Relativistic numerical models for stationary superfluid neutron stars

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Based on the article:
R. Prix, J. Novak and G. L. Comer, *Phys. Rev. D* **71**, 043005 (2005).

Pulsar Workshop, November, 25$^{\text{th}}$ 2008

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Motivations

- Neutron stars represent a very complex physical system, far beyond experimental capacities of Earth-based laboratories.
- Observations can be done in many parts of the electro-magnetic spectrum, from neutrino and, possibly with gravitational wave emission.

$\Rightarrow$ Need for realistic stationary models, used to determine some of the observable data (maximal rotation frequency, mass, ...).

$\Rightarrow$ Need for initial data for dynamical models: collapse to a black hole, oscillations and glitches (superfluidity, two-stream instability).

Inversely, having detailed models permits some "inversion" of observational data to infer composition of neutron stars and very dense matter properties (*e.g.* if gravitational waves from oscillations are observed).

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Model

Two-fluid model

NEUTRONS

Superfluid neutrons in the crust and the outer core. No viscosity, so they can flow freely through the other component.

"PROTONS"

Nuclei, electrons, muons and protons are locked together on short timescales by viscosity and magnetic field.

$\Rightarrow$these two components are coupled together by strong nuclear force.

- General Relativity for the gravitational field
- stationarity and axisymmetry
- uniform rotation of both components / common axis, but different rotation rates.

Gravitational field equations

Stationarity, axisymmetry and circularity (no meridional currents) $\Rightarrow$coordinates and metric adapted to the Killing vector fields + quasi-isotropic gauge:

$$ds^2 = -\left(N^2 - N_\varphi N^\varphi\right) dt^2 - 2N_\varphi dt d\varphi + A^2\left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta d\varphi^2$$

Set of four elliptic PDEs ($\nu = \ln N, \alpha = \ln A, \beta = \ln B$):

$$\begin{aligned}
\Delta_3 \nu &= 4\pi A^2 (E + S_i^i) + A^2 K_{ij} K^{ij} - \partial\nu\, \partial(\nu + \beta)\,, \\
\tilde{\Delta}_3 \left(r \sin\theta N^\varphi\right) &= -16\pi N A^2 \widetilde{J}^\varphi - r\sin\theta\, \partial N^\varphi \partial(3\beta - \nu)\,, \\
\Delta_2 \left[(NB - 1) r \sin\theta\right] &= 8\pi N A^2 B r \sin\theta \left(S_r^r + S_\theta^\theta\right)\,, \\
\Delta_2 (\nu + \alpha) &= 8\pi A^2 S_\varphi^\varphi + \frac{3}{2} A^2 K_{ij} K^{ij} - (\partial\nu)^2\,,
\end{aligned}$$

where E, S_i^j and J^i come from the 3+1 decomposition of the stress-energy tensor $T^{\mu\nu}$.

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Global quantities

Knowing the matter and gravitational fields, one can compute some global quantities:

- The gravitational mass M_g is determined from the asymptotic behavior of the lapse function N:

$$M_g = \int A^2 B \left[N \left(E + S_i^i \right) + 2B^2 r \sin\theta N^\varphi \widetilde{J}^\varphi \right] r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi.$$

- The angular momentum $\mathcal{J}$ is determined from the asymptotic behavior of the shift vector N^i:

$$\mathcal{J} = \int \left(A^2 B^3 r \sin\theta \widetilde{J}^\varphi \right) r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi.$$

- The 2D- and 3D-virial identities serve as useful checks of consistency and precision of numerical results: these relations are not imposed in the model.

Two-fluid hydrodynamics

from Carter, Langlois, *et al.*

- For each fluid define the conserved 4-current $n^{\mu}_{\rm n}$ and $n^{\mu}_{\rm p}$,
- The Lagrangian density $\Lambda = -\mathcal{E}$ depends only on the three possible scalar products between these 4-vectors.
- Define momenta as conjugates of currents:

$$\mathrm{d}\Lambda = p^{\rm n}_{\mu}\mathrm{d}n^{\mu}_{\rm n} + p^{\rm p}_{\mu}\mathrm{d}n^{\mu}_{\rm p}.$$

- The equations of motions (in the absence of direct dissipative forces) are:

$$n^{\mu}_{\rm n}\nabla_{[\mu}p^{\rm n}_{\mu]} = 0 \text{ and } n^{\mu}_{\rm p}\nabla_{[\mu}p^{\rm p}_{\mu]} = 0$$

- The stress-energy tensor $T_{\mu}{}^{\nu} = p^{\rm n}_{\mu}n^{\nu}_{\rm n} + p^{\rm p}_{\mu}n^{\nu}_{\rm p} + \Psi\delta_{\mu}{}^{\nu}$,
- with the generalized pressure $\Psi = -\mathcal{E} - p^{\rm n}_{\mu}n^{\mu}_{\rm n} - p^{\rm p}_{\mu}n^{\mu}_{\rm p}$.

