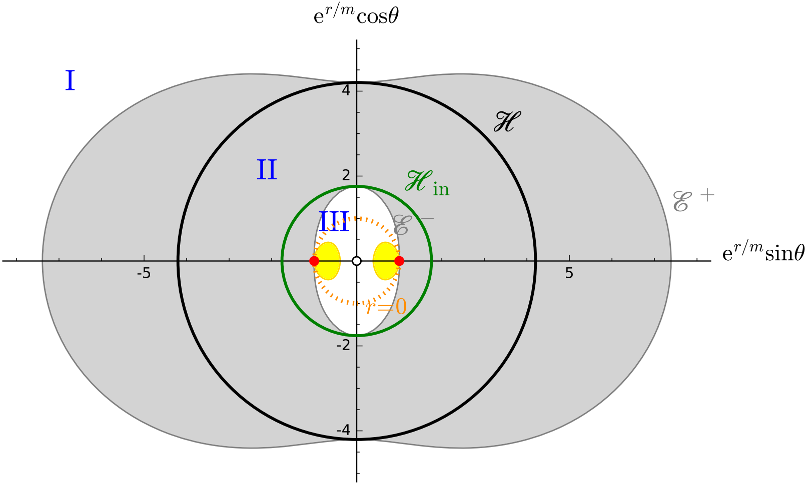
$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot} \right) \ \mathrm{km}$$

Kerr spacetime



Slice $t = \text{const}$ of the Kerr spacetime viewed in O'Neill coordinates (R, θ, φ), with $R := e^r$, $r \in (-\infty, +\infty)$

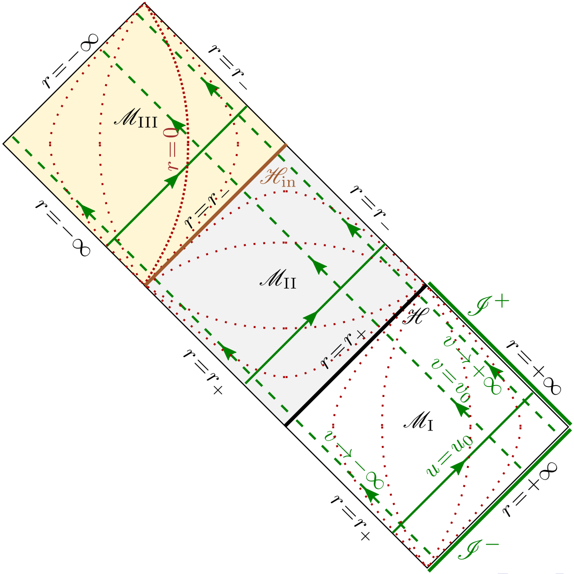
Kerr spacetime : ergoregion and Carter time machine



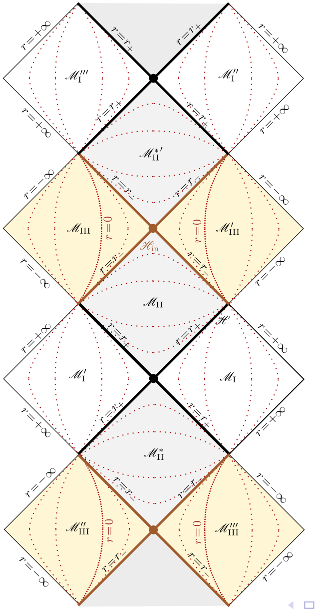
Meridional view of a section $t = \mathrm{const}$ of Kerr spacetime with $a/m = 0.90$

Conformal diagram of Kerr spacetime

with $\mathscr{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \mathscr{R}$



Carter-Penrose diagram of the maximal analytic extension



Outline

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

The no-hair theorem : the precise mathematical statement

Any spacetime $(\mathscr{M}, \boldsymbol{g})$ that

- is 4-dimensional
- is asymptotically flat
- is stationary
- is a solution of the vacuum Einstein equation
- contains a black hole with a connected regular horizon
- does not contain any closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

domain of outer communications : black hole exterior

The no-hair theorem : the precise mathematical statement

Any spacetime $(\mathscr{M}, \boldsymbol{g})$ that

- is 4-dimensional
- is asymptotically flat
- is stationary
- is a solution of the vacuum Einstein equation
- contains a black hole with a connected regular horizon
- does not contain any closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

domain of outer communications : black hole exterior

Possible improvements : remove the hypotheses of analyticity and non-existence of closed timelike curves

