Simulations of gravitational waves from core collapse events in general relativity

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Gravitational radiation from *supernovæ*

VIRGO, in Pisa, Italy
(CNRS/INFN)
Freq. range: 10 Hz $\rightarrow$ 10 kHz

Main sources for high-frequency interferometric GW detectors: coalescing binaries (black holes, neutrons stars) and *supernovæ*.

$\Rightarrow$need to know waveforms with the highest possible precision.

Within this framework, we are looking for waveforms, not for explosion scenarios. Available results only in 2D: “approximate” (Dimmelmeier *et al.*, 2001) of full General Relativity (Shibata & Sekiguchi, 2004)

Physical model

- General Relativity for gravitational field $\Rightarrow$hydrodynamics in a curved space-time;
- Perfect fluid model with hybrid ideal gas equation of state: polytropic pressure (stiffening as the density increases) and thermal pressure (after the bounce);
- Neutrinos and radiation transfers are not taken into account.

Initial model is a rotating polytrope with an effective adiabatic index $\gamma \lesssim 4/3$. During the collapse, when the density reaches the nuclear level, $\gamma \to \gamma_2 \gtrsim 2$ (Van Riper, 1978).

General relativistic hydrodynamics are written as a flux-conservative first order hyperbolic system:

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial\sqrt{\gamma}\boldsymbol{U}}{\partial t}+\frac{\partial\sqrt{-g}\boldsymbol{F}^i}{\partial x^i}\right]=\boldsymbol{Q},$$

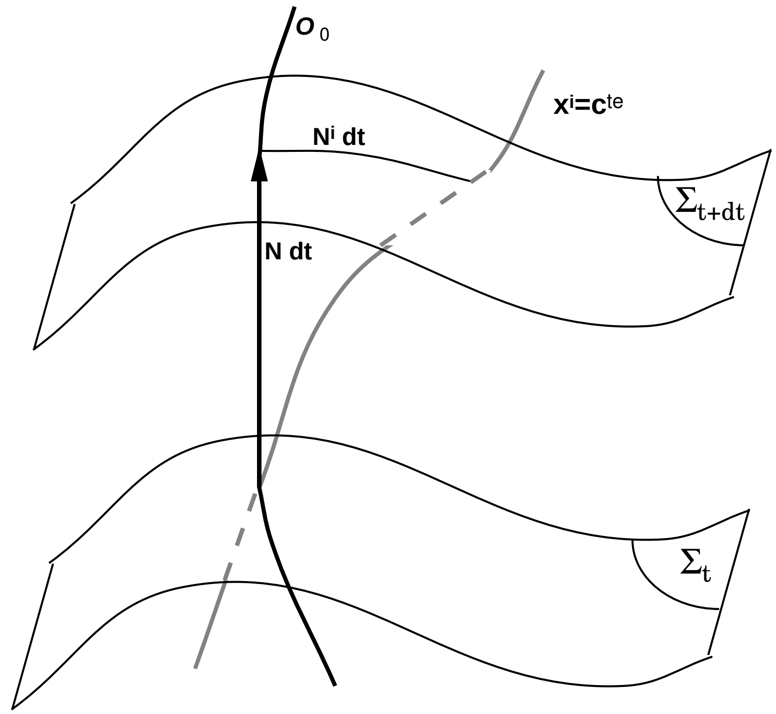
with $\boldsymbol{U} = (\rho W, \rho h W^2 v_i, \rho h W^2 - P - D)$ the conserved variables.

Approximation for the gravitational field

3+1 decomposition :

$ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

IWM approximation : $\gamma_{ij} = \psi^4 f_{ij}$



IWM approximation is exact in spherical symmetry or at first post-newtonian order but inhibits any gravitational radiation! ⇒GW are extracted using standard quadrupole formula.

⇒set of 5 coupled Poisson-like non-linear equations (instead of $\sim 7 - 10$): we are neglecting the two dynamical degrees of freedom of the gravitational field. neglecting acoustic waves...

Numerical techniques

- General relativistic simulations of core collapses were limited to 2D because of computational power $\Rightarrow$the equations for gravitational field are difficult to solve in 3D using finite diferences.

