"Mariage des maillages": A new 3D general relativistic hydro code

I. Introduction and theoretical issues

Jérôme Novak

LUTH CNRS/Observatoire de Paris

Meudon, France,

Harald Dimmelmeier and Ewald Müller

Max-Planck-Institut für Astrophysik

Garching, Germany

and

José-Antonio Font and José-M$\underline{\text{a}}$ Ibàñez

Departamento d'Astronomìa y Astrofisica

Valencia, Spain

Motivations / Physical model

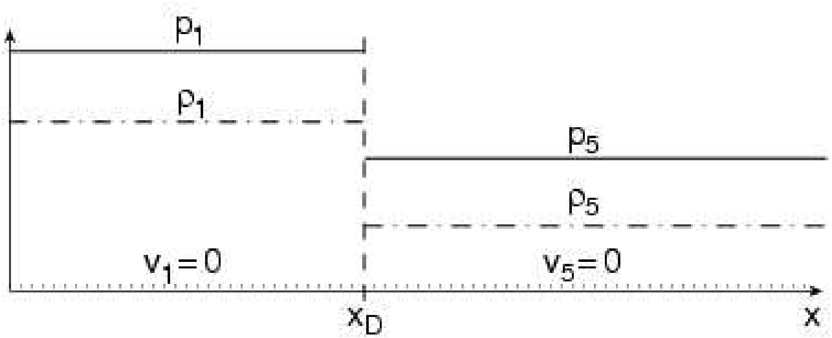
- Refine core-collapse GW simulations to have a precise catalog of waveforms;
- Bring GR into supernova core-collapse simulations.

Previous works:only in 2D: “approximate” (Dimmelmeier *et al.*, 2001) or full General Relativity (Shibata & Sekiguchi, 2004)
$\Rightarrow$need to study *bar formation* after the bounce (Imamaura *et al.*, 2003).

- General Relativity (IWM approximation: conformally-flat 3-metric) for gravitational field $\Rightarrow$hydrodynamics in a curved space-time;
- Perfect fluid model with hybrid ideal gas equation of state: polytropic pressure (stiffening as the density increases) and thermal pressure (after the bounce);
- Neutrinos and radiation transfers are not taken into account.

Initial model is a rotating polytrope with an effective adiabatic index $\gamma \lesssim 4/3$. During the collapse, $\gamma \to \gamma_2 \gtrsim 2$ at nuclear level (Van Riper, 1978).

High-Resolution Shock-Capturing Methods

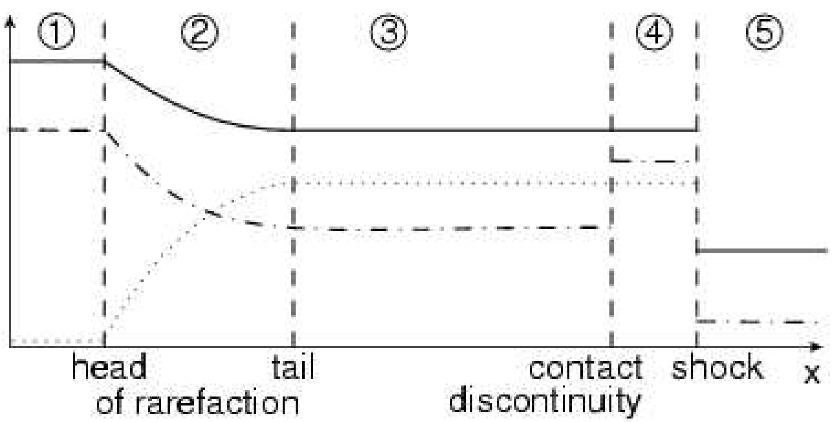


General relativistic hydrodynamics are written as a flux-conservative first order hyperbolic system:

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial\sqrt{\gamma}\boldsymbol{U}}{\partial t}+\frac{\partial\sqrt{-g}\boldsymbol{F}^i}{\partial x^i}\right]=\boldsymbol{Q},$$

with $\boldsymbol{U}=(\rho W, \rho h W^2 v_i, \rho h W^2 - P - D)$ the conserved variables.

⇒use of analytic solution of (approximate) *Riemann problems*.