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Equation of state

The EOS depends only on densities and "relative speed" Δ: $\mathcal{E}(n_{\rm n}, n_{\rm p}, \Delta^2)$, and the first law of thermodynamics reads (defining the chemical potentials μ^n and μ^p)

$$\mathrm{d}\mathcal{E} = \mu^{\rm n}\mathrm{d}n_{\rm n} + \mu^p \mathrm{d}n_{\rm p} + e\,\mathrm{d}\Delta^2,$$

and the equations of motion take the integral form:

$$\frac{N}{\Gamma_{\rm n}}\mu^{\rm n} = C^{\rm n} \text{ and } \frac{N}{\Gamma_{\rm p}}\mu^{\rm p} = C^{\rm p}$$

We have used a simple (2-fluid polytrope) EOS

$$\mathcal{E} = \rho c^2 + \frac{1}{2}\kappa_{\rm n} n_{\rm n}^2 + \frac{1}{2}\kappa_{\rm p} n_{\rm p}^2 + \kappa_{\rm np} n_{\rm n} n_{\rm p} + \kappa_\Delta n_{\rm n} n_{\rm p} \Delta^2.$$

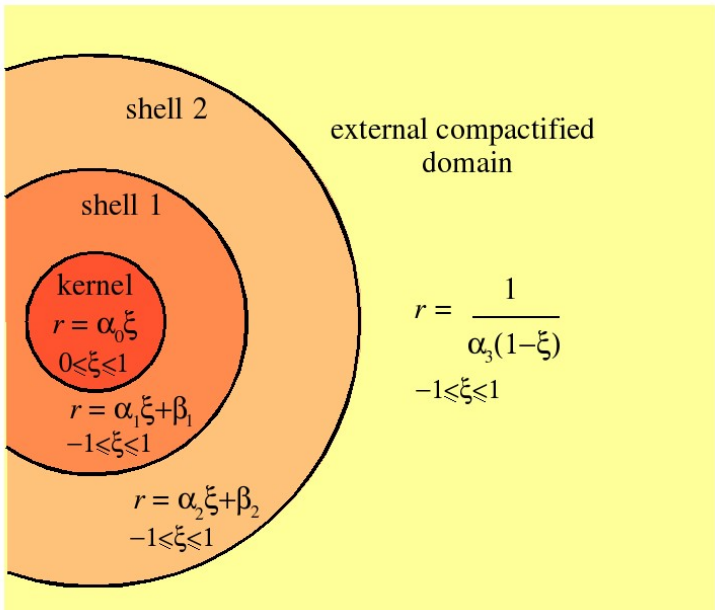
$\Rightarrow$all physical features: entrainment + symmetry energy, and the inversion $(\mu^{\rm n}, \mu^{\rm p}) \leftrightarrow (n_{\rm n}, n_{\rm p})$ is made easy (linear system).

Numerical methods

spectral, multidomain methods

Need: solve Poisson-like PDEs with sources of non-compact support.

$\Rightarrow$use a linear Poisson solver with iteration and relaxation.



Decomposition $f(r, \theta, \varphi)$:

Chebyshev polynomials for ξ,
Fourier or Y_ℓ^m for the angular part.

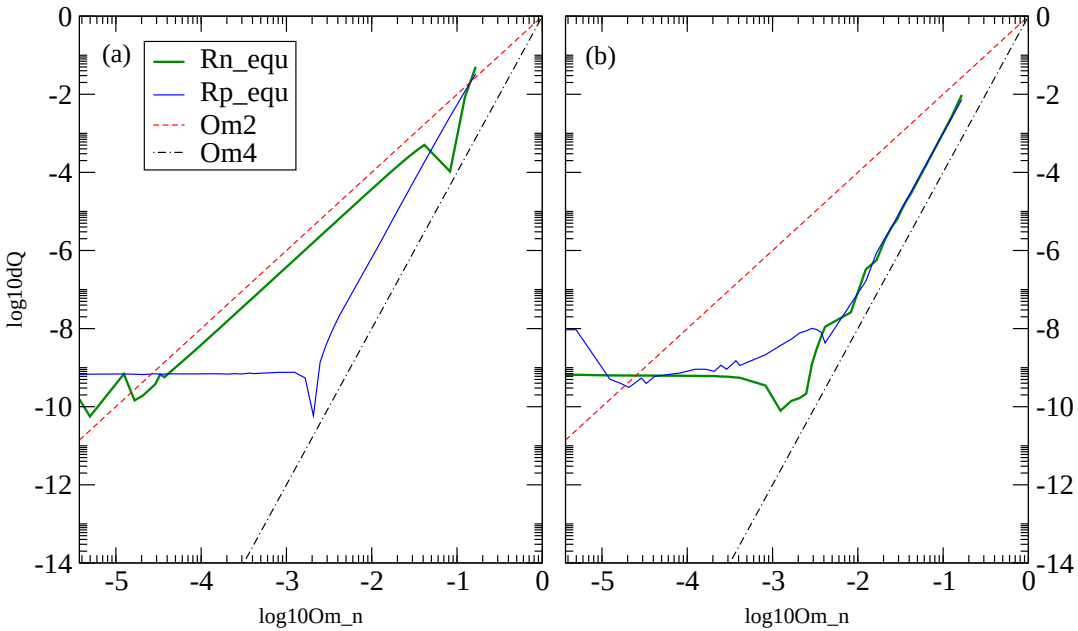
- symmetries and coordinate singularity at the origin and on the axis of spherical coordinates
- compactified variable for elliptic PDEs $\Rightarrow$boundary conditions are well imposed