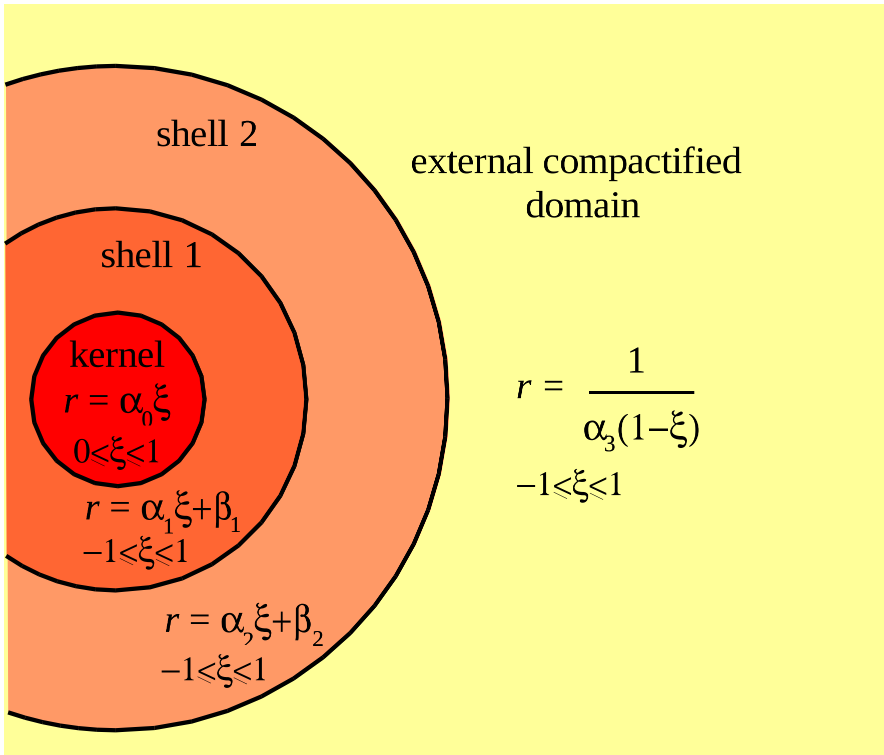
- Group of numerical relativity at LUTH uses spectral methods for Einstein equations with much less CPU and memory $\Rightarrow$unable to handle shocks in hydrodynamics.

$\Rightarrow$use of Godunov-type methods (shock-capturing) for hydro equations and spectral techniques for Einstein equations (grav. field is always smooth enough).

Use of two numerical grids (spectral and finite-difference) with sophisticated interpolation procedures, but the overall code can run in 3D on “reasonable” computers.

Spectral methods

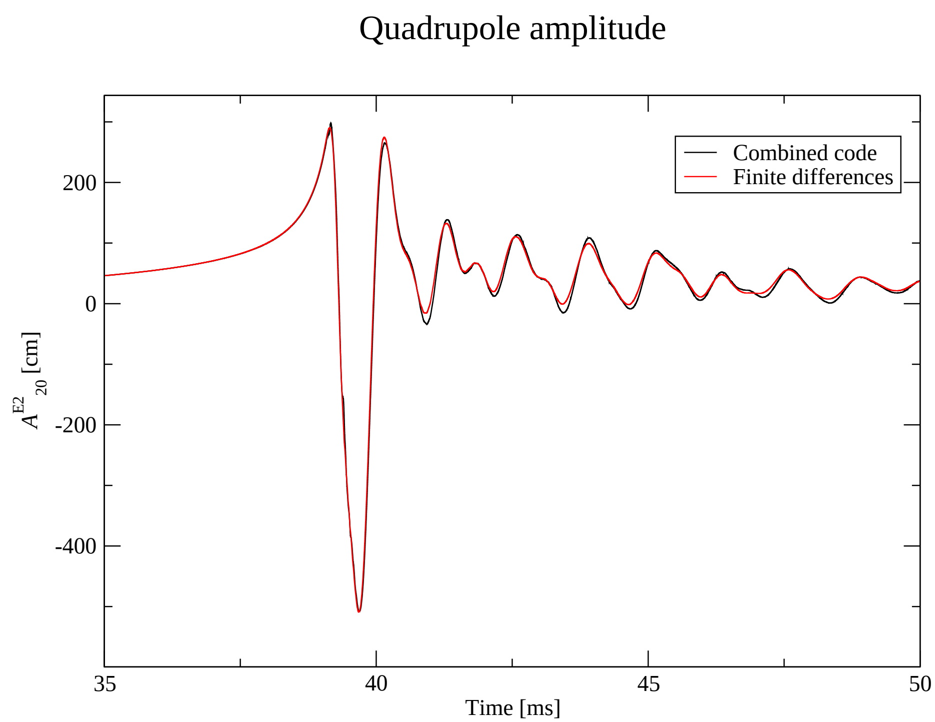
Multidomain spectral methods + spherical coordinates (and tensor components), implemented in the numerical library LORENE



Decomposition:
Chebyshev polynomials for ξ,
Fourier or Y_l^m for the angular part (θ, ϕ),
use of symmetries and regularity conditions of the fields at the origin and on the axis of spherical coordinate system.

