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Letter to the Editor

Scale-relativity and quantization of extra-solar planetary systems

L. Nottale
DAEC, CNRS et Université Paris VII, Observatoire de Paris-Meudon, F-92195 Meudon Cedex, France

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Abstract. We show in the present letter that the planetary companions recently discovered around nearby stars agree with the predictions made four years ago in the framework of the theory of scale relativity (Nottale, 1993). In the simplest case, we expect their periods to be "quantized" as $T_n = 2\pi G M n^3/w_0{}^3 \simeq 3.25\,(M/M_\odot)\,n^3$ days. In this formula, which corresponds to the peak of probability distribution of planet positions, M is the star mass, and $w_0 \simeq 144$ km/s is a universal constant having the dimension of a velocity, which manifests itself from the Solar System scale to the extragalactic scales (Tifft effect). In particular, three planetary companions, including 51 Peg B, lie at 0.05 AU from their star and so achieve the 'fundamental level' of the theory for solar-type stars.

Key-words: chaos, gravitation, relativity, planetary systems

1. Introduction

The theory of scale relativity is founded on the giving up of the arbitrary hypothesis of the differentiability of space-time. Let us briefly recall the main steps of its construction. A more detailed account can be found in the book (Nottale, 1993) and in the more recent review paper (Nottale, 1996a).

(i) We give up differentiability of space-time coordinates. This implies their explicit dependence on resolutions. In a fractal space-time, the various physical quantities, then the equations of physics, become scale-dependent.

(ii) We re-interpret resolutions as essential variables that characterize the relative state of scale of the reference system, in the same way as velocities characterize its state of motion.

(iii) We extend Einstein's principles of relativity and of covariance, in order to include the new scale transformations.

(iv) One can demonstrate that the scale laws to be constructed must combine a standard fractal (power-law) behavior at small and large scales, and a transition to scale-independence at intermediate scales. In other words, space-time, that is Riemannian at intermediate scales, becomes fractal for Δx and Δt smaller and larger than some relative transition scales.

(v) In the fractal domains, scale-covariance transforms classical mechanics into a quantum-like mechanics. We shall briefly demonstrate this result again in what follows.

The aim of the present letter is to address some observational consequences of the theory for gravitational structures. We have already shown (Nottale, 1993) that it accounts in a very constrained way for several structures observed in the solar system, including planet distances, and we have predicted that it must also apply to all extra-solar planetary systems to be discovered in the future (Nottale, 1994). Such systems have now been found and the theory can be checked on them. It is the success of this test that we report here.

2. Theory

The various physical effects of the nondifferentiable and fractal nature of space-time in the minimal theory (see Nottale, 1992, 1993, 1996a for generalizations), namely, (i) infinity of geodesics, (ii) breaking of the ($dt \rightarrow -dt$) reflection invariance, (iii) new terms in differential equations of motion due to the fractal behavior, can be reduced to the introduction of a complex, scale-covariant time-derivative (Nottale, 1993, 1996a),

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V\,.\,\nabla - i\,D\,\Delta\,, \tag{1}$$

where the mean velocity $V = dx/dt$ is now complex and D is a parameter characterizing the new scale laws.

When we apply our scale-covariant derivative to classical mechanics, it is transformed in a quantum-like mechanics. Indeed, since the mean velocity is now complex, the same is true of the Lagrange function, then of the generalized action S. Setting $\psi = e^{iS/2mD}$, Newton's equation of dynamics becomes $m\,dV/dt = -\nabla\Phi$, which can be integrated in terms of a generalized Schrödinger equation (Nottale, 1993),

$$D^2\,\Delta\psi + i\,D\,\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0\ . \tag{2}$$

with $\rho = \psi\psi^\dagger$ giving the probability density of the particle, since the imaginary part of Eq. 2 is the equation of continuity.

One of the consequences of the scale-relativistic approach is that space-time, which is fractal at small scales, must become fractal again at large scales. However, the interpretations of the small and large scale laws are very different. In the microphysical domain, nondifferentiability is unbroken toward the small scales. This allows the theory to be strictly equivalent to quantum mechanics. At large scales, the situation is reversed.

Classical mechanics is recovered toward small scales, and Eq. 2 applies only to systems such that $<v> = 0$. This allows the de Broglie length $2D/<v>$ to be infinite, so that no new transition occurs toward the largest scales.

