Imaging numerical spacetimes

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based on a collaboration with

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YITP Long-Term Workshop
Gravitational waves and numerical relativity
Yukawa Institute for Theoretical Physics
Kyoto, Japan, 23 May 2013

Outline

1. Motivations
   - A new era for black hole observations
   - New tests of gravitation will become possible
   - Visualization of gravitational-wave sources studied in numerical relativity
2. Computing geodesics in numerical spacetimes
   - Geodesic equation within the 3+1 formalism
   - The Gyoto code
3. Some preliminary results
   - Images in stationary spacetimes
   - Images in dynamical spacetimes
4. Conclusion and perspectives

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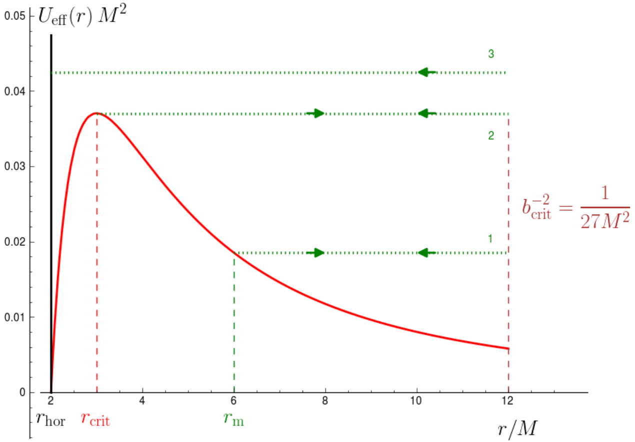
Can we see a black hole from the Earth ?

Apparent size of the event horizon of a Schwarzschild black hole:
given by the maximum of the *effective potential* $U_{\rm eff}(r)$ governing null geodesics in Schwarzschild metric:



To reach the observer, a photon emitted behind the black hole must have an impact parameter $b > b_{\rm crit} = 3\sqrt{3}M$

$\Longrightarrow$ angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d:

$$\Theta = 6\sqrt{3}\frac{M}{d} \simeq 2.60\frac{4M}{d}$$

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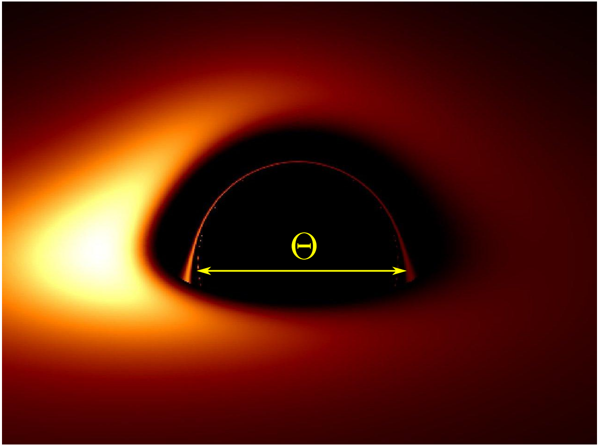


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Largest black holes (apparent size) in the Earth's sky:

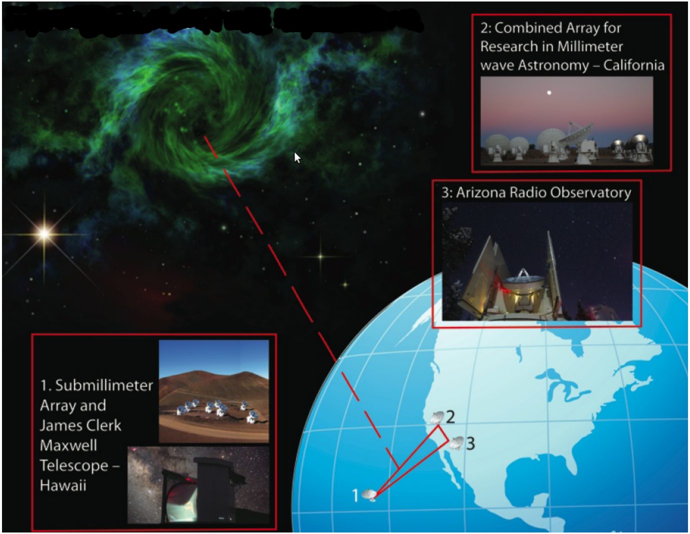
Sgr A* : $\Theta = 53\ \mu\text{as}$
M87 : $\Theta = 21\ \mu\text{as}$
M31 : $\Theta = 20\ \mu\text{as}$