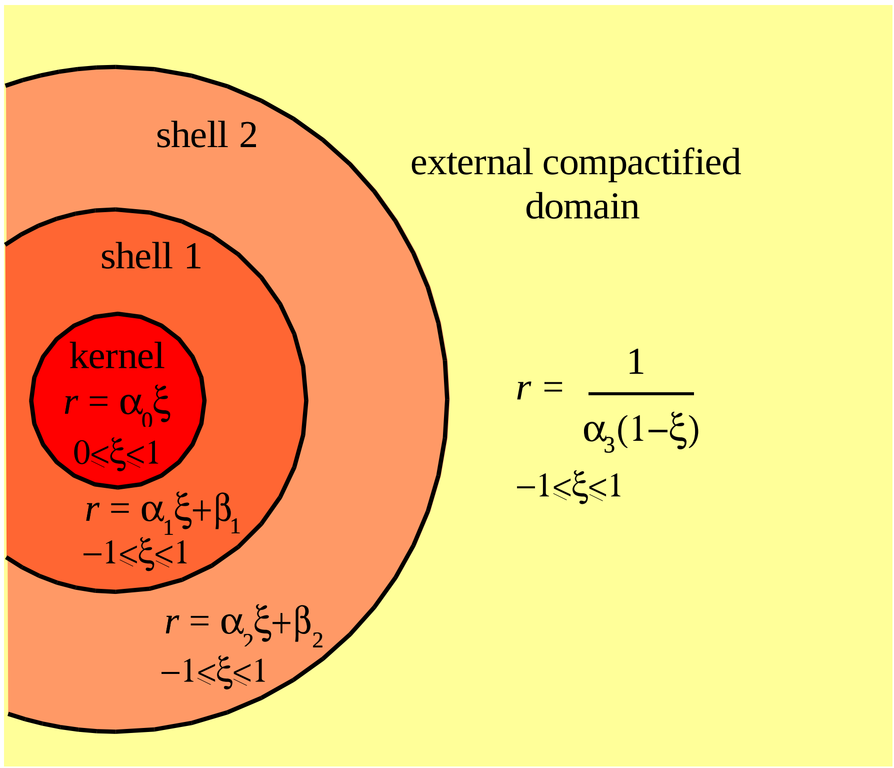
Martí & Müller Liv. Rev. Relat. 2003

⇒convergence to the physical solution

⇒sharp resolution of discontinuities

Spectral methods

Multidomain spectral methods + spherical polar coordinates, implemented in the numerical library LORENE (`http://www.lorene.obspm.fr`)



Decomposition:
Chebyshev polynomials for ξ,
Fourier or Y_l^m for the angular part (θ, ϕ),

- symmetries and regularity conditions of the fields at the origin and on the axis of spherical coordinate system.
- compactified variable for elliptic PDEs $\Rightarrow$boundary conditions are well imposed.

Drawback: Gibbs phenomenon!

Solving for the gravitational field

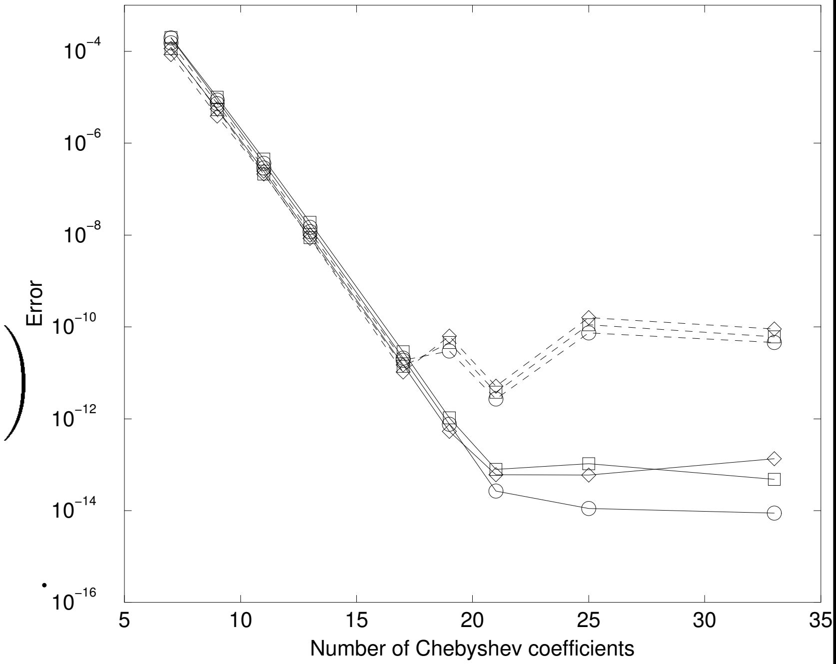
3+1 decomposition : $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$