How can a small time-scale classical description be reconciled with the large time-scale non-deterministic one ? The answer is related to the concept of chaos. The evolution of chaotic systems is highly dependent on initial conditions (e.g., an exponential divergence, $\delta x = \delta x_0\, e^{t/\tau}$, where $1/\tau$ is the Lyapunov exponent). On time-scales $\gtrsim 20\tau$, the amplification of microscopic fluctuations is so high that any predictability of individual trajectories is definitively lost. Now, we have shown (Nottale, 1993, 1994) that the description of the trajectories of a strongly chaotic system *beyond its horizon of predictability* is similar to that of the geodesics of a nondifferentiable, fractal space-time. We have then suggested that, on very large time-scales, chaotic systems must be described by the new above theory. The transition from the classical theory at intermediate time-scales to the quantum-like theory at large time-scales then corresponds to the development of a chaotic behavior, the new theory applying only beyond the horizon of predictability.

We have first applied this new approach to gravitational systems (Nottale, 1993). The study of the general problem (see Nottale, 1996a) is a vast task that we shall not consider in the present letter. We specialize our study to the Kepler two-body problem. We assume that the potential $\phi = GmM/r$ dominates, and look for stationary solutions. In this case, Eq. 2 becomes:

$$2\, D^2\, \Delta\psi \; + \; [\, \frac{E}{m} + \frac{GM}{r} \;]\, \psi \;=\; 0 \quad . \tag{3}$$

One of the main differences of our macroscopic theory compared with standard, microphysical quantum mechanics, lies in the physical meaning of the parameter D. In the microphysical situation, it is totally constrained and related to the Planck constant $\hbar$ (Nottale, 1993, 1996a). In the gravitational case, the equivalence principle implies that it must be independent of the test particle mass m and proportional to the active gravitational mass M (Nottale, 1996a,b). Therefore we write $D = GM/2w$, where w has the dimension of a velocity. Solving now Eq. 3, we find that the probability for the particle to orbit at a given distance r from the central body is given by functions similar to the hydrogen atom orbitals, i.e. $P_n(r) \propto r^{2n} e^{-2r/na}$ for circular orbits, with $a = GM/w^2$. The peaks of probability lie at distances $a_n(\text{peak}) = GMn^2/w^2$, and the mean at $a_n(\text{mean}) = GM(n^2+n/2)/w^2$. From Kepler's third law, the velocity of a particle orbiting at peak is:

$$v_n \;=\; \frac{w}{n} \;. \tag{4}$$

Our interpretation of this result is that the gravitational system will have a tendency to make structures according to the above distribution. Now a more detailed prediction depends on the system considered. For a distribution of planetesimals (as during the formation of planetary systems) or of asteroids, we expect the bodies to fill the orbitals (this is well verified in the asteroid belt of our Solar System, Hermann & Nottale, 1996). For a planet, two possibilities can be considered. (i) The system evolved from the planetesimal stage to the planet stage with negligible external perturbations. In this case, the conservation of energy implies that the planet must lie at distance $a_n(\text{mean})$. (ii) The planet motion cannot be reduced to a two-body gravitational problem, due to various perturbations (other planets, collisions, capture...). In this case we can consider the probability density $P_n(r)$ to act on short time-scale motion as a potential well, so we expect the planet to "fall" at distance $a_n(\text{peak})$.

An important feature of our equations is that they naturally make a hierarchy of structures (Nottale, 1994, 1996a). Let us summarize the argument. Consider a system of test-particles (e.g., planetesimals) in the dominant potential of the Sun. Their evolution on large time-scales is governed by Eq. 2, in terms of a constant w_{j}. The particles then form a disk whose density distribution is given in the inner region by $P_{1\mathrm{i}}(r)$. This distribution can then be fragmented in sub-structures still satisfying Eq. 2 (since the central potential remains dominant), but with a different constant w_{j+1}. We can iterate the reasoning on several hierarchy levels. The matching condition between the orbitals implies $w_{j+1} = k_j\, w_j$, with k_j integer. Our own Solar System is organized following such a hierarchy on at least 3 levels, from the Sun's radius to the outer planets (see below).

Before jumping to a comparison of observational data with our predictions, let us make a last step in the physical interpretation of the characteristic velocity w. As recalled hereabove, our equations can have two different physical interpretations. They can first be obtained in the framework of a theory of strongly chaotic systems beyond their horizon of predictability (this was our interpretation in Nottale, 1993, 1994). In this case, the theory is only approximate and the predictions depend on the specific environment of each system considered. The second, more radical interpretation, is that the geometry of space-time becomes fractal at large time-scales. In this case, the theory must apply even to classically non-chaotic systems, such as the unperturbed two-body problem, and it implies a universal, fundamental constant w_0 that characterizes the fractal geometry, and whose value is independent of the system considered.