The no-hair theorem : the precise mathematical statement

Any spacetime $(\mathscr{M}, \boldsymbol{g})$ that

- is 4-dimensional
- is asymptotically flat
- is stationary
- is a solution of the vacuum Einstein equation
- contains a black hole with a connected regular horizon
- does not contain any closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

domain of outer communications : black hole exterior

Possible improvements : remove the hypotheses of analyticity and non-existence of closed timelike curves (analyticity released recently but only for slowly rotating black holes [Alexakis, Ionescu & Klainerman, Duke Math. J. **163**, 2603 (2014)])

The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) case :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) case :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

Rotating axisymmetric case :

No Birkhoff theorem

Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL **71**, 1669 (1993)])

$\Longrightarrow$ The Kerr metric is specific to rotating black holes (in 4-dimensional general relativity)

Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a) :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}}$ $(*)$ $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a) :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}}$ $(*)$ $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

Measuring the three quantities M, J, $\mathcal{M}_2$ provides a compatibility test w.r.t. the Kerr metric, by checking $(*)$

Theoretical alternatives to the Kerr black hole

Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

Beyond general relativity

The compact object is a black hole but in a theory that differs from GR :

- Einstein-Gauss-Bonnet with dilaton
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Einstein-Yang-Mills
- ...

Is general relativity unique ?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $\boldsymbol{g}$
- field equation involving only derivatives of $\boldsymbol{g}$ up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ ($\Longrightarrow$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

Is general relativity unique ?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $\boldsymbol{g}$
- field equation involving only derivatives of $\boldsymbol{g}$ up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ ($\Longrightarrow$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

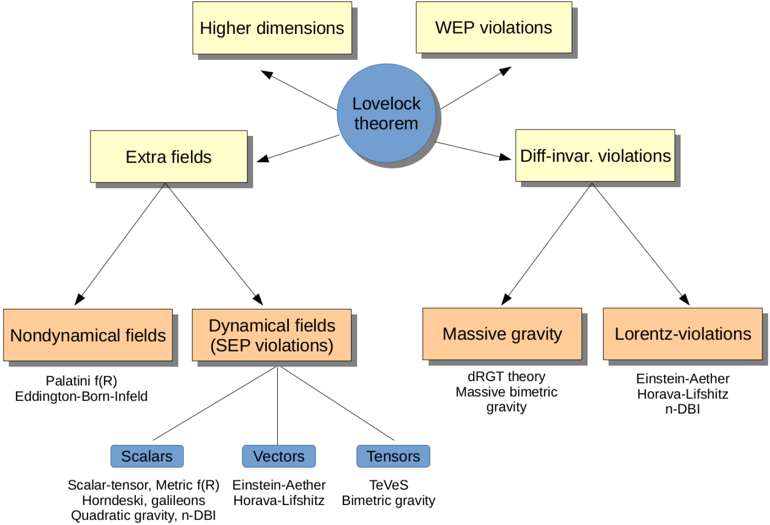
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

Extensions of general relativity



[Berti et al., CGQ **32**, 243001 (2015)]

Alternative theories of gravity

Class of metric theories of gravity, described by the action

$$S = S_{\rm grav} + S_{\rm mat}(\boldsymbol{g}, \Psi_1, \Psi_2, \ldots)$$

$\boldsymbol{g}$: spacetime metric, $\Psi_1, \Psi_2, \ldots$: matter fields

$\Longrightarrow$ test particles follow geodesics of $\boldsymbol{g}$

Alternative theories of gravity

Class of metric theories of gravity, described by the action

$$S = S_{\rm grav} + S_{\rm mat}(\boldsymbol{g}, \Psi_1, \Psi_2, \ldots)$$

$\boldsymbol{g}$: spacetime metric, $\Psi_1, \Psi_2, \ldots$: matter fields

$\Longrightarrow$ test particles follow geodesics of $\boldsymbol{g}$

General relativity :

$$S_{\rm grav} = \frac{1}{16\pi G} \int R \sqrt{-g}\, \mathrm{d}^4 x$$ (Einstein-Hilbert action)

R : scalar curvature of metric $\boldsymbol{g}$: $R := g^{\mu\nu} R^{\sigma}{}_{\mu\sigma\nu}$
$R^{\alpha}{}_{\beta\mu\nu}$: Riemann curvature tensor of $\boldsymbol{g}$

Scalar-tensor theories

Gravity action depends on a scalar field ϕ in addition to the spacetime metric $\boldsymbol{g}$:

$$S_{\rm grav} = S_{\rm grav}(\boldsymbol{g}, \phi) = \frac{1}{16\pi G} \int \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \phi^2 V \right] \sqrt{-g}\, \mathrm{d}^4 x$$

