A combined spectral/Godunov code for the simulation of gravitational waves from core collapse

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based on collaboration with
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Outline

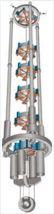
1. Introduction
2. Physical model
3. Numerical model
4. Results

Detection of gravitational waves

LIGO: USA, Louisiana

LIGO: USA, Washington

VIRGO: France/Italy (Pisa)



the arms of these Michelson-type LASERS are 3 km (VIRGO) and 4 km (LIGO) long ... with almost perfect vacuum.
⇒Starting to acquire data, with the first scientific run with 3(4) detectors.

Astrophysical sources of gravitational radiation

Gravitational luminosity from the linearized version of Einstein equations:

$$L \sim \frac{G}{c^5} s^2 \omega^6 M^2 R^4$$

(s being a factor related to non-sphericity of the matter distribution);changing the formula to

$$L \sim \frac{c^5}{G} s^2 \left(\frac{2GM}{Rc^2} \right)^2 \left(\frac{v}{c} \right)^6$$

allows to see that good sources are

- non-spherical (and dynamically changing);
- compact ($(2GM)/(Rc^2) \sim 1$);
- in relativistic motion.

⇒neutron stars and black holes in relativistic motion
⇒neutron star oscillations and *supernovæ*.

Simplified physical model of core-collapse

The phenomenon of *supernova* is too rich to be fully-modeled on a computer

- relativistic hydrodynamics ($v/c \sim 0.3$), including shocks, turbulence and rotation,
- strong gravitational field $\Rightarrow$General Relativity?
- neutrino transport (matter deleptonization)
- nuclear equation of state (EOS)
- radiative transfer and ionization of higher layers
- magnetic field?

$\Rightarrow$to track gravitational waves, some features must be neglected...and we use an effective model (not trying to make them explode)

Initial model is a rotating polytrope with an effective adiabatic index $\gamma \lesssim 4/3$. During the collapse, when the density reaches the nuclear level, $\gamma \to \gamma_2 \gtrsim 2$ (Van Riper, 1978).

Approximate gravity and extraction of gravitational waves

Einstein equations represent a set of 10 coupled non-linear second-order PDEs of mixed type (hyperbolic / elliptic).⇒when studying core-collapse, one may neglect the effect of gravitational waves onto hydrodynamics...and on the gravitational field itself!

⇒Conformally-Flat Condition (CFC)

- gravitational waves are completely discarded, no more dynamical degree of freedom in the gravitational field equations;
- with the evolution of matter, they can be (approximatively) calculated from the Newtonian quadrupole formula.

Note: even full general-relativistic codes use such formula because the signal, extracted from the the gravitation field itself, is too weak .

Equations and computational needs

hydrodynamics

General relativistic hydrodynamics are written as a flux-conservative first order hyperbolic system:

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial\sqrt{\gamma}\boldsymbol{U}}{\partial t}+\frac{\partial\sqrt{-g}\boldsymbol{F}^i}{\partial x^i}\right]=\boldsymbol{Q},$$

with $\boldsymbol{U}=(\rho W,\rho h W^2 v_i,\rho h W^2-P-D)$ the conserved variables.

The system is known to produce shocks, observed in *supernova* explosions!

⇒need for an algorithm able to treat shocks correctly.

In addition, some long-term physical instabilities can show up (much longer time-scale than the hydro one, see talk by Th.Foglizzo)

Equations and computational needs
Gravitational field

The CFC system results in 5 coupled non-linear elliptic equations, which sources are with non-compact support:

$$\hat{\Delta} \ln \phi = -4\pi\phi^4 \left(\rho h W^2 - P + \frac{K_{ij}K^{ij}}{16\pi} \right) - \hat{\nabla}^i \ln \phi \, \hat{\nabla}_i \ln \phi,$$

$$\hat{\Delta} \ln \alpha\phi = 2\pi\phi^4 \left(\rho h (3W^2 - 2) + 5P + \frac{7K_{ij}K^{ij}}{16\pi} \right) - \hat{\nabla}^i \ln \alpha\phi \, \hat{\nabla}_i \ln \alpha\phi,$$

$$\hat{\Delta}\beta^i + \frac{1}{3}\hat{\nabla}^i\hat{\nabla}_k\beta^k = 16\pi\alpha\phi^4 S^i + 2\phi^{10}K^{ij}\hat{\nabla}_j \left(\frac{\alpha}{\phi^6} \right) .$$

with $K_{ij} = \frac{1}{2\alpha}\left(\nabla_i\beta_j + \nabla_j\beta_i - \frac{2}{3}f_{ij}\nabla_k\beta^k \right)$.

⇒Either a general elliptic solver of a fast linear Poisson solver used in a iterative scheme, able to deal with spatial infinity.

All fields here are smooth, or at least $\mathcal{C}^2$.