3. Results

We report in the present Letter the success of our predictions, including this last, radical, one. We have already shown that our theory accounted for the observed distributions of position, eccentricity, angular momentum (Nottale, 1993, 1994, 1995) and mass (Nottale, 1994; Nottale et al., 1996) of planets in the Solar System. The same is true for the satellites of giant planets and for asteroids and comets (Hermann & Nottale, 1996). Since Titius and Bode, several empirical laws have been suggested for the planet and satellite distances, most of the time of a scale-invariant form (see Graner and Dubrulle, 1994). The advantage of our proposal is that it is totally constrained. This allows us to make new predictions, as e.g. the possibility of two new intramercurial small planets (Nottale et al., 1996).

The motion of isolated binary galaxies also comes under the two-body problem. This allowed us (Nottale, 1996a) to explain the effect discovered by Tifft (1977a), according to which the velocity differences in galaxy pairs are quantized in terms of ≈72 or ≈144 km/s. Moreover, we find by inserting in

Eq. 2 the potential of a uniform density that matter in the universe must structure locally, without any initial fluctuations, according to the various modes of the isotropic 3-dimensional harmonic oscillator, i.e., to the symmetries described by the SU(3) group (Nottale, 1996a,c). This result yields a new general framework for structure formation (Nottale, 1996c), and contributes to explain the Tifft (1978) effect of "global" redshift quantization (in ≈36 km/s) of galaxy redshifts, recently confirmed by Guthrie & Napier (1996). A re-analysis of their sample in the framework of our theory yields w_0 = 144.7 ± 2.2 km/s (Nottale, 1996c). The same explanation applies to the velocity structures observed in M 82 and in our galaxy (that yield w_0 = 144.8 ± 0.7 km/s, from Tifft's (1977b) data, since their potential can be taken as that of a uniform density as a first local approximation.

Let us now come to our new predictions. The various "quantizations" effects recalled above can all be expressed in terms of "Tifft's constant", $w_0 \approx$ 144 km/s. Our theory predicts that our own Solar System, as well as extra-solar planetary systems, must be structured in terms of the same constant.

Star/planet	Star mass	Period (yr)	1/2maj. axis	v (km/s)	144 / v	n δn	$n\,v$ (km/s)
Sun/Mercury	1	0.24085	0.387	47.87	3.01	3 +0.01	143.6
Sun/Venus	1	0.61521	0.723	35.06	4.11	4 +0.11	140.2
Sun/Earth	1	1.00004	1.000	29.81	4.83	5 –0.17	149.0
Sun/Mars	1	1.88089	1.524	24.14	5.96	6 –0.04	144.8
Sun/Ceres	1	4.61	2.77	17.92	8.04	8 +0.04	143.4
Sun/Cybeles	1	6.35	3.43	16.09	8.95	9 –0.05	144.8
51 Peg B	1.10	0.01158 (1)	0.053	137	1.05	1 +0.05	137
47 UMa B	1.05	3.020 (2)	2.12	20.9	6.90	7 –0.10	146
70 Vir B	1.12	0.3195 (3)	0.485	45.3	3.18	3 +0.18	136
HD114762 B	1.0	0.230 (4)	0.376	48.65	2.96	3 –0.04	146
Prox Cen B	0.11	0.211 (5)	0.170	24.0	6.00	6 +0.00	144
55 Cnc B	0.8	0.04041 (6)	0.11	81.6	1.79	2 –0.21	163
τ Boo B	1.2	0.00907 (6)	0.046	152	0.95	1 –0.05	152
υ And B	1.2	0.0126 (6)	0.058	136	1.06	1 +0.06	136

Table 1. Inner solar system planets compared to extra-solar planetary companion candidates (see text). The semi-major axes are given in AU. The star masses are in M⊙ unit and result from averages of photometric and spectroscopic studies. The new mass 1.12 M⊙ for 70 Vir comes from its new Hipparcos distance (Perryman et al., 1996). The planet candidate around Proxima Centauri comes from the observation of a 77 day periodicity in position residuals with the HST Fine Guidance Sensor (Ref. 5). The Prox Cen mass derives from the analysis of Kirkpatrick and McCarthy (1994). References for periods: (1) Mayor, Queloz, 1995; (2) Butler, Marcy, 1996; (3) Marcy, Butler, 1996a; (4) Latham et al., 1989; (5) Benedict et al., 1995; (6) Butler et al., 1996.