Crude initial guess$\longrightarrow$ E, J^φ, S_i^j $\longrightarrow$ metric $\longrightarrow$ $(\mu^{\rm n}, \mu^{\rm p})$ $\longrightarrow$ $E, J^\varphi, S_i^j \ldots$

Comparison to previous works

Most models have been devised in the "slow-rotation" approximation:

- Prix *et al.* 2002 in the Newtonian regime,
- Anderson & Comer 2001 in Relativistic theory.



In the Newtonian case, one can obtain an analytical expression for the solution and, depending on the type of EOS inversion, the behavior of the difference as a function of Ω is recovered.

In the relativistic case, the agreement on gauge-independent quantities ranges from 10^{-4} to a few percents, depending on the rotation rate.

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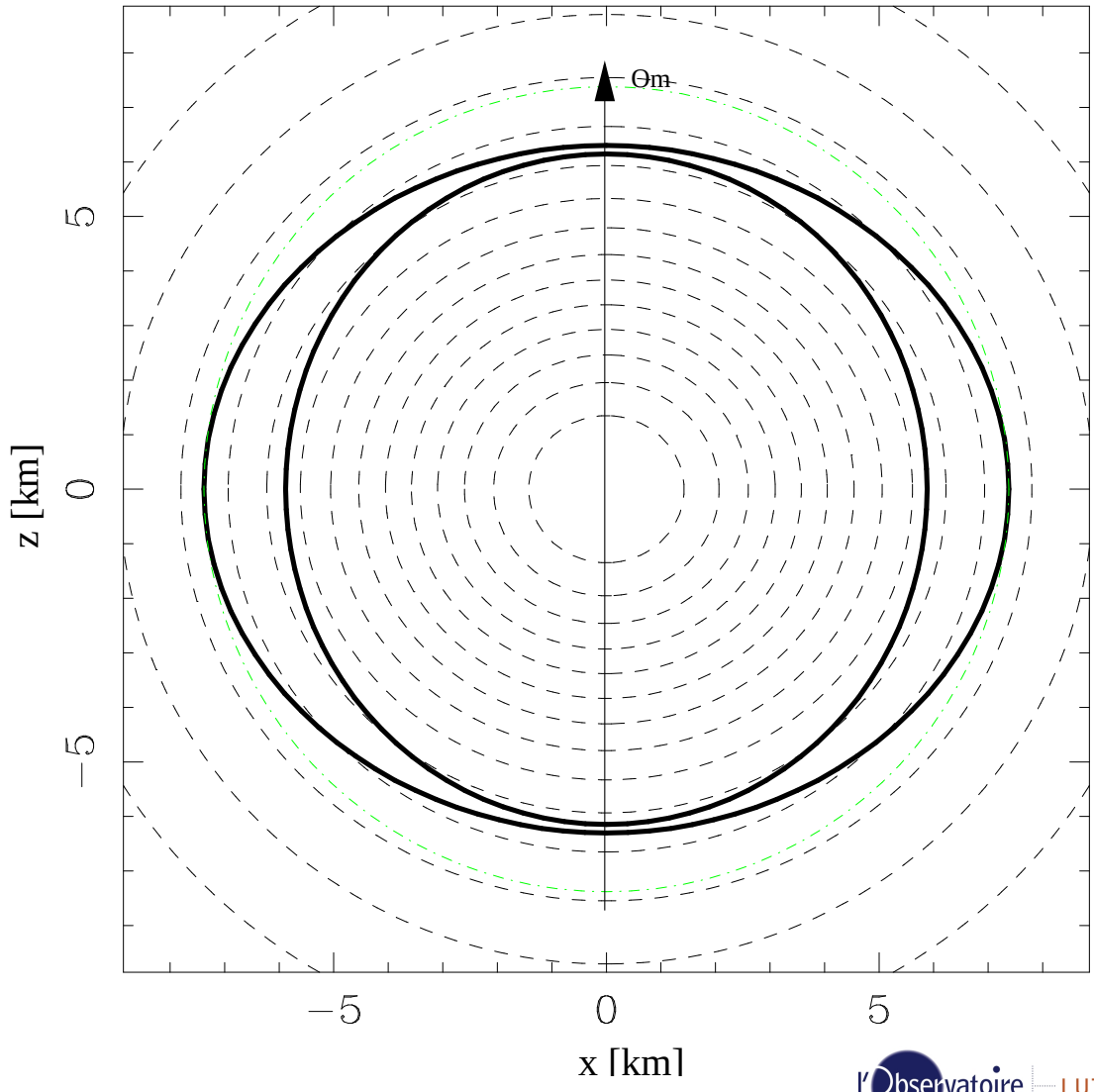
Results