Remark 1: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

Remark 2: HST angular resolution:

$$\Theta_{\rm min} \sim 10^5\ \mu\text{as} \ !$$

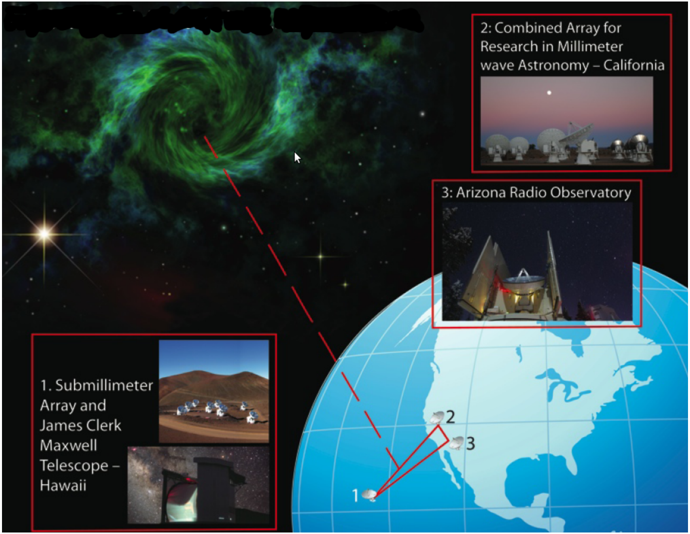
The solution to reach the μas regime: interferometry !



Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The solution to reach the μas regime: interferometry !



Existing American VLBI network [Doeleman et al. 2011]

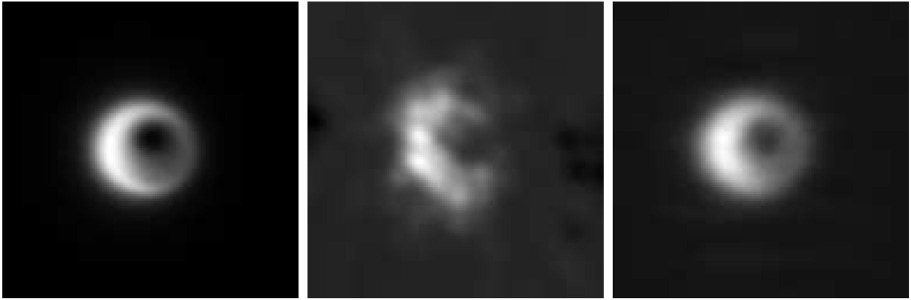
Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu\text{as}$.

The near future: the Event Horizon Telescope

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020

The near future: the Event Horizon Telescope



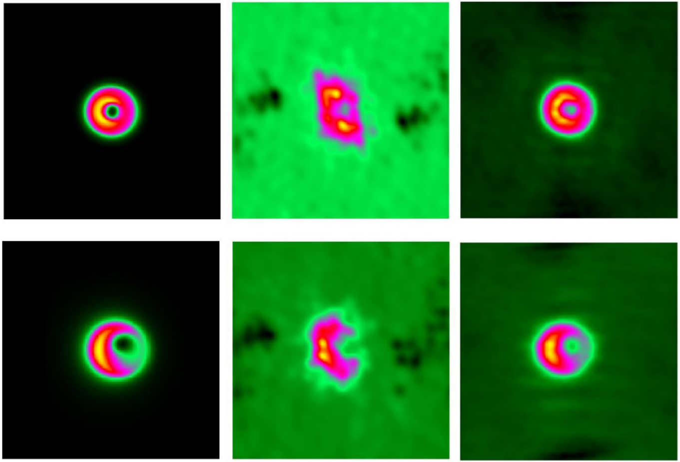
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

$a = 0, \; i = 30°$

[Fish & Doeleman, Proc. IAU Symp 261 (2010)]

The near future: the Event Horizon Telescope



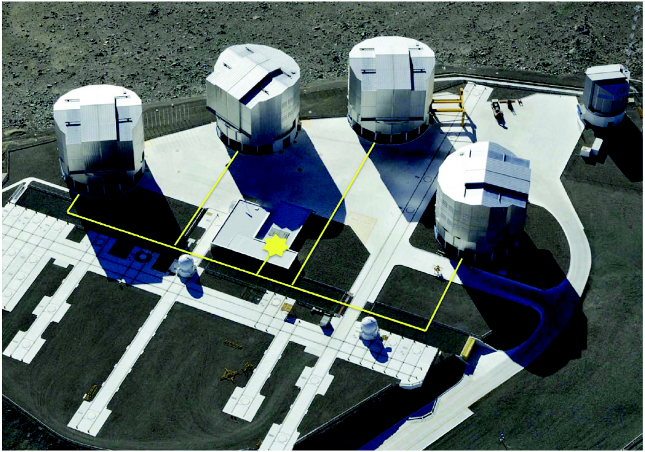
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

top: $a = 0.5$, $i = 85°$; *bottom:* $a = 0$, $i = 60°$

[Doeleman et al. (2009)]

Near-infrared optical interferometry



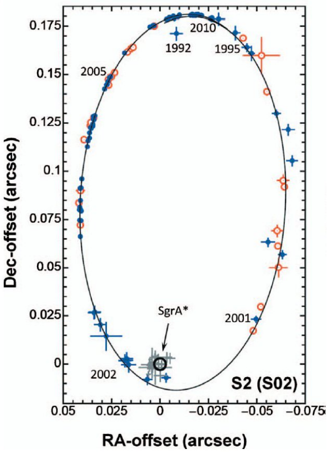
[Gillessen et al. 2010]