Use of compactified variable $\Rightarrow$boundary conditions are well imposed (grav. field is also a source of gravity)

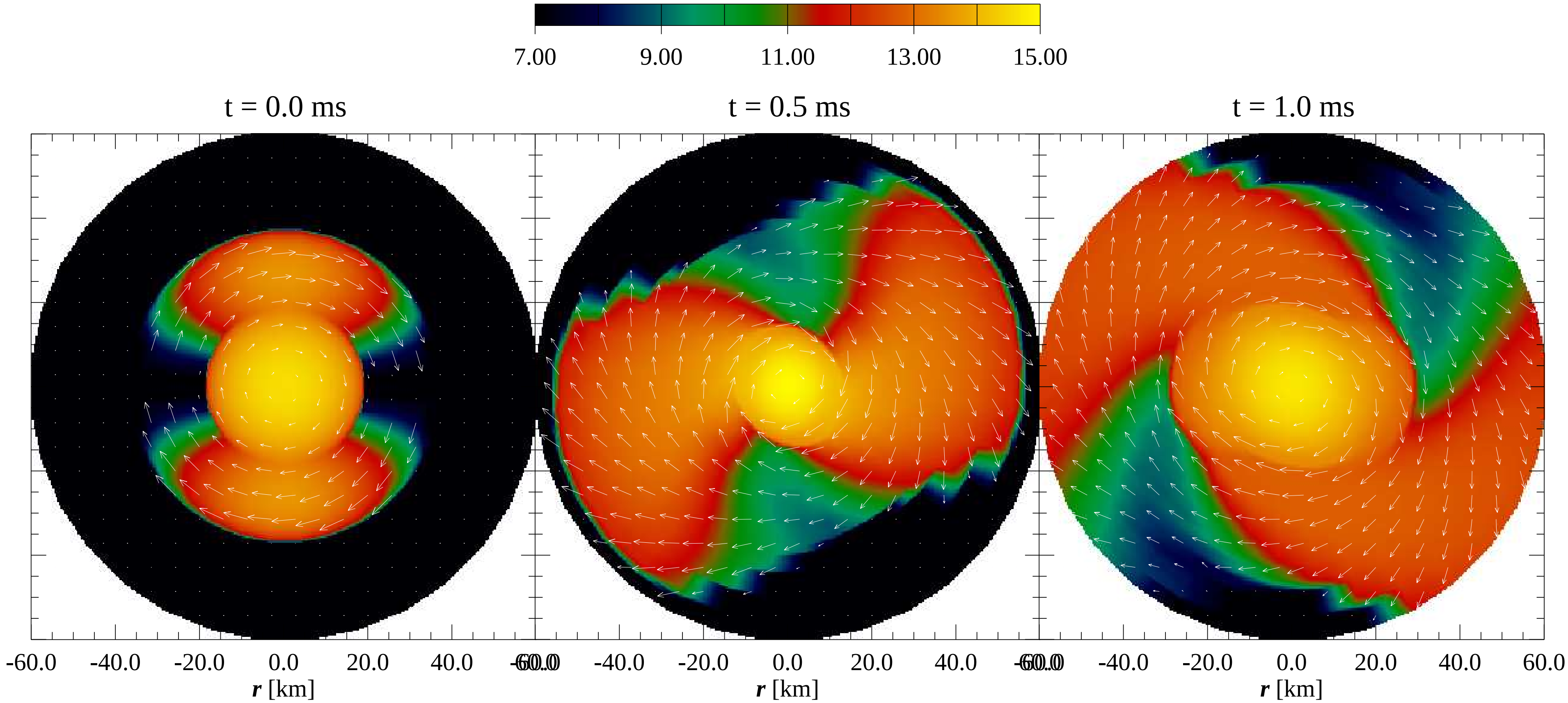
Tests



	$r_{\mathrm{s\,p}}/r_{\mathrm{s\,e}} = 0.65$	
Code	f_{F} [kHz]	f_{H1} [kHz]
current (3D)	1.20 (0.4)	3.68 (1.0)
current (2D)	1.219(2.0)	3.659 (1.6)
Cactus	1.195	3.717

- The new 3D code is able to reproduce the axisymmetric results by Dimmelmeier *et al.* (2001) obtained with pure finite-differences code;
- we retrieve the correct values for the relativistic oscillation modes for rotating neutron stars.

Long term evolution of 3D pertubed star



Initial model: uniformly rotating neutron star + non-axisymmetric ($l = 2, m = 2$, at 10% level) perturbation in density.

Summary and future work

Stable, accurate and not too CPU-consuming 3D code for the simulation of stellar core collapses and the prediction of the resulting gravitational radiation, which could be used to:

- explore 3D runs were bar-mode instabilities may occur, for strongly rotating stellar cores;
- study neutron star oscillations in General Relativity (some of which are unstable) and the resulting gravitational waves;
- add more “micro-physics” to the model: realistic equations of state, neutrino transport, ...
- ... and eventually study the *supernova* phenomenon.

But then, the computers might not be powerfull enough again!!