Combination of two numerical techniques

- hydrodynamics ⇒High-Resolution Shock-Capturing schemes (HRSC), also known as Godunov methods, here implemented in General Relativity;
- gravity ⇒multi-domain spectral solver using spherical harmonics and Chebyshev polynomials, with a compactification of type $u = 1/r$.

Use of two numerical grids with interpolation:

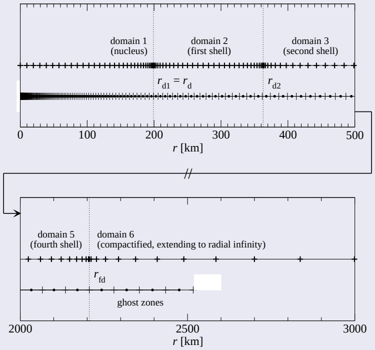
- matter sources: Godunov (HRSC) grid → spectral grid;
- gravitational fields: spectral grid → Godunov grid.

First achieved in the case of spherical symmetry, in tensor-scalar theory of gravity (Novak & Ibáñez 2000).
Spares a lot of CPU time in the gravitational sector, that can be used for other physical ingredients.

Filtering and optimization

- Godunov grid stops at a finite distance ⇒no matter outside;

Grid setting

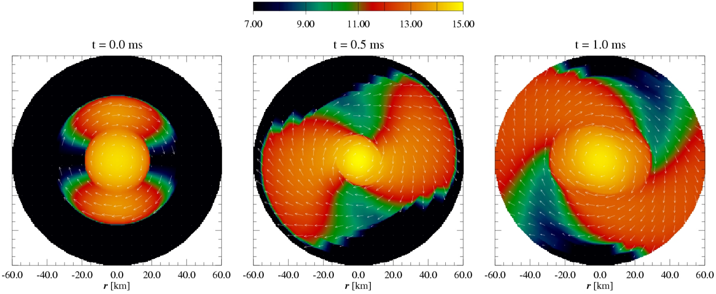


- interpolation to spectral grid using piecewise parabolic formula (many tested);
- filtering of radial coefficients (Chebyshev) by canceling the last N ones for the matter fields;
- fewest possible manipulations of these fields on spectral grid;
- partial summation technique (Orszag 1980) to gain CPU in the spectral summation.

Tests of the code
Dimmelmeier *et al.* (2005)

The 3D code is able to reproduce:

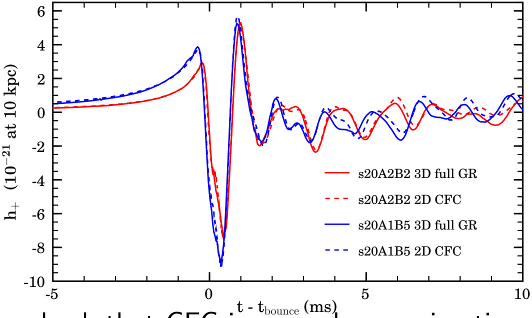
- the results and waveforms of core collapse from the axisymmetric code by Dimmelmeier *et al.* (2002);
- the frequencies of the fundamental mode, and its first harmonic, for the oscillations of rotating neutron star are recovered;
- a strongly 3D-perturbed rotating neutron star can be followed for several rotating periods.



Collapse with deleptonization and realistic EOS
Ott *et al.* (2007)

Together with the use of a purely finite-differences code in full GR, first results of realistic collapse of rotating stellar iron cores in GR

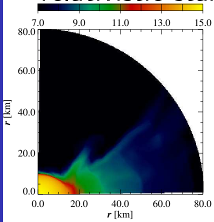
- with finite temperature EOS;
- (approximate) treatment of deleptonization.



⇒complete check that CFC is a good approximation in the case of core-collapse.

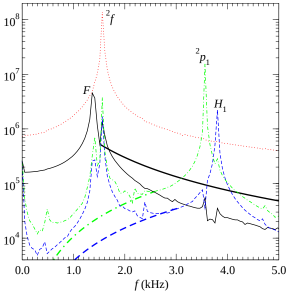
Neutron star oscillations
Dimmelmeier *et al.* (2006)

Study of non-linear axisymmetric pulsations of rotating relativistic stars



- uniformly and differentially rotating relativistic polytropes ⇒differential rotation significantly shifts frequencies to smaller values;
- mass-shedding-induced damping of pulsations, close to maximal rotation frequency.

- most powerful modes could be seen by current detectors if the source is about $\sim$ 10 kpc;
- if 4 modes are detected, information about cold nuclear matter equation of state could be extracted ⇒gravitational asterosismology.



Summary and outlook

- valid 3D code for the simulation of gravitational waves from core-collapse combining two very different numerical techniques;
- most realistic simulations today and spectrum of oscillations for rotating relativistic stars;
- improve extraction of gravitational wave signal, with the implementation of full general-relativistic equations using spectral methods (Bonazzola *et al.* 2004 formulation);
- better inclusion of micro-physics: more realistic neutrino transport, magnetic field...

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