GRAVITY instrument at VLT (2014)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes) $\Longrightarrow$ astrometric precision of $10\ \mu$as

Outline

Testing the no-hair theorem from stellar dynamics



Orbit of the star S2 around Sgr A*
[Genzel, Einsenhauer & Gillessen, RMP **82**, 3121 (2010)]

$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$

GRAVITY is expected to observe stars on relativistic orbits (closer than S2)
Measure of relativistic effects:

- periastron advance
- Lense-Thirring precession

$\Longrightarrow$ constraints on the spacetime metric in the vicinity of the central object
$\Longrightarrow$ is it really the Kerr metric (a, M) ?
[Will, ApJ **674**, L25 (2008)]

Theoretical alternatives to the Kerr black hole

Within general relativity

- boson stars
- gravastar
- Q-star
- dark stars
- ...

Beyond general relativity

black holes in

- Einstein-Yang-Mills
- Einstein-Gauss-Bonnet with dilaton
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- ...

How to test the alternatives ?

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories)
- images of the black hole shadow different from that of a Kerr black hole (EHT)

How to test the alternatives ?

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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

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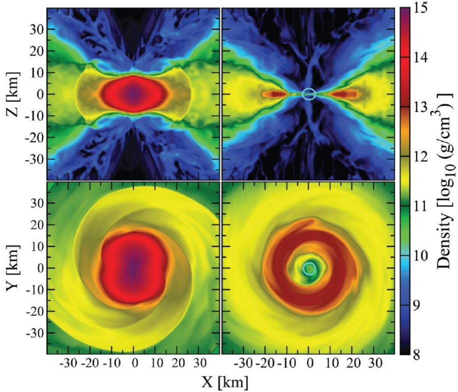
Another motivation: visualization of GW sources studied in numerical relativity



Binary neutron star merger
[Sekiguchi, Kiuchi, Kyutoku & Shibata, PTEP **2012**, 01A304 (2012)]

- Just by curiosity: what an observer passing nearby a binary merger would see ?
- From the astrophysical point of view: the ray-tracing is necessary to compute the electromagnetic counterpart or the neutrino counterpart.

Outline

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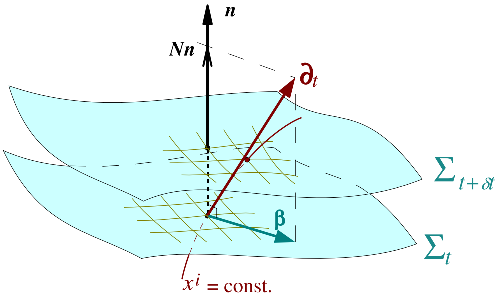
3+1 formalism for general relativity

Numerical spacetimes are generally computed within the 3+1 formalism



4-dimensional spacetime $(\mathcal{M}, \boldsymbol{g})$ foliated by spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$

Unit timelike normal: $\underline{\boldsymbol{n}} = -N\boldsymbol{\nabla}t$

Induced metric: $\boldsymbol{\gamma} = \boldsymbol{g} + \underline{\boldsymbol{n}} \otimes \underline{\boldsymbol{n}}$

Shift vector of adapted coordinates (t, x^i): vector $\boldsymbol{\beta}$ tangent to Σ_t such that $\partial/\partial t = N\boldsymbol{n} + \boldsymbol{\beta}$

$$g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t)$$

3+1 decomposition of the geodesic equation (1/2)

The geodesic equation

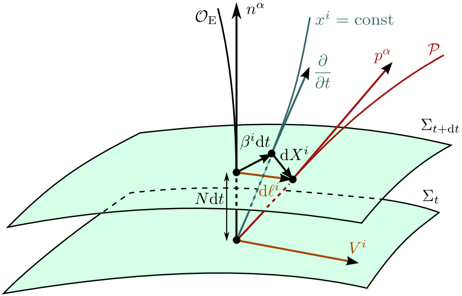
A particle $\mathcal{P}$ of 4-momentum vector $\boldsymbol{p}$ follows a geodesic iff

$$\boxed{\boldsymbol{\nabla}_{\boldsymbol{p}}\, \boldsymbol{p} = 0}$$

3+1 decomposition of $\boldsymbol{p}$: $\boxed{\boldsymbol{p} = E(\boldsymbol{n} + \boldsymbol{V})}$, with

- E : particle's energy with respect to the Eulerian observer (4-velocity $\boldsymbol{n}$)
- $\boldsymbol{V}$: vector tangent to Σ_t, representing the particle's 3-velocity with respect to the Eulerian observer

3+1 decomposition of the geodesic equation (2/2)



Equation of $\mathcal{P}$'s worldline in terms of the 3+1 coordinates : $x^i = X^i(t)$