(i) Solar System. The Solar System is structured according to the hierarchical law described above. As can be seen in Table 1 and Fig. 2, the velocities of planets in the *inner Solar System*, including the mass peaks of the asteroid belt, are given by $v_{n_2} = w_0 / n_2$, with w_0 = 144.3 ± 1.2 km/s. The whole inner system achieves the fundamental orbital of the outer system, whose peak is the Earth (n_2 = 5). We then expect the *outer system*, including the Earth at n_1 = 1, to have velocities given by $v_{n_1} = w_0/5n_1$. We find w_0 = 140 ± 3 km/s. The 'mean' formula gives a better fit for the outer system, as expected for the dominant planets. When accounting for its second order terms, we obtain w_0 = 144.8 ± 2.6 km/s. The *radius of the Sun* itself corresponds to a new sub-structure: the Kepler velocity at this distance is 436.8 = 3 x 145.6 km/s. We confirm this result on other stars (Nottale & Lefèvre, 1996). It is clearly related to the Tifft (1977b) 72.5 km/s quantization effect on the $v \sin i$'s of B stars.

(ii) Planetary system around PSR B1257+12. We have recently shown that the system of 3 planets observed around the pulsar PSR B1257+12 (Wolszczan, 1994) does follow our 'mean' law, $T_n = 2\pi GM\, w^{-3} n^3\, (1 + 2/n)^{3/2}$, with such a precision that second order terms can be checked (Nottale, 1996b and Fig. 1).

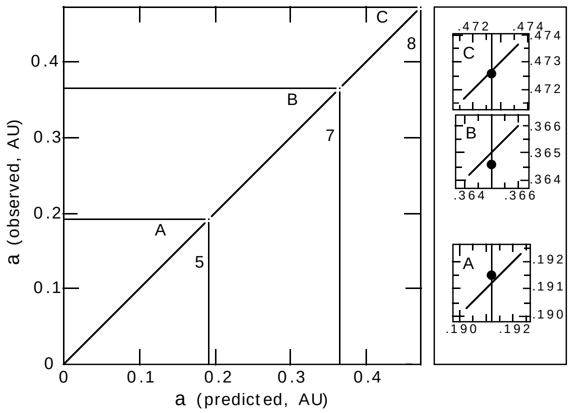

Figure 1. Comparison of our prediction, $a_n = a_0\,(n^2+n/2)$, with the observed semi-major axes for the planetary companions to the pulsar PSR B1257+12 (Wolszczan, 1994). The agreement is far better than the resolution of the diagram. The probability to obtain such an alignment by chance is < 10^{-5}. We have included three insets enlarged by a factor of ≈50 to show the small residual differences.

The planets rank n = 5, 7 and 8, having periods T_A = 0.06939(4) yrs, T_B = 0.1821696(9) yrs and T_C = 0.268925(2) yrs. This yields $(T_C/T_B)^{1/3}$ = 1.1386, to be compared to $(68/52.5)^{1/2}$ = 1.1381 and 8/7 = 1.1428, and $(T_C/T_A)^{1/3}$ = 1.5707 while $(68/27.5)^{1/2}$ = 1.5725 and 8/5 = 1.6. The agreement with the lowest order formula is already very good (relative errors –0.004 and –0.018), but *it becomes still 10 times better keeping the second order terms* (+0.0005 and –0.0011). Taking M_{PSR} = 1.4 ± 0.1 M⊙, we find w = 426 ± 10 km/s = 3 x (142 ± 3.3) km/s, which is the above Sun radius value. Conversely, using our determination w_0 = 144.7 ± 0.6 km/s (see below), we derive a pulsar mass of M_{PSR} = 1.48 ± 0.02 M⊙. We can predict with high precision the periods of possible additional planets, e.g., T_6 = 42.62 days, T_4 = 13.38 days and T_9 = 138.5 days.

(iii) Extra-Solar planets around solar-type stars. Mercury does not rank n = 1 in the inner solar system, but n = 3. This means that, in addition to the preferential values 0.39, 0.69, 1.05, 1.5 AU of semi-major axes (that agree with Mercury, Venus, Earth and Mars, see Table 1 and Fig. 2), we also expect two orbitals closer than Mercury to a solar-type star, at 0.18 AU (n = 2) and 0.05 AU (n = 1) (Nottale, 1993a,b; Nottale et al., 1996). It is then quite remarkable that the first extra-solar planet discovered around a solar-type star, 51 Peg (Mayor & Queloz, 1995) lies

precisely at 0.05 AU from its star, i.e., at the 'fundamental' level of our quantization law. Two more recent candidates, τ Boo B and υ And B (Butler et al., 1996), which also fall around 0.05 AU, confirm this result (Table 1 and Fig. 2).