IWM approximation : $\gamma_{ij} = \phi^4 f_{ij}$

$$\hat{\Delta} \ln \phi = -4\pi\phi^4 \left(\rho h W^2 - P + \frac{K_{ij}K^{ij}}{16\pi} \right) - \hat{\nabla}^i \ln \phi \, \hat{\nabla}_i \ln \phi,$$

$$\hat{\Delta} \ln \alpha\phi = 2\pi\phi^4 \left(\rho h (3W^2 - 2) + 5P + \frac{7K_{ij}K^{ij}}{16\pi} \right) - \hat{\nabla}^i \ln \alpha\phi \, \hat{\nabla}_i \ln \alpha\phi,$$

$$\hat{\Delta}\beta^i + \frac{1}{3}\hat{\nabla}^i \hat{\nabla}_k \beta^k = 16\pi\alpha\phi^4 S^i + 2\phi^{10} K^{ij} \hat{\nabla}_j \left(\frac{\alpha}{\phi^6} \right)$$

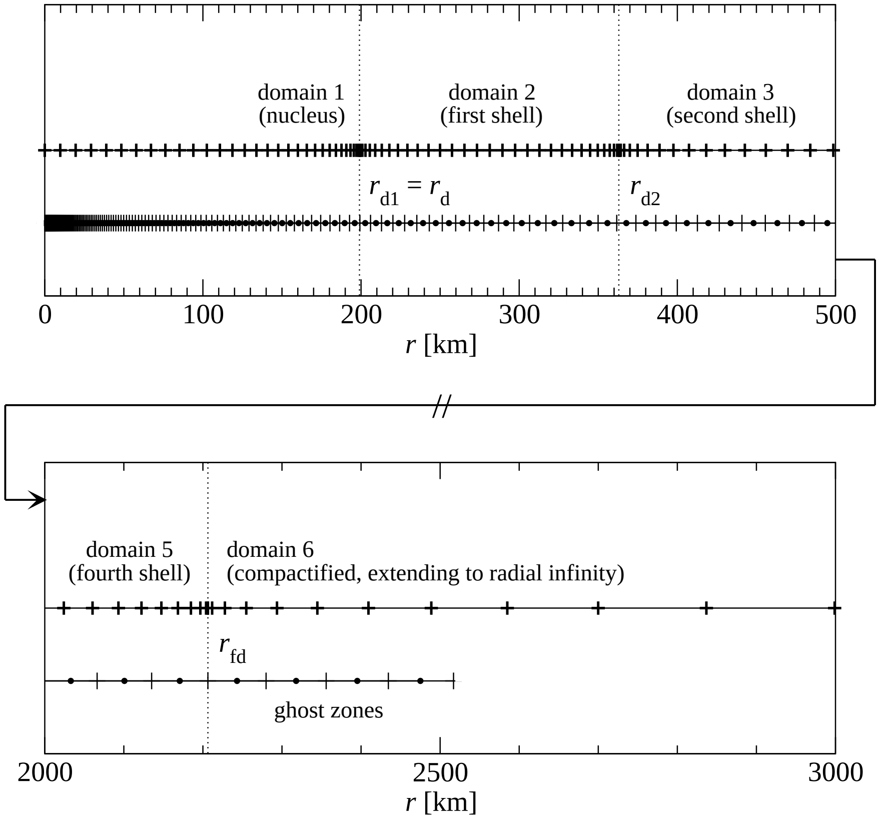


⇒set of 5 coupled Poisson-like non-linear equations (2 scalar + 1 vector) solved iteratively thanks to linear spectral scalar and vectorial Poisson solvers.

- $\forall (l,m)$, $\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \right) \phi_{lm}(r) = \sigma_{lm}(r)$ in each domain ($\sim 30 \times 30$ matrix + combination with homogeneous solutions to match across domains;
- vector Poisson equation is transformed to Cartesian *components*, turning it to 4 scalar Poisson equations.

Numerical setup

HRSC methods ⇒hydro evolution / spectral methods ⇒metric iteration



- initial profiles → HRSC evolution on finite-differences grid;
- *interpolation* of new hydro fields to spectral grid + filtering (smoothing shocks);
- iteration of metric system on spectral grid → *spectral summation* of the metric to the finite-differences grid.

⇒biggest sensitivity on domains setup (keep resolution and well-conditioned spectral matrices);

Summary and future work

Using modern high-level numerical methods (high-resolution shock-capturing for relativistic hydro and multidomain spectral ones for Einstein equations) we have constructed a 3D code, using spherical coordinates and solving the constraint equations in an affordable way during evolution, for the simulation of stellar core collapses and the prediction of the resulting gravitational radiation ... which can be improved:

GW are extracted through the Newtonian quadrupole formula:

- plugging in the *constrained evolution* of Einstein equations (presented by E.Gourgoulhon) with Dirac gauge, neglecting back-reaction of GW on hydro evolution ...
- ... or going to full Einstein;
- incorporating more detailed physical model.

Details and references in `astro-ph/0407174`