The physical 3-velocity $\boldsymbol{V}$ is related to the coordinate velocity $\dot{X}^i := \mathrm{d}x^i/\mathrm{d}t$ by

$$V^i = \frac{\mathrm{d}\ell^i}{\mathrm{d}\tau_{\rm E}} = \frac{1}{N}\frac{\mathrm{d}\ell^i}{\mathrm{d}t} = \frac{1}{N}\frac{\beta^i\mathrm{d}t + \mathrm{d}X^i}{\mathrm{d}t}$$

$$\Longrightarrow V^i = \frac{1}{N}\left(\dot{X}^i + \beta^i\right)$$

Orth. projection of $\boldsymbol{\nabla}_{\boldsymbol{p}}\,\boldsymbol{p} = 0$ along $\boldsymbol{n}$:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = E\left(NK_{jk}V^jV^k - V^j\partial_j N\right)$$

Orth. projection of $\boldsymbol{\nabla}_{\boldsymbol{p}}\,\boldsymbol{p} = 0$ onto Σ_t:

$$\begin{cases} \dfrac{\mathrm{d}X^i}{\mathrm{d}t} = NV^i - \beta^i \\ \dfrac{\mathrm{d}V^i}{\mathrm{d}t} = NV^j\left[V^i\left(\partial_j \ln N - K_{jk}V^k\right) + 2K^i_{\ j} - {}^3\Gamma^i_{jk}V^k\right] - \gamma^{ij}\partial_j N - V^j\partial_j\beta^i \end{cases}$$

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

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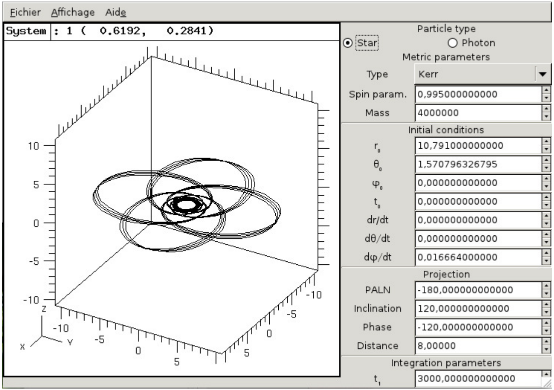
Gyoto code

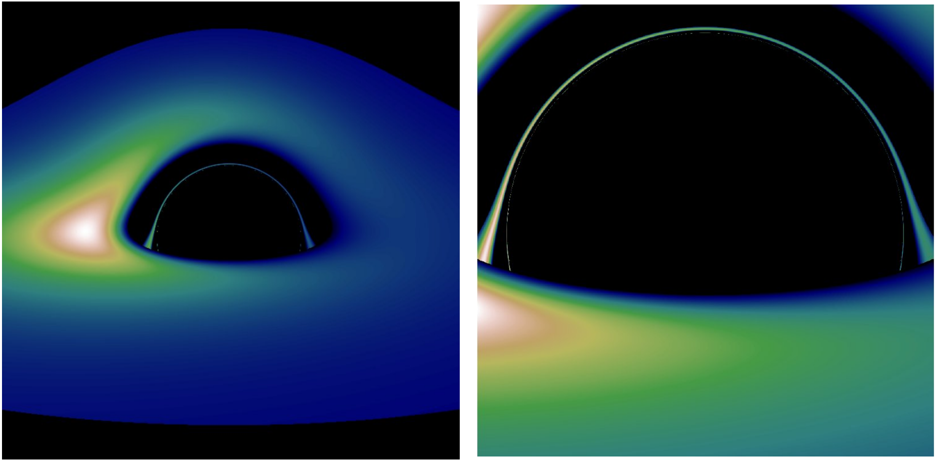


- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick interface
- Free software (GPL) : `http://gyoto.obspm.fr/`

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Gyoto code

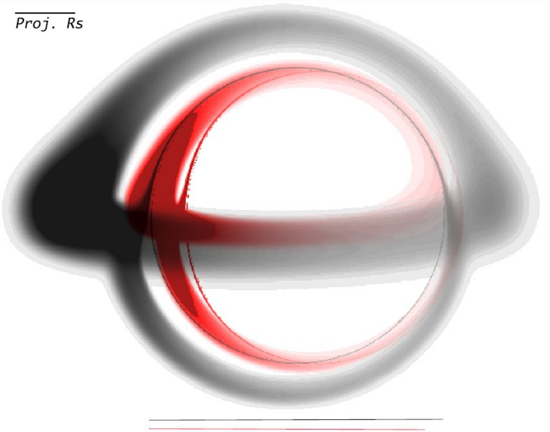


Computed images of a thin accretion disk around a Schwarzschild black hole

Measuring the spin from the black hole silhouette

Ray-tracing in the Kerr metric (spin parameter a)

Accretion structure around Sgr A* modelled as a ion torus, derived from the *polish doughnut* class [Abramowicz, Jaroszynski & Sikora (1978)]



Radiative processes included: thermal synchrotron, bremsstrahlung, inverse Compton

← Image of an ion torus computed with `Gyoto` for the inclination angle $i = 80°$:

- black: $a = 0.5M$
- red: $a = 0.9M$

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

Measuring the spin from the accretion disk spectrum

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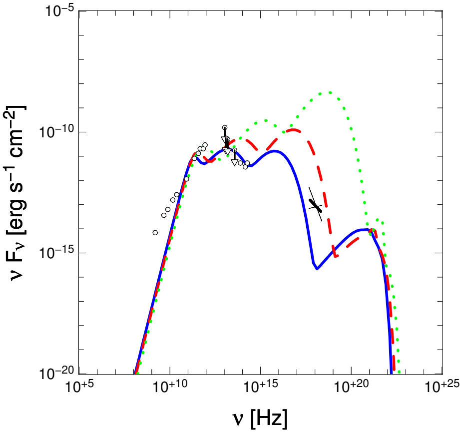


Radiative processes included: thermal synchrotron, bremsstrahlung, inverse Compton

← Spectrum of an ion torus computed with `Gyoto` for the inclination angle $i = 80°$:

- blue: $a = 0$
- red: $a = 0.5M$
- green: $a = 0.9M$

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

3+1 geodesic integration in Gyoto code (1/2)

Numerical spacetime $\Longrightarrow (N, \beta^i, \gamma_{ij}, K_{ij})$

System to be integrated

$$\begin{cases} \dfrac{\mathrm{d}E}{\mathrm{d}t} = E\left(NK_{jk}V^jV^k - V^j\partial_j N\right) \\ \dfrac{\mathrm{d}X^i}{\mathrm{d}t} = NV^i - \beta^i \\ \dfrac{\mathrm{d}V^i}{\mathrm{d}t} = NV^j\left[V^i\left(\partial_j \ln N - K_{jk}V^k\right) + 2K^i_{\ \,j} - {}^3\Gamma^i_{jk}V^k\right] - \gamma^{ij}\partial_j N - V^j\partial_j\beta^i \end{cases}$$

Integration (backward) in time: Runge–Kutta algorithm of fourth order (RK4)

Problem: the 3+1 quantities $(N, \beta^i, \gamma_{ij}, K_{ij})$ and their spatial derivatives have to be known at any point along the geodesic and not only at the grid points issued from the numerical relativity computation

3+1 geodesic integration in Gyoto code (2/2)

Solution within spectral methods: thanks to their spectral expansions, the fields $(N, \beta^i, \gamma_{ij}, K_{ij})$ are actually known at any point !

For instance, a scalar field, like N, is expanded as

$$N(t, r, \theta, \varphi) = \sum_{i,\ell,m} \hat{N}_{i\ell m}(t)\, T_i(r)\, Y_\ell^m(\theta, \varphi)$$

with

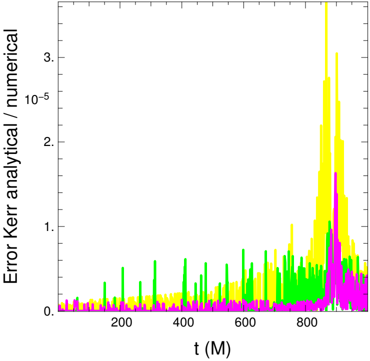
- T_i: Chebyshev polynomial of degree i
- Y_ℓ^m: spherical harmonic of index (ℓ, m)

Within spectral methods, the discretization does not occur on the values in the physical space (no grid !) but on the finite number of coefficients $\hat{N}_{i\ell m}$

The data are $(\hat{N}_{i\ell m}(t_J))$ for a finite series of time steps $(t_J)_{0\le J\le J_{\max}}$
$\Longrightarrow$ the values $(\hat{N}_{i\ell m}(t))$ at an arbitrary time t are obtained by a third order interpolation from 4 neighbouring t_J's

Test on Kerr spacetime

Integration of a null geodesic in the Kerr metric with $a = 0.5M$, using "numerical" (LORENE-prepared) 3+1 metric fields in Boyer-Lindquist coordinates



Comparison with integration using the analytical expression for the metric:

Relative difference on $r(t)$ (yellow), $\theta(t)$ (green) and $\varphi(t)$ (magenta), for

- $t = 1000M, r = 100M \to t = 0, r = 865M$
- the smallest distance $r = 4.3M$ @ $t \sim 900M$, where the error is the largest

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Stationary rotating neutron star

Rapidly rotating neutron star generated by LORENE/nrotstar

- EOS of Akmal, Pandharipande & Ravenhall
- $1.4\, M_\odot$ gravitational mass
- static *(left image)* or rotating at $f = 716$ Hz (fastest known pulsar) *(right image)*
- optically thick, emitting as a black body at 10^6 K

Map of specific intensity in $\mathrm{W\ m^{-2}\ ster^{-1}\ Hz^{-1}}$

$\Longrightarrow$ check of conservation of $p_t\ (10^{-6}), p_\varphi\ (10^{-4})$ and $p_\mu p^\mu\ (10^{-5})$ along the geodesics

Boson stars

Boson star = localized configurations of a self-gravitating complex scalar field Φ $\equiv$ "Klein-Gordon geons" [Kaup (1968), Ruffini & Bonazzola (1969)]