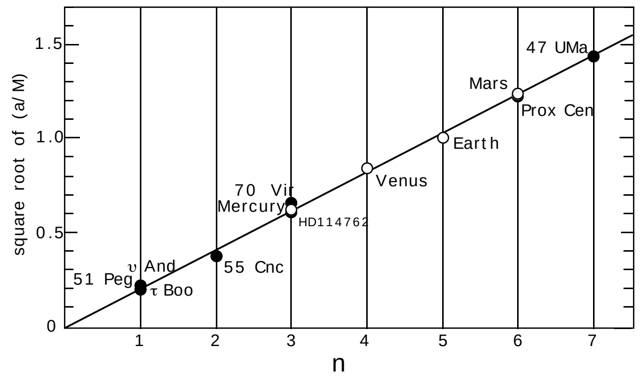

Figure 2. Square root of the ratio (a/M) (a in AU and M in $M_{\odot}$), where a is the semi-major axis of a planet and M the mass of its star, versus n integer, for inner solar system planets and extra-solar companions. The line corresponds to the law $\sqrt{a/GM} = n / w_0$, with $w_0 = 144$ km/s.

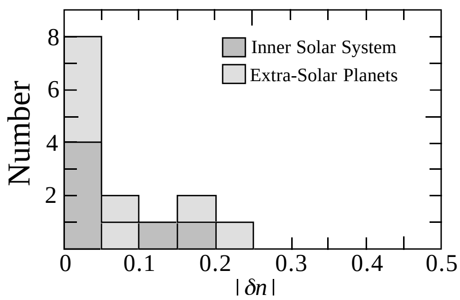

Figure 3. Distribution of $\delta n = 144/v - n$ for planetary companions around solar-type stars.

As can be seen in Table 1 and Fig. 2, the other recently discovered planets, around HD 114762 (Latham et al., 1989), 47 UMa (Butler & Marcy, 1996), 70 Vir (Marcy & Butler, 1996a), 55 Cnc (Butler et al., 1996), and Proxima Cen (Benedict et al, 1995), also agree with our prediction. The 8 new planets yield $w_0 = 143.9 \pm 3.1$ km/s. When combined with the inner Solar System planets, we find $w_0 = 144.1 \pm 1.8$ km/s. Moreover, one can demonstrate in a highly significant way that this law is a genuine quantization law, not only an average one. This can be seen from the values of δn (difference between the value of $144/v$ and its closest integer, see Table 1 and Fig. 3): the probability for δn to be drawn from a uniform distribution is $P \approx 2\ 10^{-5}$ from a χ^2 test ($\chi^2 = 39$ with 10 degrees of freedom), and $P << 10^{-4}$ from a Kolmogorov-Smirnov test.

4. Conclusion

It is noticeable that, at the time of the submission of this letter for publication, the assumed mass of 70 Vir (0.92 $M_{\odot}$) made it disagree with our law. We concluded in the first version of this paper that 70 Vir was at least 2 times more distant from the Sun than believed. This prediction has been meanwhile confirmed in a remarkable way by Hipparcos data (Perryman et al., 1996). In addition, three new planets around the stars 55 Cnc, τ Boo and υ And have also been discovered in the meantime, and have brought a new independent confirmation of our predictions (see Fig. 2). A theoretical prediction of the periods of other possible planets is now straightforward.

As stressed hereabove, a crucial test of the theory is to verify that it applies to pure two-body systems. We have shown hereabove that it does for isolated binary galaxies and for single planets such as (presumably) the 51 Peg system. Similar results are obtained for double stars. For example, the average velocity of eclipsing binaries in the Brancewicz catalog is $289.4 \pm 3.0 = 2 \times (144.7 \pm 1.5)$ km/s, and the distribution of velocities of binary systems including a pulsar shows a periodicity of $(145.8 \pm 3.0) \div 3$ km/s (Nottale, Lefèvre & Schumacher, 1996).

We have mainly considered velocity structures in the present short letter, but clearly our theory predicts also structures in position, angular momentum, obliquities, inclinations..., and provides us with a new general framework to study the formation and evolution of gravitational structures (Nottale, 1996c). Let us conclude by remarking that the various redshift quantization effects are explained here in terms of normal, Doppler velocities (only their probability distribution is changed with respect to the standard theory). This is confirmed by our discovery that the same effect occurs in our own Solar System, in terms of the directly observable motion of planets. The universal status of the Tifft constant (which we find equal to $w_0 = 144.7 \pm 0.55$ km/s from an average of the various determinations quoted above) is corroborated by the fact that it acts on at least 16 decades, from the Sun scale to hundred Mpcs.

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