Special case : Jordan-Fierz-Brans-Dicke theory : $\omega(\phi) = \mathrm{const}$

No-hair theorem : for a *real* scalar field ϕ, the only black hole solution is Kerr

However, for *complex* scalar fields, hairy black hole solutions exist [Herdeiro & Radu, arXiv:1403.2757 (2014)]

Einstein-Gauss-Bonnet with dilaton

Gravity action is quadratic in the curvature :

$$
\begin{aligned}
S_{\rm grav} \quad = \quad & \frac{1}{16\pi G} \int \Big[R + e^{\gamma\phi} \left(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right) \\
& - \frac{\beta}{2} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2 V(\phi) \right) \Big] \sqrt{-g}\, \mathrm{d}^4 x
\end{aligned}
$$

Low energy expansion of string theory

Chern-Simons gravity

Gravity action is quadratic in the curvature :

$$S_{\rm grav} = \frac{1}{16\pi G}\int\left[R + \frac{\alpha}{4}\phi R^{*}_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{\beta}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)\right)\right]\sqrt{-g}\,\mathrm{d}^4x$$

Low energy expansion of string theory or loop quantum gravity

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)
- gravitational waves :
  - ring-down phase of binary black hole mergers (LIGO, Virgo)
  - extreme-mass-ratio binaries (LISA)

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

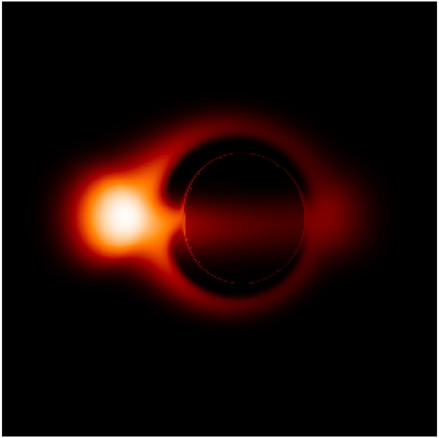
Observational tests

Search for

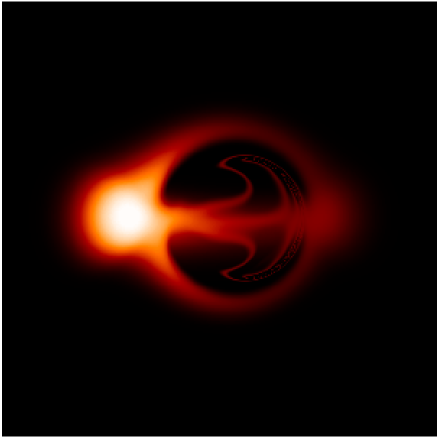
- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)
- gravitational waves :
  – ring-down phase of binary black hole mergers (LIGO, Virgo)
  – extreme-mass-ratio binaries (LISA)
- pulsar orbiting Sgr A* : the Holly Grail !

An example : comparison with a boson star

Kerr BH $a/M = 0.9$



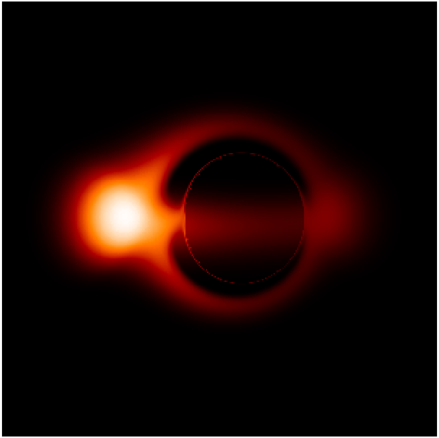
Boson star $k = 1$, $\omega = 0.70\, m/\hbar$



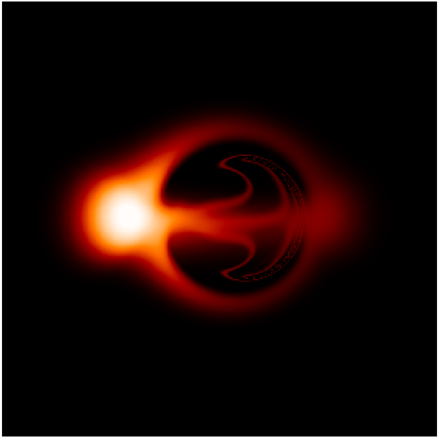
[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

An example : comparison with a boson star

Kerr BH $a/M = 0.9$



Boson star $k = 1,\ \omega = 0.70\, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

More examples next Wednesday...