- Lagrangian of the scalar field: $\mathcal{L} = -\frac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right]$
- Field equation: $\nabla_\mu \nabla^\mu \Phi = V'(|\Phi|^2)\,\Phi$
- Einstein equation: $R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta}$

  with $T_{\alpha\beta} = \nabla_{(\alpha} \bar{\Phi} \nabla_{\beta)} \Phi - \frac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right] g_{\alpha\beta}$

Examples:

- free field: $V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2$

  $\Longrightarrow$ field equation = Klein-Gordon equation : $\nabla_\mu \nabla^\mu \Phi = \frac{m^2}{\hbar^2} \Phi$
- a standard self-interacting field: $V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2 + \lambda |\Phi|^4$

Boson stars could behave as black-hole mimickers

Rotating boson stars

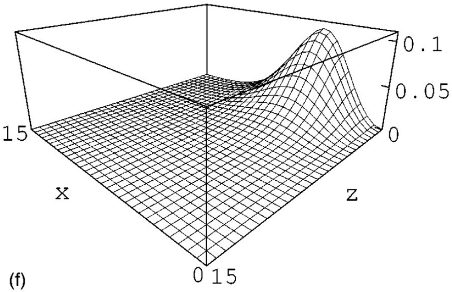
Ansatz for stationary and axisymmetric spacetimes:

$$\Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$$

with $\Phi_0(r, \theta)$ real function, $\omega \in \mathbb{R}$ and $k \in \mathbb{N}$ (regularity on the rotation axis)

Solutions:

- $k = 0$: static and spherically symmetric boson stars
  $\Longrightarrow$ exterior spacetime = Schwarzschild (or close to it if Φ never vanishes)
- $k \geq 1$: stationary rotating "stars" with toroidal topology
  $\Longrightarrow$ exterior spacetime expected to be significantly different from Kerr



$\leftarrow$ Profile of $\Phi_0(r, \theta)$ for a free field with $k = 2$

z-axis = rotation axis:
$z = r\cos\theta$, $x = r\sin\theta\cos\varphi$

[Yoshida & Eriguchi, PRD **56**, 762 (1997)]

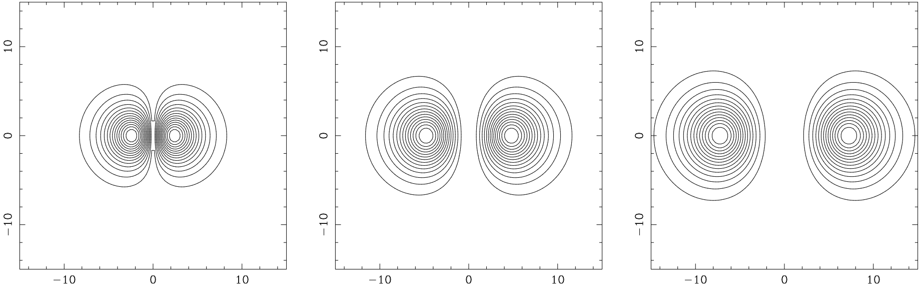
Rotating boson stars

Solutions computed by means of Kadath [Grandclément, JCP **229**, 3334 (2010)]

http://luth.obspm.fr/~luthier/grandclement/kadath.html

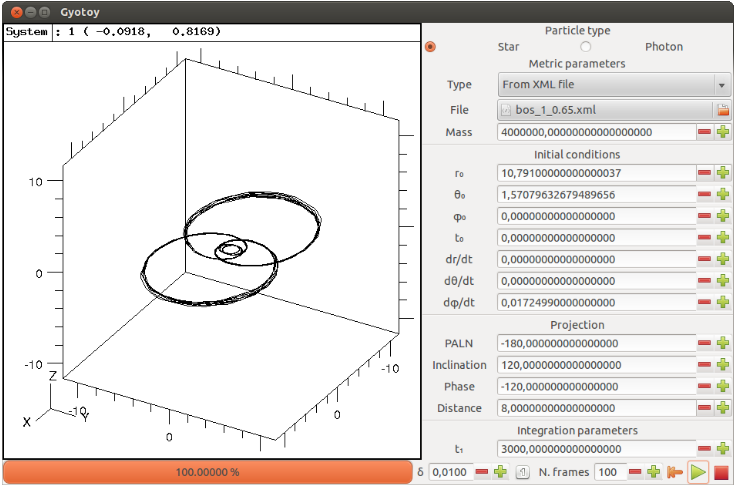
Isocontours of $\Phi_0(r, \theta)$ in the plane $\varphi = 0$ for $\omega = 0.8 \frac{m}{\hbar}$:

$k = 1$ $\quad$ $k = 2$ $\quad$ $k = 3$



Orbits in a rotating-boson-star spacetime

Rotating boson star computed by Kadath
Integration of timelike geodesics performed in 3+1 form by Gyoto