oblate / prolate configurations

- It is possible with non-realistic parameters to get a configuration with one fluid surface having oblate shape, while the other has a prolate one.
- made possible by counter-rotation and the effective interaction potential, which tends to "separate" both fluids.



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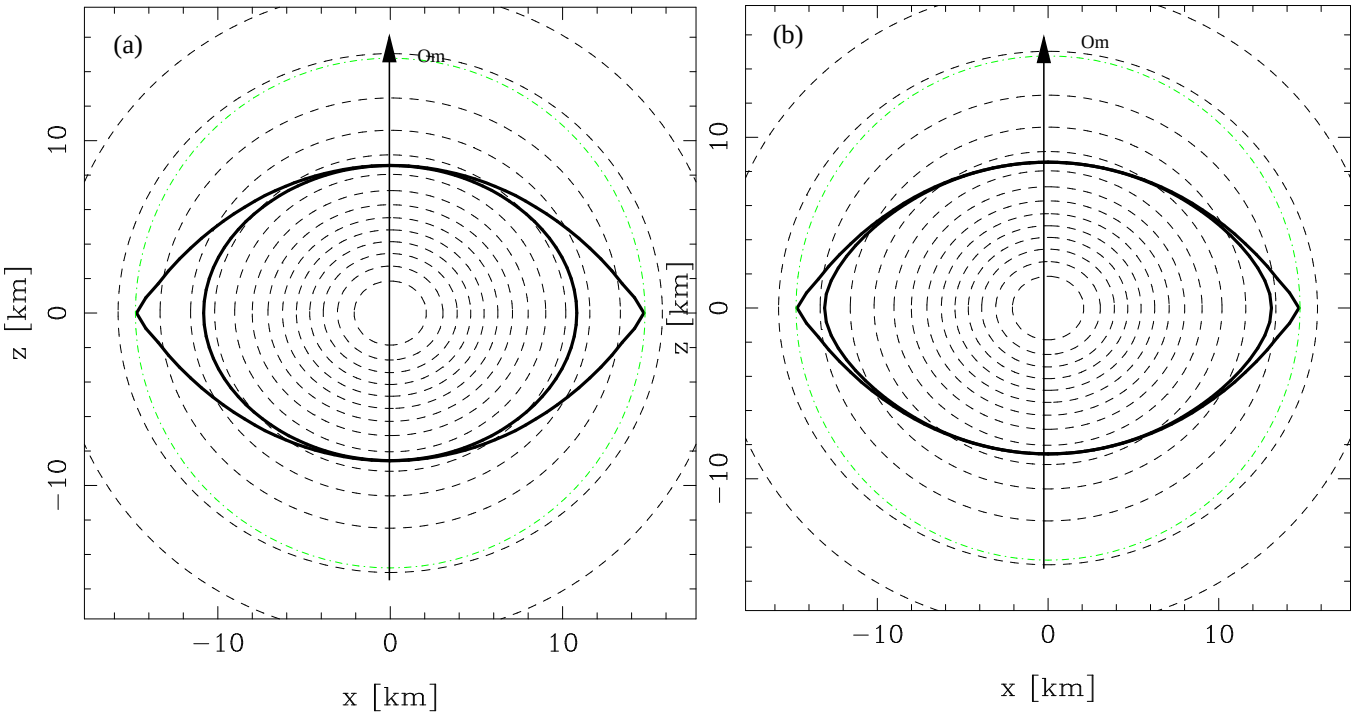
Results

Kepler limit



- Slow-rotation approximations overestimate the Kepler frequencies by $\sim 15\%$.
- If no chemical equilibrium at the center, the Kepler limit is determined by the outer fluid, even if it is rotating slower than the inner one.

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Outlook

- Allow for differential rotation of superfluid component.
- Need for more realistic nuclear-physics EOS, particularly for the entrainment term.
- Add a solid crust...
- Study the dynamical evolution: oscillation modes and gravitational wave emission.
- What about mutual friction in such situations?

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