$k = 1$, $\omega = 0.65\, m/\hbar$

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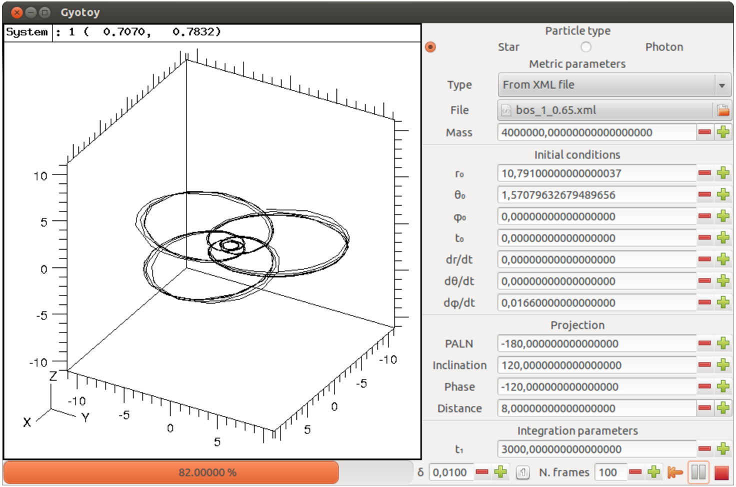

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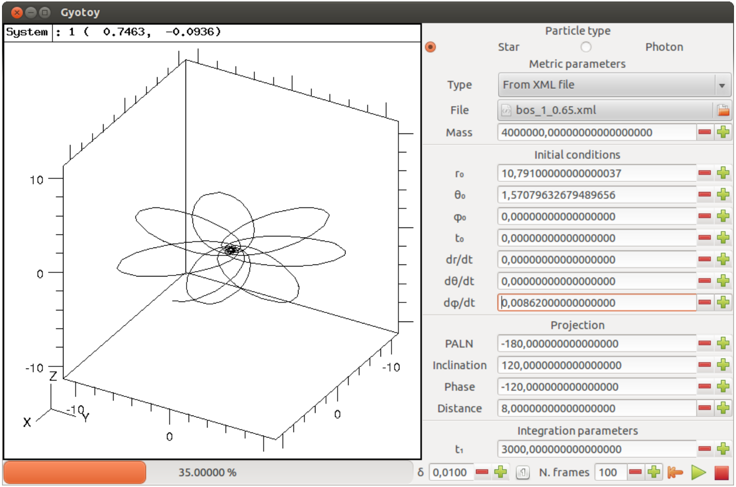

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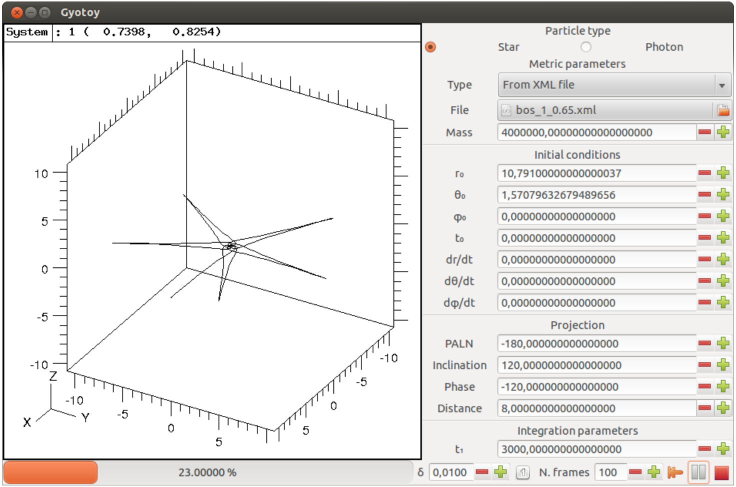

$k = 1$, $\omega = 0.65\, m/\hbar$, $\ell = 0$

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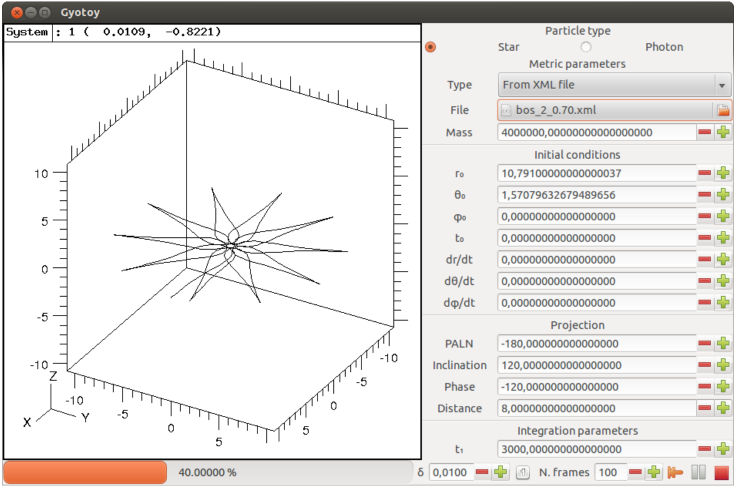

$k = 2$, $\omega = 0.70\, m/\hbar$, $\ell = 0$

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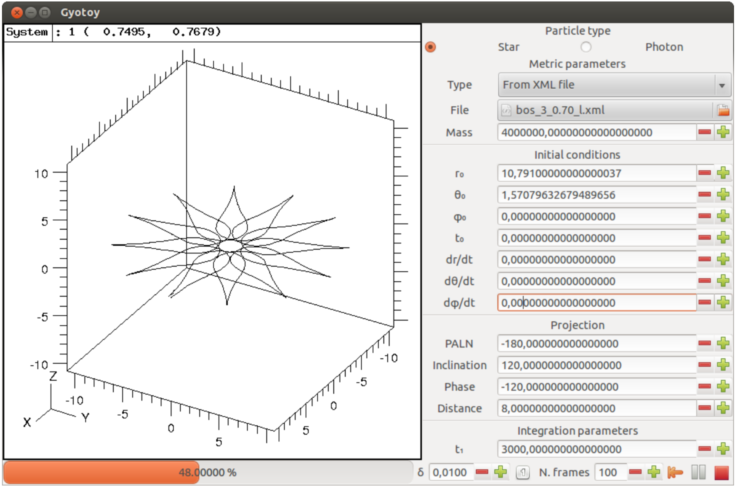

$k = 3$, $\omega = 0.70\, m/\hbar$, $\ell = 0$

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Collapse to a black hole (1/2)

Spacetime generated by the CoCoNuT code

Initial data:

- spherically symmetric neutron star on the unstable branch
- polytropic EoS, $\gamma = 2$, $M_{\rm grav} = 1.62 M_\odot$, $M_{\rm bar} = 1.77 M_\odot$
- initial perturbation $\rho \to \rho \left[1 + 0.01 \sin\left(\frac{\pi r}{10\ {\rm km}}\right)\right]$

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Initial data:

- spherically symmetric neutron star on the unstable branch
- polytropic EoS, $\gamma = 2$, $M_{\rm grav} = 1.62 M_\odot$, $M_{\rm bar} = 1.77 M_\odot$
- initial perturbation $\rho \to \rho \left[1 + 0.01 \sin\left(\frac{\pi r}{10 \ {\rm km}}\right)\right]$

sent to CoCoNuT, run with 500 radial cells.

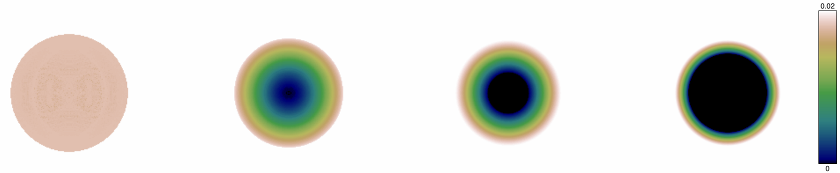
- at $t = 0.438$ ms, appearance of the apparent horizon
- at $t = 0.495$ ms, 99.99% of matter is inside the AH
- run is stopped when too strong gradients appear on metric (maximal slicing)

$\Longrightarrow$ 3+1 metric $(N, \beta^i, \gamma_{ij})$, K_{ij}, fluid velocity u^α, radius of the star and/or AH exported at every time-step to Gyoto

Collapse to a black hole (2/2)

Integration backward until reaching the star's surface or the apparent horizon

Surface of the star: black body at 10^6 K. Intensity given in logarithmic scale



- coordinate radius of the star 7 km (left) $\rightarrow$ 2.9 km (right)
- relativistic bending of light rays $\Longrightarrow$ apparent radius larger (cf. the magnification factor 2.60 seen in Sec. 1)
- event horizon first appear at the centre, closer to the observer

Images of other dynamical spacetimes

- rotating stellar collapse: in progress
- binary coalescence: computer demanding; a good stragegy could be set up first a spectral representation of the data: i.e. to export data from the Cartesian grid to the spectral grid.

Outline

1. Motivations
   - A new era for black hole observations
   - New tests of gravitation will become possible
   - Visualization of gravitational-wave sources studied in numerical relativity
2. Computing geodesics in numerical spacetimes
   - Geodesic equation within the 3+1 formalism
   - The Gyoto code
3. Some preliminary results
   - Images in stationary spacetimes
   - Images in dynamical spacetimes
4. Conclusion and perspectives

Conclusion and perspectives

- We have developed a ray-tracing code, `Gyoto`, capable of integrating timelike and null geodesics in any spacetime, either provided in analytical form (e.g. Kerr spacetime) or in 3+1 numerical form.
- This code is free and downloadable at `http://gyoto.obspm.fr/`
- It can be used to devise observational tests using data the near-future high angular resolution observations of Sgr A* or the core of M 87. These tests address the nature of the central object or the theory of gravity.
- Computations are currently in progress for boson stars and black holes in Hořava-Lifshitz gravity
- Another application of `Gyoto` is the visualization of dynamical spacetimes generated by numerical relativity, especially spacetimes of gravitational-wave sources. Suggestions are welcome !